CP violation in the scalar sector

Scalars 2015

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Goal of study

- Find "simple" model which provides:
- CP violation and
- Dark Matter candidate

Standard Model scalar sector

- One SU(2) doublet, two complex fields
- 3 real fields "removed" (Goldstone)
- 1 real field left, Higgs
- CP conserved in the scalar sector

Two-Higgs-Doublet Model

- Two SU(2) doublets, four complex fields
- 3 real fields “removed” (Goldstone)
- 1 complex field left (Charged Higgs)
- 3 real fields left.
- If these mix, we have CP violation

Two-Higgs-Doublet Model, cont

- CP violation may be spontaneous (real potential), or
- CP violation may be “explicit” (complex, but Hermitian potential).
- Conditions for CP violation expressed in terms of “invariants” formed from coefficients in potential

2HDM notation

$$\boxed{V(\Phi_1,\Phi_2)} = -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\}$$
$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\}$$
$$\boxed{\equiv Y_{a\bar{b}}\Phi_{\bar{a}}^\dagger\Phi_b + \frac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi_{\bar{a}}^\dagger\Phi_b)(\Phi_{\bar{c}}^\dagger\Phi_d)}$$

Allow CPV: $m_{12}^2, \lambda_5, \lambda_6, \lambda_7$ complex

No FCNC: $\lambda_6 = 0; \quad \lambda_7 = 0$

CP conservation:

Define:

$$\mathrm{Im}\,J_1 = -\frac{2}{v^2}\mathrm{Im}\left[\hat{v}^*_{\bar{a}} Y_{a\bar{b}} Z^{(1)}_{b\bar{d}} \hat{v}_d\right]$$

$$\longrightarrow \quad = -\frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)\mathrm{Im}\,\lambda_5$$

$$\mathrm{Im}\,J_2 = \frac{4}{v^4}\mathrm{Im}\left[\hat{v}^*_{\bar{b}} \hat{v}^*_{\bar{c}} Y_{b\bar{e}} Y_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right]$$

$$\longrightarrow \quad = -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2\right) v_1^4 + 2(\lambda_1 - \lambda_2)\mathrm{Re}\,\lambda_5 v_1^2 v_2^2 - \left((\lambda_2 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2\right) v_2^4\right] \mathrm{Im}\,\lambda_5$$

$$\mathrm{Im}\,J_3 = \mathrm{Im}\left[\hat{v}^*_{\bar{b}} \hat{v}^*_{\bar{c}} Z^{(1)}_{b\bar{e}} Z^{(1)}_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right]$$

$$\longrightarrow \quad = \frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)(\lambda_1 + \lambda_2 + 2\lambda_4)\mathrm{Im}\,\lambda_5$$

Second lines valid in “2HDM5”

CPC: $\quad \mathrm{Im}\,J_1 = \mathrm{Im}\,J_2 = \mathrm{Im}\,J_3 = 0$

On the other hand, if

$$\boxed{\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = \mathrm{Im}\, J_3 = 0}$$

is violated, we have CP violation

What is the physical content?

Can

$$\operatorname{Im} J_1, \ \operatorname{Im} J_2, \ \operatorname{Im} J_3$$

be rephrased in terms of "physical" quantities?

The physical content

$$\mathrm{Im}\, J_1 = \frac{1}{v^5} \begin{vmatrix} q_1 & q_2 & q_3 \\ e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \end{vmatrix}$$

$$\mathrm{Im}\, J_2 = \frac{2}{v^9} \begin{vmatrix} e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \\ e_1 M_1^4 & e_2 M_2^4 & e_3 M_3^4 \end{vmatrix}$$

$$\mathrm{Im}\, J_{30} = \frac{1}{v^5} \begin{vmatrix} e_1 & e_2 & e_3 \\ q_1 & q_2 & q_3 \\ q_1 M_1^2 & q_2 M_2^2 & q_3 M_3^2 \end{vmatrix}$$

Footnote: $\mathrm{Im}\, J_3 = \mathrm{Im}\, J_{30} + \text{ terms } \propto \mathrm{Im}\, J_1, \mathrm{Im}\, J_2$

This result was published by
Lavoura and Silva in 1994
(see also Botella and Silva, 1995)

Revisited by Grzadkowski et al, 2014

Couplings:

antisymmetric

$i, j, k : 1, 2, 3$

$$Z^\mu H_i H_j : \quad \frac{g}{2v\cos\theta_{\mathrm{W}}} \epsilon_{ijk} e_k (p_i - p_j)^\mu$$

$$H^+ H^- H_i : \quad -iq_i$$

rotation matrix

$$\rightarrow \quad e_i = v_1 R_{i1} + v_2 R_{i2}$$

$$\rightarrow \quad q_i = \dots$$ more complicated

Recall CP-conserving 2HDM

Let $$H_1 = h \quad \text{Discovered 2012, 125 GeV}$$
$$H_2 = H$$
$$H_3 = A$$

Then

(ZHA)	$e_1 \neq 0$	(hH^+H^-)	$q_1 \neq 0$
(ZhA)	$e_2 \neq 0$	(HH^+H^-)	$q_2 \neq 0$
(ZhH)	$e_3 = 0$	(AH^+H^-)	$q_3 = 0$

The invariants $\mathrm{Im}\, J_1,\ \mathrm{Im}\, J_2,\ \mathrm{Im}\, J_3$ **vanish**

LHC

- Discovered Higgs particle is practically CP even
- Within 2HDM, $\mathrm{Im}\, J_1,\ \mathrm{Im}\, J_2$ vanish
- But $\mathrm{Im}\, J_{30}$ could be non-zero
- However, it is very hard to measure

Dark Matter

- Add a third doublet, zero vev
- Studied in 2009, 2011: "IDM2"
- Extension of popular IDM, but allowing CPV

3 doublets

Motivations for three Higgs doublets

- Three fermion generations may suggest three doublets
- Interesting scenario for dark matter
- Possibility of having a discrete symmetry and still having spontaneous CP violation
- Rich phenomenology

Motivation for imposing discrete symmetries

- Symmetries reduce the number of free parameters
- Symmetries help to control FCNC
- Symmetries are needed to stabilise dark matter

Footnote

- The first derivative of a potential (when set to zero) defines the vacuum expectation value(s)
- The second derivatives of a potential define a mass-squared matrix

“Problems”

- Vacuum: When many fields, get many coupled equations (cubic and trigonometric)
- Mass matrices: When many fields, get large matrices to diagonalize

Simpler approach

- Pick a vacuum of interest (must identify possibilities)
- Pick a mass spectrum of interest
- Construct potential
- Check consistency (positivity etc)

Advantages

- Control of physical content
- Linear equations!

End of footnote

Three $SU(2) \times U(1)$-symmetric doublets

Most general potential has 46 parameters (counted by Olaussen et al, 2011)

Consider S_3 symmetric potential

Basic papers:

(a) Pakvasa & Sugawara, 1978
(b) Derman, 1979
(c) Kubo, Okada, Sakamaki, 2004

(a,c): irreducible reps, (b): reducible rep

Two “Frameworks”

May work with the

- reducible representation (Derman) or the
- irreducible representations (Pakvasa & Sugawara, Das & Dey)

There is a linear map from one framework to the other

Reducible representation

$$\phi_1, \quad \phi_2, \quad \phi_3$$

$$\phi_i = \begin{pmatrix} \varphi_i^+ \\ (\rho_i + \eta_i + i\chi_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, 3$$

$$V = V_2 + V_4$$

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \tfrac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \text{h.c.}],$$

$$\begin{aligned} V_4 = {} & A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{ C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \overline{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \tfrac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \text{h.c.}]\} \\ & + \tfrac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \text{h.c.}] + \sum_{i \neq j \neq k \neq i, j<k} \{ \tfrac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \text{h.c.}] \\ & + \tfrac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \text{h.c.}] + \tfrac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \text{h.c.}]\} \end{aligned}$$

10 parameters

Irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 - \phi_2) \\ \frac{1}{\sqrt{6}}(\phi_1 + \phi_2 - 2\phi_3) \end{pmatrix} \qquad h_S = \frac{1}{\sqrt{3}}(\phi_1 + \phi_2 + \phi_3)$$

$$h_i = \begin{pmatrix} h_i^+ \\ (w_i + \tilde{\eta}_i + i\tilde{\chi}_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, \quad h_S = \begin{pmatrix} h_S^+ \\ (w_S + \tilde{\eta}_S + i\tilde{\chi}_S)/\sqrt{2} \end{pmatrix}$$

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2)$$

$$\begin{aligned} V_4 = {} & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_S^\dagger h_S)^2 \end{aligned}$$

10 parameters

Note that irreducible representation chooses a particular “direction” among

$$\phi_1, \quad \phi_2, \quad \phi_3$$

Not unique — convention

This potential exhibits

$h_1 \to -h_1$ symmetry

but not $h_2 \to -h_2$

Equivalent doublet representation

$$\begin{pmatrix} \tilde{\chi}_1 \\ \tilde{\chi}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

the above symmetry becomes

$$\tilde{\chi}_1 \leftrightarrow \tilde{\chi}_2$$

In the irreducible-rep framework

the case $\lambda_4 = 0$ SPECIAL

or, in the reducible-rep framework

$$4A - 2(C + \overline{C} + D) - E_1 + E_2 + E_3 + E_4 = 0$$

leads to a continuous SO(2) symmetry

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Massless states!

At this stage, the two frameworks are equivalent

However, introducing Yukawa couplings, for example, in terms of

$$\phi_1, \quad \phi_2, \quad \phi_3$$

or

$$h_1, \quad h_2, \quad h_S$$

they would naturally be different

The vevs are related

$$w_1 = \frac{1}{\sqrt{2}}(\rho_1 - \rho_2)$$
$$w_2 = \frac{1}{\sqrt{6}}(\rho_1 + \rho_2 - 2\rho_3)$$
$$w_S = \frac{1}{\sqrt{3}}(\rho_1 + \rho_2 + \rho_3)$$

$$\rho_1 = \frac{1}{\sqrt{3}}w_S + \frac{1}{\sqrt{2}}w_1 + \frac{1}{\sqrt{6}}w_2$$
$$\rho_2 = \frac{1}{\sqrt{3}}w_S - \frac{1}{\sqrt{2}}w_1 + \frac{1}{\sqrt{6}}w_2$$
$$\rho_3 = \frac{1}{\sqrt{3}}w_S - \frac{\sqrt{2}}{\sqrt{3}}w_2$$

Vacua

Derivatives of potential wrt (complex) fields must vanish

Three complex derivatives = 0 or

Five real derivatives (3 moduli, 2 relative phases) = 0

The minimisation conditions must be consistent.
This is an important constraint on the potential.

May work in either framework

But a particular vacuum may look simpler in one framework than in the other.

Classical (real) vacua

The early literature focused on fermion masses and real vacua (no CPV):

Examples:

$\rho_2 = \rho_3$ Derman 1979

$w_1 = \sqrt{3} w_2$ Das & Dey 2014

Classical (real) vacua

In the reducible-representation framework,
we may equally well take

$$\rho_1 = \rho_2 \quad \text{or} \quad \rho_1 = \rho_3$$

They correspond to different vacua in the irreducible-representation framework, one case has

$$w_1 = 0$$

Complex vacua

Complex vacua may allow CP violation

Examples:

C-0 $\quad (\rho_1, \rho_2, \rho_3) = x(1, e^{2i\pi/3}, e^{-2i\pi/3}) \quad \Rightarrow w_S = 0$

C-I-a $\quad (\rho_1, \rho_2, \rho_3) = x(1, e^{i\tau}, e^{i\tau}) \quad \Rightarrow w_1 = \sqrt{3} w_2$

C-I-a1 $\quad (\rho_1, \rho_2, \rho_3) = x(e^{-i\tau}, 1, 1) \quad \Rightarrow w_1 = \sqrt{3} w_2$

C-I-a2 $\quad (\rho_1, \rho_2, \rho_3) = x(1, e^{-i\tau}, 1) \quad \Rightarrow w_1 = -\sqrt{3} w_2$

C-I-a3 $\quad (\rho_1, \rho_2, \rho_3) = x(1, 1, e^{-i\tau}) \quad \Rightarrow w_1 = 0$

C-I-a violates CP, C-0 does not

Complex vacua

Here, an overall phase rotation brings us from vacuum C-I-a to C-I-a1

$$\text{C-I-a} \quad \xrightarrow{\exp(-i\tau)} \quad \text{C-I-a1}$$

Next:

$$\text{C-I-a1} \quad \xrightarrow{\rho_1 \leftrightarrow \rho_2} \quad \text{C-I-a2} \quad \xrightarrow{\rho_2 \leftrightarrow \rho_3} \quad \text{C-I-a3}$$

These are all different names for one and the same vacuum

Complex vacua

Spontaneous CP violation

C-0	$(\rho_1, \rho_2, \rho_3) = x(1, e^{2i\pi/3}, e^{-2i\pi/3})$	$\Rightarrow w_S = 0$
C-I-a	$(\rho_1, \rho_2, \rho_3) = x(1, e^{i\tau}, e^{i\tau})$	$\Rightarrow w_1 = \sqrt{3} w_2$
C-I-a1	$(\rho_1, \rho_2, \rho_3) = x(e^{-i\tau}, 1, 1)$	$\Rightarrow w_1 = \sqrt{3} w_2$
C-I-a2	$(\rho_1, \rho_2, \rho_3) = x(1, e^{-i\tau}, 1)$	$\Rightarrow w_1 = -\sqrt{3} w_2$
C-I-a3	$(\rho_1, \rho_2, \rho_3) = x(1, 1, e^{-i\tau})$	$\Rightarrow w_1 = 0$

C-I-a violates CP, C-0 does not

- Complex vevs are no guarantee for SCPV
- The symmetry of the Lagrangian could "hide" the complex conjugation

Example: $\text{C-0:} \quad (\rho_1, \rho_2, \rho_3) = x(1, e^{2i\pi/3}, e^{-2i\pi/3})$

Complex conjugation:

$$x(1, e^{2i\pi/3}, e^{-2i\pi/3}) \Rightarrow x(1, e^{-2i\pi/3}, e^{2i\pi/3})$$

But the Lagrangian has a symmetry:

$$\phi_2 \leftrightarrow \phi_3 \quad \text{and} \quad \rho_2 \leftrightarrow \rho_3$$

which will undo the complex conjugation

Such geometrical phases,
and their relation to CP violation, have been explored
by Branco, Gerard and Grimus (1984)

Complex vacua

Complex vacua may allow CP violation

More:

$$\text{C-II-a} \quad (\rho_1, \rho_2, \rho_3) = \hat{\rho}, \hat{\rho}, \hat{\rho}' e^{i\tau} \quad \Rightarrow w_1 = 0$$

$$\text{C-II-b} \quad (\rho_1, \rho_2, \rho_3) = \hat{\rho}, \hat{\rho}' e^{i\tau}, \hat{\rho} \quad \Rightarrow w_1 = -\sqrt{3} w_2$$

$$\text{C-II-c} \quad (\rho_1, \rho_2, \rho_3) = \hat{\rho}' e^{i\tau}, \hat{\rho}, \hat{\rho} \quad \Rightarrow w_1 = \sqrt{3} w_2$$

$$\text{C-III} \quad (\rho_1, \rho_2, \rho_3) = \hat{\rho}_1, \hat{\rho}_2 e^{i\tau_2}, \hat{\rho}_3 e^{i\tau_3} \quad \Rightarrow w_1, w_2, w_S$$

Complex vacua

Spontaneous CP violation

C-II-a	$(\rho_1, \rho_2, \rho_3) = \hat{\rho}, \hat{\rho}, \hat{\rho}' e^{i\tau}$	$\Rightarrow w_1 = 0$
C-II-b	$(\rho_1, \rho_2, \rho_3) = \hat{\rho}, \hat{\rho}' e^{i\tau}, \hat{\rho}$	$\Rightarrow w_1 = -\sqrt{3} w_2$
C-II-c	$(\rho_1, \rho_2, \rho_3) = \hat{\rho}' e^{i\tau}, \hat{\rho}, \hat{\rho}$	$\Rightarrow w_1 = \sqrt{3} w_2$
C-III	$(\rho_1, \rho_2, \rho_3) = \hat{\rho}_1, \hat{\rho}_2 e^{i\tau_2}, \hat{\rho}_3 e^{i\tau_3}$	$\Rightarrow w_1, w_2, w_S$

Complex vacua $\lambda_4 = 0$

C-II-a	$(w_1, w_2, w_S) = (0, \hat{w}e^{i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho, \rho, \rho')$
C-II-b	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, -\hat{w}e^{i\sigma}/\sqrt{3}, \hat{w}_S)$	$\Rightarrow (\rho, \rho', \rho)$
C-II-c	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}/\sqrt{3}, \hat{w}_S)$	$\Rightarrow (\rho', \rho, \rho)$
C-II-d	$(w_1, w_2, w_S) = (\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-II-PS	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{-i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-II-IN	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-III	$(w_1, w_2, w_S) = (\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$

Spontaneous CP violation

C-II-a	$(w_1, w_2, w_S) = (0, \hat{w}e^{i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho, \rho, \rho')$
C-II-b	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, -\hat{w}e^{i\sigma}/\sqrt{3}, \hat{w}_S)$	$\Rightarrow (\rho, \rho', \rho)$
C-II-c	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}/\sqrt{3}, \hat{w}_S)$	$\Rightarrow (\rho', \rho, \rho)$
C-II-d	$(w_1, w_2, w_S) = (\hat{w}_1e^{i\sigma_1}, \hat{w}_2e^{i\sigma_2}, 0)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-II-PS	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{-i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-II-IN	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-III	$(w_1, w_2, w_S) = (\hat{w}_1e^{i\sigma_1}, \hat{w}_2e^{i\sigma_2}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$

Note that $\mathrm{C\text{-}II\text{-}PS}$ does not violate CP

$$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{-i\sigma}, \hat{w}_S) \quad \xrightarrow{\text{c. c.}} \quad (\hat{w}e^{-i\sigma}, \hat{w}e^{i\sigma}, \hat{w}_S)$$

When $\lambda_4 = 0$ have symmetry

$$h_1 \leftrightarrow h_2 \quad \text{and} \quad w_1 \leftrightarrow w_2$$

Sometimes there are problems

When $\lambda_4 = 0$

there are massless states

Add a soft SO(2)-breaking term:

$$V \to V + \frac{1}{2}\nu^2(h_2^\dagger h_1 + h_1^\dagger h_2)$$

Vacuum conditions are changed

Our Aims

Determine whether Spontaneous CP violation in S_3 is compatible with a good inert dark matter candidate and what are the properties

Challenges include:

- Determine necessary and sufficient vacuum stability conditions
- Obey unitarity constraints for the potential
- Obtain correct dark matter density
- Identify realistic Yukawa structures

Concluding comments

- The S_3-symmetric scalar sector is very rich
- Two different (equivalent) frameworks
- Spontaneous CP violation can take place
- Room for Dark Matter
- Next: Yukawa couplings

Apology

This was not meant to be an overview of what is known about an S_3-symmetric potential, only elements of what we have understood so far

Back-up

For more details on these vacua, see talks at

- Multi-Higgs models, Lisboa Sep 2016
- Scalars 2017
- Multi-Higgs models, Lisboa 2018
- Scalars 2019
- etc

also by other authors