Higgs-R^2 inflation and cosmological collider physics

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Based mainly on 2002.11739, 2102.12501, 2309.xxxxx
with R. Jinno, K. Nakayama, K. Mukaida, J. van de Vis, S. Verner
See also 1609.05209, 1701.07665, 1907.00993 and 2008.01096.

Outline

1. Higgs inflation, unitarity and R^2

2. Cosmological collider signatures

3. Summary

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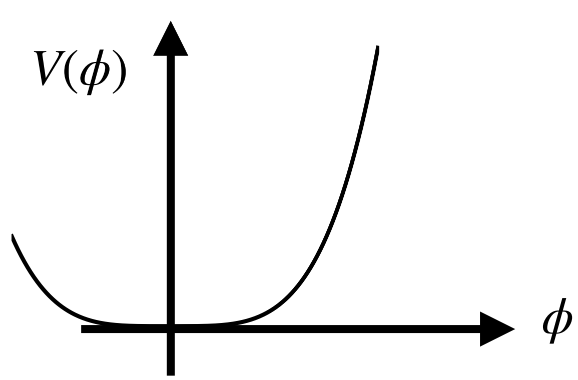
Higgs inflation

- Higgs inflation: standard model Higgs = inflaton. [Bezrukov, Shaposhnikov 07; ...]

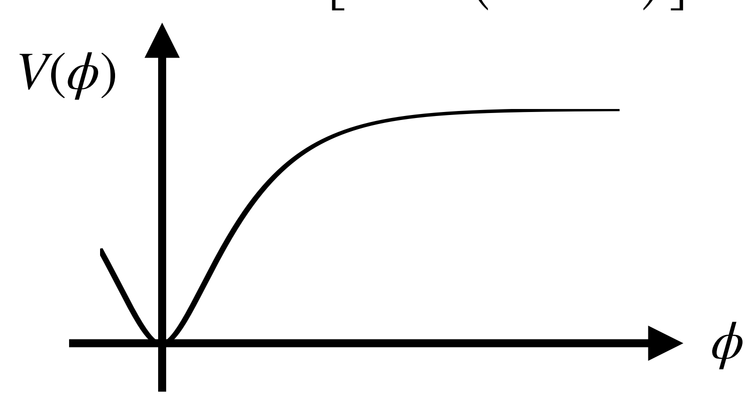
$$\mathscr{L} = \frac{M_P^2}{2}R + |D_\mu \Phi|^2 + \underline{\xi |\Phi|^2 R} - \lambda |\Phi|^4$$ with Φ: Higgs doublet.

- Non-minimal Higgs-gravity coupling "ξ" flattens the potential.

$$V(\phi) \simeq \frac{\lambda}{4}\phi^4$$



$$V(\phi) \simeq \frac{\lambda M_P^4}{4\xi^2}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right)\right]^2$$



CMB: $\xi \simeq 4\sqrt{\lambda} \times 10^4 \gg 1.$

(Focusing on metric formalism)

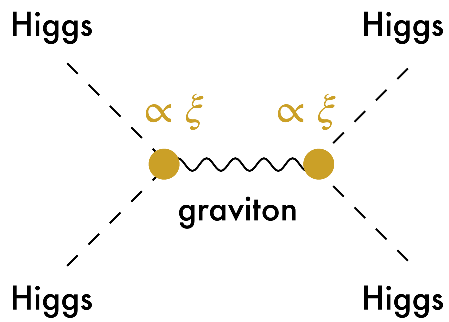
- Inflationary predictions: $n_s = 1 - \dfrac{2}{N_e}, \quad r = \dfrac{12}{N_e^2}.$

Reheating determines N_e, essential for inflationary prediction,

e.g. to distinguish between Higgs and R^2 inflation.

Unitarity issue

- Higgs inflation is not consistent for $E > M_P/\xi \sim 10^{14}\,\mathrm{GeV}$. [Burgess+ 09,10; Barbon+09, Hertzberg10; Bezrukov+10]

$i\mathcal{M} =$ 

$\sim \dfrac{\xi^2 E^2}{M_P^2}$, $|\mathcal{M}|^2 \sim$ (scattering probablity) > 1 for $E > M_P/\xi$.

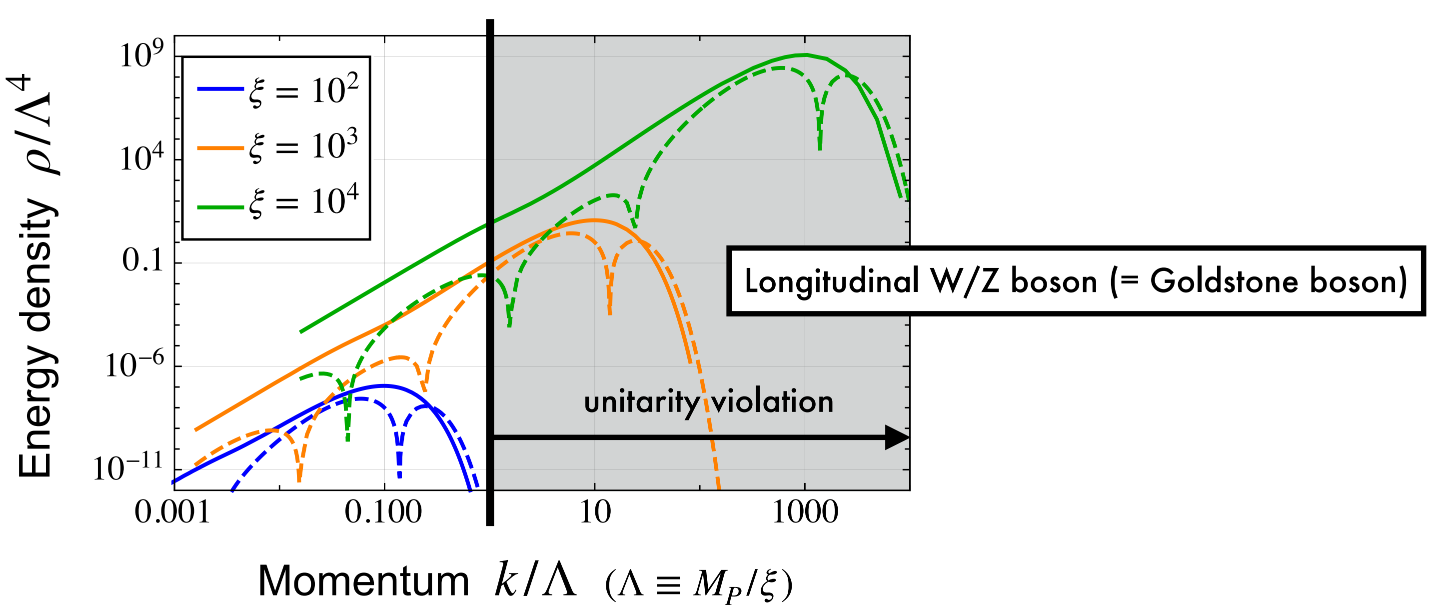
- Inflation energy scale exceeds this scale

$$\frac{M_P}{\xi} \sim 10^{14}\,\mathrm{GeV} < V_{\rm inf}^{1/4} \sim 10^{16}\,\mathrm{GeV}$$ (fixed from CMB),

but still debates on validity during inflation.

- Higgs inflation indeed violates unitarity during (p)reheating.

[YE, Jinno, Mukaida, Nakayama 16; Sfakianakis+ 18; YE, Jinno, Nakayama, van de Vis 21]



→ theoretically inconsistent, UV completion necessary to determine T_R.

Higgs-R^2 inflation

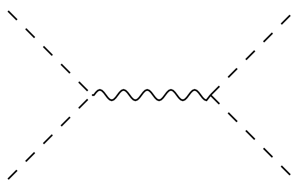
- Unitarity issue of Higgs inflation healed by R^2 term : "Higgs-R^2 inflation".

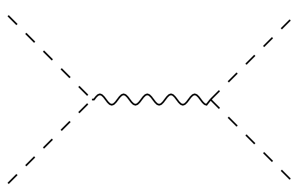
[YE 17, 19; Gorbunov+ 18]

$$\mathscr{L} = \frac{M_P^2}{2}R + \alpha R^2 + |D_\mu \Phi|^2 + \xi |\Phi|^2 R - \lambda |\Phi|^4 .$$

$R^2 \sim (\partial^2 g)^2$ introduces new degree of freedom "scalaron." $\begin{cases} \partial^2 \phi = 0 \to 2 \text{ initial conditions,} \\ \partial^4 \phi = 0 \to 4 \text{ initial conditions.} \end{cases}$

(no ghost here)

$i\mathscr{M}_{\text{Higgs}} =$  $\sim \dfrac{\xi^2 E^2}{M_P^2} > 1$ for $E > M_P/\xi$: unitarity violation.

$i\mathscr{M}_{\text{Higgs}+R^2} =$  $+$ 

$\sim \dfrac{\xi^2 m_s^2}{M_P^2}\dfrac{E^2}{m_s^2 - E^2} < 1$: unitarity always conserved

where scalaron mass $m_s^2 = \dfrac{M_P^2}{12\alpha} \lesssim \dfrac{M_P^2}{\xi^2}$, or $\alpha \gtrsim \xi^2$, needed.

- Higgs-R^2 inflation: no other operators generated below M_P (except for Higgs mass and CC).

[YE 19; YE, Mukaida, van de Vis 20]

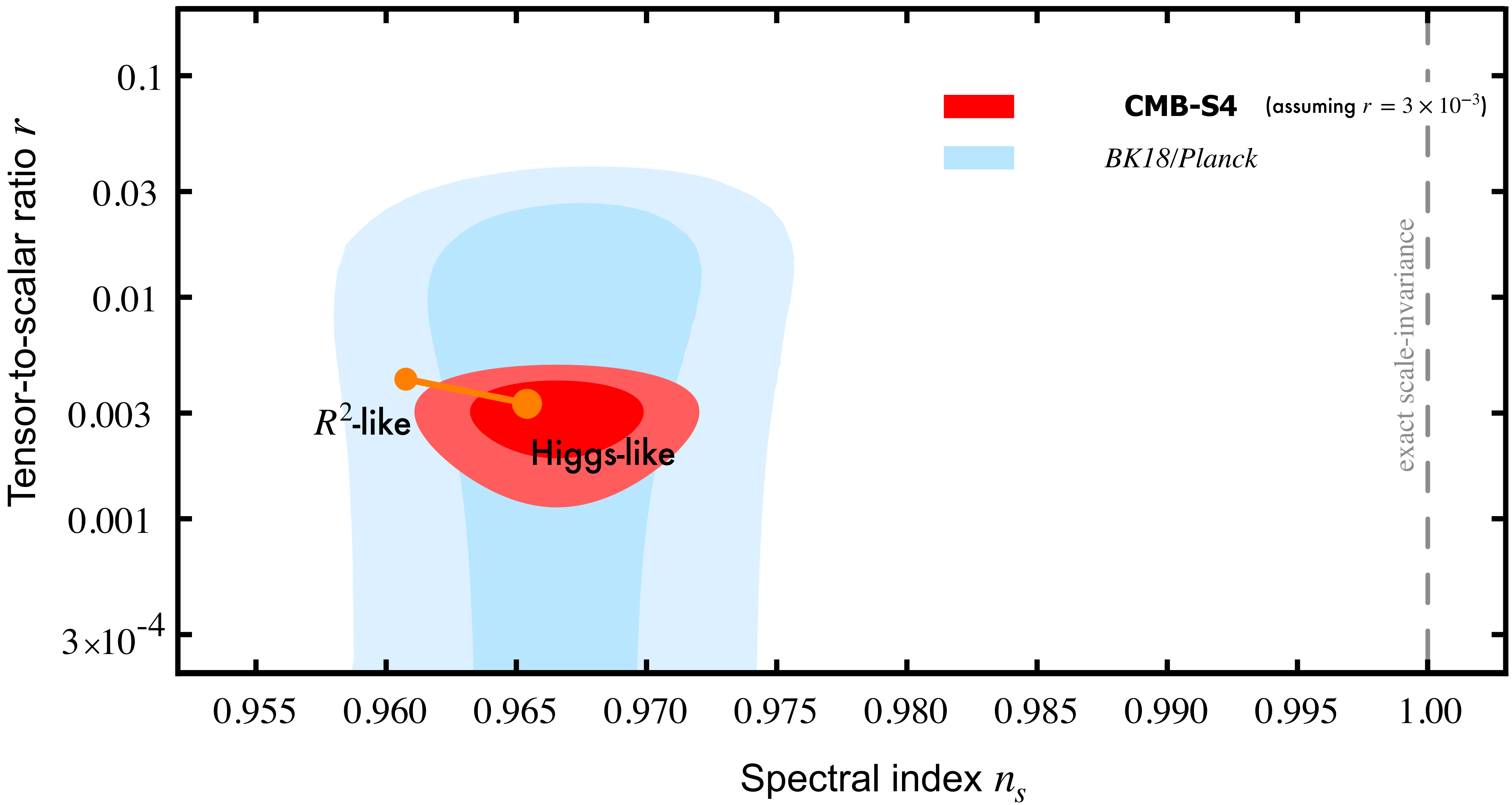
$\mathscr{L}_{\text{Higgs}} \xrightarrow{\text{quantum correction}} \mathscr{L}_{\text{Higgs}+R^2}$ but $\mathscr{L}_{\text{Higgs}+R^2} \xrightarrow{\text{quantum correction}} \mathscr{L}_{\text{Higgs}+R^2}$.

($M_P \to \infty$ with M_P/ξ : fixed, renormalizability of (spin-0 part of) quadratic gravity)

Higgs-R^2 inflation: prediction

CMB requires $\dfrac{\xi^2}{\lambda} + 4\alpha \simeq 2 \times 10^9\,.$ $\begin{cases} \xi^2/\lambda \gg \alpha \to \text{inflaton: Higgs-like,} \\ \xi^2/\lambda \ll \alpha \to \text{inflaton: scalaron-like.} \end{cases}$

[Snowmass2021 Cosmic Frontier: CMB Measurements White Paper] (adapted by YE)



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1. Higgs inflation, unitarity and R^2
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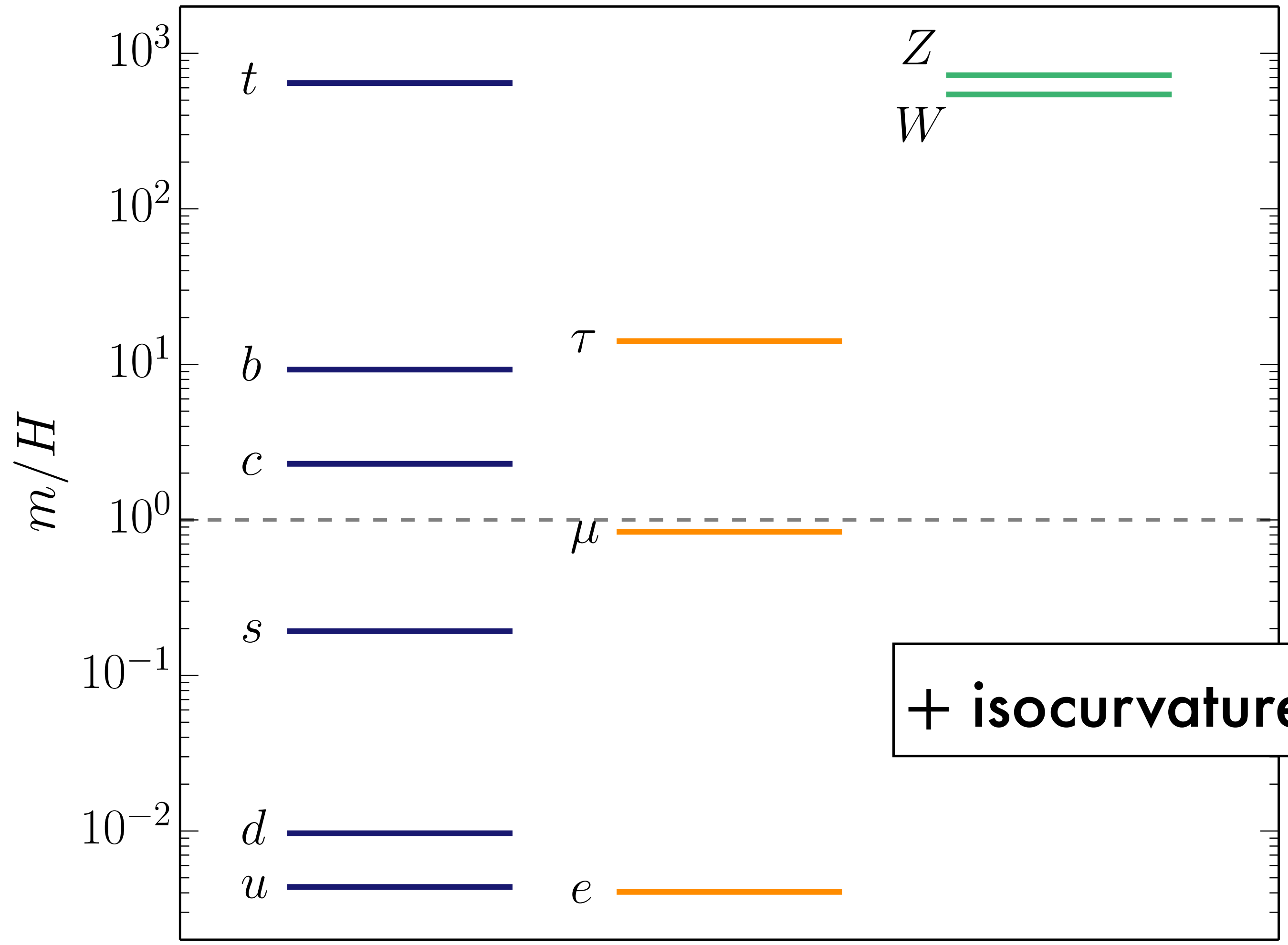
SM mass spectrum

- SM mass spectrum rich and some as light as H even though $h_0 \gg H$ during inflation.

[Chen+ 16]



+ isocurvature (= scalaron + Higgs).

- Inflaton couples to SM particles through Higgs/conformal factor in Higgs-R^2 inflation.

Any observable cosmological collider signatures?

SPOILER ALERT!

Yes, but only in somewhat a corner of parameter space.

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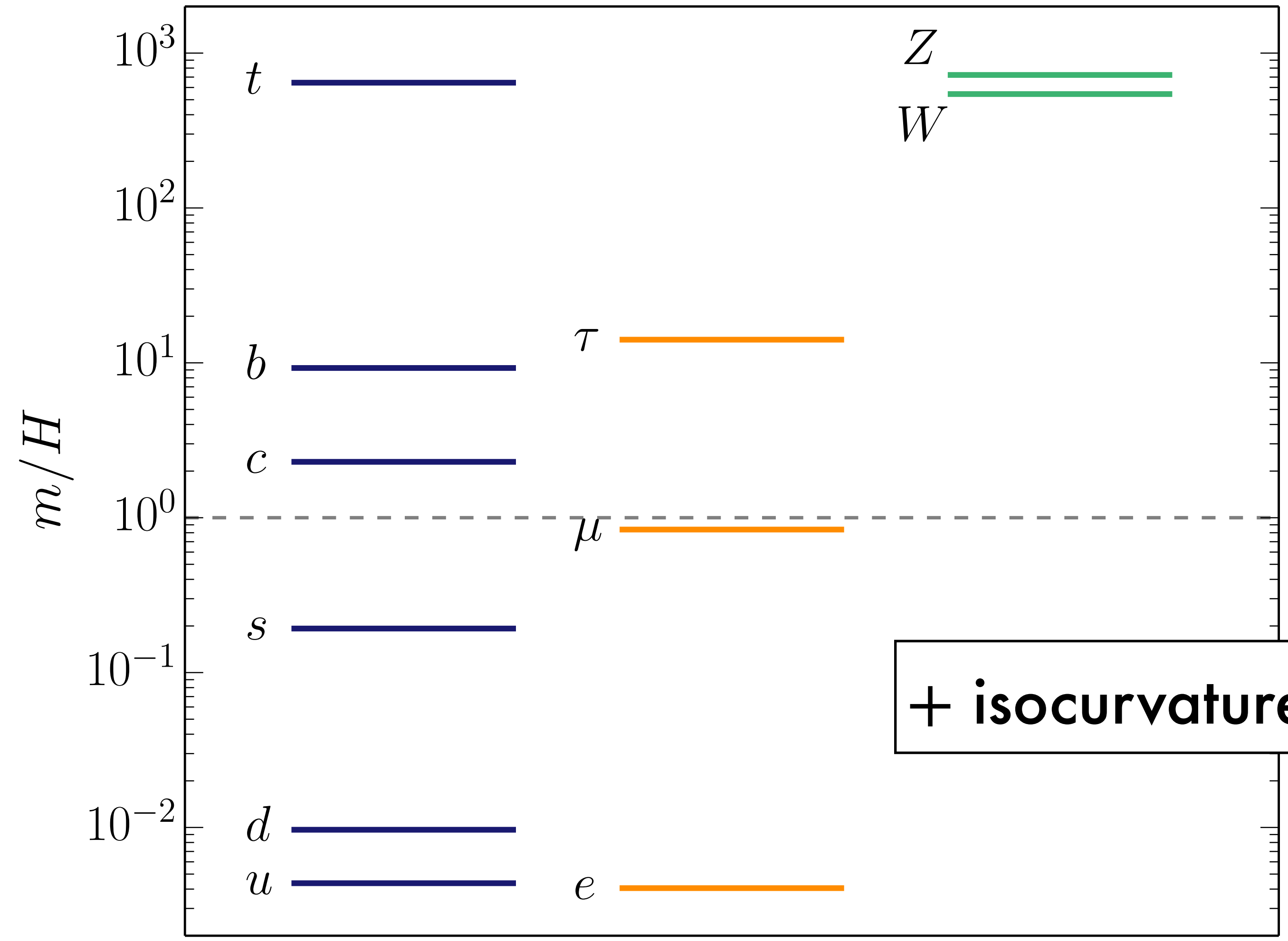

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Yes, but only in somewhat a corner of parameter space.

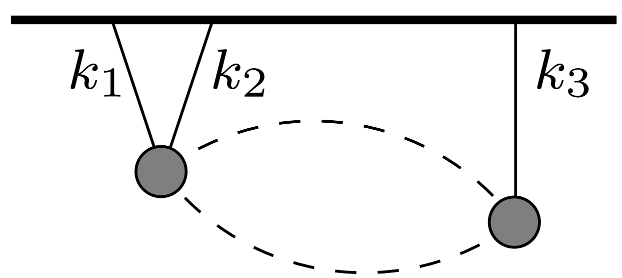
Fermion and gauge boson

- Inflaton couples to SM fermion and gauge boson through the mass term:

$$S_{\rm int} = \int d^4x \sqrt{-g}\,\mathcal{O}_{\rm SM}\left[1 + c_1 \frac{\varphi}{N_e M_P} + c_2 \frac{\varphi^2}{2M_P^2}\right], \quad \mathcal{O}_{\rm SM} = -\, m\bar{\psi}\psi, \quad \frac{m^2}{2} g^{\mu\nu} A_\mu A_\nu,$$

with $(c_1, c_2) = (\sqrt{6}/16, 1/6)$ for ψ and $(\sqrt{6}/8, 1/3)$ for A_μ.

- Cosmological collider signatures diagrammatically given by

$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle' =$ 

$\equiv (2\pi)^4 \dfrac{P_\zeta^2}{k_1^2 k_2^2 k_3^2} \times S_{\rm NG}\,.$

- Squeezed limit given by

$$S_{\rm NG} = \begin{cases} -\dfrac{3c_1 c_2 H\dot{\phi}_0}{8\pi^4 N_e M_P^3} \dfrac{m^2}{H^2} C_{1/2}(\nu_-) \left(\dfrac{k_3}{k_1}\right)^{4-2\nu_-} + (\nu_- \to \nu_+) : \ \text{fermion}, \\ -\dfrac{3c_1 c_2}{8\pi^4} \dfrac{H\dot{\phi}_0}{N_e M_P^3} \dfrac{m^4}{H^4} C_1(\nu) \left(\dfrac{k_3}{k_1}\right)^{2-2\nu} + (\nu \to -\nu) : \ \text{gauge boson}\,. \end{cases}$$

➡ $S_{\rm NG} \lesssim \dfrac{H\dot{\phi}_0}{(2\pi)^4 N_e M_P^3} \sim 10^{-17}$: far too small to be observable. [YE, Verner 23]

Isocurvature mode

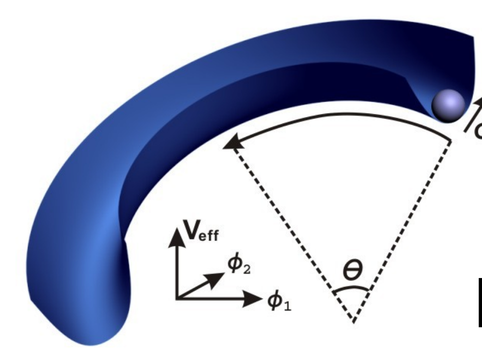
- Action of adiabatic mode ζ and isocurvature mode χ given by

$$S = \int dt d^3x a^3 \left[\frac{1}{2}\frac{\dot{\phi}_0^2}{H^2}\left(\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right) + \frac{1}{2}\left(\dot{\chi}^2 - \frac{1}{a^2}(\partial_i\chi)^2 - m_\chi^2\chi^2\right) - \frac{2\dot{\theta}\dot{\phi}_0}{H}\dot{\zeta}\chi \right]$$

where $m_\chi^2 \simeq \frac{\xi(24\lambda\alpha + \xi(1+6\xi))}{\lambda\alpha} H^2 > 24\xi H^2, \quad \frac{\dot{\theta}}{H} \simeq \sqrt{\frac{3\xi}{2(4\lambda\alpha + \xi^2)}}.$

- Isocurvature mode heavy in “standard” case $\xi \gg 1$ but light for $\xi \sim \lambda\alpha \lesssim \mathcal{O}(0.1)$.

➡ Realizing the idea of “quasi-single field inflation”:



[Chen+ 09]

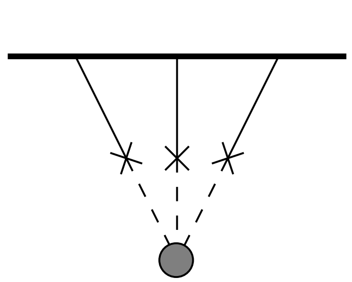
- Three point couplings from potential dominate:

$$S_{\text{cubic}} = \int d\tau d^3x a^4 \left[-\frac{1}{6}V_{N^3}\chi^3 - \frac{1}{2}V_{T^2N}\varphi^2\chi \right], \quad V_{N^3} \sim V_{T^2N} \sim \frac{H}{\sqrt{\alpha}} \quad \text{(for } \xi \sim \lambda\alpha \sim \mathcal{O}(1)\text{)}.$$

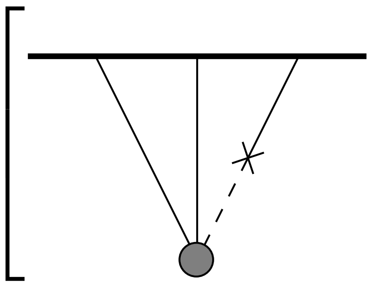
Signal of isocurvature mode

$$\boxed{-\frac{1}{6}V_{N^3}\chi^3 - \frac{1}{2}V_{T^2N}\varphi^2\chi}$$

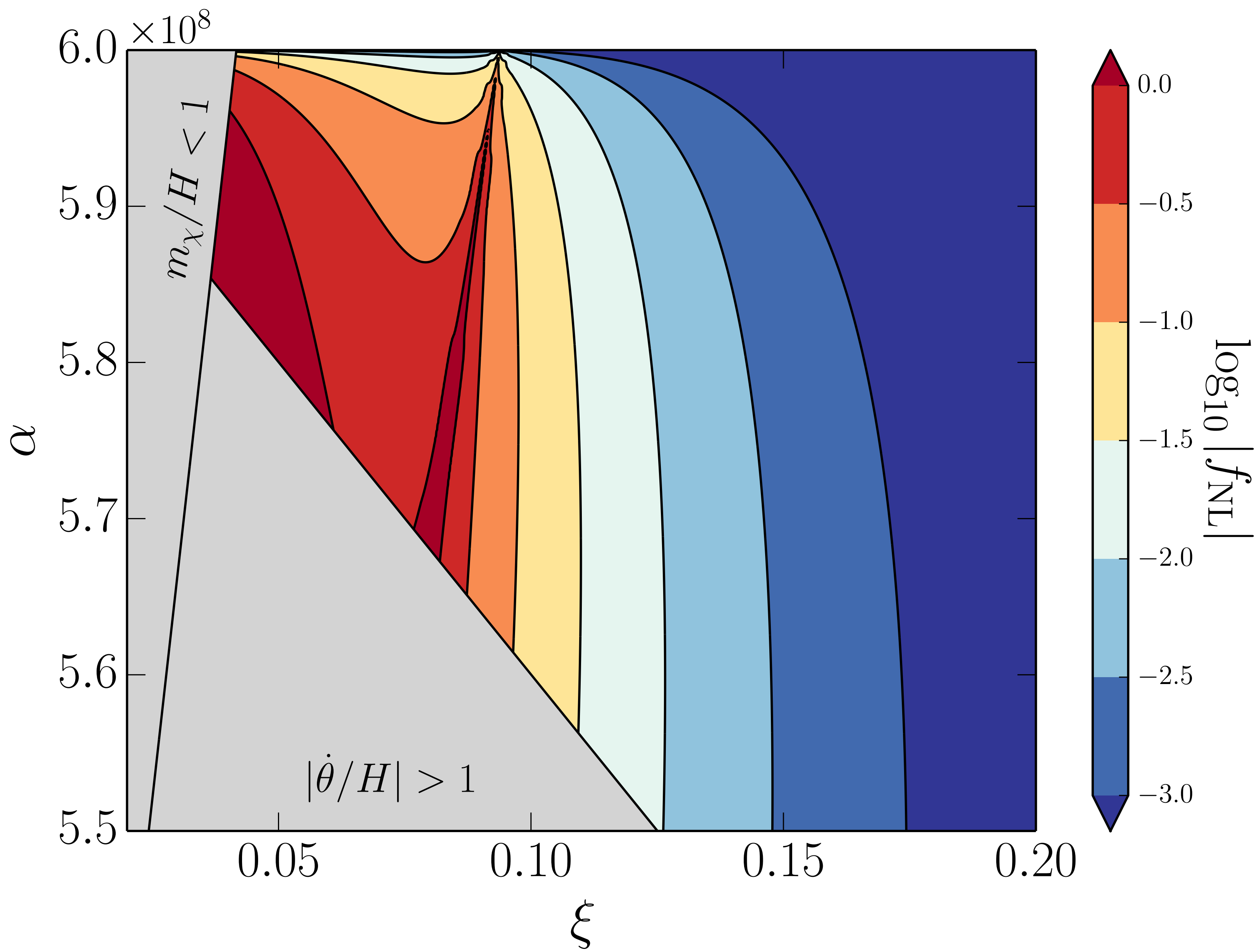
$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle' =$



$+ \Bigg[$



$+ \text{(perms.)} \Bigg] \sim P_\zeta^{-1/2}\,\frac{\dot\theta^3}{H^3}\frac{V_{N^3}}{H}, \quad P_\zeta^{-1/2}\frac{\dot\theta}{H}\frac{V_{T^2N}}{H}$



[YE, Verner 23]

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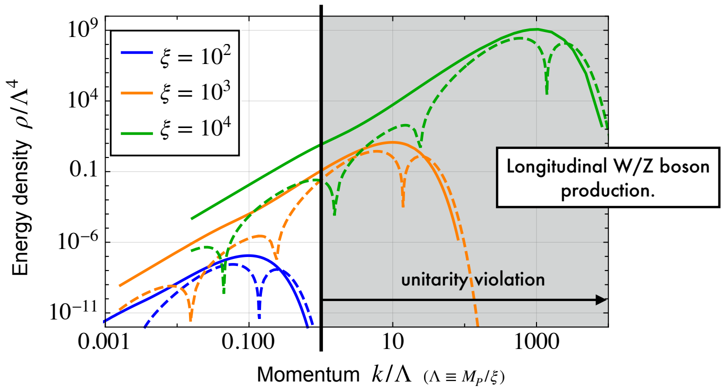
Summary

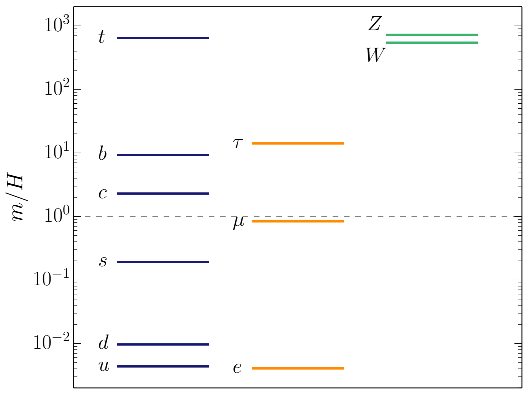
- Higgs inflation is appealing at first sight, as Higgs is the only scalar field within SM.

BUT unitarity violated during reheating:

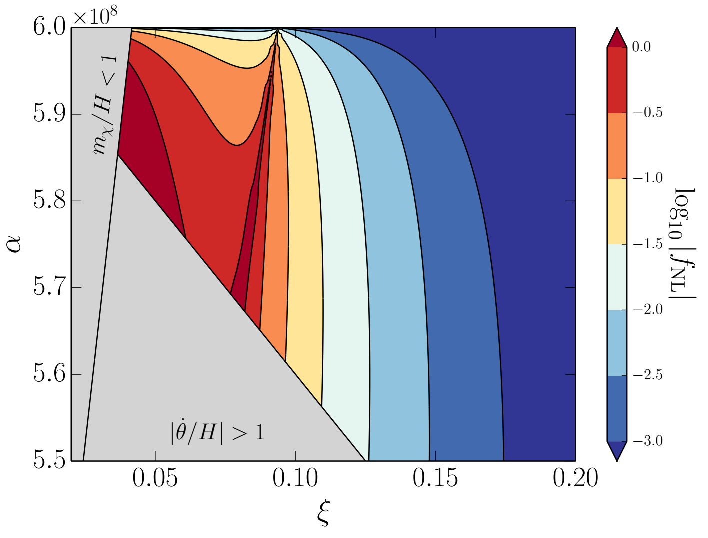


- Higgs-R^2 inflation UV-completes, with scalaron playing the role of "σ-meson".

- Rich SM mass spectrum and inflaton naturally couples to SM particles.



- Cosmological collider signature from the isocurvature mode can be sizable:

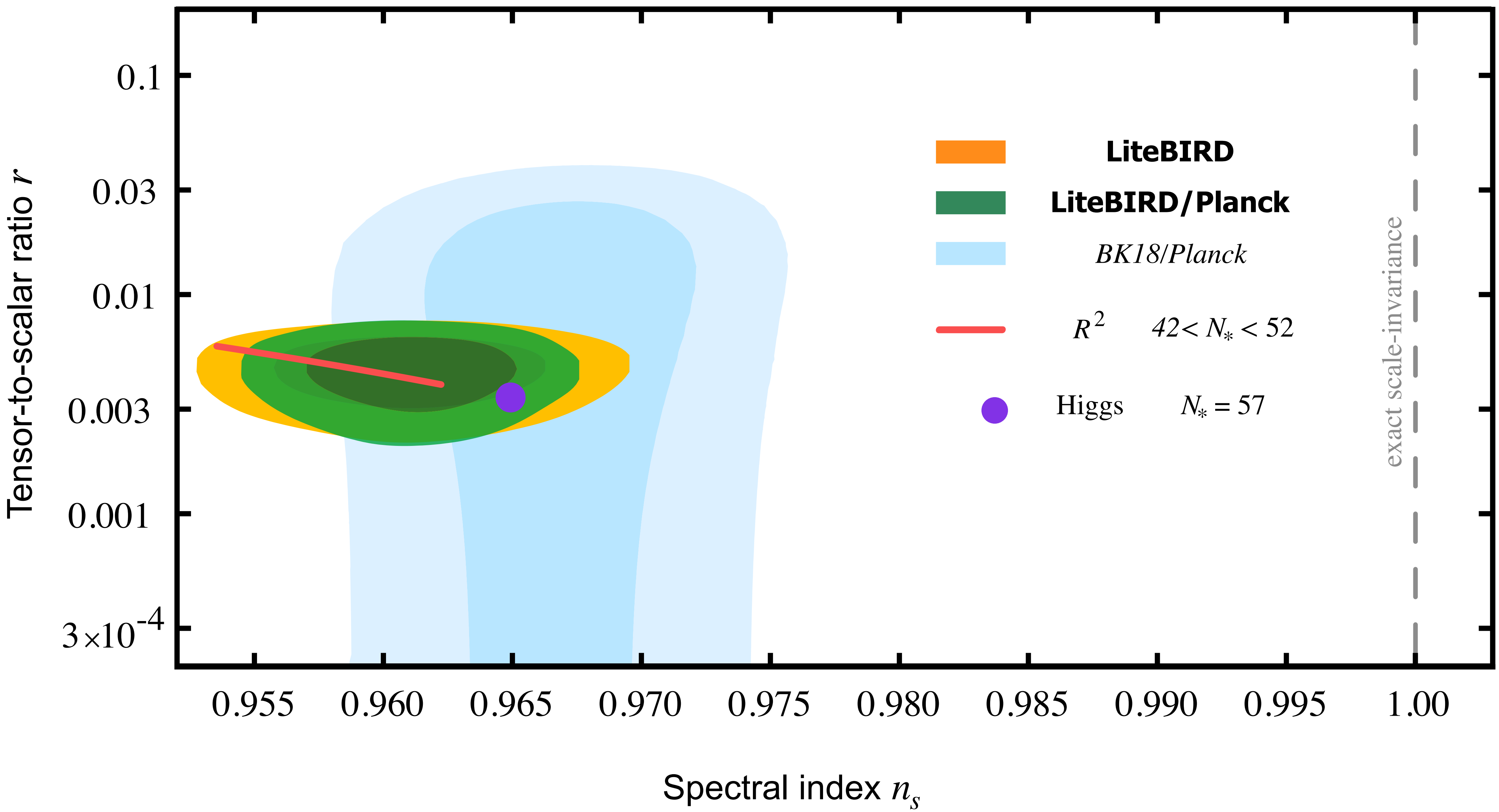


Back up

LiteBIRD

True values: $n_s = 0.961,\ r = 0.0046$ assumed.

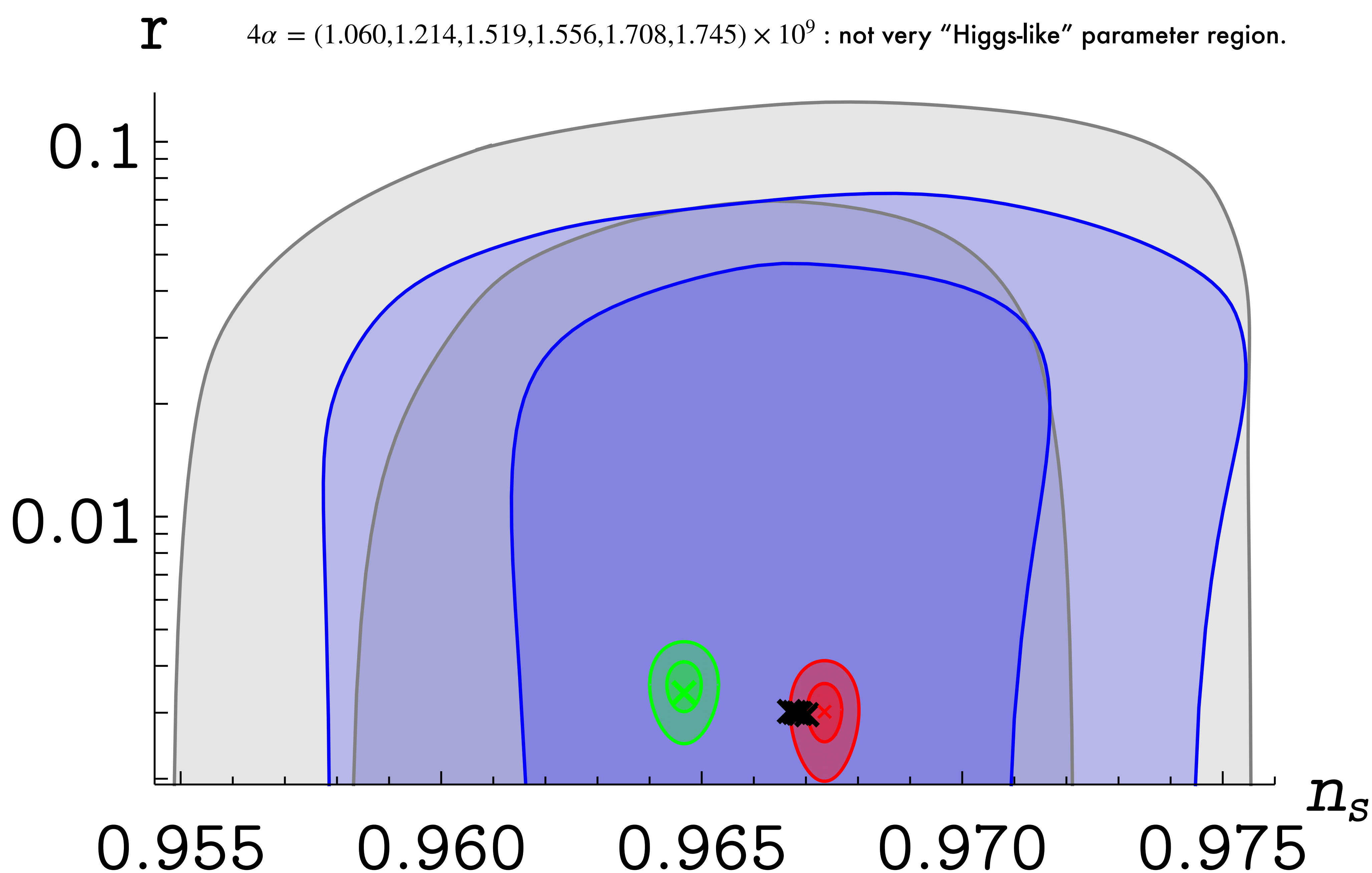
[LiteBIRD Collaboration 22] (adapted by YE)



Higgs-R^2 inflation: reheating

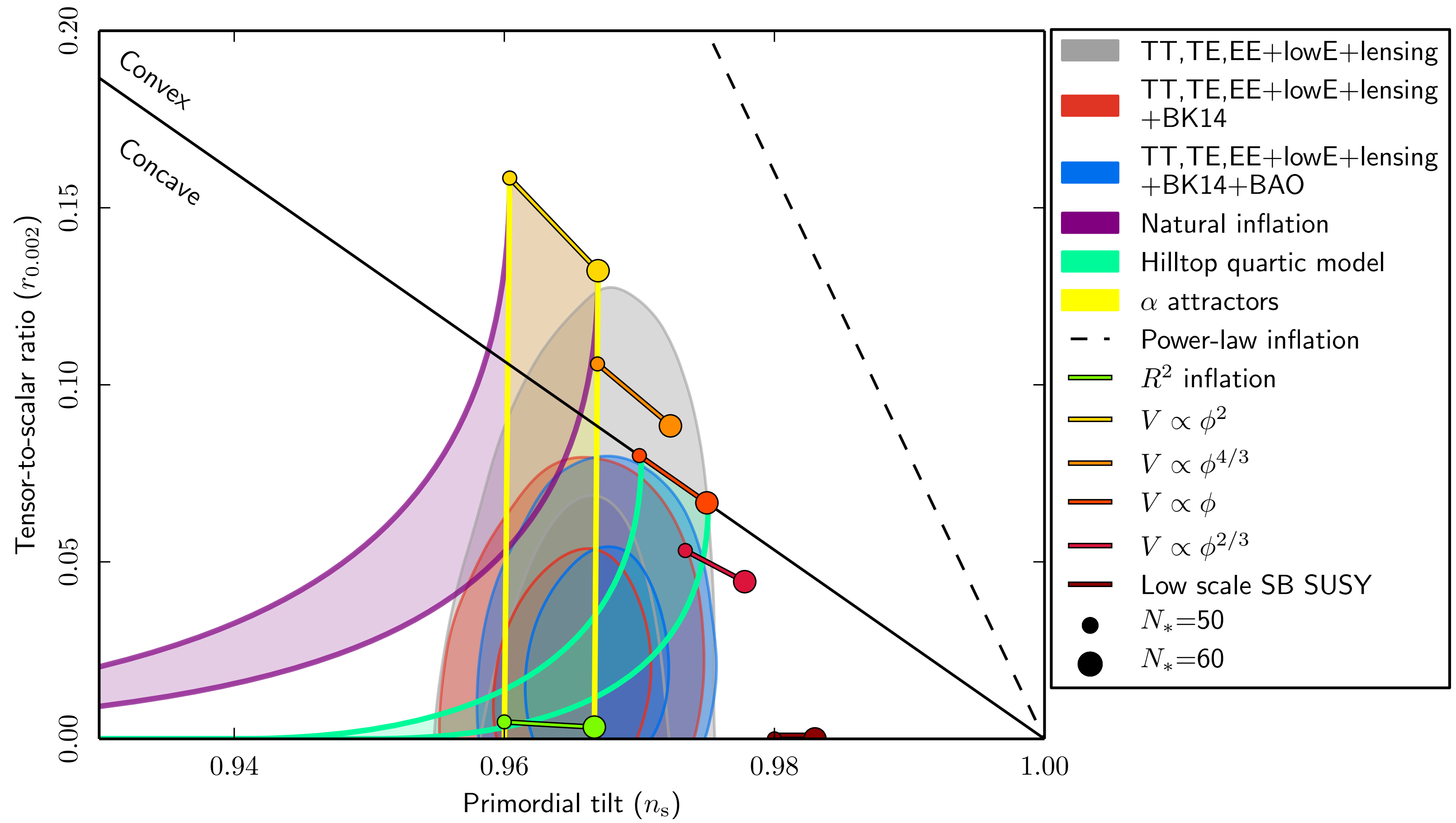
Reheating tends to be efficient → predicts large N_e.

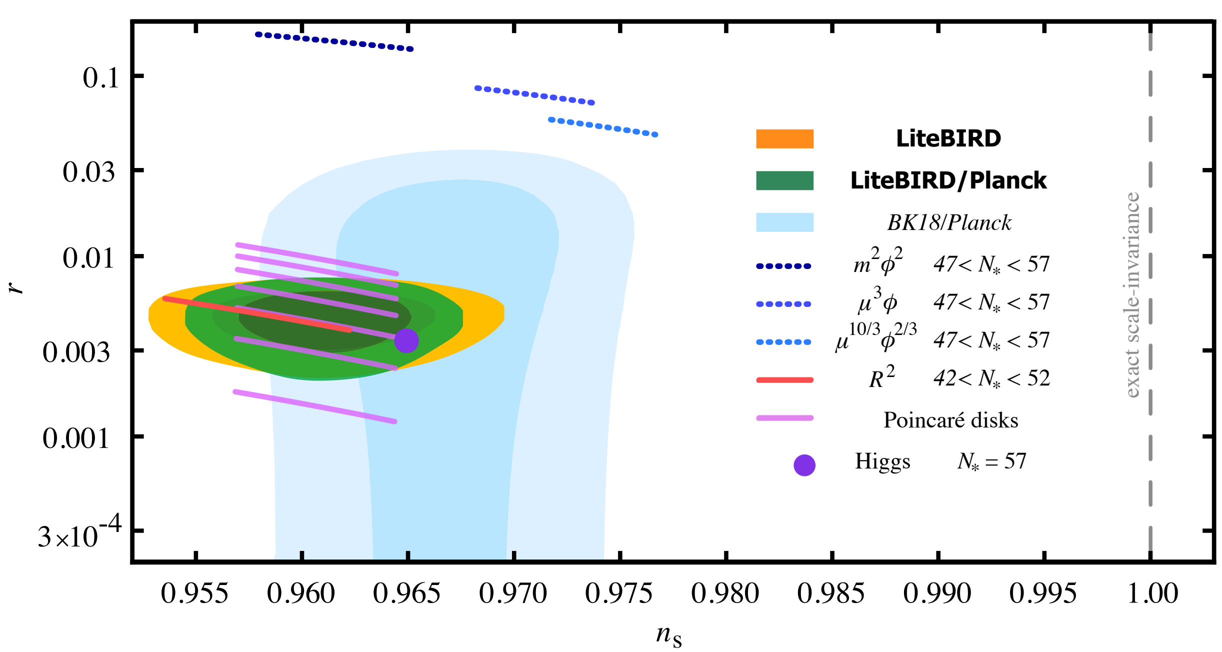
$4\alpha = (1.060, 1.214, 1.519, 1.556, 1.708, 1.745) \times 10^9$: not very "Higgs-like" parameter region.

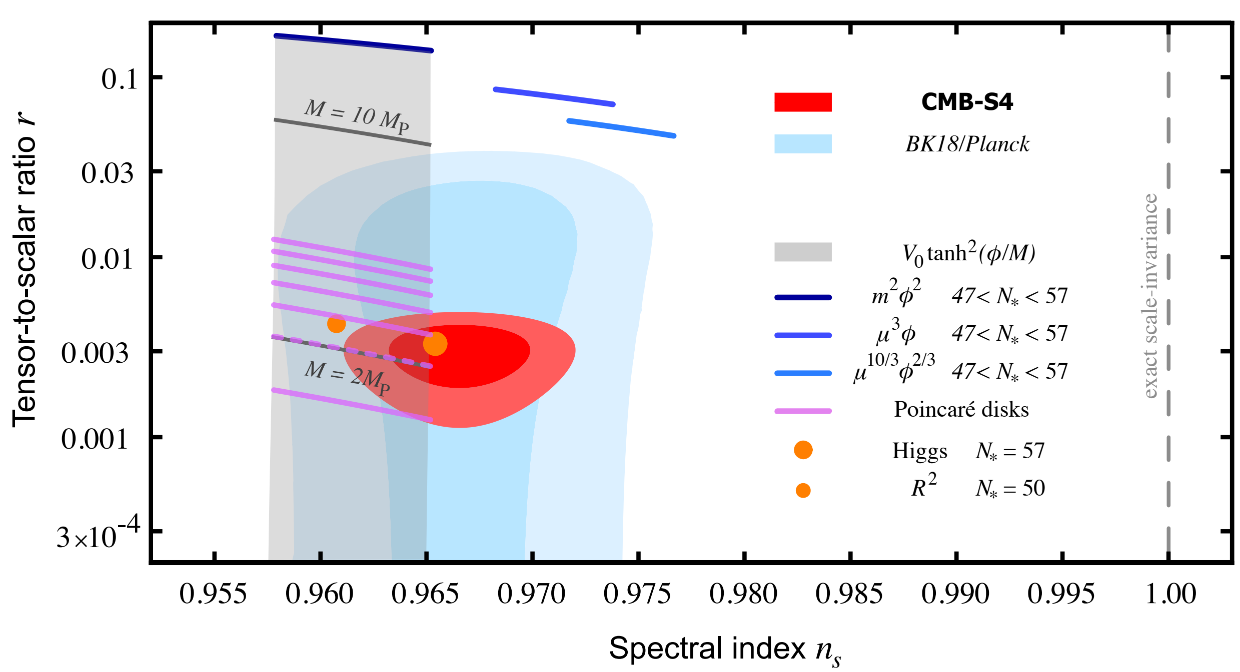


[Figure from Bezrukov, Shephered 20]

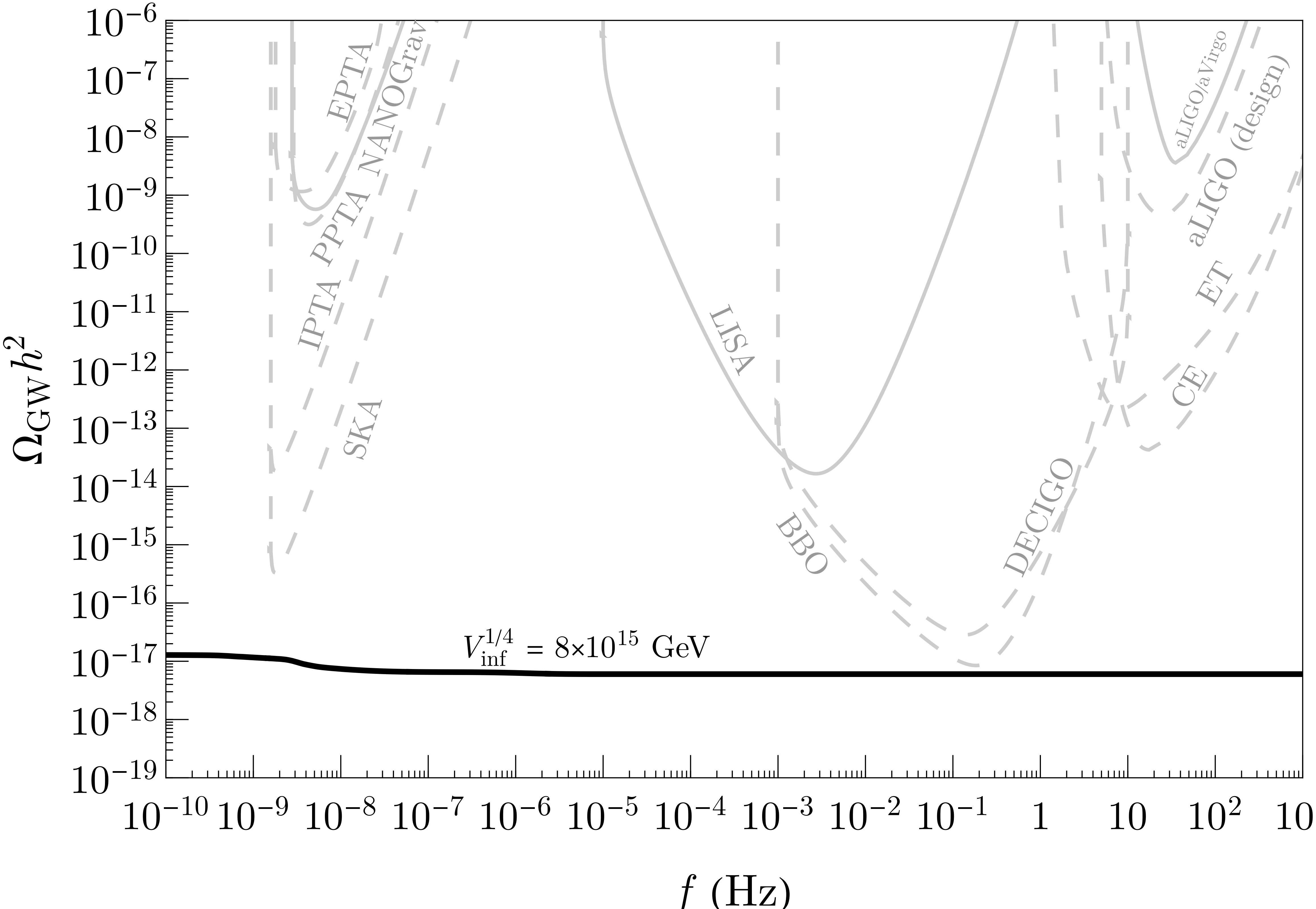
Other inflation models





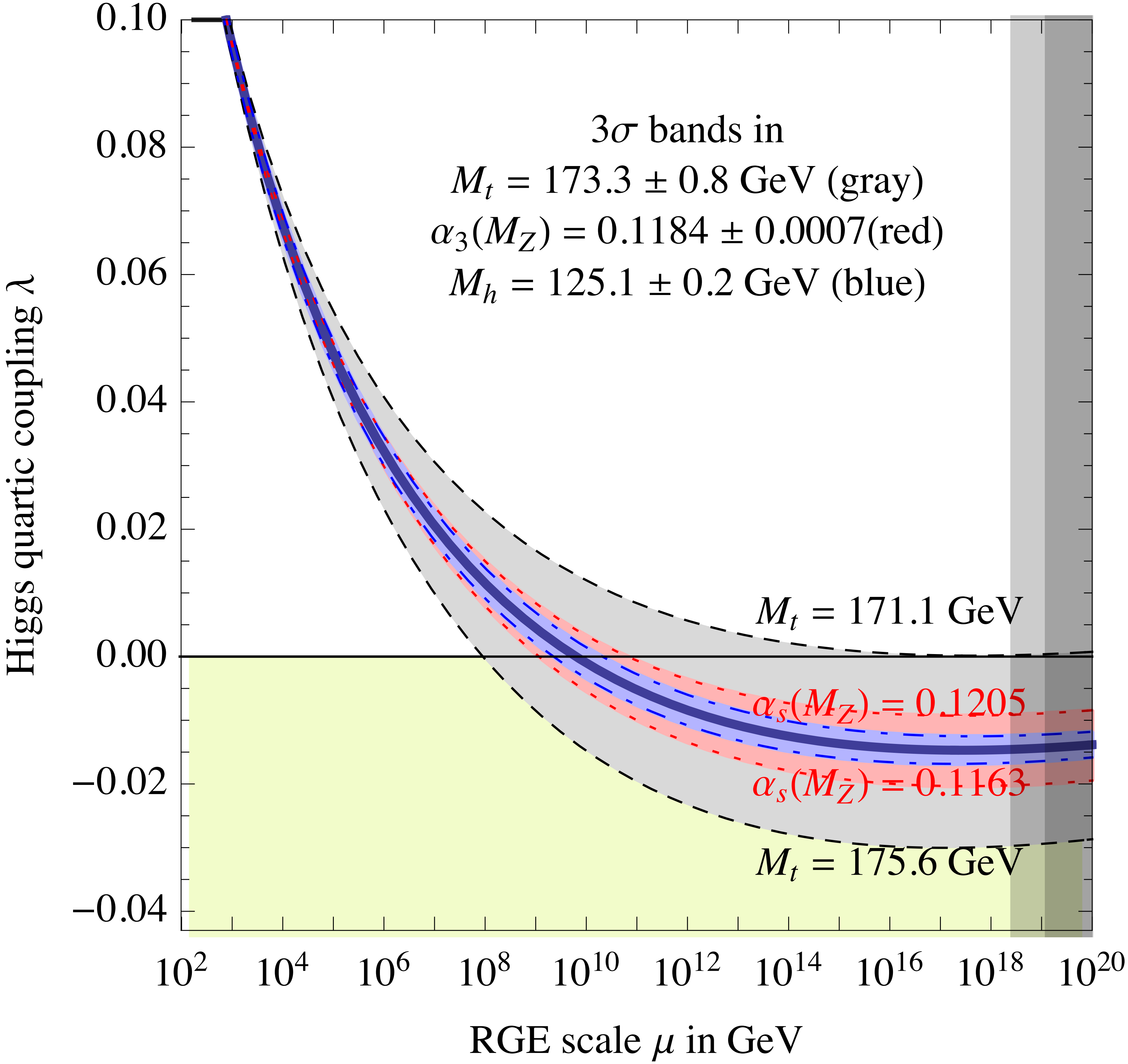


Gravitational wave experiments



[Figure credit: Raymond Co]

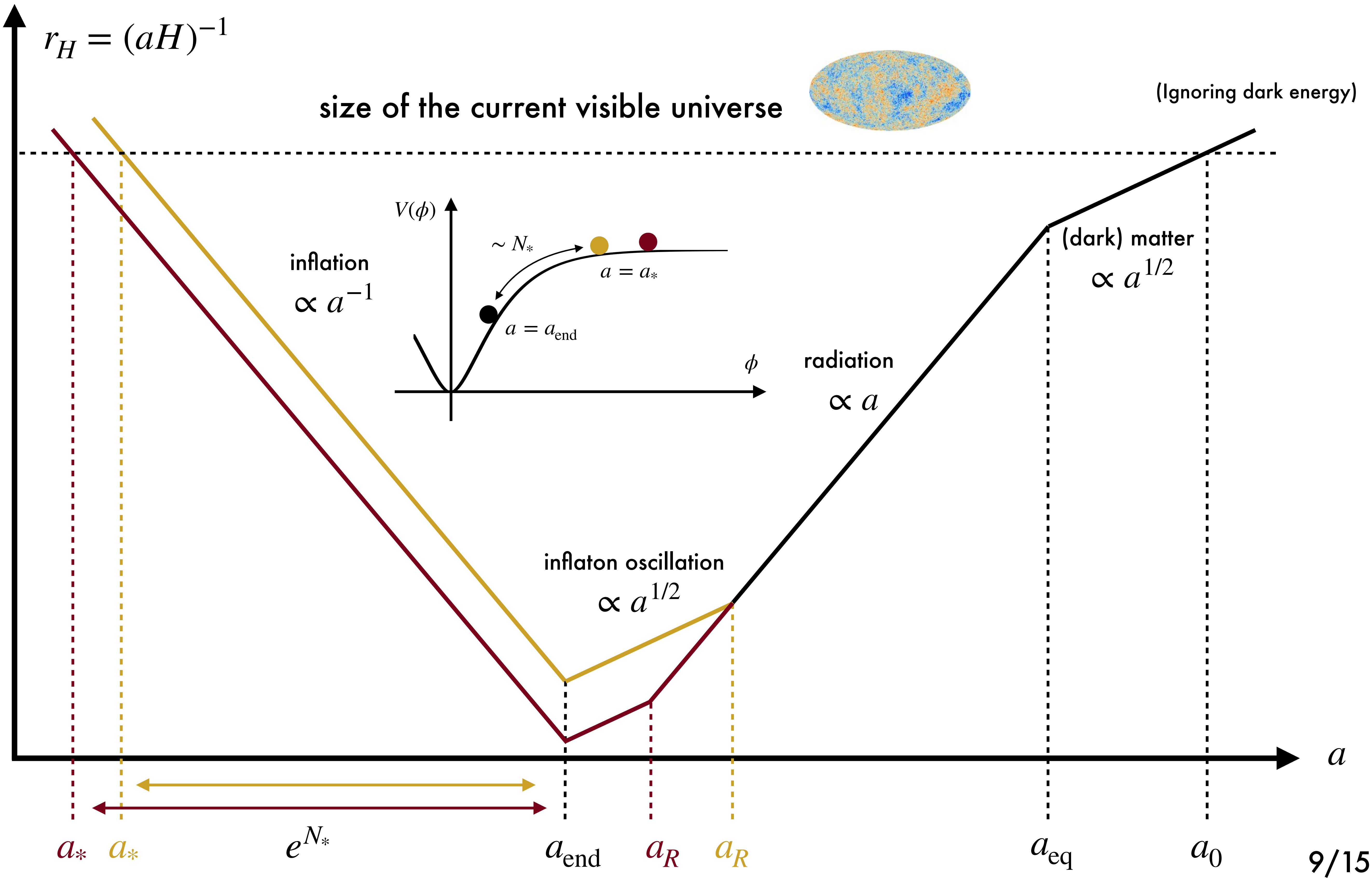
Running of Higgs quartic coupling



[Buttazzo+ 13]

Reheating

Cosmological perturbation depends on reheating temperature T_R through N_* .



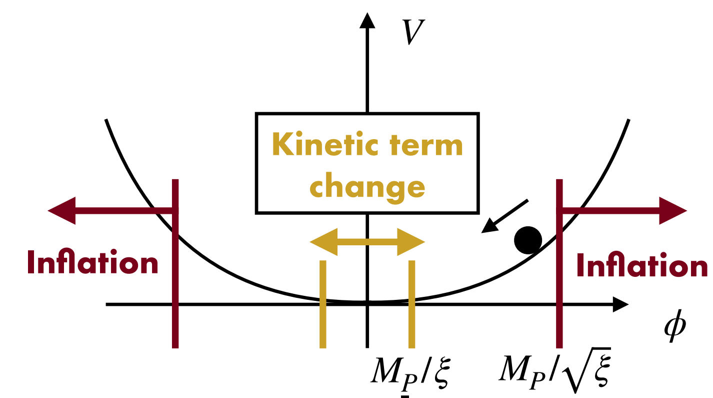
Spiky oscillation after inflation

- Higgs fields have a non-trivial target space in Einstein frame:

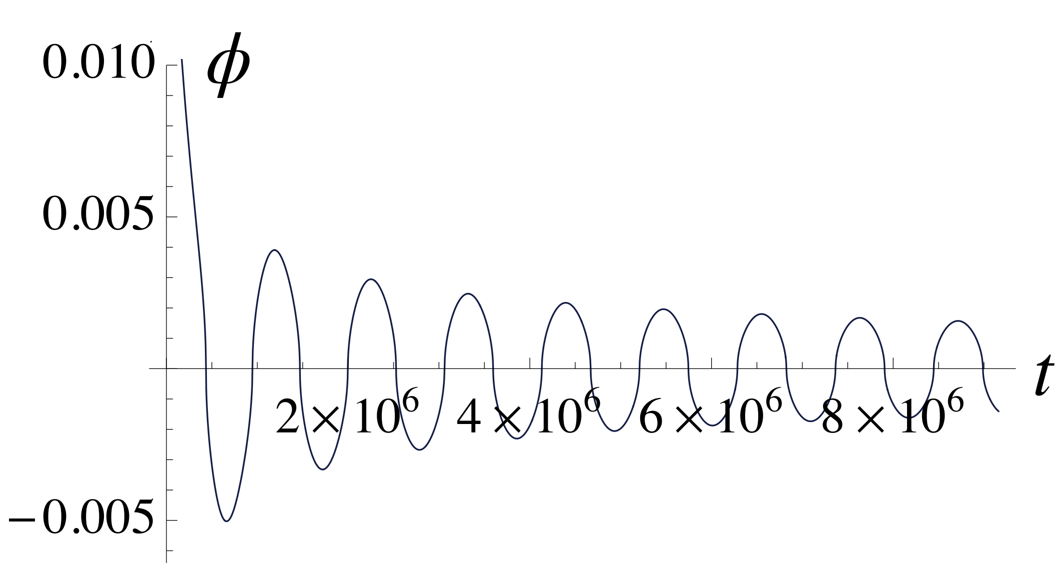
$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \frac{1}{2}h_{ab}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

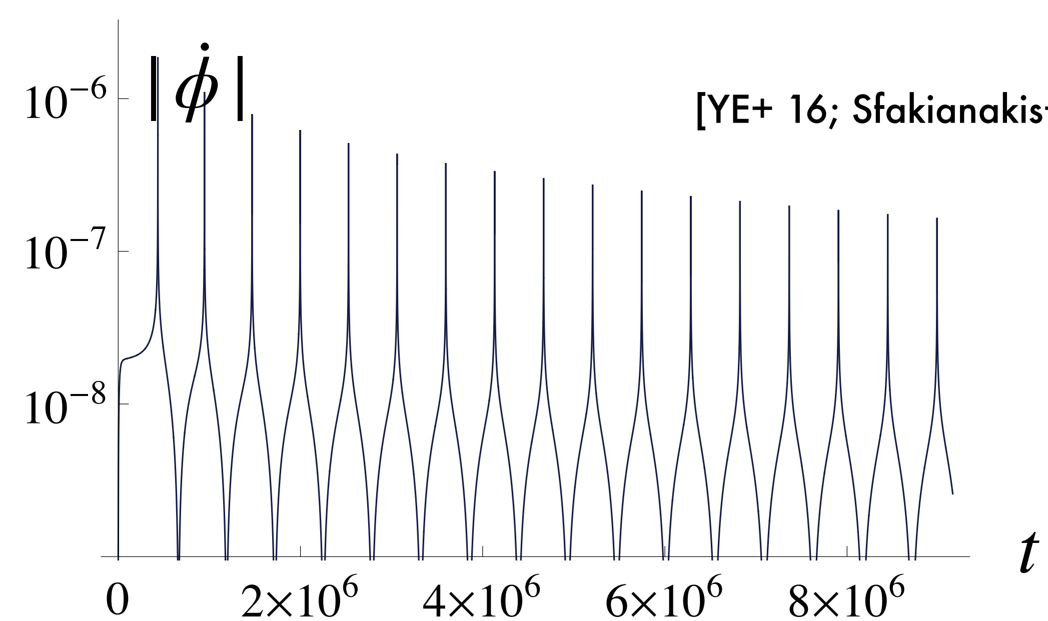
with $h_{ab} = \dfrac{1}{\Omega^4}\begin{pmatrix} \Omega^2 + \boxed{\dfrac{6\xi^2\phi^2}{M_P^2}} & \dfrac{6\xi^2\phi\chi}{M_P^2} \\ \dfrac{6\xi^2\phi\chi}{M_P^2} & \Omega^2 + \dfrac{6\xi^2\chi^2}{M_P^2} \end{pmatrix}$ with χ : NG mode(s) and $\Omega^2 = 1 + \xi\dfrac{\phi^2+\chi^2}{M_P^2}$ for HI.

- Kinetic term drastically changes for $|\phi| \lesssim M_P/\xi$,



➡ a "spiky" feature for $|\phi| \lesssim M_P/\xi$, causing unitarity violation.





[YE+ 16; Sfakianakis+ 18; Nguyen+ 19]

Target space and unitarity

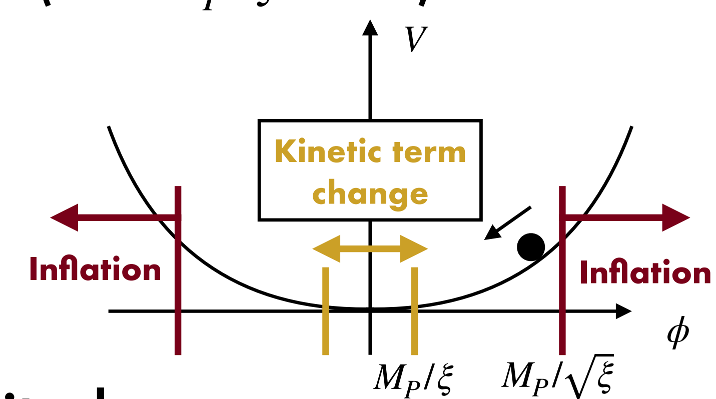
An easy-to-use condition of unitarity violation from target space

[YE, Jinno, Nakayama, van de Vis 21]

- NG boson has mass from target space curvature → feels spikes:

$$m_\chi^2 = \nabla^\chi V_\chi - \dot{\phi}^2 R^\chi{}_{\phi\phi\chi}\,, \qquad \text{e.g.} \left(1 + \frac{\chi^2}{\Lambda^2}\right)(\partial\phi)^2 \to m_\chi^2 = -\frac{\dot{\phi}^2}{\Lambda^2}.$$

- Inflaton motion changes for $|\phi| \lesssim \Lambda$ with curvature $R[h] \sim \Lambda^{-2}$ ($\Lambda \sim M_P/\xi$ for HI).



➡ typical momentum scale: $k_{\rm spike} \sim (\Lambda/\dot{\phi}_{\rm origin})^{-1}$.

- Cut-off also $\sim \Lambda$ since the curvature affects e.g. scattering amplitudes.

- With energy cons. $\dot{\phi}^2_{\rm origin} \sim V_{\rm inf}$, unitarity violation $k_{\rm spike} \gtrsim \Lambda$ translates to

$\boxed{V_{\rm inf} \gtrsim \Lambda^4\text{: simply compare inflation energy scale and cut-off.}}$

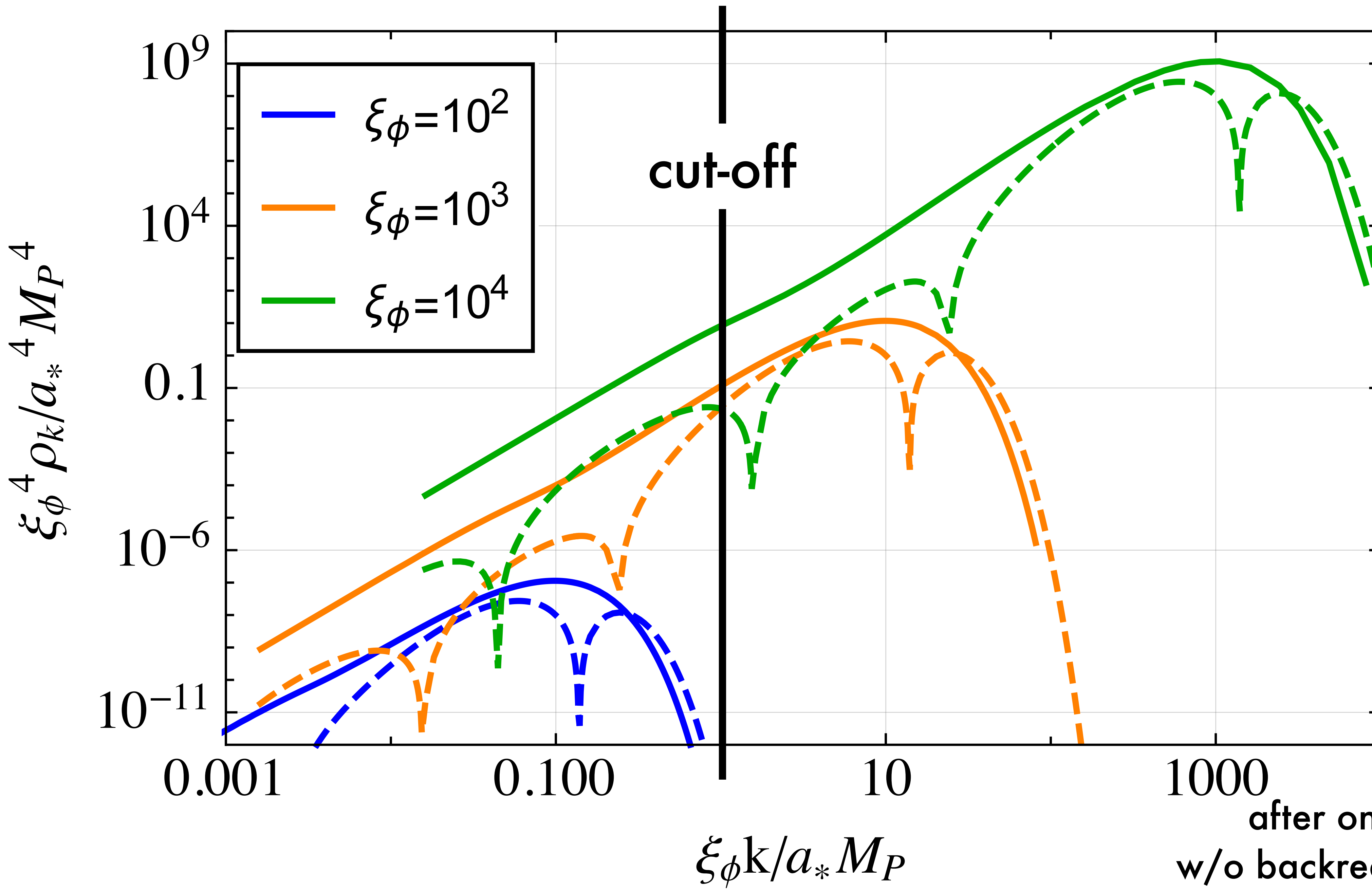
e.g. $V_{\rm inf}/\Lambda^4 \sim \lambda\xi^2 \sim 10^{-9}\xi^4$ for HI → unitarity violation for $\xi \gtrsim 10^2$.

- Applicable to other inflation models (can see e.g. running kinetic inflation violates unitarity).

Numerical result

Numerical result confirms the previous estimation.

[YE, Jinno, Nakayama, van de Vis 21]



after one oscillation,
w/o backreaction to inflaton.

$\rho_\chi \sim k_{\rm spike}^4 \sim V_{\rm inf}^2/\Lambda^4 > V_{\rm inf}$ for $V_{\rm inf} > \Lambda^4$ → this production is fatal.

Linear σ-model

[YE, Mukaida, van de Vis 20]

$$\mathscr{L} = \frac{M_P^2}{2}\left(1 + \frac{\xi\phi_i^2}{M_P^2}\right)R + \alpha R^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4 .$$

Auxiliary field introduction + Weyl transformation

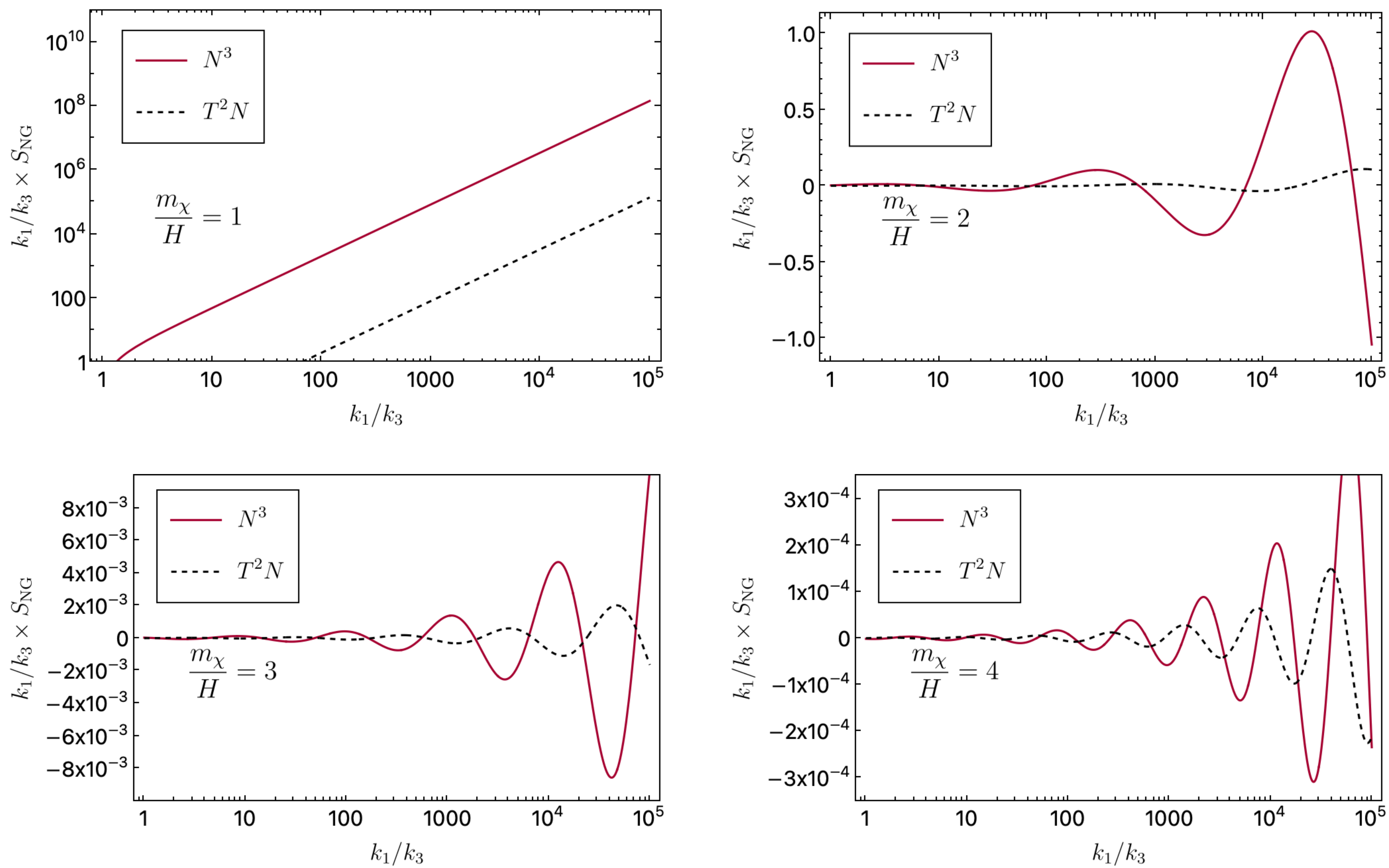
$$\mathscr{L} = \underline{\frac{M_P^2}{2}\left(1 - \frac{\sigma^2}{6M_P^2} - \frac{\phi_i^2}{6M_P^2}\right)R + \frac{1}{2}\left[\left(\partial\phi_i\right)^2 + (\partial\sigma)^2\right]} - \frac{\lambda}{4}\phi_i^4 - \frac{1}{144\alpha}\left[\frac{3M_P^2}{2} - \left(\sigma + \sqrt{\frac{3}{2}}M_P\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2 .$$

Flat kinetic term in the conformal frame!

$\mathrm{O}(N)$ NLSM	Higgs inflation
pions π_i	Higgs fields ϕ_i, conformal mode of metric Φ
target space: $\pi_i^2 + h^2 = v^2, \quad \left(\pi_i, h\right) \in \mathbb{R}^{(N+1)}$	target space: $\frac{6\xi+1}{2}\phi_i^2 + \left(h + \frac{\Phi}{2}\right)^2 = \frac{\Phi^2}{4}, \quad \left(\Phi, \phi_i, h\right) \in \mathbb{R}^{(1,N+1)}$
sigma meson σ	scalaron σ

Cosmo collider signatures

[YE, Verner 23]



Renormalizability of LSM

[YE, Mukaida, van de Vis 20]

- The LSM with the Higgs mass and the cosmological constant is renormalizable.

(= renormalizability of (spin-0 part of) quadratic gravity)

One can compute the RGEs without any ambiguity!

1-loop:

$$\beta^{(1)}_{g_1} = \frac{41}{10} g_1^3, \quad \beta^{(1)}_{g_2} = -\frac{19}{6} g_2^3, \quad \beta^{(1)}_{g_3} = -7 g_3^3,$$

$$\beta^{(1)}_{y_t} = y_t \left[\frac{9 y_t^2}{2} - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2 \right],$$

$$\beta^{(1)}_{\lambda} = \left(8\bar{\xi}^2 - 8\bar{\xi} + 2\right) \bar{\xi}^2 \lambda_\alpha^2 + 24 \bar{\xi}^2 \lambda \lambda_\alpha + 24 \lambda^2 - 6 y_t^4 + \frac{27 g_1^4}{200} + \frac{9 g_2^4}{8} + \frac{9}{20} g_1^2 g_2^2 + \left[12 y_t^2 - \frac{9 g_1^2}{5} - 9 g_2^2 \right] \lambda,$$

$$\beta^{(1)}_{\lambda_m} = 2\bar{\xi}\left(2\bar{\xi} - 1\right) \lambda_\alpha^2 - 8 \bar{\xi} \lambda_m^2 + \lambda_m \left[4 \bar{\xi}^2 \lambda_\alpha + 8 \bar{\xi} \lambda_\alpha - 3 \lambda_\alpha + 12 \lambda + 6 y_t^2 - \frac{9 g_1^2}{10} - \frac{9 g_2^2}{2} \right],$$

$$\beta^{(1)}_{\bar{\xi}} = \bar{\xi} \left[\left(4\bar{\xi}^2 + 4\bar{\xi} - 3\right) \lambda_\alpha + 12 \lambda + 6 y_t^2 - \frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 \right],$$

$$\beta^{(1)}_{\lambda_\alpha} = \left(8\bar{\xi}^2 + 5\right) \lambda_\alpha^2,$$

$$\beta^{(1)}_{\lambda_\Lambda} = \frac{\lambda_\alpha^2}{2} - 2 \lambda_\alpha \lambda_\Lambda - 16 \bar{\xi} \lambda_\Lambda \lambda_m + 2 \lambda_m^2.$$

* See 2008.01096 for an explicit form up to 2-loop.

- The Higgs mass and the CC are naturally at the scalaron mass scale = hierarchy problem.

They do not affect inflationary dynamics, but (p)reheating??

- EW scale parameters can be related to inflationary scale parameters (with ξ and α).

Spin-two sector

- Vacuum pol. diagrams contain divergences.

  Renormalized by $\mathscr{L}_{\text{c.t.}} = \alpha R^2 + \alpha_2 \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right).$

  * We have no choice but including these terms.

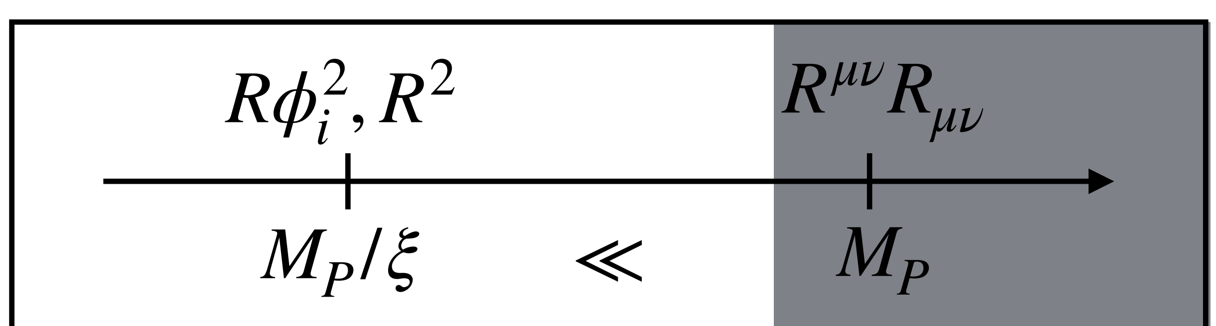
- Renormalization group equations:

$$\beta_\alpha \equiv \frac{d\alpha}{d \ln \mu} = - \frac{N_s}{1152\pi^2} \left(1 + 6\xi\right)^2, \quad \beta_{\alpha_2} \equiv \frac{d\alpha_2}{d \ln \mu} = - \frac{N_s}{960\pi^2}.$$

  The hierarchy $\alpha \sim \mathcal{O}(\xi^2) \gg \alpha_2 \sim \mathcal{O}(1)$ naturally exits.

- Alternatively, the coupling for the spin-2 is suppressed:

  $T_{\mu\nu} \ni \xi \left(\partial_\mu \partial_\nu - \eta_{\mu\nu} \Box \right) \phi_i^2$ ⇨ $h^{\perp}_{\mu\nu} T^{\mu\nu}$: independent of ξ.



Frame-independent target space

[YE, Mukaida, van de Vis 20]

- Naive definition solely by scalar fields is frame-dependent.

$$\begin{cases} \mathscr{L}_J = \frac{M_P^2}{2}\Omega^2 R + \underline{\frac{1}{2}\left(\partial\phi_i\right)^2} + \cdots, \quad \Omega^2 = 1 + \frac{\xi\phi_i^2}{M_P^2}, \\ \mathscr{L}_E = \frac{M_P^2}{2} R + \underline{\frac{1}{2\Omega^4}\left(\Omega^2\delta_{ij} + \frac{6\xi^2\phi_i\phi_j}{M_P^2}\right)\partial\phi_i\partial\phi_j} + \cdots . \end{cases}$$

Physics is frame-independent → a frame-independent definition is desirable.

- Frame-independent definition by including the conformal mode.

Metric decomposition: $g_{\mu\nu} = e^{2\varphi}\tilde{g}_{\mu\nu}, \;\; \mathrm{Det}\left[\tilde{g}_{\mu\nu}\right] = -1.$

$\Phi = \sqrt{6}M_P e^{\varphi}$: conformal mode.

Target space defined by $\left(\phi_i, \Phi\right)$: frame-independent!

∵ Weyl transformation = redefinition of Φ = coordinate transf. of target space.

Higgs inflation as NLSM

[YE, Mukaida, van de Vis 20]

- Focus on the conformal mode of the metric as $g_{\mu\nu} = e^{2\varphi}\eta_{\mu\nu}$.

$$S = \int d^4x \left[-\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{6\xi+1}{2}\left(\frac{\Box\Phi}{\Phi}\right)\phi_i^2 - \frac{\lambda}{4}\phi_i^4 \right].$$

- Can be simplified by field redefinitions as

$$S = \int d^4x \left[-\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}\left(\partial h\right)^2 - \frac{\lambda}{4}\phi_i^4 \right],$$

where $h\left(\Phi, \phi_i\right) = \frac{1}{2}\left[\sqrt{\Phi^2 - 2\left(6\xi+1\right)\phi_i^2} - \Phi\right]$.

Interpreted frame-independently as NLSM.

- Φ is ghost-like but harmless.

* Similar to A_0 of $\mathrm{U}(1)$ gauge boson in the Lorentz gauge $\partial_\mu A^\mu = 0$:

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{2}\eta^{\alpha\beta}\partial^\mu A_\alpha \partial_\mu A_\beta = -\frac{1}{2}\left(\partial A_0\right)^2 + \frac{1}{2}\left(\partial A_i\right)^2.$$

Scalaron as σ-meson

- Higgs inflation as NLSM: [YE, Mukaida, van de Vis 20]

$$\mathscr{L}_{\rm NLSM} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}\left(\partial h\right)^2 - \frac{\lambda}{4}\phi_i^4, \quad h = \frac{1}{2}\left[\sqrt{\Phi^2 - 2\left(6\xi+1\right)\phi_i^2} - \Phi\right].$$

Naturally imply σ-meson that linearizes the NLSM:

$$\mathscr{L}_{\rm LSM} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{144\alpha}\left[\frac{\Phi^2}{4} - \left(\sigma + \frac{\Phi}{2}\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2 - \frac{\lambda}{4}\phi_i^4.$$

- It is identified as the scalaron:

$$\mathscr{L} = \frac{M_P^2}{2}\left(1 + \frac{\xi\phi_i^2}{M_P^2}\right)R + \alpha R^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4.$$

Auxiliary field introduction + Weyl transformation

$$\mathscr{L} = \underbrace{\frac{M_P^2}{2}\left(1 - \frac{\sigma^2}{6M_P^2} - \frac{\phi_i^2}{6M_P^2}\right)R + \frac{1}{2}\left[\left(\partial\phi_i\right)^2 + (\partial\sigma)^2\right]}_{\text{Flat kinetic term in the conformal frame!}} - \frac{\lambda}{4}\phi_i^4 - \frac{1}{144\alpha}\left[\frac{3M_P^2}{2} - \left(\sigma + \sqrt{\frac{3}{2}}M_P\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2.$$

$g_{\mu\nu} = e^{2\varphi}\eta_{\mu\nu}$ + rescaling fields

$$\mathscr{L} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{144\alpha}\left[\frac{\Phi^2}{4} - \left(\sigma + \frac{\Phi}{2}\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2 - \frac{\lambda}{4}\phi_i^4.$$

* Remember this identification is frame-independent.

Why no ghost with R^2?

- Higher derivative in general involves ghost:

$$\pm\frac{1}{p^2(1-p^2/M^2)}=\pm\left(\frac{1}{p^2}-\frac{1}{p^2-M^2}\right).$$

two choices $\begin{cases} + : \text{high energy additional pole is ghost-like,} \\ - : \text{low energy pole is ghost-like.} \end{cases}$

- In the case of R^2, it is the low-energy pole that is ghost

This mode is unphysical, thanks to gauge (BRST) symmetry.

c.f. photon propagator in Feynman gauge: $-\frac{\eta_{\mu\nu}}{p^2}=\begin{cases} -\frac{1}{p^2} & (\mu,\nu)=(0,0), \\ +\frac{1}{p^2} & (\mu,\nu)=(i,i)\,. \end{cases}$

- In the case of $R_{\mu\nu}R^{\mu\nu}$, low-energy pole is physical ⇒ spin-2 ghost problem.