SM Background in Rare B-Meson Decays

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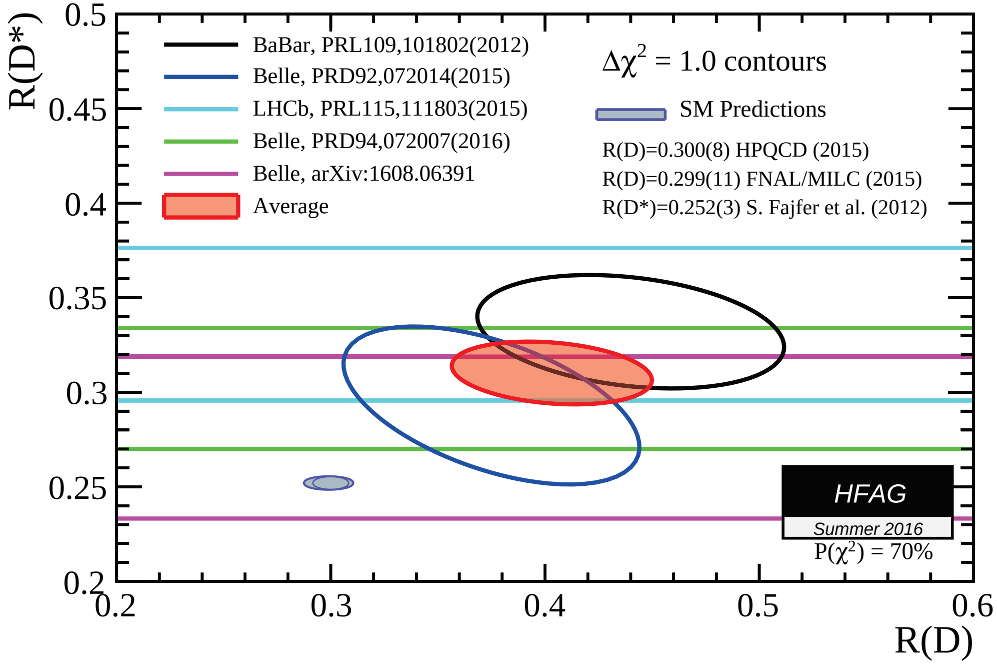
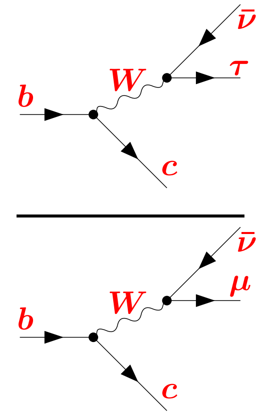
HARMONIA meeting, April 26-30th 2018, Warsaw

1. B-physics "anomalies"
2. $\bar{B} \to X_s \gamma$ - progress in perturbative calculations
3. $B_{s,d} \to \ell^+ \ell^-$ - a phenomenological update
4. Charm-quark loops $\bar{B} \to X_s \ell^+ \ell^-$
5. Summary

$R(D)$ and $R(D^*)$ "anomalies" [HFAG, arXiv:1612.07233] (3.9σ)

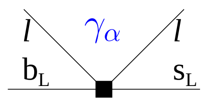


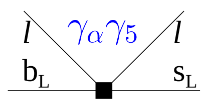


$$R(D^{(*)}) = \mathcal{B}(B \to D^{(*)}\tau\bar{\nu})/\mathcal{B}(B \to D^{(*)}\mu\bar{\nu})$$

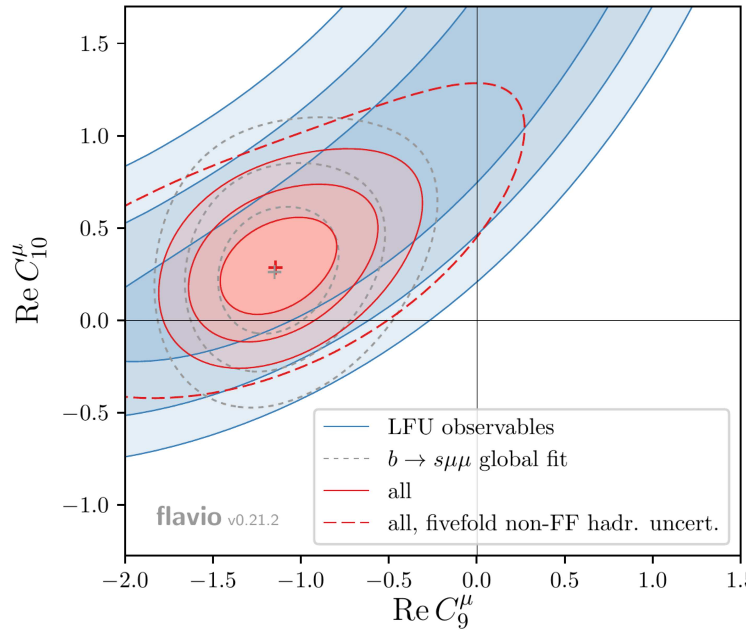
$b \to s\ell^+\ell^-$ "anomalies" $(> 5\sigma)$

[W. Altmanshofer, February 2018, talk at the Munich workshop]

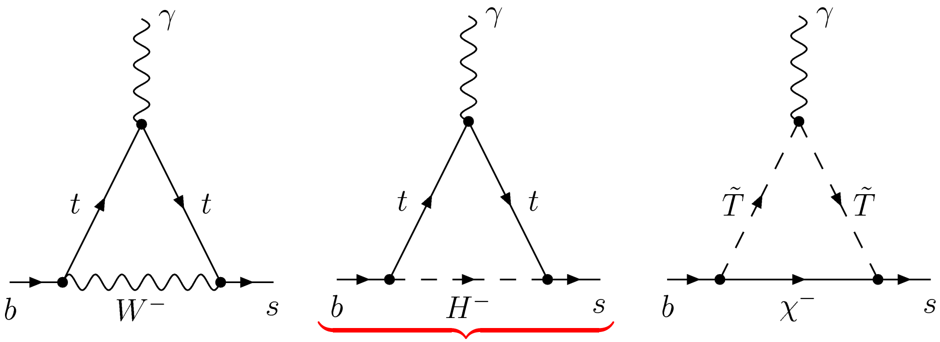
$Q_9^\ell =$ 

$Q_{10}^\ell =$ 

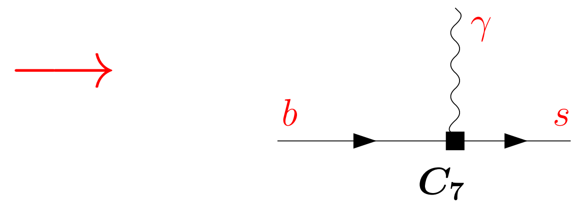
$\ell = e$ or μ



Information on electroweak-scale physics in the $b \to s\gamma$ transition is encoded in an effective low-energy local interaction:



$\Rightarrow M_{H^\pm} > \sim 500\,\mathrm{GeV}$
in the 2HDM-II



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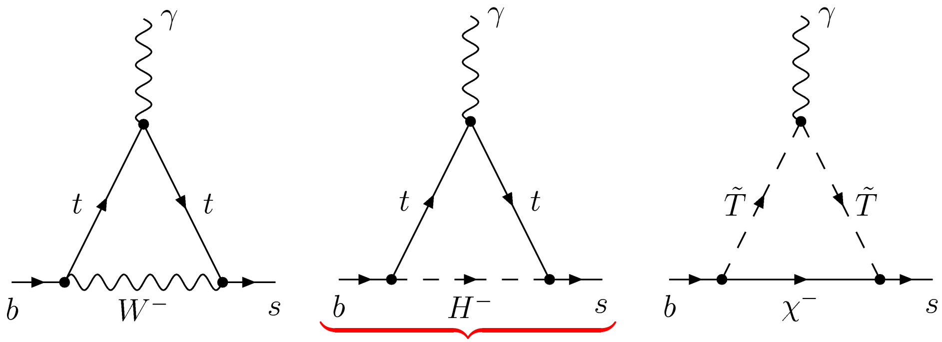

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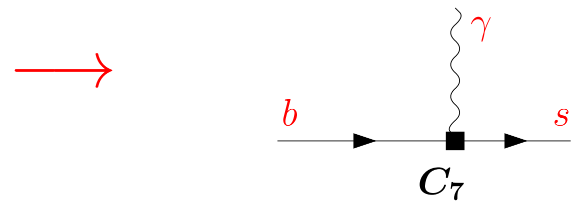

$b \in \bar{B} \equiv (\bar{B}^0 \text{ or } B^-)$

The inclusive $\bar{B} \to X_s \gamma$ decay rate for $E_\gamma > E_0$ is well approximated by the corresponding perturbative decay rate of the b-quark:

$$\Gamma(\bar{B} \to X_s \gamma) \;=\; \Gamma(b \to X_s^p \gamma) \;+ \begin{pmatrix} \text{non-perturbative effects} \\ (3 \pm 5)\% \end{pmatrix}$$

[G. Buchalla, G. Isidori and S.-J. Rey, Nucl. Phys. B511 (1998) 594]
[M. Benzke, S.J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099]
(BLNP)

provided E_0 is large ($E_0 \sim m_b/2$)
but not too close to the endpoint ($m_b - 2E_0 \gg \Lambda_{\text{QCD}}$).

Conventionally, $E_0 = 1.6\,\text{GeV} \simeq m_b/3$ is chosen.

Updated SM estimate for the CP- and isospin-averaged branching ratio of $\bar{B} \to X_s\gamma$ [arXiv:1503.01789, arXiv:1503.01791]:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.36 \underbrace{\pm\, 0.23}_{\pm 6.9\%}) \times 10^{-4} \qquad \text{for } E_\gamma > 1.6\,\text{GeV}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, **3%** from the interpolation in m_c

3% higher order $\mathcal{O}(\alpha_s^3)$, **2%** parametric

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$\Rightarrow$ Strong bound on the $H^\pm$ mass in the Two-Higgs-Doublet-Model II:

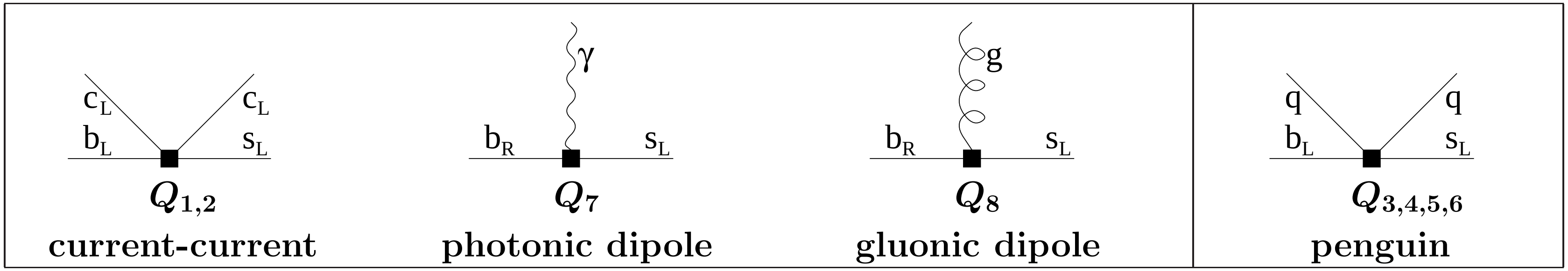
$$M_{H^\pm} > 580\,\text{GeV at } 95\%\text{C.L.}$$

[MM, M. Steinhauser, EPJC 77 (2017) 201]

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

$$L_{\text{weak}} \sim \sum_i C_i Q_i$$

Eight operators Q_i matter for $\mathcal{B}_{s\gamma}^{\text{SM}}$ when the NLO EW and/or CKM-suppressed effects are neglected:



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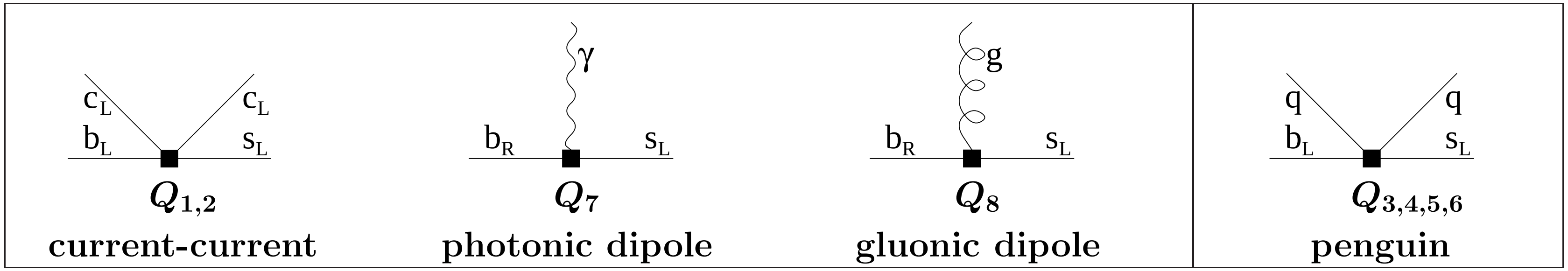
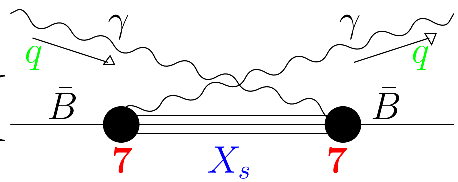


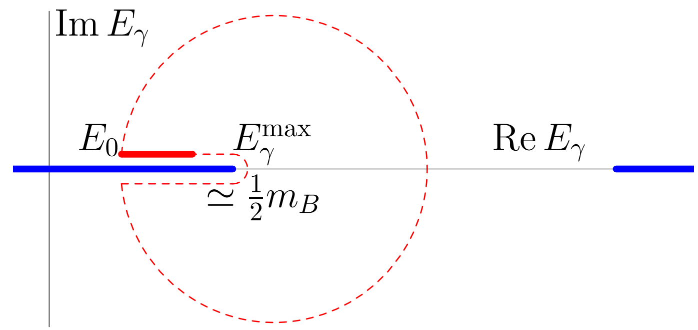
$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = |C_7(\mu_b)|^2\, \Gamma_{77}(E_0) + (\text{other}) \qquad (\mu_b \sim m_b/2)$$

Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \text{Im}\{\ \ \} \equiv \text{Im}\, A$$



Integrating the amplitude A over E_γ:



OPE on the ring $\Rightarrow$ Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\rm QCD}}{m_b}$ and α_s that begins with

$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3},\ldots;\ \frac{\alpha_s\mu_\pi^2}{(m_b - 2E_0)^2},\ \frac{\alpha_s\mu_G^2}{m_b(m_b - 2E_0)};\ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\rm QCD})$ are extracted from the semileptonic $\bar{B} \to X_c e\bar{\nu}$ spectra and the B–$B^\star$ mass difference.

NNLO QCD corrections to $\bar{B} \to X_s \gamma$

The relevant perturbative quantity $P(E_0)$:

$$\frac{\Gamma[b \to X_s \gamma]_{E_\gamma > E_0}}{\Gamma[b \to X_u e \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{ub}} \right|^2 \frac{6\alpha_{\rm em}}{\pi} \underbrace{\sum_{i,j} C_i(\mu_b)\, C_j(\mu_b)\, K_{ij}}_{P(E_0)}$$

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Expansions of the Wilson coefficients and K_{ij} in $\widetilde{\alpha}_s \equiv \frac{\alpha_s(\mu_b)}{4\pi}$:

$$C_i(\mu_b) = C_i^{(0)} + \widetilde{\alpha}_s\, C_i^{(1)} + \widetilde{\alpha}_s^2\, C_i^{(2)} + \ldots$$

$$K_{ij} = K_{ij}^{(0)} + \widetilde{\alpha}_s\, K_{ij}^{(1)} + \widetilde{\alpha}_s^2\, K_{ij}^{(2)} + \ldots$$

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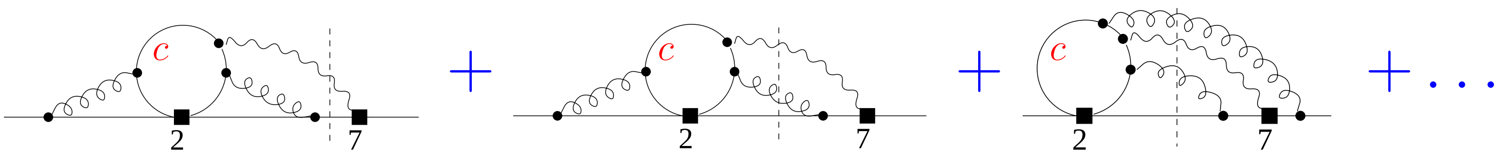
Most important at the NNLO: $K_{77}^{(2)}$, $K_{27}^{(2)}$ and $K_{17}^{(2)}$.

They depend on $\frac{\mu_b}{m_b}$, $\delta = 1 - \frac{2E_0}{m_b}$ and $z = \frac{m_c^2}{m_b^2}$.

Towards complete $K_{17}^{(2)}$ and $K_{27}^{(2)}$ for arbitrary m_c

[MM, A. Rehman, M. Steinhauser, ...]
in progress



1. Generation of diagrams and performing the Dirac algebra to express everything in terms of 585309 four-loop two-scale scalar integrals with unitarity cuts (437 families).

2. Reduction to master integrals with the help of Integration By Parts (IBP).
   Available public C++ codes: REDUZE [C. Studerus, arXiv:0912.2546], FIRE [A.V. Smirnov, arXiv:1408.2372].
   A useful Mathematica code: LiteRed [R.N. Lee, arXiv:1212.2685] (symmetries...).
   At the moment (MM), 147 families (166509 integrals) still await for reduction.
   Expected needs for the most difficult families: 100 GB RAM & 1 month CPU.

3. Extending the set of master integrals I_n so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations
$$\frac{d}{dz} I_n = \Sigma_k \, w_{nk}(z,\epsilon) \, I_k, \qquad (*)$$
   where w_{nk} are rational functions of their arguments.

4. Calculating boundary conditions for $(*)$ using automatized asymptotic expansions at $m_c \gg m_b$.

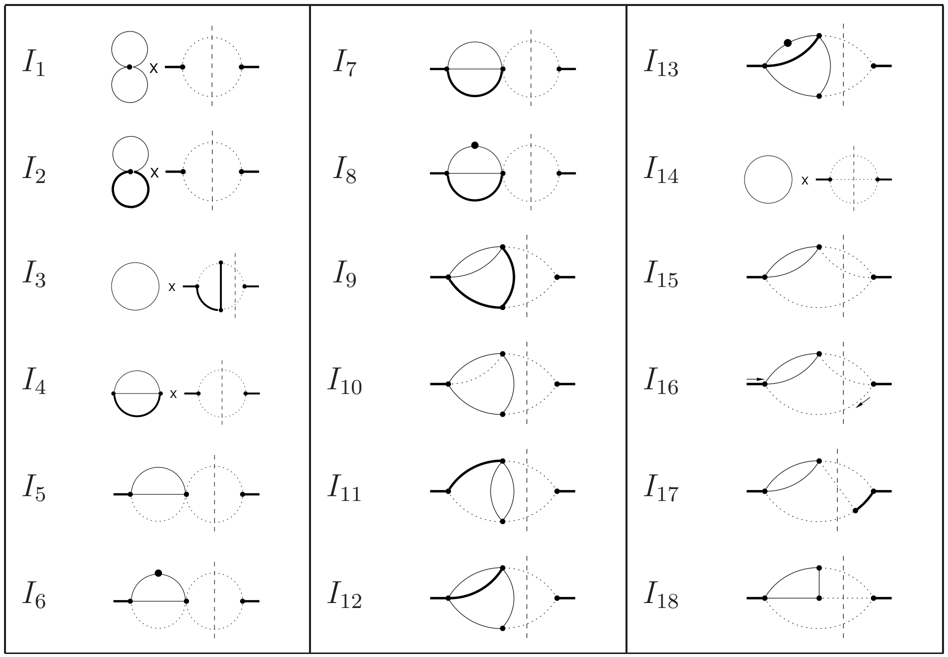
5. Calculating three-loop single-scale master integrals for the boundary conditions. Methods ...

6. Solving the system $(*)$ numerically [A.C. Hindmarsch, `http://www.netlib.org/odepack`] along an ellipse in the complex z plane. Doing so along several different ellipses allows us to estimate the numerical error.

The same method has been applied to the 3-loop counterterm diagrams

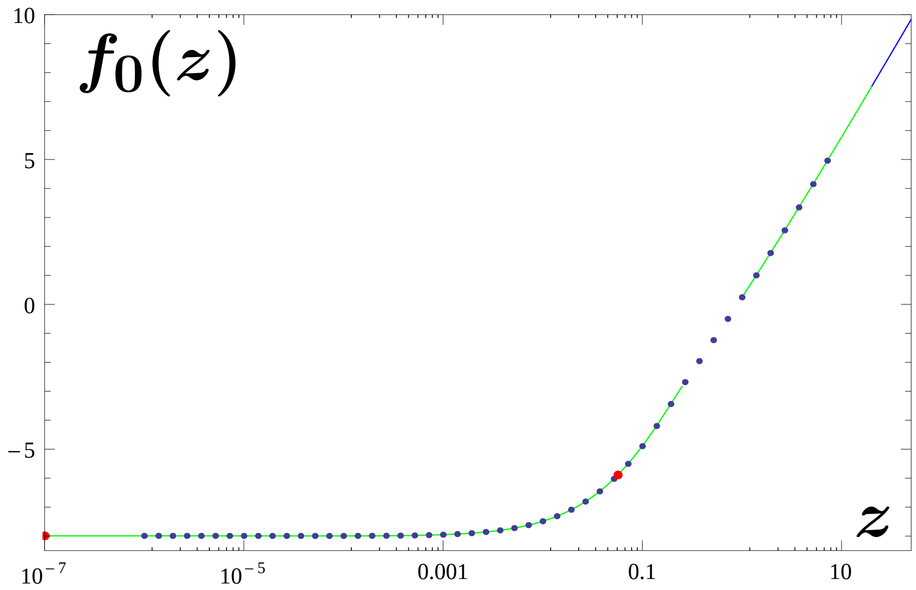
[MM, A. Rehman, M. Steinhauser, PLB 770 (2017) 431]

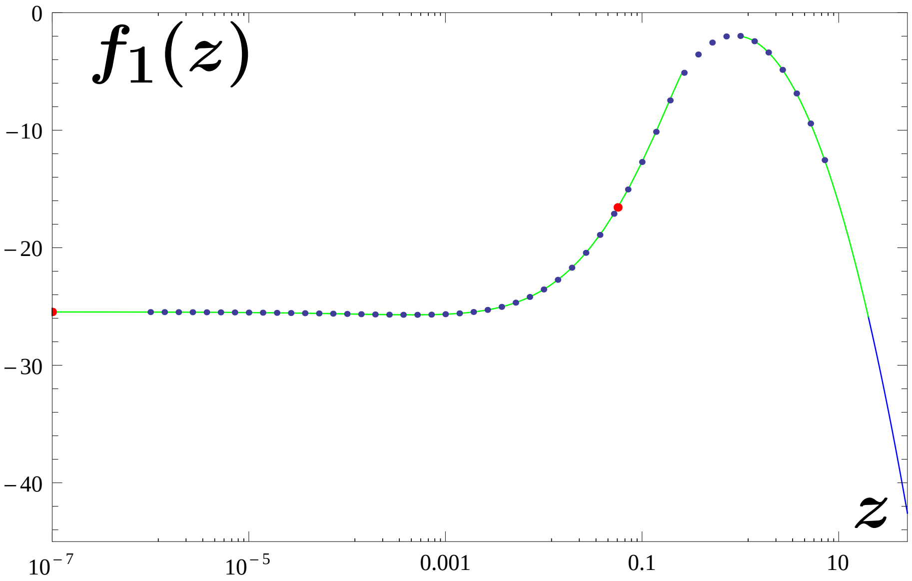
Master integrals:



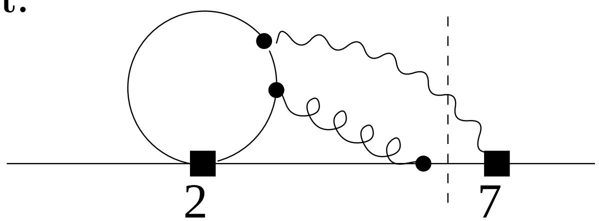
Results for the bare NLO contributions up to $\mathcal{O}(\epsilon)$:

$$\hat{G}_{27}^{(1)2P} = -\frac{92}{81\epsilon} + f_0(z) + \epsilon f_1(z) \xrightarrow{z\to 0} -\frac{92}{81\epsilon} - \frac{1942}{243} + \epsilon\left(-\frac{26231}{729} + \frac{259}{243}\pi^2\right)$$





Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.
Lines: large- and small-z asymptotic expansions



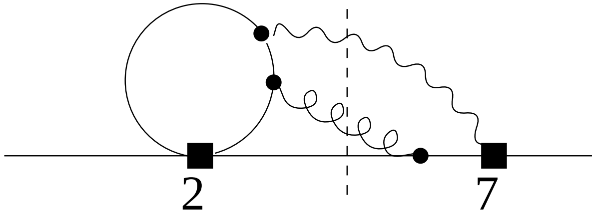
Small-z expansions of $\hat{G}_{27}^{(1)2P}$:

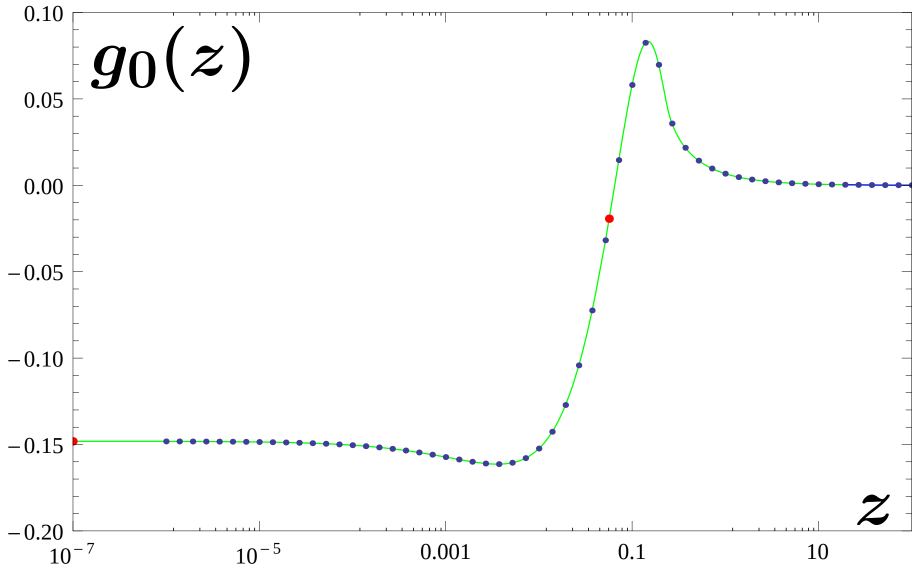
f_0 from C. Greub, T. Hurth, D. Wyler, hep-ph/9602281, hep-ph/9603404,
A. J. Buras, A. Czarnecki, MM, J. Urban, hep-ph/0105160,

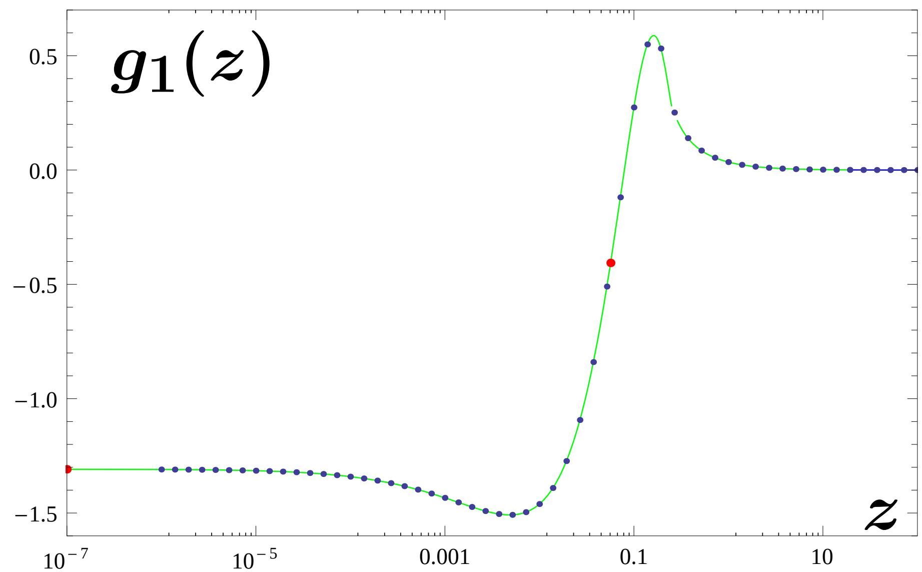
f_1 from H.M. Asatrian, C. Greub, A. Hovhannisyan, T. Hurth and V. Poghosyan, hep-ph/0505068.

Analogous results for the 3-body final state contributions ($\delta = 1$):

$$\hat{G}_{27}^{(1)3P} = g_0(z) + \epsilon g_1(z) \xrightarrow{z\to 0} -\frac{4}{27} - \frac{106}{81}\epsilon$$







Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.
Lines: exact result for g_0, as well as large- and small-z asymptotic expansions for g_1.

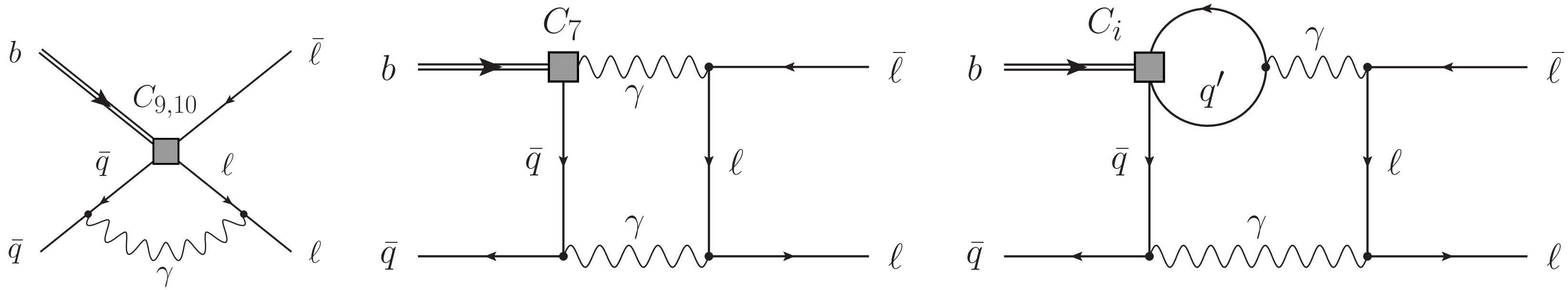
$$g_0(z) = \begin{cases} -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, s\, L + \frac{16}{9}z(6z^2-4z+1)\left(\frac{\pi^2}{4} - L^2\right), & \text{for } z \le \frac{1}{4}, \\ -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, t\, A + \frac{16}{9}z(6z^2-4z+1)\, A^2, & \text{for } z > \frac{1}{4}, \end{cases}$$

where $s = \sqrt{1-4z}$, $L = \ln(1+s) - \frac{1}{2}\ln 4z$, $t = \sqrt{4z-1}$, and $A = \arctan(1/t)$.

Enhanced QED effects in $B_q \to \ell^+\ell^-$

The leading contribution to the decay rate is proportonal to $f_{B_q}^2 \sim \frac{\Lambda^3}{M_{B_q}}$.

As observed by M. Beneke, C. Bobeth and R. Szafron in arXiv:1708.09152, some of the QED corrections scale like Λ^2:



Consequently, the relative QED correction scales like $\frac{\alpha_{em}}{\pi} \frac{M_{B_q}}{\Lambda}$.

Their explicit calculation implies that the previous results for all the $B_q \to \ell^+\ell^-$ branching ratios need to be multiplied by

$$0.993 \pm 0.004.$$

Thus, despite the $\frac{M_{B_q}}{\Lambda}$-enhancement, the effect is well within the previously estimated $\pm 1.5\%$ non-parametric uncertainty.

However, it is larger than $\pm 0.3\%$ stemming from scale-variation of the Wilson coefficient $C_A(\mu_b)$.

SM predictions for all the branching ratios $\overline{\mathcal{B}}_{q\ell} \equiv \overline{\mathcal{B}}(B_q^0 \to \ell^+\ell^-)$

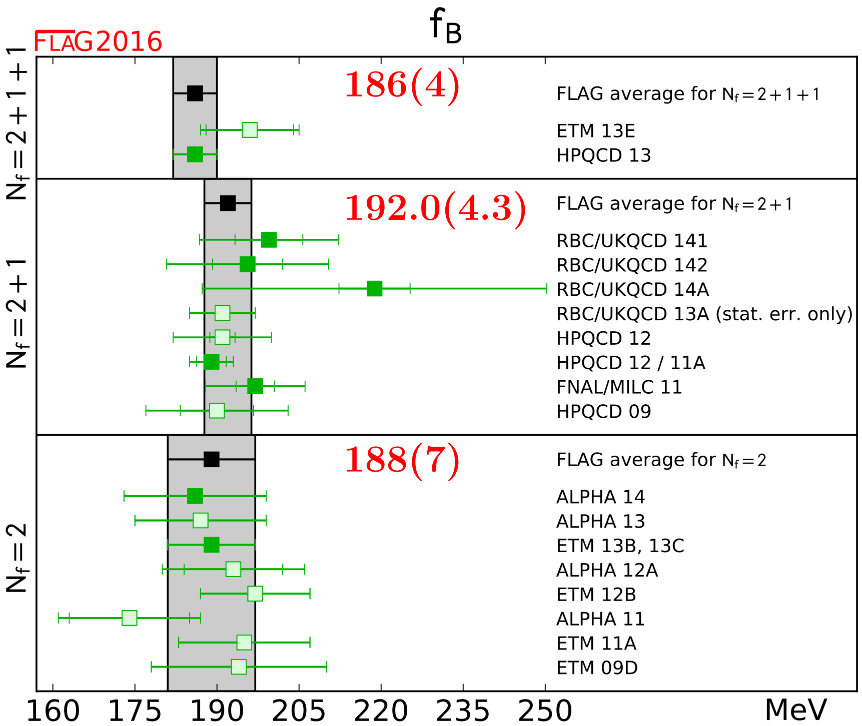
[C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou, M. Steinhauser, PRL 112 (2014) 101801]

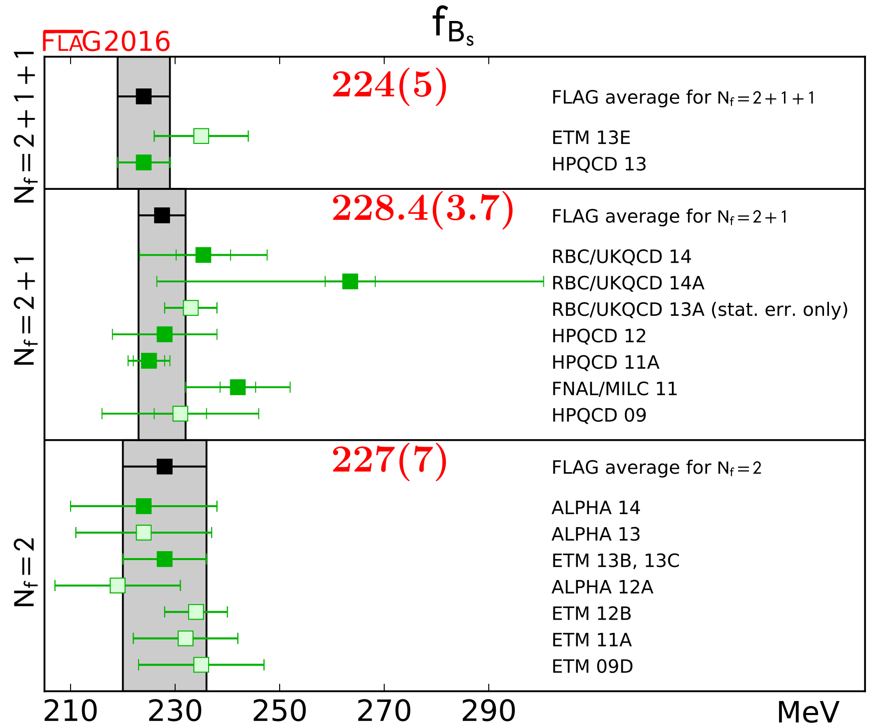
$$
\begin{aligned}
\overline{\mathcal{B}}_{se} \times 10^{14} &= (8.54 \pm 0.13)\, R_{t\alpha}\, R_s, \\
\overline{\mathcal{B}}_{s\mu} \times 10^{9} &= (3.65 \pm 0.06)\, R_{t\alpha}\, R_s, \\
\overline{\mathcal{B}}_{s\tau} \times 10^{7} &= (7.73 \pm 0.12)\, R_{t\alpha}\, R_s, \\
\overline{\mathcal{B}}_{de} \times 10^{15} &= (2.48 \pm 0.04)\, R_{t\alpha}\, R_d, \\
\overline{\mathcal{B}}_{d\mu} \times 10^{10} &= (1.06 \pm 0.02)\, R_{t\alpha}\, R_d, \\
\overline{\mathcal{B}}_{d\tau} \times 10^{8} &= (2.22 \pm 0.04)\, R_{t\alpha}\, R_d,
\end{aligned}
$$

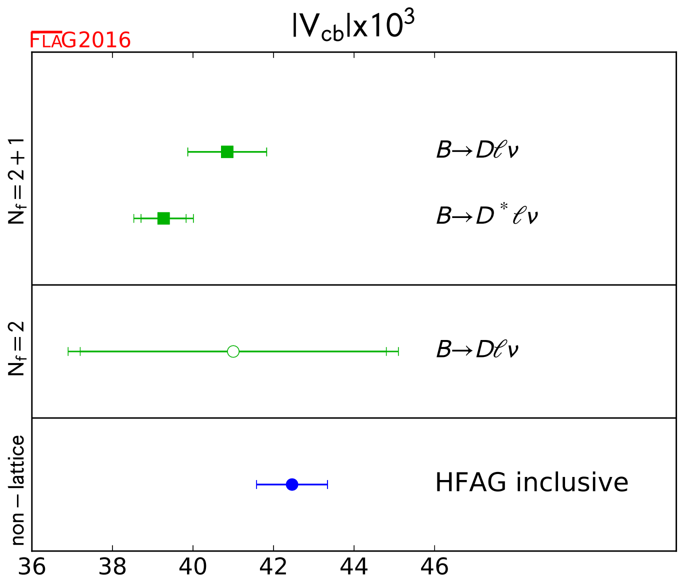
where

$$
\begin{aligned}
R_{t\alpha} &= \left(\frac{M_t}{173.1\ \mathrm{GeV}}\right)^{3.06} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.18}, \\
R_s &= \left(\frac{f_{B_s}[\mathrm{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\ [\mathrm{ps}]}{1.615}, \\
R_d &= \left(\frac{f_{B_d}[\mathrm{MeV}]}{190.5}\right)^2 \left(\frac{|V_{tb}^\star V_{td}|}{0.0088}\right)^2 \frac{\tau_d^{\mathrm{av}}\ [\mathrm{ps}]}{1.519}.
\end{aligned}
$$

Inputs from FLAG, arXiv:1607.00299, Figs. 20 and 30 (+ web page update)







0.041(1)

0.03927(76) **(2.7σ tension with the inclusive)**

$\longrightarrow$ **0.04200(64)** **from P. Gambino, K. J. Healey and S. Turczyk Phys.Lett.B 763 (2016) 60.**

Update of the input parameters

	2014 paper	this talk	source
M_t [GeV]	173.1(9)	174.30(65)	CDF & D0, arXiv:1608.01881
$\alpha_s(M_Z)$	0.1184(7)	0.1182(12)	PDG 2016
f_{B_s} [GeV]	0.2277(45)	0.2240(50)	FLAG 2016
f_{B_d} [GeV]	0.1905(42)	0.1860(40)	FLAG 2016
$\|V_{cb}\|$	0.04240(90)	0.04089(44)	naive average excl. & incl.
$\|V_{tb}^* V_{ts}\|/\|V_{cb}\|$	0.9800(10)	0.9819(4)	derived from CKMfitter 2016
$\|V_{tb}^* V_{td}\|$	0.0088(3)	0.0087(2)	derived from CKMfitter 2016
τ_H^s [ps]	1.615(21)	1.619(9)	HFLAV 2017
τ_H^d [ps]	1.519(7)	1.518(4)	HFLAV 2017
$\overline{\mathcal{B}}_{s\mu} \times 10^9$	3.65(23)	3.35(18)	
$\overline{\mathcal{B}}_{d\mu} \times 10^{10}$	1.06(9)	1.00(7)	

Sources of uncertainties	f_{B_q}	CKM	τ_H^q	M_t	α_s	other parametric	non-parametric	$\sum$
$\overline{\mathcal{B}}_{s\ell}$	4.5%	2.2%	0.6%	1.2%	0.1%	$< 0.1\%$	1.5%	5.4%
$\overline{\mathcal{B}}_{d\ell}$	4.3%	4.6%	0.3%	1.2%	0.1%	$< 0.1\%$	1.5%	6.7%

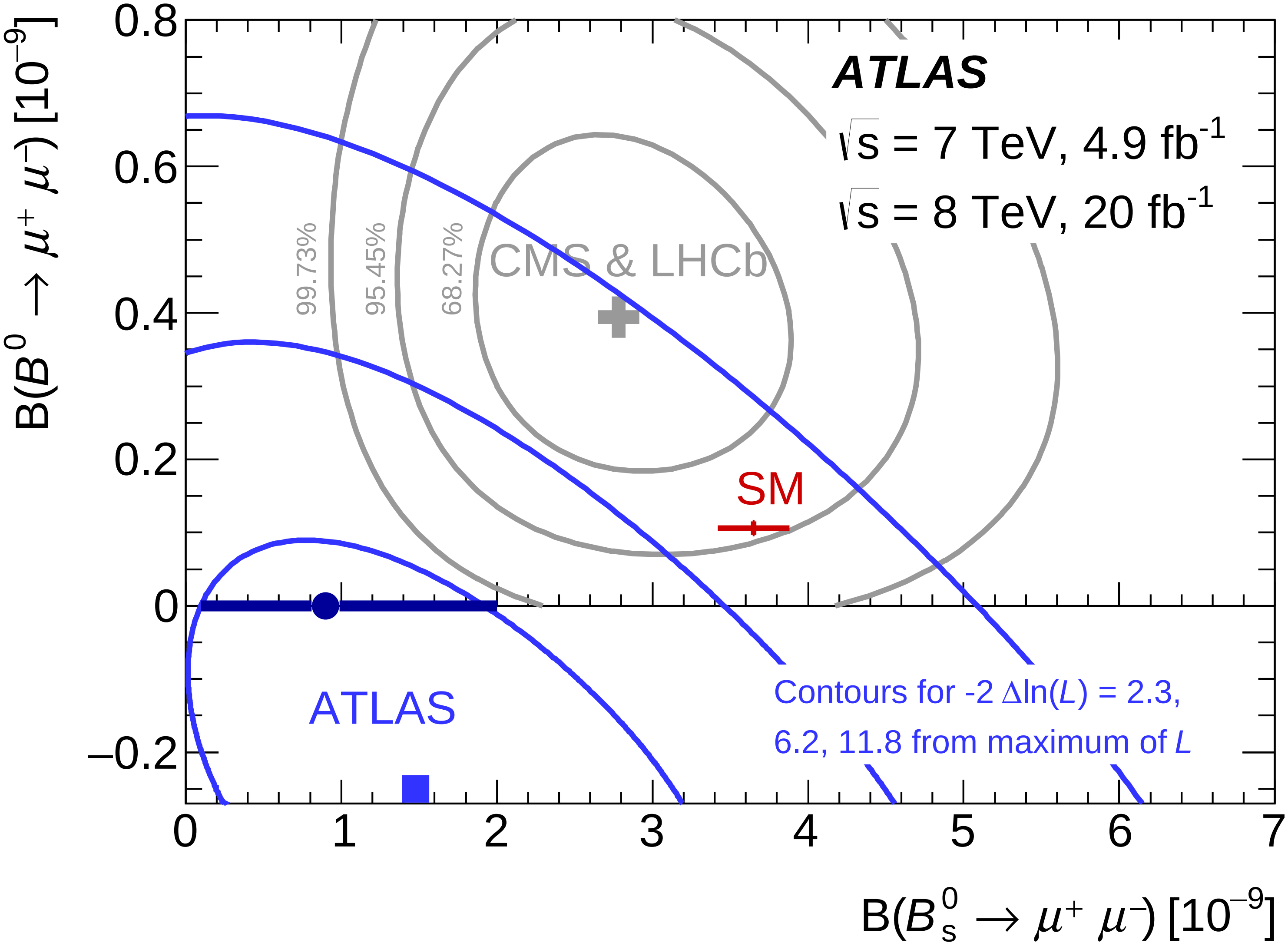
If the inclusive $|V_{cb}| = 0.04200(64)$ alone is used instead of the naive average, then $\overline{\mathcal{B}}_{s\mu} \times 10^9 = 3.54(21)$.

Comparison with the measurements

Previous averages, CMS and LHCb, Nature 522 (2015) 68: $\overline{\mathcal{B}}_{s\mu} = \left(2.8^{+0.7}_{-0.6}\right) \times 10^{-9}$, $\overline{\mathcal{B}}_{d\mu} = \left(3.9^{+1.6}_{-1.4}\right) \times 10^{-10}$.

New results of LHCb, PRL 118 (2017) 191801: $\overline{\mathcal{B}}_{s\mu} = \left(3.0 \pm 0.6^{+0.3}_{-0.2}\right) \times 10^{-9}$, $\overline{\mathcal{B}}_{d\mu} = \left(1.5^{+1.2+0.2}_{-1.0-0.1}\right) \times 10^{-10}$.

ATLAS in EPJC 76 (2016) 513 gives 95% C.L. bounds: $\overline{\mathcal{B}}_{s\mu} < 3.0 \times 10^{-9}$ **and** $\overline{\mathcal{B}}_{d\mu} < 4.2 \times 10^{-10}$.



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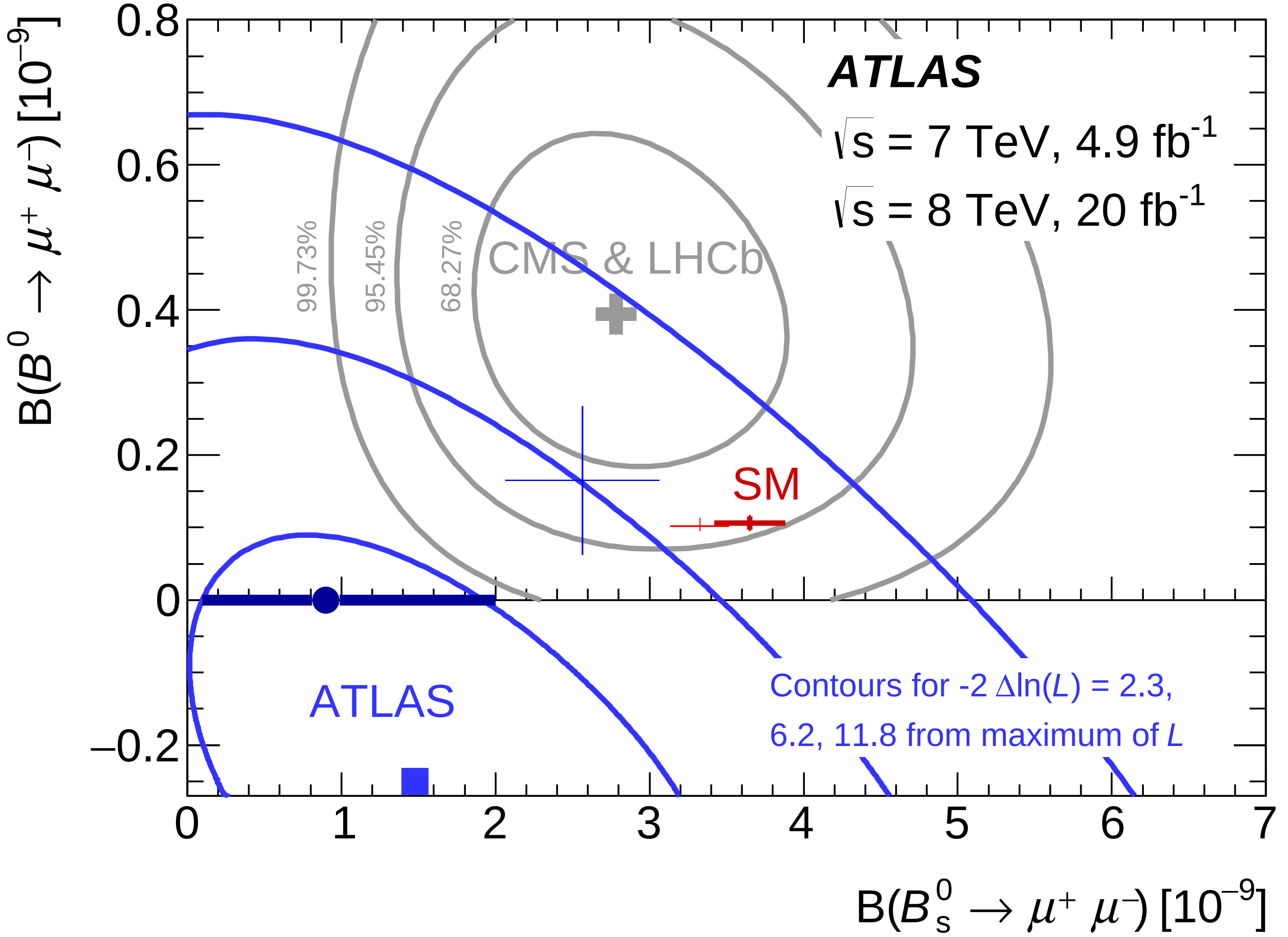
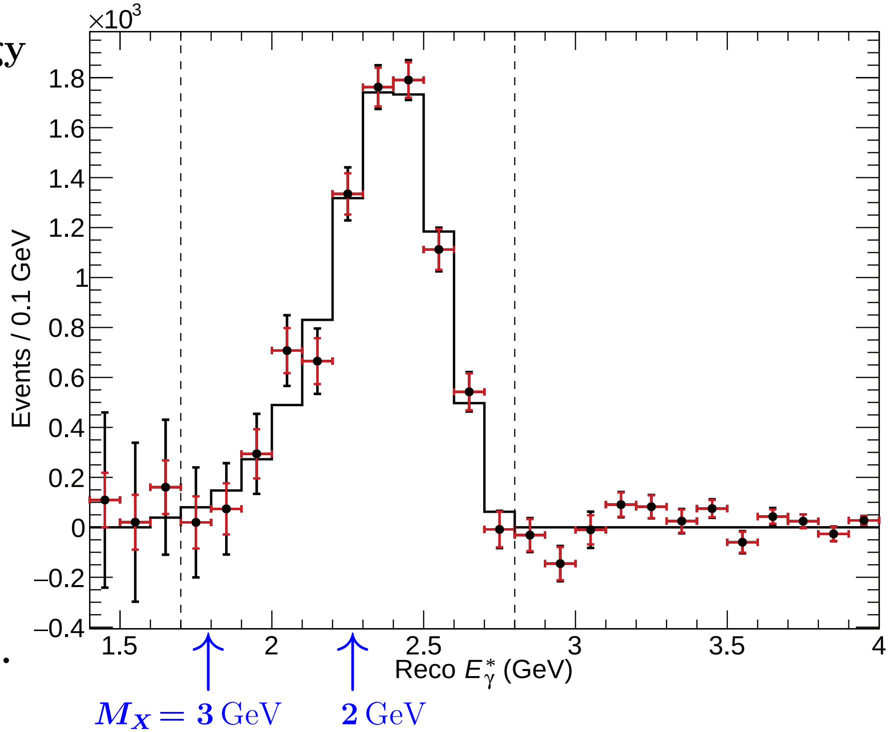


Non-local charm loops in $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s\ell^+\ell^-$

Background-subtracted $\bar{B} \to X_{s+d}\gamma$ photon energy spectrum in the $\Upsilon(4S)$ rest frame, from Fig. 1 of the Belle analysis in arXiv:1608.02344.



For $M_X \lesssim 3\,\text{GeV}$ and in the absence of 4-quark ops, we have local OPE $\Rightarrow \mathcal{O}(\Lambda^2/m_b^2)$.

In the presence of 4-quark ops:
Light quark loops – suppressed by $C_{3,\ldots,6}$ or CKM;
Charm loops – factorizable or local if m_c^2 is sufficiently large w.r.t. $m_b\Lambda$. Numerically, $\mathcal{O}(3\%)$ non-fact. effects found in $\mathcal{B}(\bar{B} \to X_s\gamma)$ and $\mathcal{B}(\bar{B} \to X_s\ell^+\ell^-)$ with $q^2 \in [1,6]\,\text{GeV}$.
[Buchalla, Isidori, Rey, NPB 511 (1998) 594]

However, m_c^2 is not sufficiently large $\Rightarrow$ Treat it as $\mathcal{O}(m_b\Lambda)$ and use SCET, so far up to $\mathcal{O}(\Lambda/m_b)$:
For $\mathcal{B}(\bar{B} \to X_s\gamma)$ with $E_\gamma > 1.6\,\text{GeV}$ [Benzke, Lee, Neubert, Paz, JHEP 1008 (2010) 099] $[-4.8\%, +5.6\%]$ uncert. range.
For $\mathcal{B}(\bar{B} \to X_s\ell^+\ell^-)$ with $q^2 \in [1,6]\,\text{GeV}$ [Benzke, Hurth, Turczyk, JHEP 1710 (2017) 031] $[-2.7\%, +1.8\%]$ range.
(On the top of the factorizable and/or local effects, including the Λ^2/m_c^2 ones.)

Corrections not involving Q_7 and Q_8 are of higher order, i.e. $\mathcal{O}\left[\left(\frac{\Lambda}{m_b}\right)^a\right]$ with $a \geq \frac{3}{2}$ and/or $\mathcal{O}\left(\frac{\alpha_s\Lambda}{m_b}\right)$.
That's what we miss using the purely perturbative expression for $\left|C_9^{\text{eff}}(q^2)\right|^2$ and the local $1/m_c^2$ effects.
However, the applied SCET power counting works only for small M_X and small q^2 – verte.

SCET power counting in the $\bar{B} \to X_s \ell^+ \ell^-$ analysis of Benzke, Hurth and Turczyk, JHEP 1710 (2017) 031.

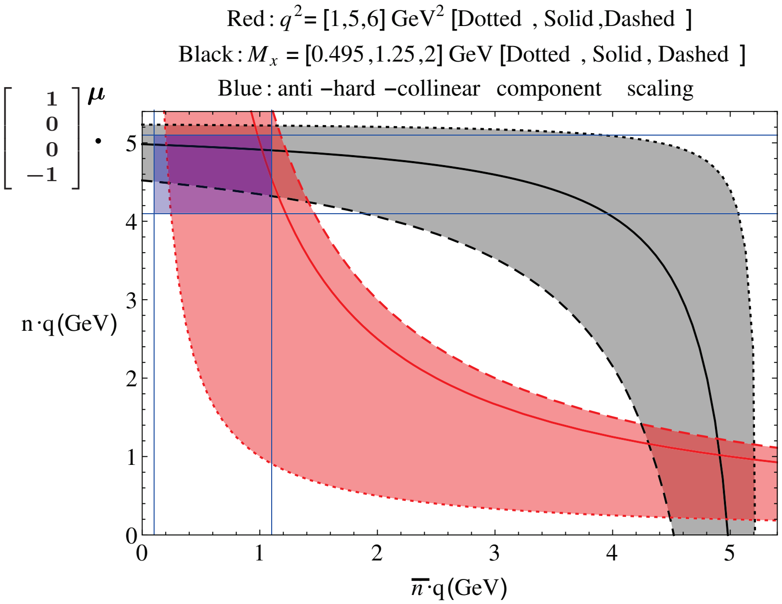
$\lambda = \frac{\Lambda}{M_B}, \quad M_X \lesssim \sqrt{M_B \Lambda} = M_B \sqrt{\lambda} \sim m_c.$

$a^\mu = \frac{1}{2}(na)\bar{n}^\mu + \frac{1}{2}(\bar{n}a)n^\mu + a_\perp^\mu,$ with $n^\mu = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}^\mu, \quad \bar{n}^\mu = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}^\mu.$

↑ any vector

In the plot $q_\perp = 0 \Rightarrow q^2 = (\bar{n}q)(nq)$.

hard:	$p \sim (1, 1, 1)$,
hard-collinear:	$p \sim (\lambda, 1, \sqrt{\lambda})$,
anti-hard-collinear:	$p \sim (1, \lambda, \sqrt{\lambda})$,
soft:	$p \sim (\lambda, \lambda, \lambda)$



Plot ⇒ Our cuts would better fit into the SCET-accessible region if we restricted to $q^2 \in [1, 5]$ GeV. On the other hand, the cut on M_X could be somewhat larger than 2 GeV.

The factorization formula:

$$d\Gamma = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} + \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

Remarks:

1. Not proven. Contradictions observed in the $Q_8 - Q_8$ case, claimed to be phenomenologically irrelevant.
2. Relates unknowns to unknowns. Models of soft functions needed (constraints available).
3. Corrections beyond $\mathcal{O}(\Lambda/m_b)$ are likely to be relevant because $|C_{9,10}/C_7| \sim 13$ (work in progress) [BHT].
4. Other observables – after including the above corrections.
5. Different power counting than in the previous SCET analyses where no "resolved photons" were included [Lee, Stewart, PRD74 (2006) 014005], [Bell, Beneke, Huber, Li, NPB843 (2011) 143].

Sample (previous) SM predictions for $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-) \times 10^6$ with $q^2 \in [1,6]$ GeV and no cut on M_X:

$\left.\begin{array}{l} 1.64 \pm 0.11, \;\; \ell = e \\ 1.59 \pm 0.11, \;\; \ell = \mu \end{array}\right\}$ parametric and perturbative uncert. only [Huber, Lunghi, MM, Wyler, NPB 740 (2006) 105]

$\left.\begin{array}{l} 1.67 \pm 0.10, \;\; \ell = e \\ 1.62 \pm 0.09, \;\; \ell = \mu \end{array}\right\}$ param. update + Krüger-Sehgal, [Huber, Hurth, Lunghi, JHEP 1506 (2015) 176]
PLB380 (1996) 199
PRD55 (1997) 2799

The corresponding semi-inclusive experimental results, averaged over $\ell = e, \mu$:

$1.60^{+0.41+0.17}_{-0.39-0.13} \pm 0.18$, Babar, PRL112 (2014) 211802, $471 \times 10^6 B\bar{B}$, extrapolated from $M_X < 1.8$ GeV,

$1.493 \pm 0.504^{+0.411}_{-0.321}$, Belle, PRD72 (2005) 092005, $152 \times 10^6 B\bar{B}$.

Remarks:

1. The Krüger-Sehgal (factorizable) contribution should be retained even after the SCET estimates for the resolved photon contributions are included in the future. What about $\mathcal{O}(\alpha_s)$?
2. Given the presence of resolved photon contributions, neither $\bar{B} \to X_s \gamma$ nor $\bar{B} \to X_s \ell^+ \ell^-$ are useful for precise determination of the HQET parameters. The semileptonic observables alone should be sufficient.
3. Suggestion: use $M_X < 3$ GeV as a default cut to which the experimental $\bar{B} \to X_s \ell^+ \ell^-$ results are being extrapolated, similarly to $E_\gamma > 1.6$ GeV in the $\bar{B} \to X_s \gamma$ case. The leading shape function is identical in both processes.

Summary

- **Large deviations from the SM are observed in tree-level LFU-violating observables $R_{D^{(*)}}$, as well as in the loop generated transition $b \to s\ell^+\ell^-$. On the other hand, several sensitive loop processes like $\bar{B} \to X_s\gamma$ or $B_s \to \mu^+\mu^-$ remain in good agreement with the SM. Certain leptoquark models can accommodate such a situation.**

- **Perturbative calculations of $\bar{B} \to X_s\gamma$ require further optimization of software/hardware for the IBP reduction.**

- **In the case of $B_s \to \mu^+\mu^-$, resolving the inclusive-exclusive tension in $|V_{cb}|$ would help a lot.**

- **Charm quark loops in the inclusive $\bar{B} \to X_s\ell^+\ell^-$ decay seem to be under better control than in the corresponding exlusive decay channels.**