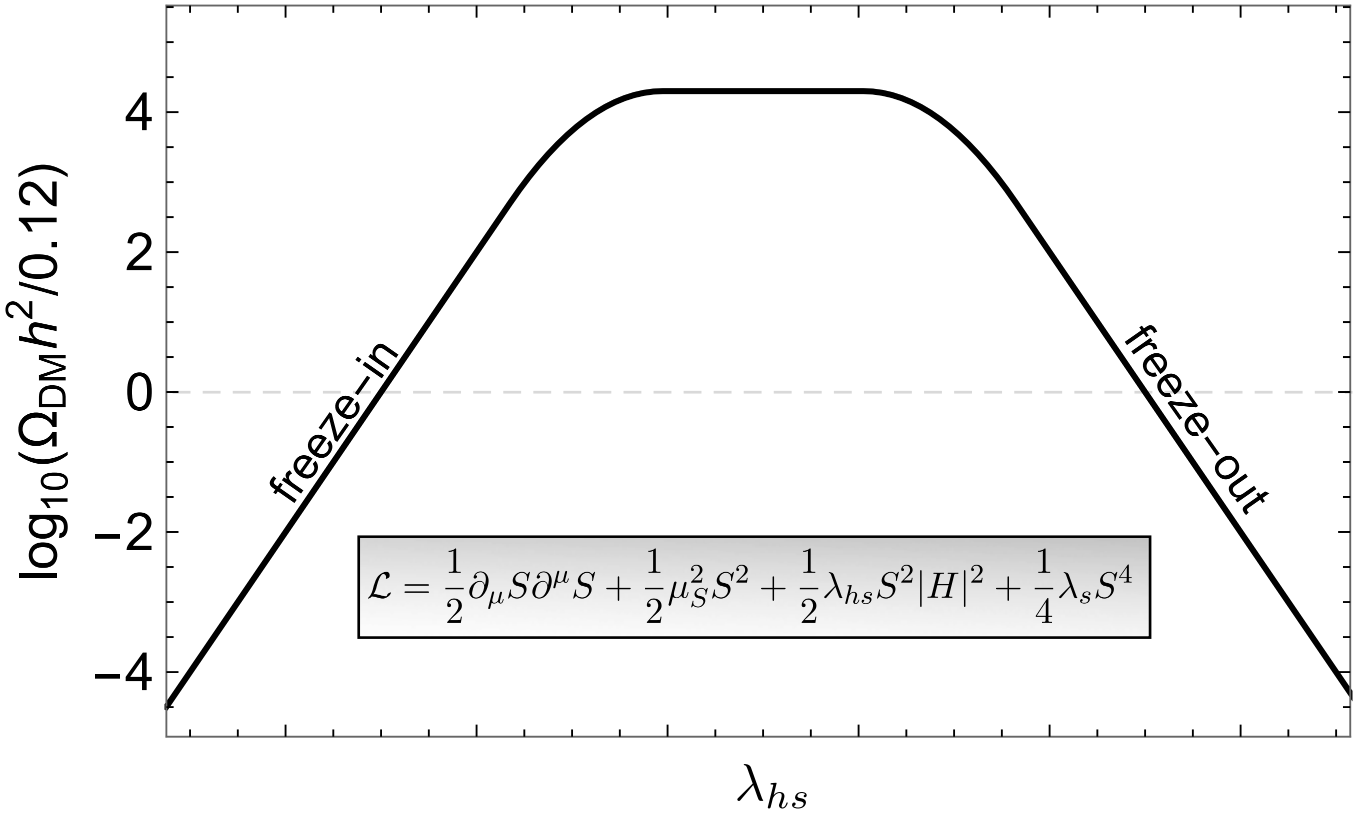
Scalar singlet dark matter

Updates on freeze-out and freeze-in production

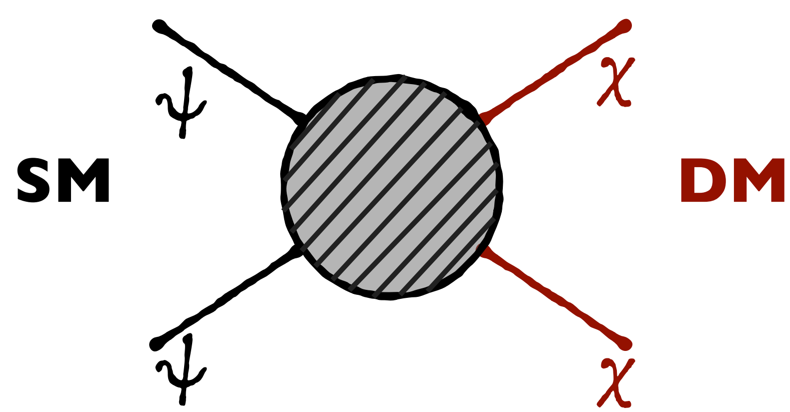
Torsten Bringmann



UiO : University of Oslo

The origin of dark matter

- Existence of (particle) DM = evidence for BSM physics
- Precision measurement of its abundance $\boxed{\Omega_{\rm CDM} h^2 = 0.1188 \pm 0.0010}$
  Ade+ [Planck Coll.], A&A '16
- Any convincing model for dark matter must include a production mechanism to explain the observed abundance!
- Most often considered interactions with primordial heat bath:
  - [Z_2 symmetry not strictly necessary, but automatically guarantees stability of DM]



Boltzmann equation

- Evolution of DM phase-space density:

$$\boxed{L[f_\chi] = C[f_\chi]}$$

$$\text{“} = \frac{\mathrm{d}f_\chi}{\mathrm{d}t}\text{”} = E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f_\chi$$ **for FRW metric**

- Collision term (for $f_\chi \ll 1$):

$$C[f_\chi] = \frac{1}{2g_\chi}\int\frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}}\int\frac{d^3k}{(2\pi)^3 2\omega}\int\frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}}(2\pi)^4\delta^{(4)}(\tilde{p}+p-\tilde{k}-k)$$
$$\times\left[\underbrace{|\mathcal{M}|^2_{\bar{\chi}\chi\leftarrow\bar{f}f}\, f_\psi(\omega) f_\psi(\tilde{\omega})}_{\text{‘production’}} - \underbrace{|\mathcal{M}|^2_{\bar{\chi}\chi\to\bar{f}f}\, f_\chi(E) f_\chi(\tilde{E}) \{1 \pm f_\psi(\omega)\}\{1 \pm f_\psi(\tilde{\omega})\}}_{\text{‘annihilation’}}\right]$$

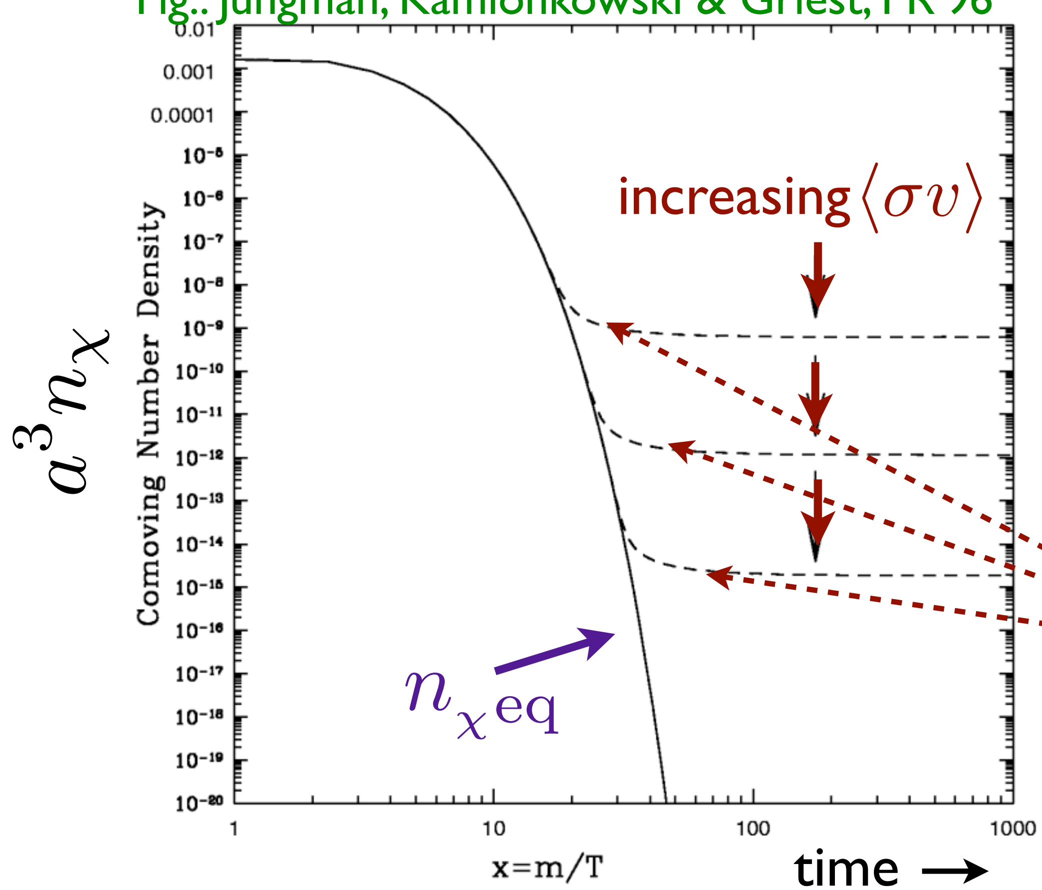
- Detailed balance: ‘production’ = ‘annihilation’ *in equilibrium*
  - ⇝ allows to re-write everything in terms of a *would-be* equilibrium population of DM

- Pauli suppression / Bose enhancement
  - ⇝ not relevant for production of non-relativistic ***DM*** [energy conservation!]

Weakly Interacting Massive Particles

- well-motivated from particle physics [SUSY, EDs, …]
- thermal production in early universe:

Fig.: Jungman, Kamionkowski & Griest, PR'96



$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle \left(n_\chi^2 - n_{\chi\,\mathrm{eq}}^2\right)$$

$\langle\sigma v\rangle$: $\chi\chi \to \mathrm{SM\ SM}$

(thermal average)

"Freeze-out" when annihilation rate falls behind expansion rate

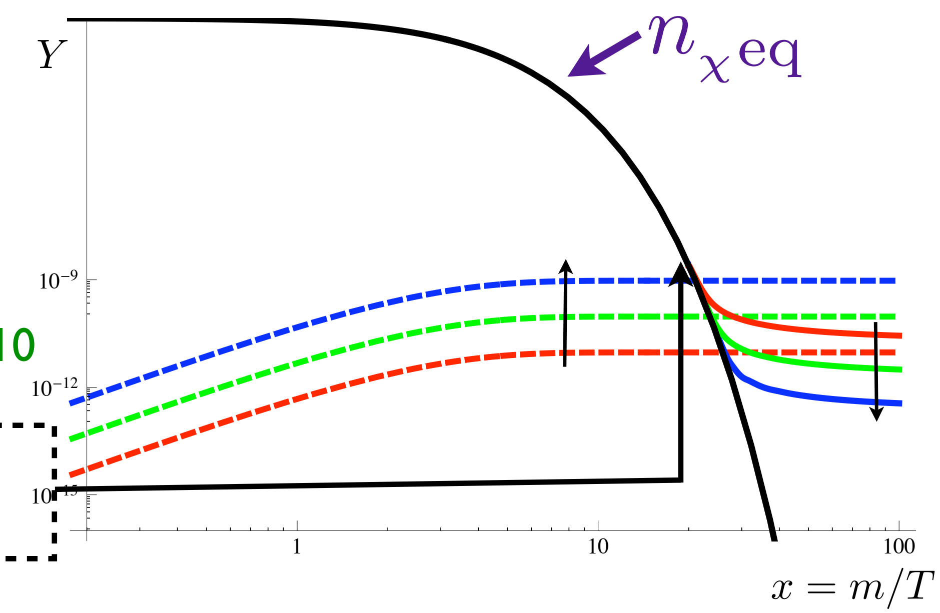
for weak-scale interactions!

➡ Relic density (today): $\Omega_\chi h^2 \sim \frac{3 \cdot 10^{-27}\mathrm{cm}^3/\mathrm{s}}{\langle\sigma v\rangle} \sim \mathcal{O}(0.1)$

Feably Interacting Massive Particles

- alternative production from thermal bath with much smaller interactions
  - DM never equilibrates in this case

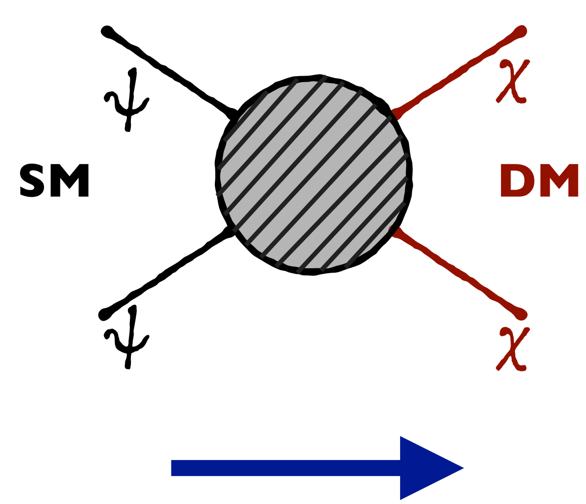
Hall+, JHEP '10



***Increasing* interaction rate for freeze-in**

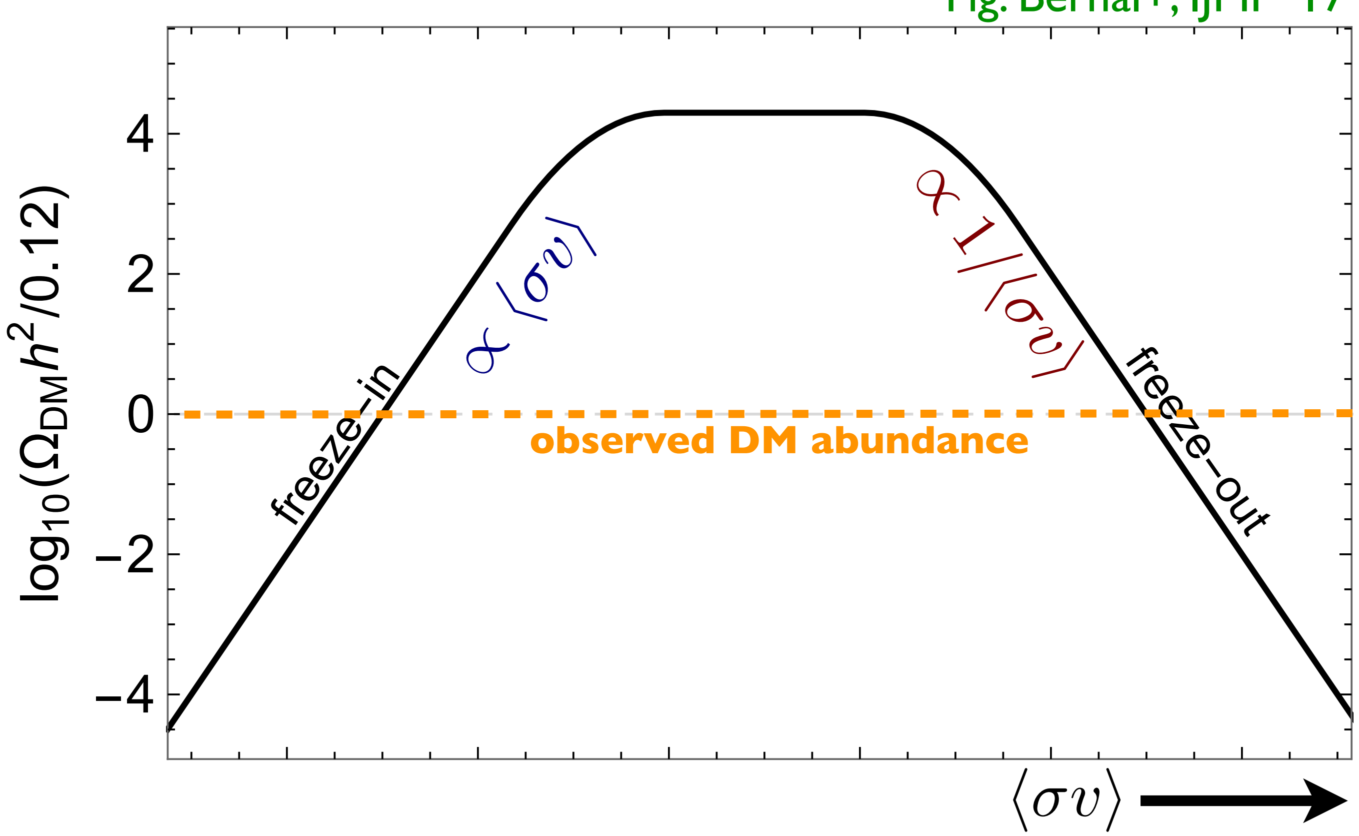
- Smooth transition between two regimes:

freeze-in

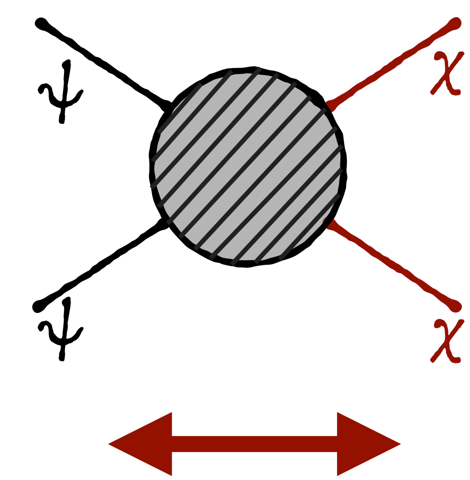


depends on initial conditions

Fig: Bernal+, IJMP '17



freeze-out



insensitive to initial conditions

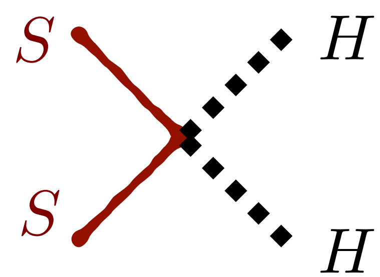
Still the most economic model…

- **Scalar Singlet DM**

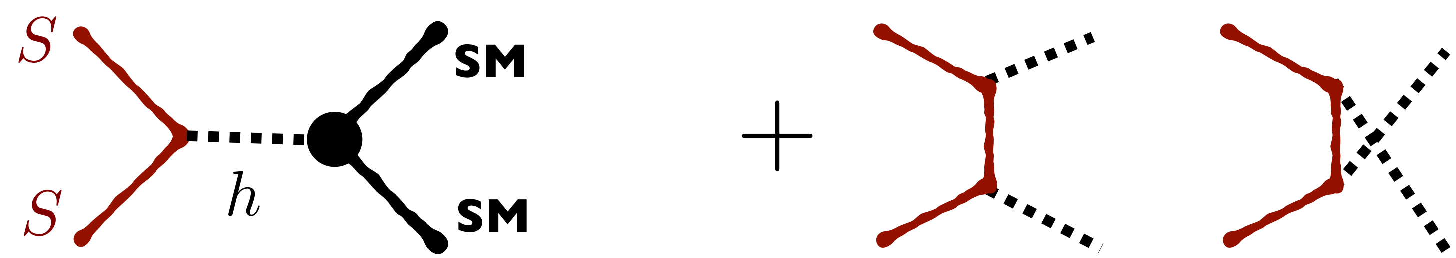
$$\mathcal{L} = \frac{1}{2}\partial_\mu S \partial^\mu S + \frac{1}{2}\mu_S^2 S^2 + \frac{1}{2}\lambda_{hs} S^2 |H|^2 + \frac{1}{4}\lambda_s S^4$$

Silveira & Zee, PLB '85

- before EWSB:

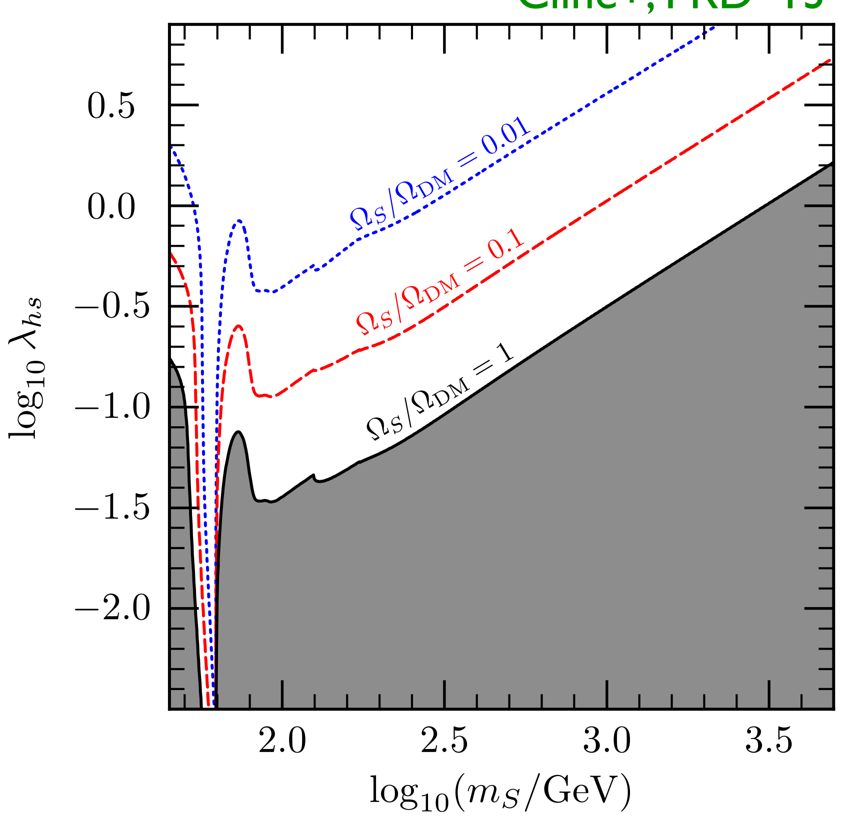


- after EWSB:



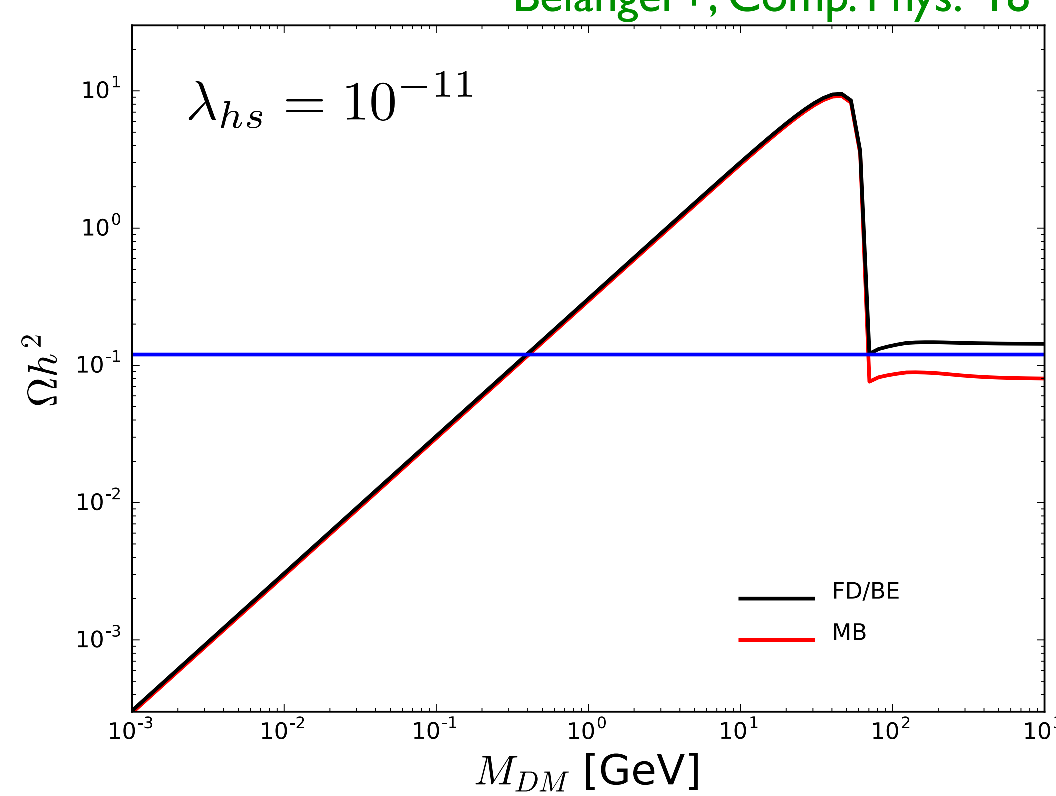
- Well established DM candidate in both regimes
  - freeze-out:

Cline+, PRD '13



  - freeze-in:

Belanger+, Comp. Phys. '18



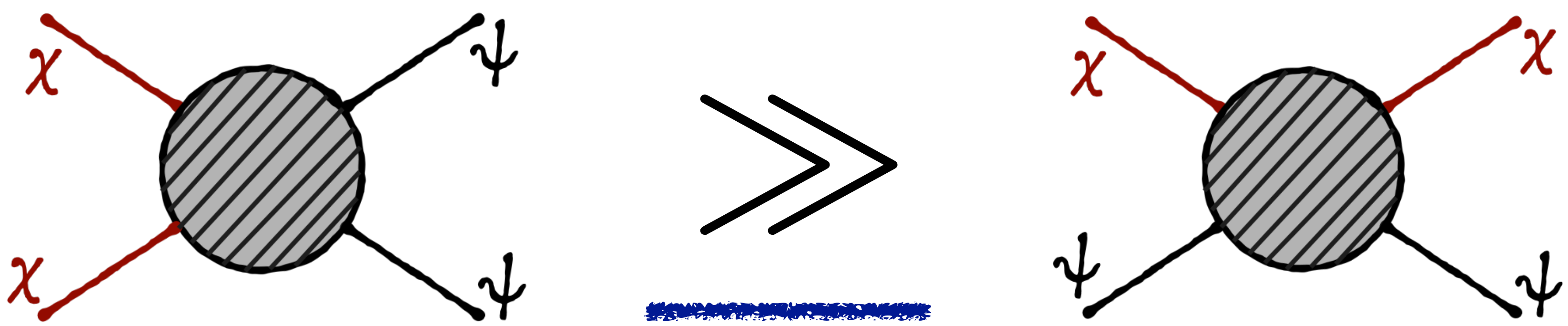
I.

A closer look at

freeze-out

Freeze-out $\neq$ decoupling

- Typical situation:



i.e. kinetic decoupling much later than chemical decoupling [review: TB, NJP '09]

- Recall the 'standard' Boltzmann equation:

$$\boxed{\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - n_{\chi\,\mathrm{eq}}^2\right)} \quad \Leftarrow \quad \boxed{L[f_\chi] = C[f_\chi]}$$

- CP invariance + detailed balance
- $m_\chi \gg T$
- $f_\chi \propto e^{-E/T}$

Gondolo & Gelmini, NPB '91

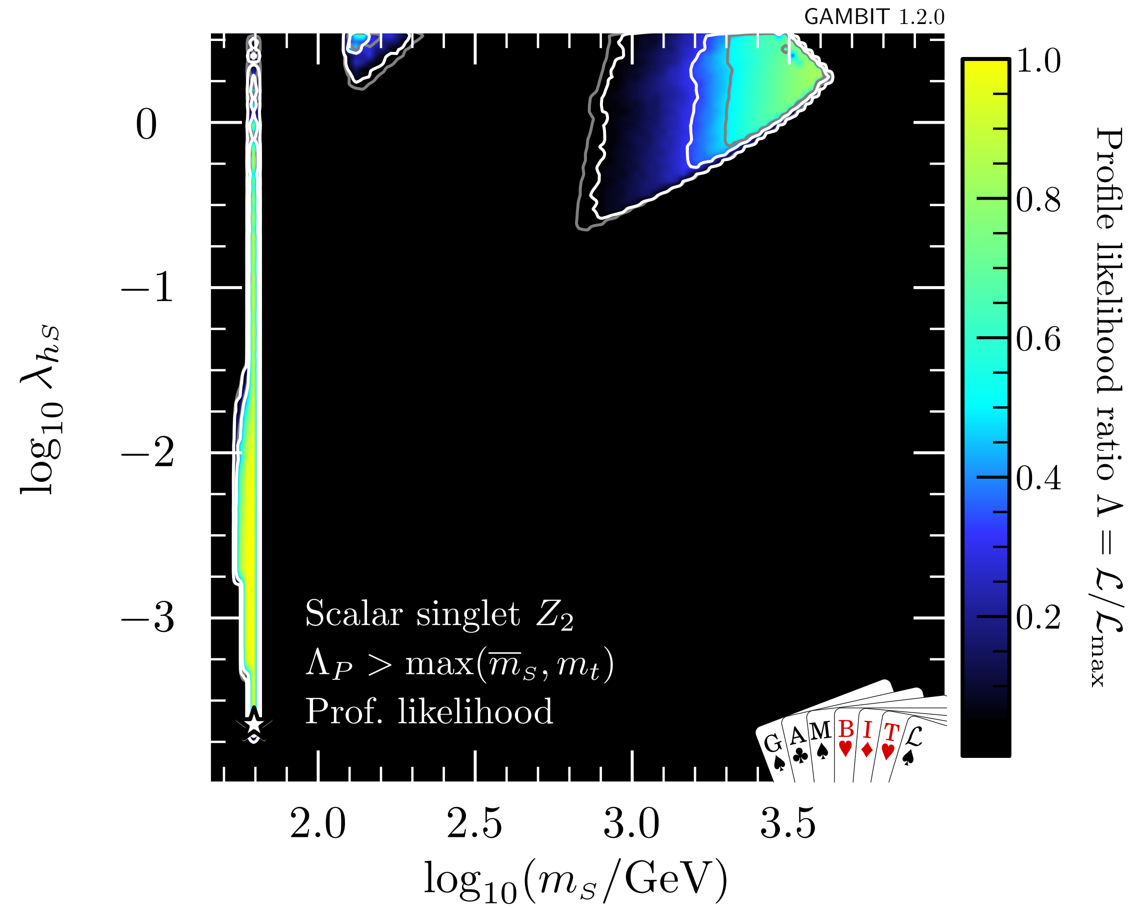
kinetic equilibrium is a crucial *assumption* !

Relevant parameter space ?

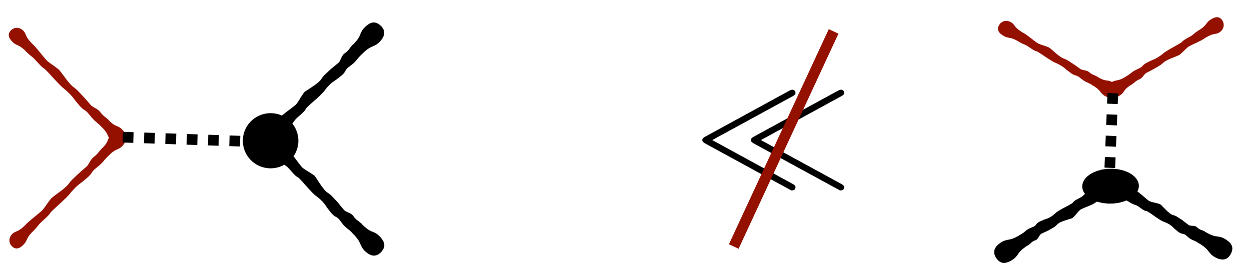
- Only two regions surviving complementary constraints :

  Athron+, EPJC '17
  Athron+, EPJC '18

  - $m_S \sim 3\,\mathrm{TeV}$
  - $m_S \sim m_h/2 \sim 62\,\mathrm{GeV}$



- Focus on resonance region:



  - $\langle \sigma v \rangle \sim 10^{-26} \mathrm{cm}^3 \mathrm{s}^{-1}$
  - *need* small λ_{hs}

  - *suppressed* by small λ_{hs}
  - *no* enhancement $n_{\mathrm{SM}} \gg n_\chi$

    (Yukawa coupling favours scattering on heavy particles!)

→ *difficult to maintain kinetic equilibrium !*

Back to the drawing board…

$$\boxed{L[f_\chi] = C[f_\chi]}$$

- solve 2D coupled Boltzmann equations

  van den Aarssen, TB & Goedecke, PRD '12

  Binder+, PRD '17

  $$1. \int \frac{d^3p}{E} \rightsquigarrow n_\chi$$

  $$2. \int \frac{d^3p}{E} p^2 \rightsquigarrow T_\chi \equiv N \int \frac{d^3p}{E} p^2 f_\chi$$

  - based on assumption that Boltzmann hierarchy closes (higher moments can be neglected)
  - E.g. for large self-interactions λ_s
  - numerically (relatively) straightforward

    DarkSUSY since 6.2.5

- direct (numerical) integration at phase-space level
  - Numerically much more challenging
  - Public code:

    DRAKE

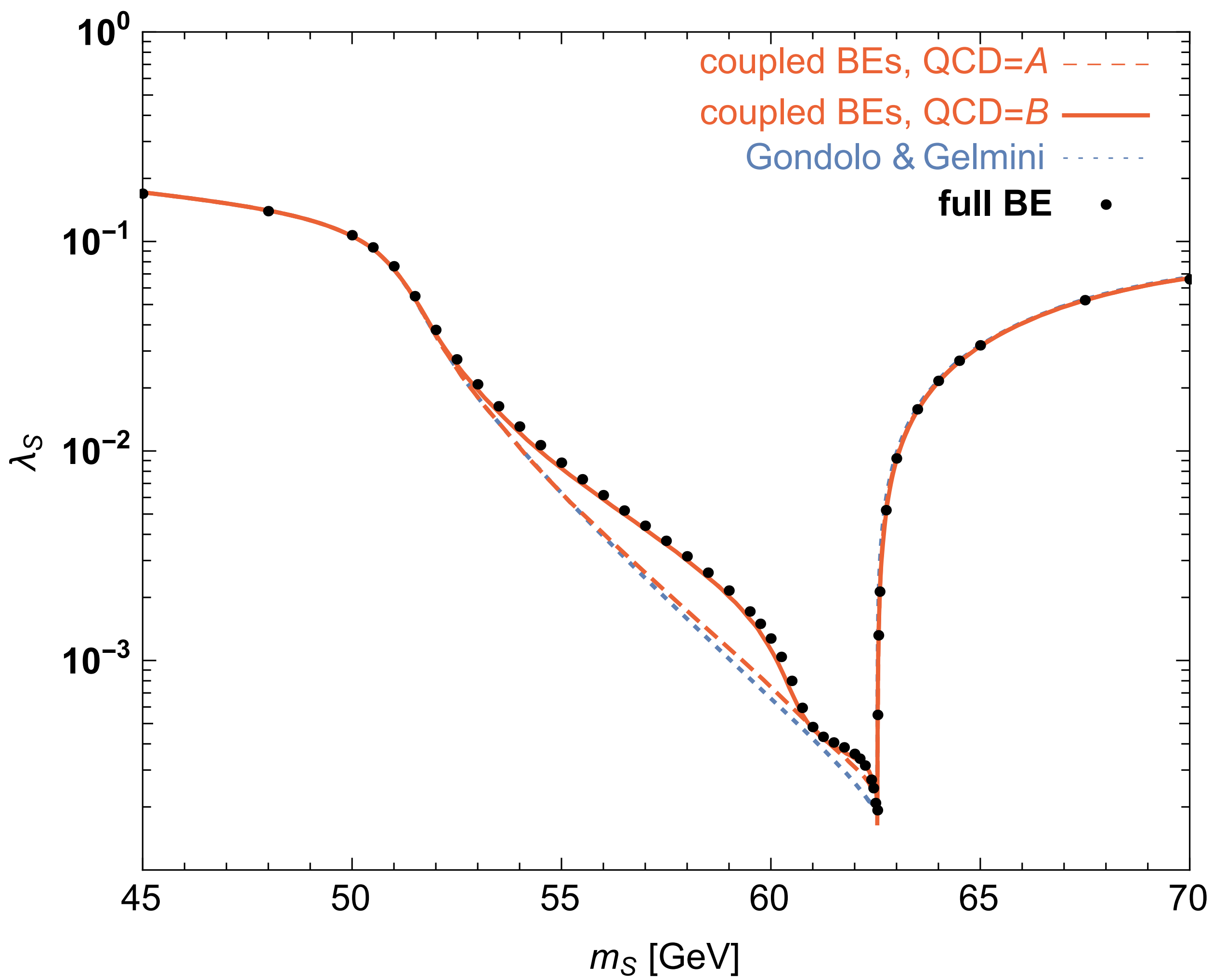
    also nBE, cBE…

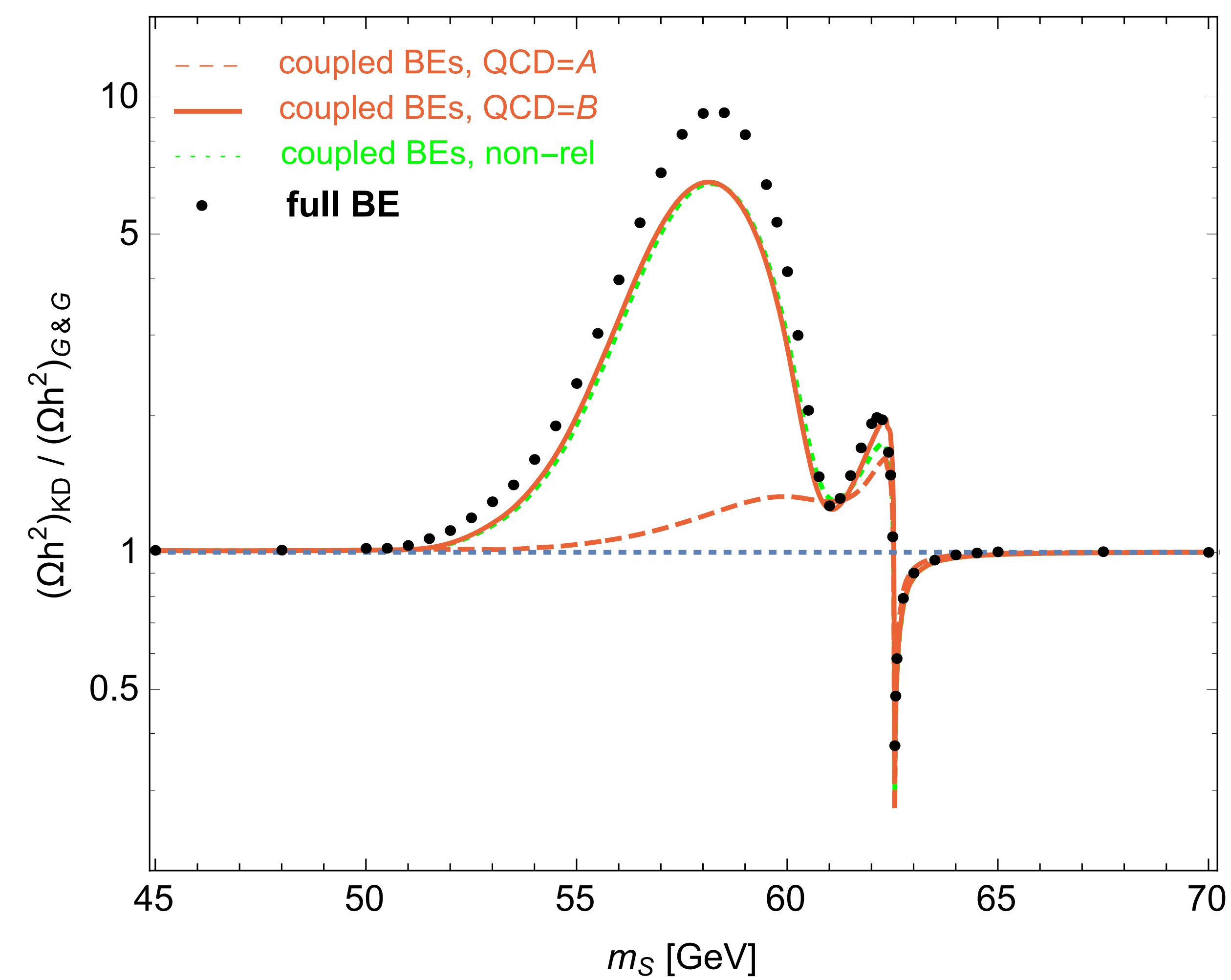
    Binder+, EPJC '21
  - Including self-interactions is even harder

    But see Hryzcuk & Laletin, PRD '22 !

Results

Binder, TB, Gustafsson & Hryczuk, PRD '17





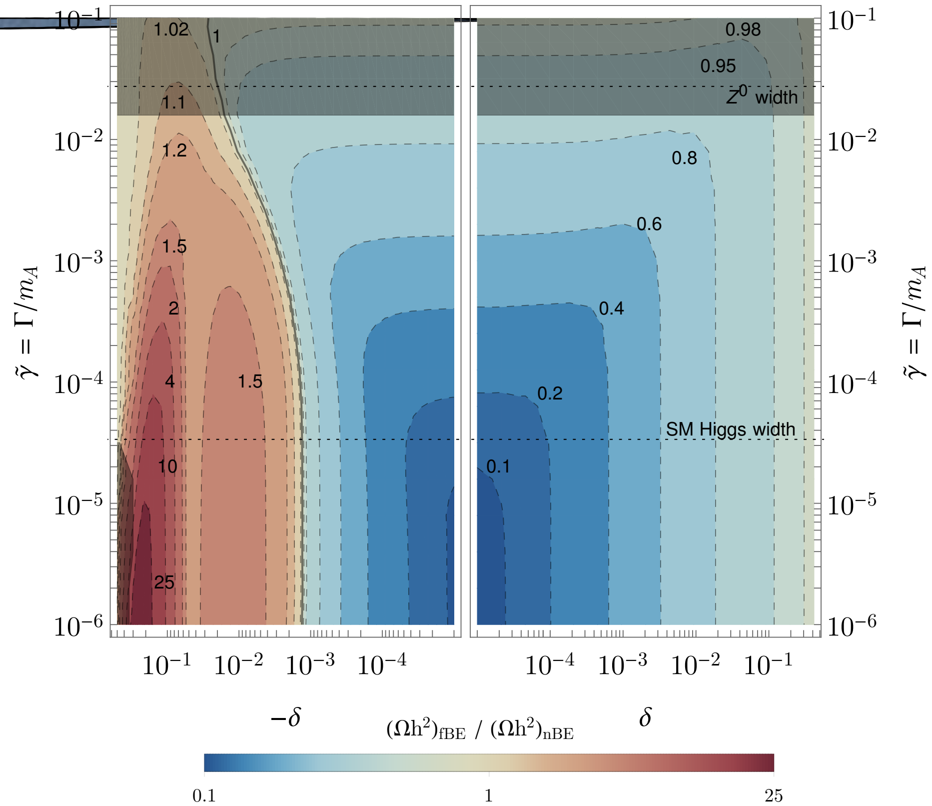
- Effect is highly significant (cf. $\Delta\Omega_{\rm DM}h^2/\Omega_{\rm DM}h^2 \lesssim 0.01$)
- cBE works surprisingly (?) well, despite its simplicity
  - NB: Langevin simulations independently confirm that momentum distribution remains Gaussian

Kim & Laine, JCAP '23

Comments

- general feature: mind your $\Omega_{\rm DM}h^2$ calculation in the presence of (narrow) resonances !

Binder+, EPJC '21



- Various detailed studies about how to implement scattering

Ala-Mattinen & Kainulainen, JCAP '20
Ala-Mattinen+, PRD '22
Du+, JCAP '22
Laine, JHEP '23
...

  - Some differences due to different scattering amplitudes
    [don't take the overly aggressive 'QCD B' too literally…!]
  - Some differences due to different modelling of relativistic collision term
  - Qualitatively good agreement

II.

A closer look at

freeze-in

Collision term for FIMPs

$$C[f_\chi] = \frac{1}{2g_\chi}\int \frac{d^3\tilde p}{(2\pi)^3 2\tilde E}\int \frac{d^3 k}{(2\pi)^3 2\omega}\int \frac{d^3\tilde k}{(2\pi)^3 2\tilde\omega}(2\pi)^4\delta^{(4)}(\tilde p + p - \tilde k - k)$$

$$\times\left[\underbrace{|\mathcal{M}|^2_{\bar\chi\chi\leftarrow\bar\psi\psi} f_\psi(\omega) f_\psi(\tilde\omega)}_{\text{'production'}} - \underbrace{\cancel{|\mathcal{M}|^2_{\bar\chi\chi\rightarrow\bar\psi\psi} f_\chi(E) f_\chi(\tilde E)\{1\pm f_\psi(\omega)\}\{1\pm f_\psi(\tilde\omega)\}}}_{\text{'annihilation'}}\right]$$

- Only 2 integrals after exploiting spherical symmetry:

[γ : Lorenz boost to CMS; $\tilde s = s/(4m_\chi^2)$]

$$\boxed{\langle\sigma v\rangle_{\chi\chi\to\psi\psi} = \frac{8x^2}{{K_2}^2(x)}\int_1^\infty d\tilde s\,\tilde s\,(\tilde s - 1)\underbrace{\int_1^\infty d\gamma\,\sqrt{\gamma^2-1}e^{-2\sqrt{\tilde s}x\gamma}}_{\to K_1(2\sqrt{\tilde s}x)/(2\sqrt{\tilde s}x)\ ✔}\sigma_{\chi\chi\to\psi\psi}(s,\cancel{\gamma})}$$

TB, Heeba, Kahlhoefer & Vangsnes, JHEP '22

(see also Lebedev & Toma, PLB '19 Arcadi+, JHEP '19)

$$\sigma_{\chi\chi\to\psi\psi}(p,\tilde p) = \frac{(2\pi)^4}{4N_\psi E\tilde E\, v_{\text{Møl}}}\int \frac{d^3 k}{(2\pi)^3 2\omega}\int \frac{d^3\tilde k}{(2\pi)^3 2\tilde\omega}\,\delta^{(4)}(\tilde p + p - \tilde k - k)\,|\overline{\mathcal{M}}|^2\,\underbrace{\{1\pm f_\psi(\omega)\}\{1\pm f_\psi(\tilde\omega)\}}_{\to 1}$$

➡ $\boxed{C[f_\chi] = \langle\sigma v\rangle_{\chi\chi\to\psi\psi}\left(n_\chi^{\text{MB}}\right)^2}$ **annihilation** of ***would-be*** MB population → ***actual*** (in eq)

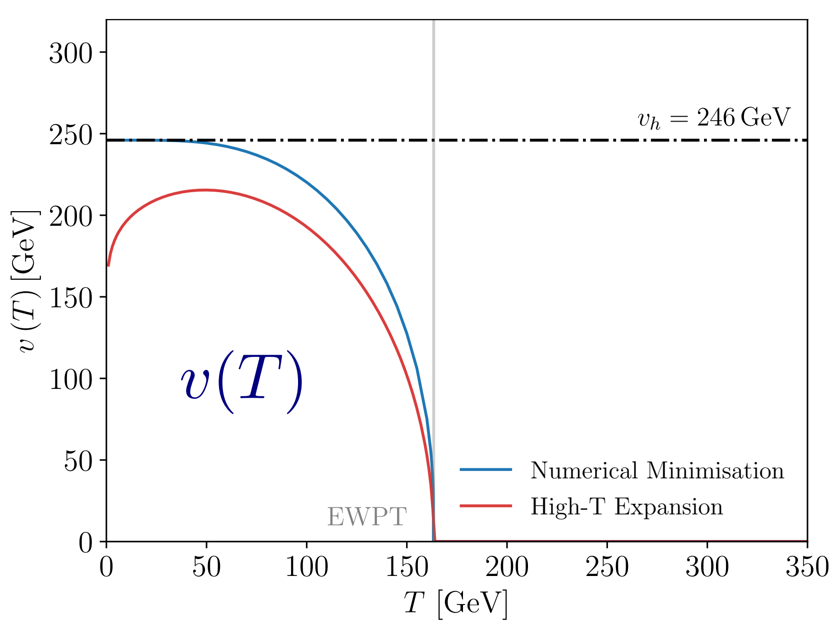
- In *this* formulation, direct analogy with WIMP case!
  - ➡ Can recycle sophisticated numerical tools for thermal averages
  - Easier to estimate higher-order corrections

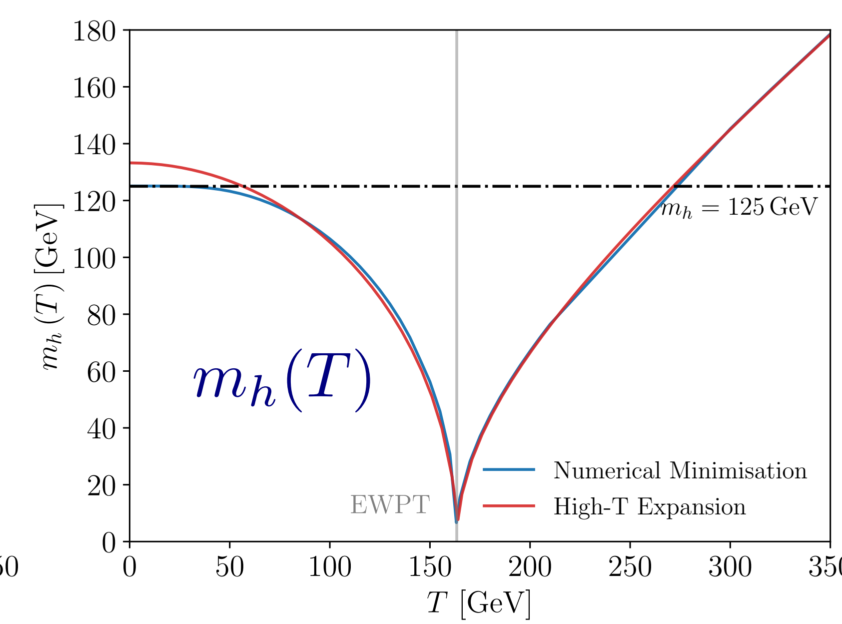
(Further) Finite temperature effects

- FIMPs are dominantly produced at higher temperatures than WIMPs, and over a larger range

➡ Not only quantum statistics matters, but also
  - Thermal masses of heat bath particles $\delta m^2 \sim g^2 T^2$
  - Phase transitions:
    - Potentially *additional* $m(T)$
    - Spectrum of states changes

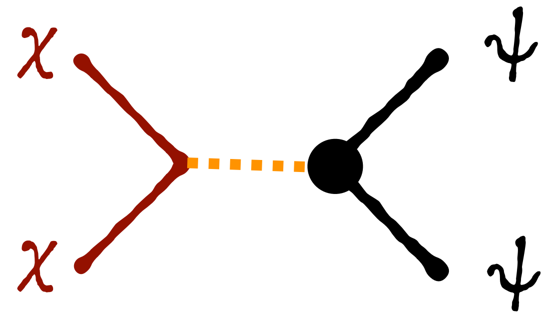
- For example the EWPT:





DM from decays

- 2→2 or 1→2 ?
  - For $\Gamma \to 0$, rates should be identical at (N)LO !



- For a fully thermalized mediator *A*, this requires

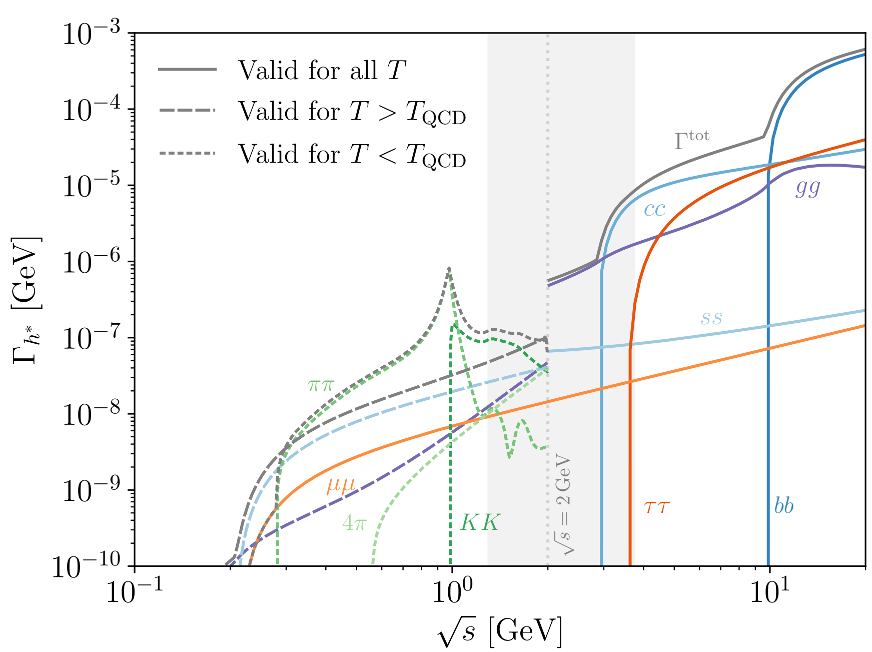
$$\Gamma_{\rm BW} = \frac{1}{1+f_A(\omega')} \sum_{\psi_1\psi_2} \Gamma_{\psi_1\psi_2} \overline{G}_{\psi_1\psi_2}(\gamma, m_A^2)$$

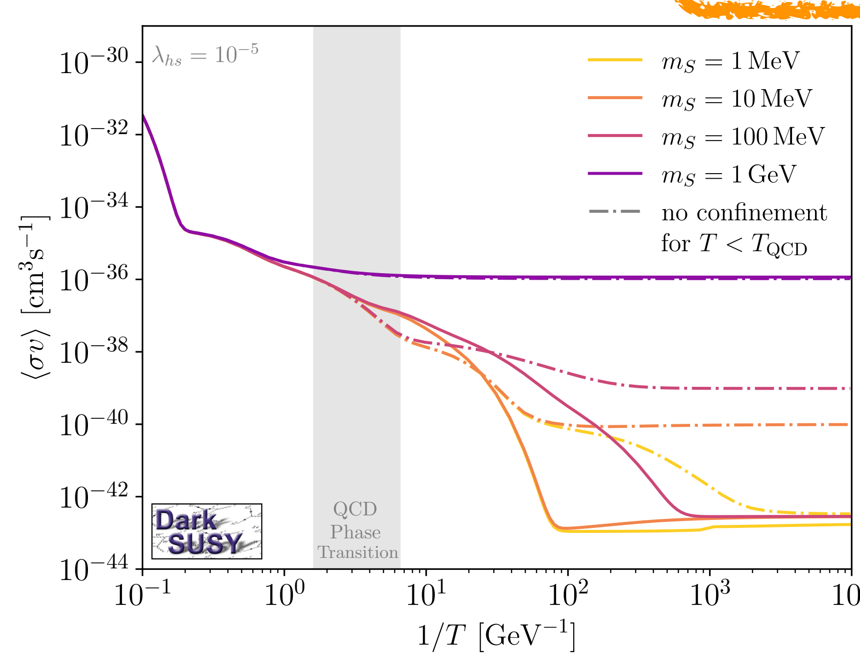
width in propagator **width in vacuum** **final state quantum enhancement/suppression**

TB, Heeba, Kahlhoefer & Vangsnes, JHEP '22

- For a SM Higgs mediator, e.g., we can directly use tabulated partial widths, both below and above QCD PT

for "→", but not for "←"!

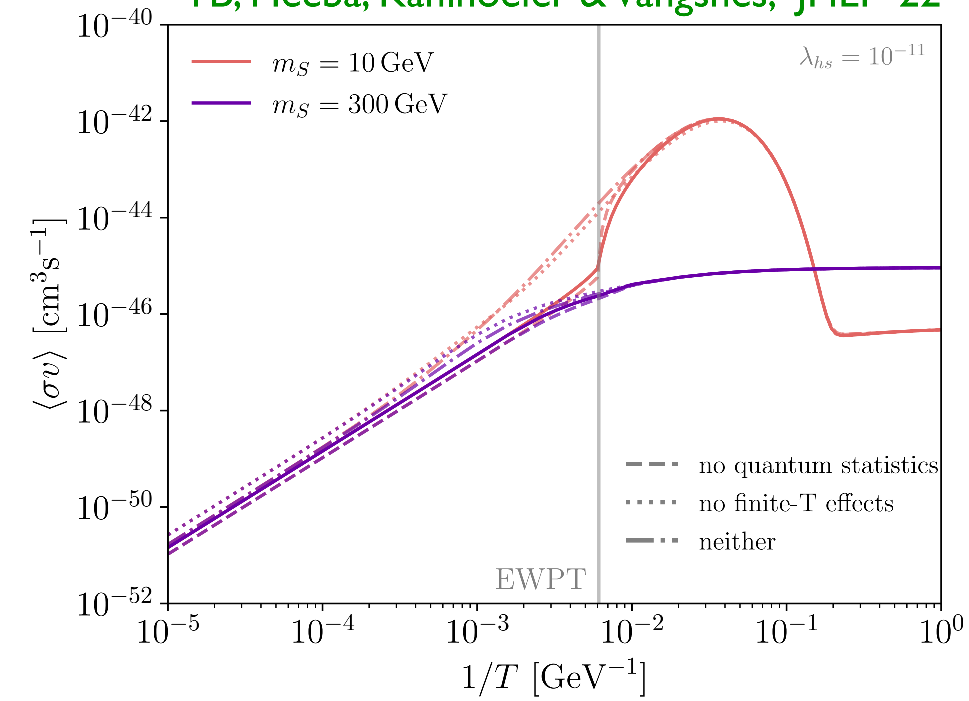




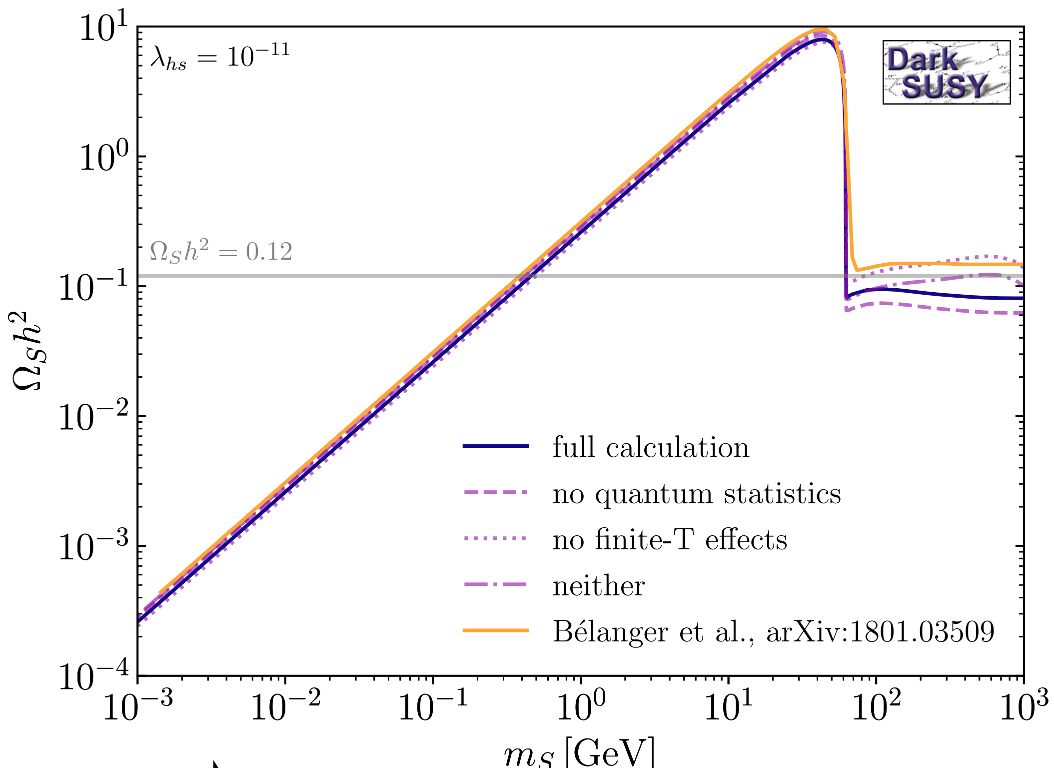
Results

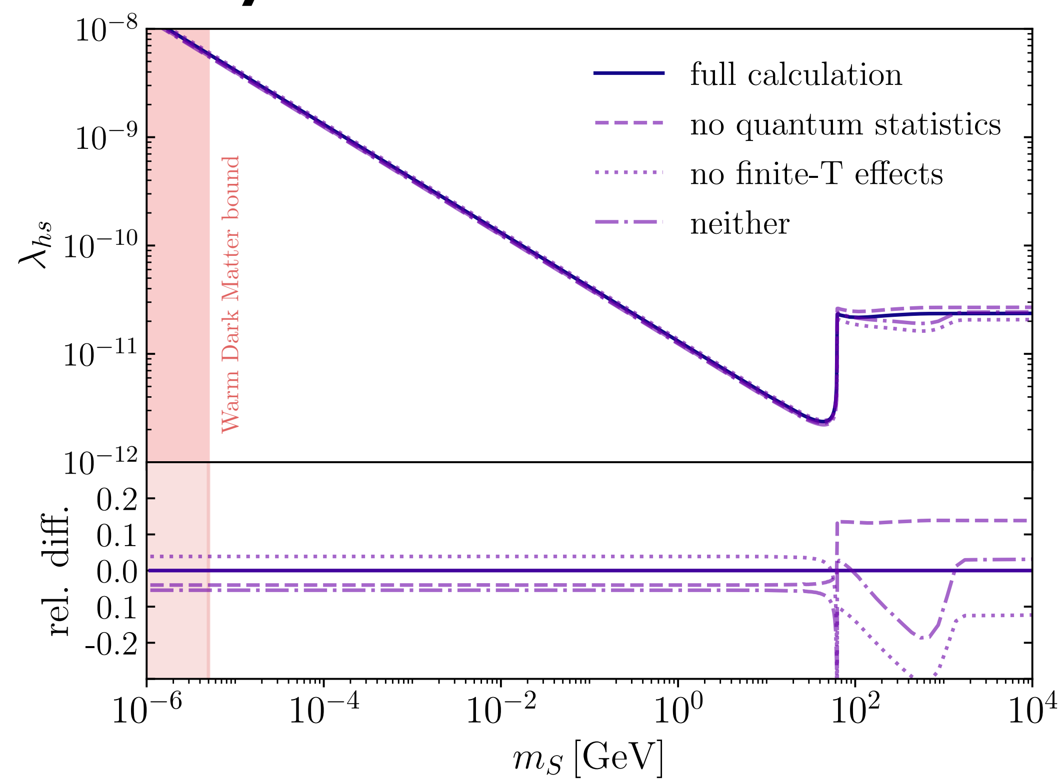
- Direct integration of Boltzmann equation — much easier than for freeze-out: $\boxed{\frac{dY_\chi}{dx} = \frac{\left(n_\chi^{\mathrm{MB}}\right)^2}{x s \tilde{H}} \langle \sigma v \rangle}$ $\quad x \equiv m_\chi / T$

TB, Heeba, Kahlhoefer & Vangsnes, JHEP ‘22



- Precision determination of relic density





⇒ unlike for (non-rel.) freeze-out, both quantum statistics and other finite-*T* effects do impact final result $[@\ \mathcal{O}(10\%)]$!

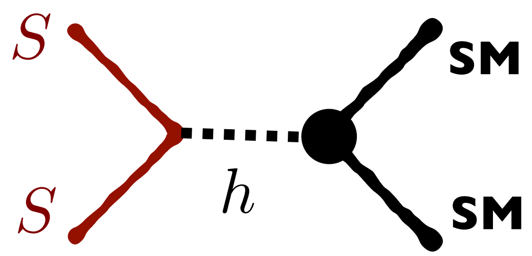
Variations of a theme

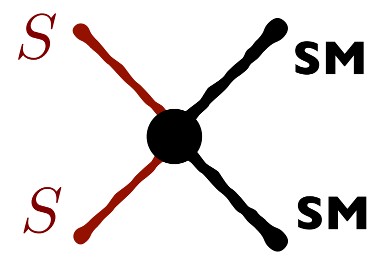
- Scalar Singlet DM

$$\mathcal{L} = \frac{1}{2}\partial_\mu S \partial^\mu S + \frac{1}{2}\mu_S^2 S^2 + \frac{1}{2}\lambda_{hs} S^2 |H|^2 + \frac{1}{4}\lambda_s S^4$$

renormalizable theory ⇒ **freeze-in is UV insensitive**

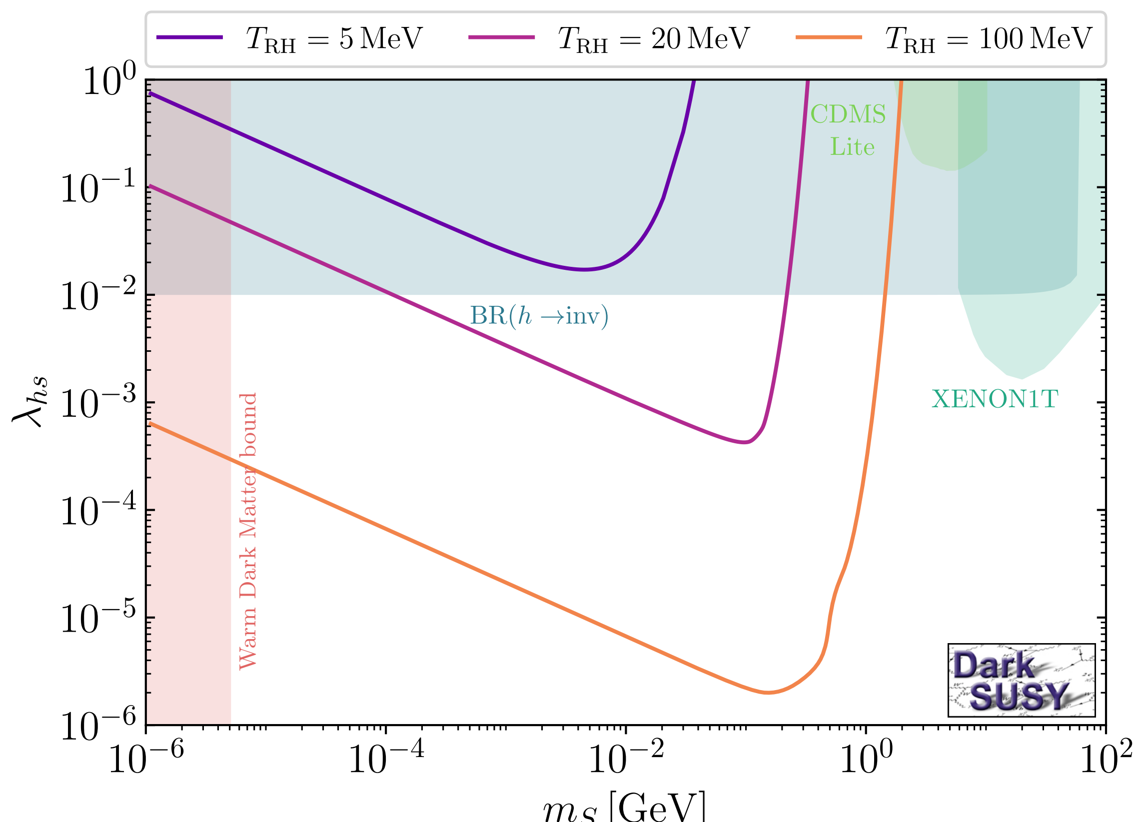
- But not at low temperatures:



⇒


(dim 5) ⇒ **freeze-in becomes UV sensitive**

- Freeze-in requires *large(ish) couplings* for low reheating temperatures



- Freeze-in *may* actually be directly testable !

TB, Heeba, Kahlhoefer & Vangsnes, JHEP '22

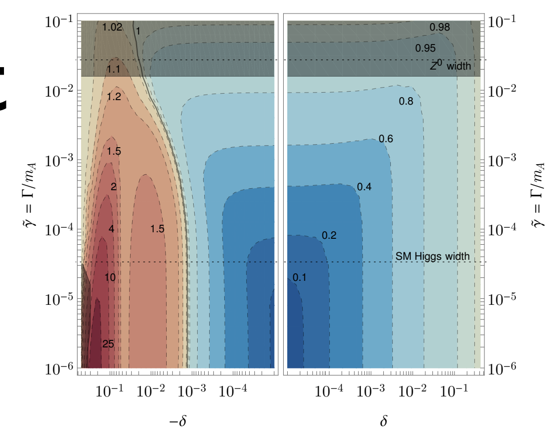
- In fact, *generally* true for $m_\chi \gg T_{\rm RH}$

Cosme, Costa & Lebedev, 2306.13061

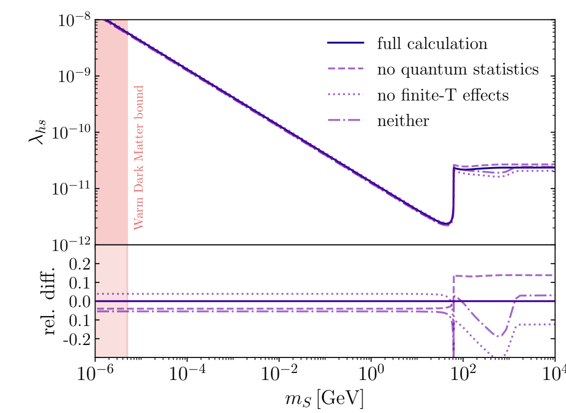
Conclusions

- Surprising subtleties in relic density calculations
  - even for a simple dark matter candidate like the scalar singlet!

- Lesson 1: careful with freeze-out calculations near resonances



- Lesson 2: for freeze-in, full temperature dependence of production rate must be included



- Freeze-in with testable couplings ?

- Want to explore these effects yourself (and much more)? Download DarkSUSY… 😉
  - Or DRAKE for freeze-out at phase-space level!

Thanks for your attention!

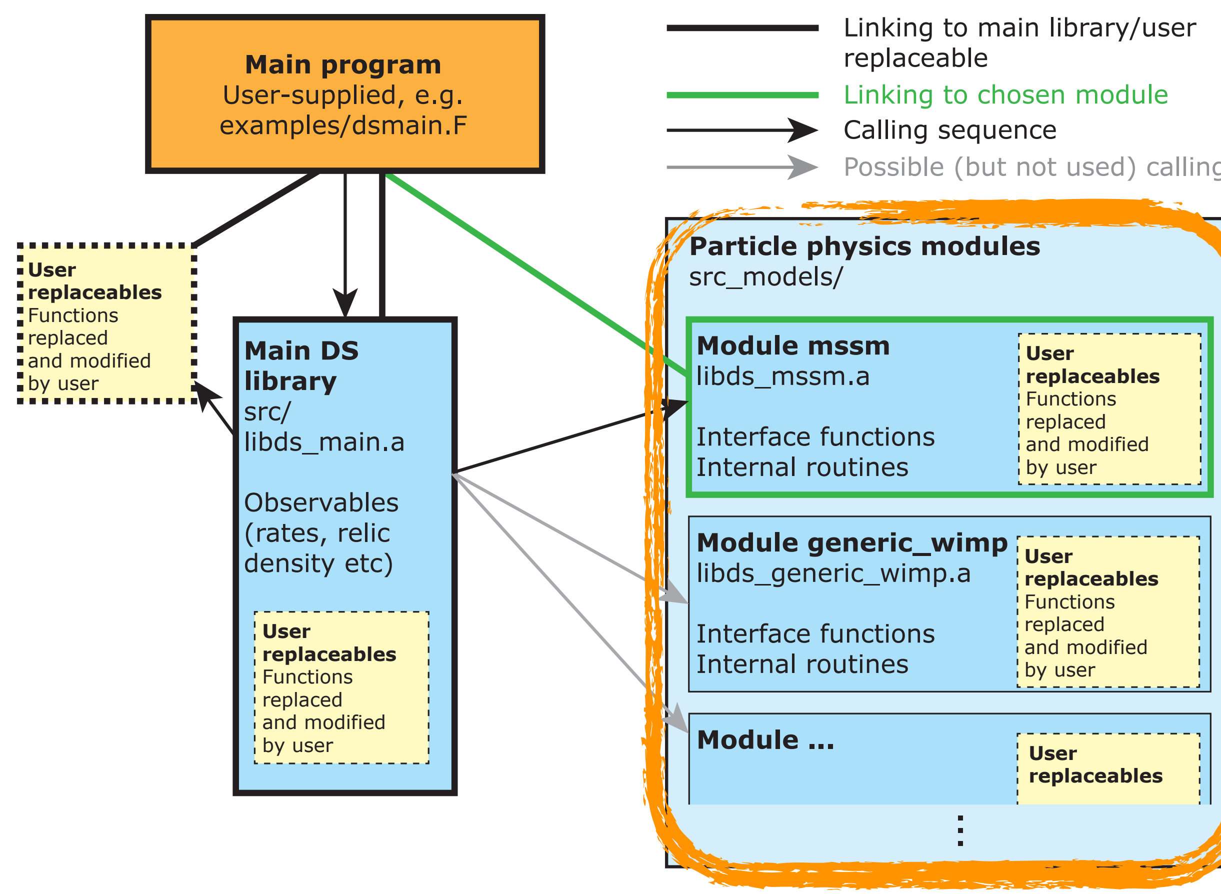
DarkSUSY

TB, Edsjö, Gondolo,
Ullio & Bergström,
JCAP '18

http://darksusy.hepforge.org

Since version 6:
no longer restricted to supersymmetric DM !

- Numerical package to calculate 'all' DM related quantities:
  - *relic density + kinetic decoupling* **(also for $T_{\rm dark} \neq T_{\rm photon}$)**
  - *generic SUSY models + laboratory constraints implemented*
  - *cosmic ray propagation*
  - particle yields for generic DM annihilation or decay
  - *indirect detection rates: gammas, positrons, antiprotons, neutrinos*
  - *direct detection rates*
  - …



since 6.1: DM self-interactions

since 6.2: 'reverse' direct detection
(also Q^2 -dependent scattering!)

since 6.3: freeze-in