Gravitational wave energy budget in strongly supercooled phase transitions

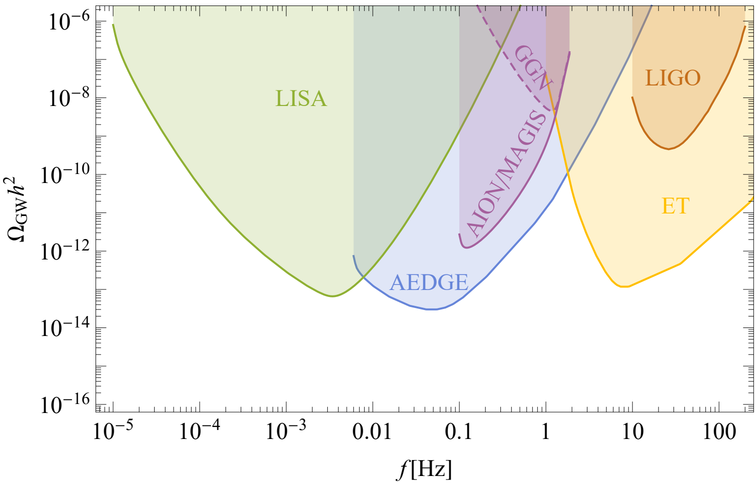
Marek Lewicki

Kings College London

SCALARS 2019, 14 Sep 2019, Warsaw

Based on:
J. Ellis, ML, J. M. No, V Vaskonen arXiv:1903.09642
J. Ellis, ML, J. M. No arXiv:1809.08242
J. Ellis, M. Fairbairn, ML, J. M. No, V. Vaskonen, A Wickens arXiv:1907.04315

Experimental outlook



phase transition dynamics

Bubble: static field configuration passing the barrier (excited through thermal fluctuations)

- decay rate

$$\Gamma(T) \approx T^4 \exp\left(-\frac{S_3(T)}{T}\right),$$

- $\mathcal{O}(3)$ symmetric action

$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right].$$

- EOM $\rightarrow$ bubble profile

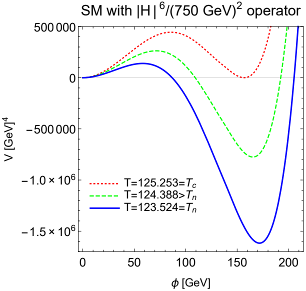
$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi, T)}{\partial \phi} = 0,$$

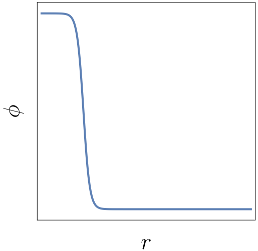
$$\phi(r \rightarrow \infty) = 0 \quad \text{and} \quad \dot{\phi}(r = 0) = 0.$$

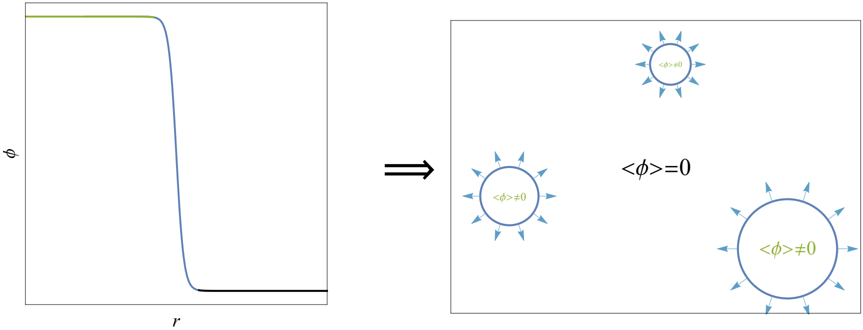
- nucleation temperature

$$N(T_n) = \int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} = 1$$

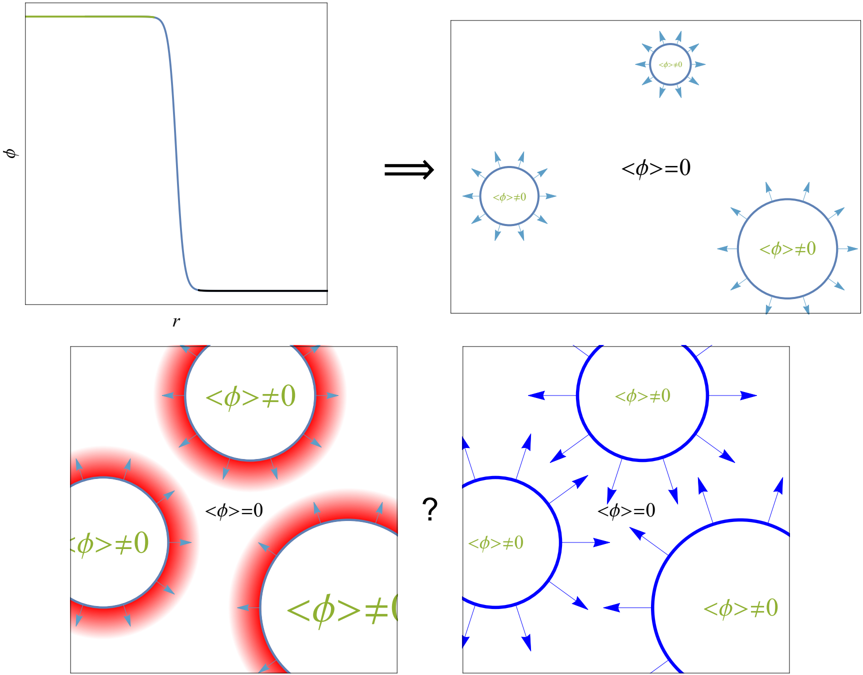
Linde '81 '83







ϕ
r
$<\phi>\neq 0$
$<\phi>=0$
$<\phi>\neq 0$
$<\phi>\neq 0$



ϕ
r
$\Longrightarrow$
$<\phi>\neq 0$
$<\phi>=0$
?

- Energy of the bubble

$$\mathcal{E} = 4\pi\gamma\sigma R^2 - \frac{4\pi}{3} R^3 p\,, \qquad \gamma = \frac{1}{\sqrt{1-\dot{R}^2}}$$

- Vacuum pressure on the wall
Coleman '73

$$p_0 = \Delta V$$

- Energy of the bubble

$$\mathcal{E} = 4\pi\gamma\sigma R^2 - \frac{4\pi}{3} R^3 p\,, \qquad \gamma = \frac{1}{\sqrt{1-\dot{R}^2}}$$

- Vacuum pressure on the wall
  Coleman '73

$$p_0 = \Delta V$$

- Leading order plasma contribution
  Bodeker '09 Caprini '09

$$p_1 = \Delta V - \Delta P_{\rm LO} \approx \Delta V - \frac{\Delta m^2 T^2}{24}\,,$$

- Energy of the bubble

$$\mathcal{E} = 4\pi\gamma\sigma R^2 - \frac{4\pi}{3} R^3 p\,, \qquad \gamma = \frac{1}{\sqrt{1-\dot{R}^2}}$$

- Vacuum pressure on the wall
  Coleman '73

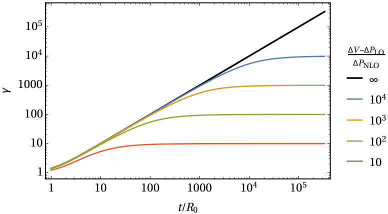
$$p_0 = \Delta V$$

- Leading order plasma contribution
  Bodeker '09 Caprini '09

$$p_1 = \Delta V - \Delta P_{\rm LO} \approx \Delta V - \frac{\Delta m^2 T^2}{24}\,,$$

- Next-To-Leading order plasma contribution
  Bodeker '17

$$p = \Delta V - \Delta P_{\rm LO} - \gamma \Delta P_{\rm NLO} \approx \Delta V - \frac{\Delta m^2 T^2}{24} - \gamma\, g^2 \Delta m_V T^3\,.$$



- γ factor at which the bubble stops accelerating and the value it would reach if we neglected $\Delta P_{\rm NLO}$

$$\gamma_{\rm eq} \equiv \frac{\Delta V - \Delta P_{\rm LO}}{\Delta P_{\rm NLO}} \, , \qquad \gamma_* \equiv \frac{2}{3}\frac{R_*}{R_0} \, ,$$

- Finally the efficiency factors read

$$\kappa_{\rm col} = \frac{E_{\rm wall}}{E_V} = \begin{cases} \frac{\gamma_{\rm eq}}{\gamma_*}\left[1 - \frac{\Delta P_{\rm LO}}{\Delta V}\left(\frac{\gamma_{\rm eq}}{\gamma_*}\right)^2\right] \, , & \gamma_* > \gamma_{\rm eq} \\ 1 - \frac{\Delta P_{\rm LO}}{\Delta V} \, , & \gamma_* \leq \gamma_{\rm eq} \, , \end{cases}$$

$$\kappa_{\rm sw} = \frac{\alpha_{\rm eff}}{\alpha} \frac{\alpha_{\rm eff}}{0.73 + 0.083\sqrt{\alpha_{\rm eff}} + \alpha_{\rm eff}} \, , \quad \text{with} \quad \alpha_{\rm eff} = \alpha(1 - \kappa_{\rm col}) \, .$$

Espinosa '10

Gravitational waves from a PT

- Strength of the transition

$$\alpha \approx \left.\frac{\Delta V}{\rho_R}\right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

- Characteristic scale

$$HR_* = (8\pi)^{\frac{1}{3}} \left(\frac{\beta}{H}\right)^{-1} = H_* N_b^{-\frac{1}{3}} = H_* \left(\int dt' \left(\frac{a(t')}{a(t)}\right)^3 \Gamma(t') P(t')\right)^{-\frac{1}{3}}$$

- Signals are produced by three main mechanisms:
  - collisions of bubble walls: $\Omega_{\text{col}} \propto \left(\kappa_{\text{col}} \frac{\alpha}{\alpha+1}\right)^2 (HR_*)^2$

    Kamionkowski '93, Huber '08, Hindmarsh '18,
  - sound waves: $\Omega_{\text{sw}} \propto \left(\kappa_{\text{sw}} \frac{\alpha}{\alpha+1}\right)^2 (HR_*)(H\tau_{sw})$

    Hindmarsh '13 '15 '17 Ellis '18
  - turbulence $\Omega_{\text{turb}} \propto \left(\kappa_{\text{sw}} \frac{\alpha}{\alpha+1}\right)^{\frac{3}{2}} (HR_*)(1 - H\tau_{sw})$

    Caprini '09 Ellis '19
- Sound wave period lasts $H\tau_{\text{sw}} \equiv \min\left[1, \frac{HR_*}{U_f}\right]$
- The frequency of the signal changes as $f \propto \frac{T_*}{HR_*}$

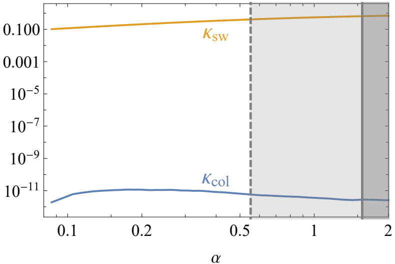
Particle physics examples

- Two models with very different potential cosmological evolution

$$H = \frac{1}{3M_{\rm pl}^2}\left(\rho_R + \Delta V\right) = H_R + \frac{\Delta V}{3M_{\rm pl}^2}$$

Standard Model $+|H|^6$ operator

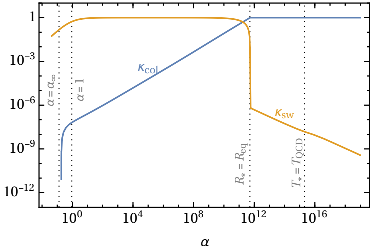
$$V(\phi) \simeq m^2\phi^2 + \lambda\phi^4 + \frac{\phi^6}{\Lambda^2}$$

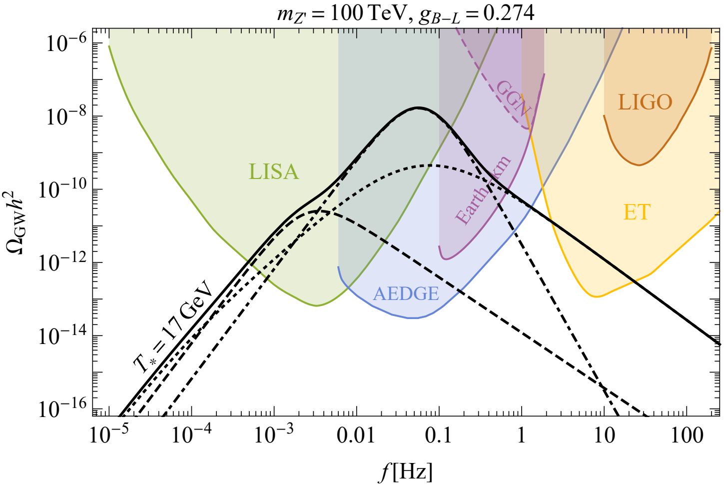


$U(1)_{B-L}$ extension of SM

$$V(\varphi) \simeq \frac{3g_{\rm B-L}^4\varphi^4}{4\pi^2}\left[\log\left(\frac{\varphi^2}{v_\varphi^2}\right) - \frac{1}{2}\right] + g_{\rm B-L}^2 T^2\varphi^2$$

$$m_{Z'} = 2g_{\rm B-L}v_\varphi$$





$m_{Z'} = 100\,\mathrm{TeV},\, g_{B-L} = 0.274$
$\Omega_{\rm GW}h^2$
$f\,[\mathrm{Hz}]$
LISA
AEDGE
Earth km
GGN
LIGO
ET
$T_* = 17\,\mathrm{GeV}$

Plasma related GW sources

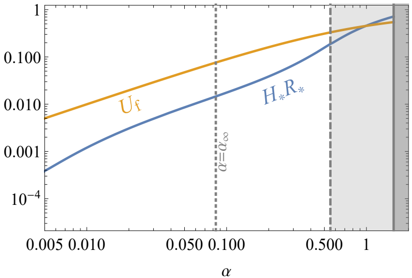
- Sound wave period last a fraction of the Hubble time

$$H\tau_{\rm sw} \equiv \min\left[1, \frac{HR_*}{U_f}\right]$$

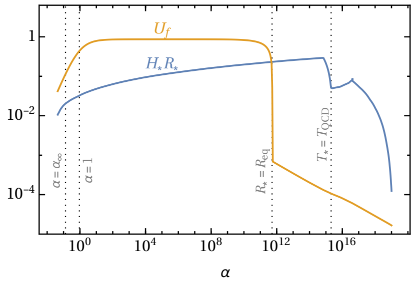
- Root-mean-square four-velocity of the plasma

$$U_f \approx \sqrt{\frac{3}{4}\frac{\kappa_{\rm sw}\alpha}{1+\alpha}} \xRightarrow{v_w\approx 1} \frac{\sqrt{3}\alpha}{2(1+\alpha)\sqrt{0.73+0.083\sqrt{\alpha}+\alpha}}\,.$$

Standard Model $+|H|^6$ operator

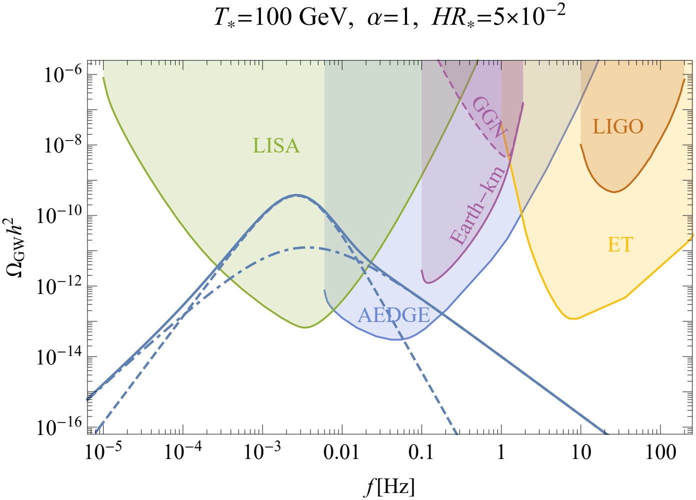


$U(1)_{B-L}$ extension of SM



- Sound wave spectrum reduction and earlier onset of turbulence

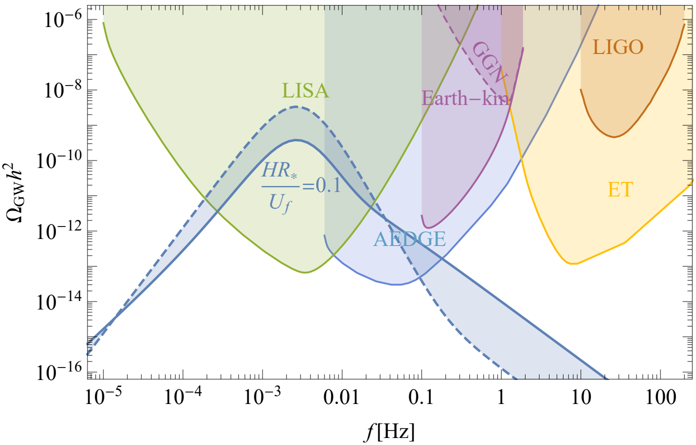
$$\Omega_{\rm sw} \propto H\tau_{\rm sw} = \frac{HR_*}{U_f}\,, \quad \Omega_{\rm turb} \propto 1 - H\tau_{\rm sw} = 1 - \frac{HR_*}{U_f}$$



- Sound wave spectrum reduction and earlier onset of turbulence

$$\Omega_{\rm sw} \propto H\tau_{\rm sw} = \frac{HR_*}{U_f}\,, \quad \Omega_{\rm turb} \propto 1 - H\tau_{\rm sw} = 1 - \frac{HR_*}{U_f}$$

T_*=100 GeV, α=1, HR_*=5 10^{-2}



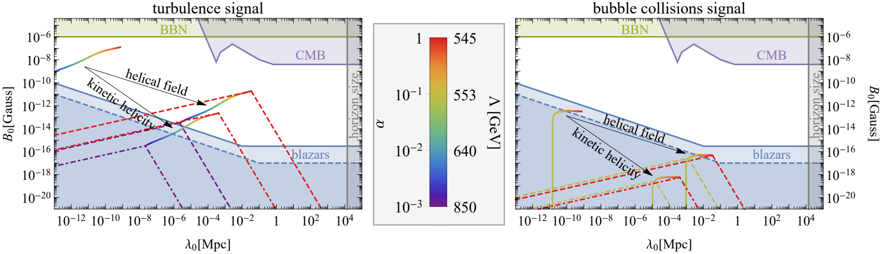
Primordial Magnetic Fields

- Energy density and correlation length of produced Magnetic Field

  Durrer ‘13 Brandenburg ‘17 Vachaspati ‘19

$$\rho_{B,*} = 0.1\kappa_{\mathrm{col/sw}}\frac{\alpha}{1+\alpha}\rho_* \qquad \lambda_* = R_*$$

- In the $SM + H^6$ model

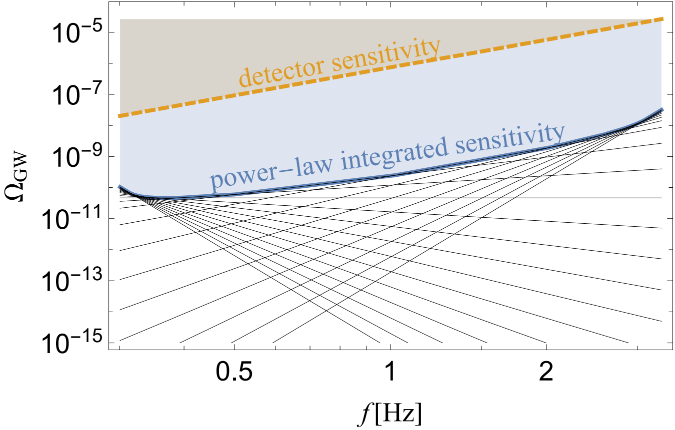


Conclusions

- Observable bubble collision GW signal requires significant supercooling. We derived the efficiency factor quantifying this requirement precisely.
- Sound wave period generically last less than a Hubble time. This leads to a much weaker sound wave sourced GW signal and potentially a significant increase in the signal sourced by turbulence.
- PTs an also have leave other complimentary signals including for example production of primordial magnetic fields.

Power-law integrated sensitivity

$$\Omega_{\mathrm{GW}}^{\mathrm{noise}} = \frac{2\pi}{3}\frac{f^3 S_h}{H_0^2}, \qquad \mathrm{SNR} = \sqrt{\mathcal{T}\int df \left(\frac{\Omega_{\mathrm{GW}}^{\mathrm{signal}}}{\Omega_{\mathrm{GW}}^{\mathrm{noise}}}\right)^2}$$



Thrane, Romano '13