A Simple Method to detect spontaneous CP Violation in multi-Higgs models

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In memory of Maria

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In memory of Maria

Maria Krawczyk

Reception, Discrete 2016, Warsaw, 28 November 2016

Mead (Miód pitny) in the Old Town, 3 December 2016

The Red Hog, Warsaw, 5 December 2016

Portoroz, 18 April 2017

SCALARS 2011, WARSAW

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Two Higgs doublet Models are very well motivated

as shown in the previous talk by G. C. Branco

and have very interesting phenomenological implications

Despite several good motivations,
there is the need to suppress potentially dangerous FCNC:

Without HFCNC

NFC — Weinberg, Glashow (1977); Paschos (1977)

aligned two Higgs doublet model — Pich, Tuzon (2009)

With HFCNC

assume existence of suppression factors, e.g., suppression by small elements of VCKM: Minimal Flavour Violation

Branco, Grimus, Lavoura (1996)

BGL models, gBGL models

Notice that:

NFC, i.e., natural flavour conservation with MHDM, consists on imposing some extra symmetry on the Lagrangian constraining the Yukawa interactions of the neutral scalars in such a way that there are no FCNC

The only way is to ensure that only one Higgs doublet has Yukawa interactions with SM quark singlets of a given charge:

Glashow, Weinberg , Phys.Rev. D15 (1977) 1958; E.A. Paschos, Phys.Rev. D15 (1977) 1966

The case of two Higgs doublets with an exact reflection symmetry

$$Z_2 : \ \Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2,$$

$$\begin{aligned} V(\Phi_1, \Phi_2) \ &= \ m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ &+ \ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.} \end{aligned}$$

No CP violation in the scalar sector, neither explicit nor spontaneous.

At least three Higgs doublets required for CP violation in the scalar sector in the context of NFC with exact reflection symmetry

Motivation for three Higgs doublets

Three fermion generations may suggest three doublets

Interesting scenario for dark matter

Possibility of having a discrete symmetry and still having spontaneous CP violation

Rich phenomenology

Motivation for imposing discrete symmetries

Symmetries reduce the number of free parameters leading to (testable) predictions

Symmetries help to control HFCNC

Example: NFC, no HFCNC due to Z_2 symmetry(ies)

Example: MFV suppression of HFCNC, BGL models

Symmetries are needed to stabilise dark matter

Three Higgs Doublets NFC and CP Violation

Early motivation, Weinberg 1976, NFC with explicit CP violation and four quarks

Phys.Rev.Lett. 37 (1976) 657

NFC, at most two Higgs doublets couple to the quarks: one couples to the up sector only, the other to the down sector only

Lagrangian invariant under separate reflections under which any one of the doublets changes sign

$$
\begin{aligned}
V = {} & m_1 \phi_1^\dagger \phi_1 + m_2 \phi_2^\dagger \phi_2 + m_3 \phi_3^\dagger \phi_3 + a_1 \left(\phi_1^\dagger \phi_1\right)^2 + a_2 \left(\phi_2^\dagger \phi_2\right)^2 + a_3 \left(\phi_3^\dagger \phi_3\right)^2 \\
& + b_1 \left(\phi_2^\dagger \phi_2\right)\left(\phi_3^\dagger \phi_3\right) + b_2 \left(\phi_3^\dagger \phi_3\right)\left(\phi_1^\dagger \phi_1\right) + b_3 \left(\phi_1^\dagger \phi_1\right)\left(\phi_2^\dagger \phi_2\right) \\
& + c_1 \left(\phi_2^\dagger \phi_3\right)\left(\phi_3^\dagger \phi_2\right) + c_2 \left(\phi_3^\dagger \phi_1\right)\left(\phi_1^\dagger \phi_3\right) + c_3 \left(\phi_1^\dagger \phi_2\right)\left(\phi_2^\dagger \phi_1\right) \\
& + d_1 \left[e^{i\varepsilon_1}\left(\phi_2^\dagger \phi_3\right)^2 + e^{-i\varepsilon_1}\left(\phi_3^\dagger \phi_2\right)^2\right] + d_2 \left[e^{i\varepsilon_2}\left(\phi_3^\dagger \phi_1\right)^2 + e^{-i\varepsilon_2}\left(\phi_1^\dagger \phi_3\right)^2\right] \\
& + d_3 \left[e^{i\varepsilon_3}\left(\phi_1^\dagger \phi_2\right)^2 + e^{-i\varepsilon_3}\left(\phi_2^\dagger \phi_1\right)^2\right],
\end{aligned}
\tag{22.45}
$$

Three independent phases in V, two relative phases in the vevs

There is explicit CP violation if the product of the three complex coefficients is not real

CP violation in charged Higgs mediated flavour currents

Three Higgs Doublets NFC and CP Violation

Arbitrary number of quark generations, spontaneous CP breaking with NFC: a minimal number of three Higgs doublets required

Gustavo C. Branco Phys.Rev. D22 (1980) 2901

VCKM is real and there is no CP violation mediated by charged gauge bosons

Now we know that VCKM is complex

crucial rôle played by the angle γ

F.J. Botella, G.C. Branco, M. Nebot and MNR, Nucl.Phys. B725 (2005) 155-172

This fact rules out SCPV with NFC: whenever only one Higgs doublet gives mass to each quark sector the phase of its vev can be rotated away

Inert Higgs

Initial proposal: 2 Higgs doublets, Unbroken Z_2 symmetry $\Phi_2 \to -\Phi_2$

all other Standard Model particles are invariant under Z_2

E. Ma; R. Barbieri, L. J. Hall, and V. S. Rychkov, 2006

L.L. Honorez, E. Nezri, J. F. Oliver, M. H. G. Tytgat , 2006

Φ_2- , the inert Higgs, does not couple to matter and acquires no vev, NFC

Notice that this is different from going to the Higgs basis

The Z_2 symmetry is left unbroken, as a result the lightest inert particle will be stable and will contribute to dark matter density

Inert scalars can be produced at colliders through their couplings to the EW gauge bosons subject to Z_2 constraints and participate in cubic and quartic Higgs couplings

Many works on Dark matter with an Inert Higgs doublet

N. Darvishi, Mikael Dhen, I. F. Ginzburg, Thomas Hambye, K.A. Kanishev, M. Krawczyk, D. Sokolowska, P. Swaczyna, B. Swiezewska, many many more authors

The Inert doublet model has been extended by several authors to include three Higgs Doublets

Possibility of having CP Violation and a stable DM candidate

B. Grzadkowski, O. M. Ogreid, P. Osland, G.M. Pruna , A. Pukhov, M. Purmohammadi

A. Cordero-Cid, J. Hernández-Sánchez, V. Keus, S.F. King, S. Moretti, D. Rojas, D. Sokołowska

How can we test whether or not there is SCPV in multi-Higgs models?

Important Tool

most general CP transformation

$$\Phi_i \xrightarrow{\text{CP}} U_{ij}\Phi_j^*$$

together with assumption that vacuum is invariant

$$\text{CP}|0\rangle = |0\rangle$$

leads to the following condition

$$\mathcal{L}(U\phi) = \mathcal{L}(\phi)$$

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

G. C. Branco, G. M. Gerard and W. Grimus (1984)

Very simple and powerful relation

Symmetries, if present, play a crucial role

However, in some cases construction of matrix U may not be obvious

Three Higgs doublet models with S_3 Symmetry

(extended to flavour)

Despite

many works aiming at explaining neutrino masses and leptonic mixing

Ma, Koide, Kubo, Mondragon, Rodriguez-Jauregui, Chen, Wolfenstein, Mohapatra, Nasri, Yu, Harrison, Scott, Frigerio, Grimus, Lavoura, Branco, Silva-Marcos…

several works addressing masses and mixing in the quark sector

Harari, Haut, Weyers, Meloni, Teshima, Melic, Canales, S Salazar, Velasco-Sevilla ,…

a lot of work already done analysing the Higgs potential

Derman, Tsao, Pakvasa, Sugawra, Wyler, Branco, Gerard, Grimus, Das, Dey, Bhattacharyya, Leser, Pas, Ivanov, Nishi…

inert dark matter candidates from S_3 3HDM considered

Fortes, Machado, Montano, Pleitez…

Interesting open questions still remain!

The Scalar potential

S_3 is the permutation group involving three objects, ϕ_1, ϕ_2, ϕ_3

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \frac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \mathrm{hc}]$$

$$V_4 = A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \mathrm{hc}]\}$$
$$+\frac{1}{2}E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \mathrm{hc}] + \sum_{i \neq j \neq k \neq i, j<k} \{\frac{1}{2}E_2[(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \mathrm{hc}]$$
$$+\frac{1}{2}E_3[(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \mathrm{hc}] + \frac{1}{2}E_4[(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \mathrm{hc}]\}$$

Derman, 1979

here all fields appear on equal footing

this representation is not irreducible, for instance, the combination

$$\phi_1 + \phi_2 + \phi_3$$

remains invariant, it splits into two irreducible representations,

doublet and singlet: $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad h_S$

Decomposition into these two irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

This definition does not treat equally ϕ_1, ϕ_2, ϕ_3 , they could be interchanged

Notice similarity with tribimaximal mixing: **Harrison, Perkins and Scott, 1999**

$$(F =) \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

The matrix F diagonalizes the democratic matrix , Δ

$$F'^T \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} F' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad \Delta = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

The democratic mass matrix can be obtained from S_3 flavour symmetries

S_{3L} x S_{3R}: $M_l = \lambda' \ \Delta \quad ; \qquad M_D = \lambda \ \Delta \quad ; \qquad M_R = \mu \ \ (\Delta + a \ \mathbb{1})$

Very interesting alternative, democratic with phases (USY)

The scalar potential in terms of fields from irreducible representations

$$
\begin{aligned}
V_2 &= \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \\
V_4 &= \lambda_8 (h_S^\dagger h_S)^2 + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) + \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 \\
&\quad + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\
&\quad + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] \\
&\quad + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\
&\quad + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}]
\end{aligned}
$$

Das and Dey

no symmetry under the interchange of h_1 and h_2

however there is symmetry for $h_1 \to -h_1$

equivalent doublet representation $\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$

now there is symmetry for $\chi_1 \leftrightarrow \chi_2$

In the special case $\lambda_4 = 0$ **the potential has SO(2) symmetry:**

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Danger: massless scalar!

Constraining the potential by the vevs

Possibility of SCPV - real parameters

Let us start with real vacua (no CP violation)

Three minimisation conditions:

can be solved to give μ_0^2 and μ_1^2 in terms of the quartic coefficients:

$$\mu_0^2 = \frac{1}{2w_S}\left[\lambda_4(w_2^2 - 3w_1^2)w_2 - (\lambda_5 + \lambda_6 + 2\lambda_7)(w_1^2 + w_2^2)w_S - 2\lambda_8 w_S^3\right], \tag{4.2a}$$

$$\mu_1^2 = -\frac{1}{2}\left[2(\lambda_1 + \lambda_3)(w_1^2 + w_2^2) + 6\lambda_4 w_2 w_S + (\lambda_5 + \lambda_6 + 2\lambda_7)w_S^2\right], \tag{4.2b}$$

$$\mu_1^2 = -\frac{1}{2}\left[2(\lambda_1 + \lambda_3)(w_1^2 + w_2^2) - 3\lambda_4(w_2^2 - w_1^2)\frac{w_S}{w_2} + (\lambda_5 + \lambda_6 + 2\lambda_7)w_S^2\right]. \tag{4.2c}$$

Eqs (4.2b) and (4.2c) obtained dividing by w_1 and w_2 respectively

$$\lambda_4 = 4A - 2(C + \bar{C} + D) - E_1 - E_2 + E_4 = 0$$

Consistency requires:

- for $w_1 = 0$ the corresponding derivative is zero - no clash
- or else $\lambda_4(3w_2^2 - w_1^2)w_S = 0$ i. e., $\lambda_4 = 0$ or $w_1 = \pm\sqrt{3}w_2$ or $w_S = 0.$
- for $w_S = 0.$ special condition: $\lambda_4 w_2(3w_1^2 - w_2^2) = 0,$ i. e., in addition:

$$\lambda_4 = 0 \text{ or } w_2 = \pm\sqrt{3}w_1, \text{ or else } w_2 = 0.$$

SSB, real vacua, residual symmetries

Derman, Tsao Phys. Rev. D20 (1979) 1207:

(x, x, x) S_3 ; (x, x, y) S_2 ; (x, y, z) = (x, -x, 0) S_2 $\lambda_4 \neq 0$

Translation into doublet singlet notation

(x, x, x) $\longrightarrow$ (0,0, ω_S) $w_1 = 0$ (also verifies $w_1 = \pm\sqrt{3}w_2$)

(x, -x, 0) $\longrightarrow$ $(\omega_1, 0, 0)$ $w_S = 0$ together with $w_2 = 0.$

(x, 0, -x) $\longrightarrow$ $(\omega_1, \omega_2, 0)$ $w_S = 0$ together $w_2 = \sqrt{3}w_1$

(0,x, -x) $\longrightarrow$ $(\omega_1, \omega_2, 0)$ $w_S = 0$ together with $w_2 = -\sqrt{3}w_1$

(x, x, y) translates into $(0, w_2, w_S)$; consistency condition: $w_1 = 0.$

(x, y, x) translates into $(w_1, -\frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = -\sqrt{3}w_2$

(y, x, x) translates into $(w_1, \frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = \sqrt{3}w_2$

For $\lambda_4 = 0$ SO(2) symmetry implies (x, y, z) possible solution

Vacuum	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	Comment
R-0	$0, 0, 0$	$0, 0, 0$	Not interesting
R-I-1	x, x, x	$0, 0, w_S$	$\mu_0^2 = -\lambda_8 w_S^2$
R-I-2a	$x, -x, 0$	$w, 0, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_1^2$
R-I-2b	$x, 0, -x$	$w, \sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-I-2c	$0, x, -x$	$w, -\sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-II-1a	x, x, y	$0, w, w_S$	$\mu_0^2 = \frac{1}{2}\lambda_4 \frac{w_2^3}{w_S} - \frac{1}{2}\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2 + \frac{3}{2}\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1b	x, y, x	$w, -w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1c	y, x, x	$w, w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-2	$x, x, -2x$	$0, w, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2,\ \lambda_4 = 0$
R-II-3	$x, y, -x-y$	$w_1, w_2, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right), \lambda_4 = 0$
R-III	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	$\mu_0^2 = -\frac{1}{2}\lambda_a(w_1^2 + w_2^2) - \lambda_8 w_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right) - \frac{1}{2}\lambda_a w_S^2,$ $\lambda_4 = 0$

$$\lambda_a = \lambda_5 + \lambda_6 + 2\lambda_7,$$

$$\lambda_b = \lambda_5 + \lambda_6 - 2\lambda_7.$$

Complex vacua

Table 2: Complex vacua. Notation: $\epsilon = 1$ and -1 for C-III-d and C-III-e, respectively; $\xi = \sqrt{-3\sin 2\rho_1/\sin 2\rho_2}$, $\psi = \sqrt{[3+3\cos(\rho_2-2\rho_1)]/(2\cos\rho_2)}$. With the constraints of Table 4 the vacua labelled with an asterisk (*) are in fact real.

	IRF (Irreducible Rep.)	RRF (Reducible Rep.)
	w_1, w_2, w_S	ρ_1, ρ_2, ρ_3
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x+iy, x-iy, x$
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho}-\frac{y}{2}, -xe^{i\rho}-\frac{y}{2}, y$
C-III-d,e	$\pm i\hat{w}_1, \epsilon\hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho}\pm ix, re^{i\rho}\mp ix, \frac{3}{2}re^{-i\rho}-\frac{1}{2}re^{i\rho}$
C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho}\pm ix, re^{-i\rho}\mp ix, \frac{3}{2}re^{i\rho}-\frac{1}{2}re^{-i\rho}$
C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2\sigma_1)}{1+9\tan^2\sigma_1}}\hat{w}_2 e^{i\sigma_1},$ $\pm\hat{w}_2 e^{-i\arctan(3\tan\sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho}+x, -re^{i\rho}+x, x$
C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho}+x, -re^{-i\rho}+x, -re^{i\rho}+re^{-i\rho}+x$
C-IV-c	$\sqrt{1+2\cos^2\sigma_2}\hat{w}_2,$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho}+r\sqrt{3(1+2\cos^2\rho)}+x,$ $re^{i\rho}-r\sqrt{3(1+2\cos^2\rho)}+x, -2re^{i\rho}+x$
C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm\hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho}+x, (r_2-r_1)e^{i\rho}+x, -r_2 e^{i\rho}+x$
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}}\hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2}+re^{i\rho_1}\xi+x, re^{i\rho_2}-re^{i\rho_1}\xi+x,$ $-2re^{i\rho_2}+x$
C-IV-f	$\sqrt{2+\frac{\cos(\sigma_1-2\sigma_2)}{\cos\sigma_1}}\hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1}+re^{i\rho_2}\psi+x,$ $re^{i\rho_1}-re^{i\rho_2}\psi+x, -2re^{i\rho_1}+x$
C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

Constraints

Vacuum	Constraints
C-I-a	$\mu_1^2 = -2\left(\lambda_1 - \lambda_2\right)\hat{w}_1^2$
C-III-a	$\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2 \lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \frac{4\cos\sigma_2 \hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-b	$\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_1^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\lambda_b \hat{w}_S^2,$ $\lambda_4 = 0$
C-III-c	$\mu_1^2 = -(\lambda_1 + \lambda_3)(\hat{w}_1^2 + \hat{w}_2^2),$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0$
C-III-d,e	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{(\hat{w}_1^2 - \hat{w}_2^2)^2}{\hat{w}_S^2} - \epsilon\lambda_4 \frac{(\hat{w}_1^2 - \hat{w}_2^2)(\hat{w}_1^2 - 3\hat{w}_2^2)}{4\hat{w}_2 \hat{w}_S}$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \epsilon\lambda_4 \frac{\hat{w}_S(\hat{w}_1^2 - \hat{w}_2^2)}{4\hat{w}_2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_7 = \frac{\hat{w}_1^2 - \hat{w}_2^2}{\hat{w}_S^2}(\lambda_2 + \lambda_3) - \epsilon\frac{(\hat{w}_1^2 - 5\hat{w}_2^2)}{4\hat{w}_2 \hat{w}_S}\lambda_4$
C-III-f,g	$\mu_0^2 = -\frac{1}{2}\lambda_b\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\lambda_b \hat{w}_S^2, \lambda_4 = 0$
C-III-h	$\mu_0^2 = -2\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2 \lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \mp\frac{2\cos\sigma_2 \hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-i	$\mu_0^2 = \frac{16\left(1-3\tan^2\sigma_1\right)^2}{(1+9\tan^2\sigma_1)^2}(\lambda_2 + \lambda_3)\frac{\hat{w}_2^4}{\hat{w}_S^2} \pm \frac{6\left(1-\tan^2\sigma_1\right)(1-3\tan^2\sigma_1)}{(1+9\tan^2\sigma_1)^{\frac{3}{2}}}\lambda_4 \frac{\hat{w}_2^3}{\hat{w}_S}$ $-\frac{2(1+3\tan^2\sigma_1)}{1+9\tan^2\sigma_1}(\lambda_5 + \lambda_6)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\frac{4(1+3\tan^2\sigma_1)}{1+9\tan^2\sigma_1}(\lambda_1 - \lambda_2)\hat{w}_2^2 \mp \frac{\left(1-3\tan^2\sigma_1\right)}{2\sqrt{1+9\tan^2\sigma_1}}\lambda_4 \hat{w}_2 \hat{w}_S$ $-\frac{1}{2}(\lambda_5 + \lambda_6)\hat{w}_S^2,$ $\lambda_7 = -\frac{4\left(1-3\tan^2\sigma_1\right)\hat{w}_2^2}{(1+9\tan^2\sigma_1)\hat{w}_S^2}(\lambda_2 + \lambda_3) \mp \frac{\left(5-3\tan^2\sigma_1\right)\hat{w}_2}{2\sqrt{1+9\tan^2\sigma_1}\hat{w}_S}\lambda_4$

Vacuum	Constraints
C-IV-a*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_1^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = 0$
C-IV-b	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)^2}{\hat{w}_S^2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = -\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-c	$\mu_0^2 = 2\cos^2\sigma_2\left(1 + \cos^2\sigma_2\right)\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$ $-\left(1 + \cos^2\sigma_2\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left[2\left(1 + \cos^2\sigma_2\right)\lambda_1 - \left(2 + 3\cos^2\sigma_2\right)\lambda_2 - \cos^2\sigma_2 \lambda_3\right]\hat{w}_2^2$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = -\frac{2\cos\sigma_2 \hat{w}_2}{\hat{w}_S}\left(\lambda_2 + \lambda_3\right), \lambda_7 = \frac{\cos^2\sigma_2 \hat{w}_2^2}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-d*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = 0$
C-IV-e	$\mu_0^2 = \frac{\sin^2(2(\sigma_1 - \sigma_2))}{\sin^2(2\sigma_1)}\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$ $-\frac{1}{2}\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_1 - \lambda_2\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = -\frac{\sin(2(\sigma_1 - \sigma_2))\hat{w}_2^2}{\sin 2\sigma_1 \hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-f	$\mu_0^2 = -\frac{(\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1)\cos(\sigma_2 - \sigma_1)}{2\cos^2\sigma_1}\lambda_4 \frac{\hat{w}_2^3}{\hat{w}_S}$ $-\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{2\cos\sigma_1}\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{\cos\sigma_1}\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2$ $-\frac{3\cos 2\sigma_1 + 2\cos(2(\sigma_1 - \sigma_2)) + \cos 2\sigma_2 + 4}{4\cos(\sigma_1 - \sigma_2)\cos\sigma_1}\lambda_4 \hat{w}_2 \hat{w}_S - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_2 + \lambda_3 = -\frac{\cos\sigma_1 \hat{w}_S}{2\cos(\sigma_2 - \sigma_1)\hat{w}_2}\lambda_4, \lambda_7 = -\frac{\cos(\sigma_2 - \sigma_1)\hat{w}_2}{2\cos\sigma_1 \hat{w}_S}\lambda_4$
C-V*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0, \lambda_7 = 0$

The case of $\lambda_4 = 0$

Potential has additional continuous SO(2) symmetry

$$\lambda_4 = 4A - 2(C + \bar{C} + D) - E_1 - E_2 + E_4 = 0$$

Derman (1979), “unnatural”

Spontaneous breaking of this SO(2) symmetry leads to massless particles

Possible solution: break the symmetry softly, the most general quadratic potential can be written:

$$V = \mu_0^2 h_S^\dagger h_S + \mu_1^2(h_1^\dagger h_1 + h_2^\dagger h_2) + \mu_2^2(h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{1}{2}\nu^2(h_2^\dagger h_1 + h_1^\dagger h_2)$$
$$+ \mu_3^2(h_S^\dagger h_1 + h_1^\dagger h_S) + \mu_4^2(h_S^\dagger h_2 + h_2^\dagger h_S)$$

Complex vacua, Spontaneous CP Violation

Table 1: Spontaneous CP violation

Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV
C-I-a	X	no	C-III-f,g	0	no	C-IV-c	X	yes
C-III-a	X	yes	C-III-h	X	yes	C-IV-d	0	no
C-III-b	0	no	C-III-i	X	no	C-IV-e	0	no
C-III-c	0	no	C-IV-a	0	no	C-IV-f	X	yes
C-III-d,e	X	no	C-IV-b	0	no	C-V	0	no

Next we present a few illustrative examples. Important tool:

most general CP transformation

$$\Phi_i \xrightarrow{\mathrm{CP}} U_{ij}\Phi_j^*$$

together with assumption that vacuum is invariant

$$\mathrm{CP}|0\rangle = |0\rangle$$

leads to the following condition

$$\mathcal{L}(U\phi) = \mathcal{L}(\phi) \qquad U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

G. C. Branco, J. M. Gerard and W. Grimus (1984)

Vacuum C-I-a

$$x, \;\; xe^{\frac{2\pi i}{3}}, \;\; xe^{-\frac{2\pi i}{3}}$$

geometrical phases

G. C. Branco, J. M. Gerard and W. Grimus (1984)

calculable non-trivial phases, fixed by symmetry of V, no explicit dependence on parameters of the potential

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

CP is conserved

For new models with geometrical phases and the possibility of having CP violation with geometrical phases see

Ivo de Medeiros Varzielas, JHEP 1208 (2012) 055

Vacuum C-III-c

$$\hat{w}_1 e^{\imath\sigma_1}, \hat{w}_2 e^{\imath\sigma_2}, 0 \qquad \lambda_4 = 0$$

SO(2) rotation

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \qquad \tan 2\theta = \frac{\hat{w}_1^2 - \hat{w}_2^2}{2\hat{w}_1\hat{w}_2\cos\sigma}.$$

$$(ae^{i\delta_1}, ae^{i\delta_2}, 0)$$

followed by overall phase rotation

$$(ae^{i\delta}, ae^{-i\delta}, 0)$$

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

symmetry for interchange: $h_1' \leftrightarrow h_2'$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ae^{i\delta} \\ ae^{-i\delta} \\ 0 \end{pmatrix}^* = \begin{pmatrix} ae^{i\delta} \\ ae^{-i\delta} \\ 0 \end{pmatrix}$$

CP is conserved

$$U = e^{i(\delta_1+\delta_2)} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

Very simple and powerful relation. However, in some cases construction of matrix U may not be obvious

Simple Alternative Test

- Go to a basis where only one Higgs field acquires a vev different from zero and real

- If the coefficients of scalar potential can be made real by rephasing the fields with zero vev, there is no CP violation

Inspect the potential

C-III-c vaccum

$$\begin{pmatrix} h'_1 \\ h'_2 \\ h'_S \end{pmatrix} = \begin{pmatrix} \frac{1}{N_1}(\hat{w}_1 & \hat{w}_2 & \hat{w}_S) \\ \frac{1}{N_2}(\hat{w}_2 & -\hat{w}_1 & 0) \\ \frac{1}{N_3}(\hat{w}_1 & \hat{w}_2 & X) \end{pmatrix} \begin{pmatrix} e^{-i\sigma_1} & 0 & 0 \\ 0 & e^{-i\sigma_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix}$$

Example T. D. Lee Model, 2HDM

$$
\begin{aligned}
V(\phi) = & -\lambda_1 \phi_1^\dagger \phi_1 - \lambda_2 \phi_2^\dagger \phi_2 \\
& + A(\phi_1^\dagger \phi_1)^2 + B(\phi_2^\dagger \phi_2)^2 + C(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \bar{C}(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\
& + \frac{1}{2}[(\phi_1^\dagger \phi_2)(D\phi_1^\dagger \phi_2 + E\phi_1^\dagger \phi_1 + F\phi_2^\dagger \phi_2) + \text{h.c.}].
\end{aligned}
$$

CP is violated spontaneously by vevs of the form $(\rho_1 e^{i\theta}, \rho_2)$,

in the region of parameters

of the potential where ρ_1 and ρ_2 are different from zero and $e^{i\theta} \neq 1$

Change of basis:

$$
\begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \frac{1}{v} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\chi} \end{pmatrix} \begin{pmatrix} \rho_1 & \rho_2 \\ -\rho_2 & \rho_1 \end{pmatrix} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad v^2 = \rho_1^2 + \rho_2^2.
$$

bilinear part of the potential is only real if $\sin\chi = 0$ or $\lambda_1 = \lambda_2$.

in either case requiring the quartic part of the potential to be real leads to special conditions on the parameters and therefore does not hold in general

Models with Two Higgs doublets

$$\left(\begin{array}{cc} \rho_1 & \rho_2 \\ -\rho_2 & \rho_1 \end{array}\right)\left(\begin{array}{cc} e^{-i\theta} & 0 \\ 0 & 1 \end{array}\right)\left(\begin{array}{c} \phi_1 \\ \phi_2 \end{array}\right)$$

"the Higgs basis" (up to a sign ambiguity)

Models with more than two Higgs doublets (n)

$$\left(\begin{array}{c} h_1' \\ h_2' \\ h_S' \end{array}\right) = \left(\begin{array}{ccc} \frac{1}{N_1}(\hat{w}_1 & \hat{w}_2 & \hat{w}_S) \\ \frac{1}{N_2}(\hat{w}_2 & -\hat{w}_1 & 0) \\ \frac{1}{N_3}(\hat{w}_1 & \hat{w}_2 & X) \end{array}\right)\left(\begin{array}{ccc} e^{-i\sigma_1} & 0 & 0 \\ 0 & e^{-i\sigma_2} & 0 \\ 0 & 0 & 1 \end{array}\right)\left(\begin{array}{c} h_1 \\ h_2 \\ h_S \end{array}\right)$$

there are infinite bases where only one doublet acquires vev different from zero, freedom associated to a U matrix (n-1)x(n-1)

each choice is "a" different Higgs basis

- an SMA basis (SMA - standard model aligned)

Final Remarks

Models with three Higgs doublets have rich phenomenology

Aims and challenges

Exploit possible dark matter candidates in this context, beyond cases where the singlet plays the role of the SM Higgs doublet

Study how to generate realistic fermion masses and mixing with the fermions transforming non trivially under S_3

Look for viable models in the context of spontaneous CP violation

Look for interesting scenarios with the potential of being tested at the LHC