Asymptotically free scaling solutions in nonabelian Higgs models

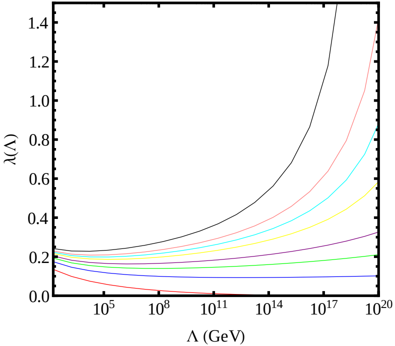
L.Zambelli (Jena, TPI)

talk based on: Gies, LZ '15
and previous work: Gies, Rechenberger, Scherer, LZ '13

SCALARS 2015, Warsaw, December 6, 2015

Invitation

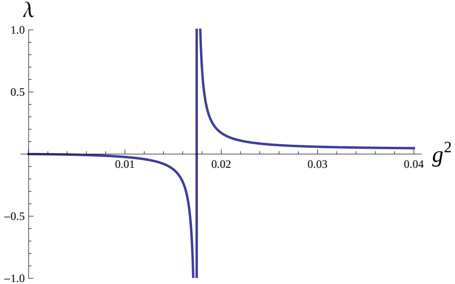
Light Higgs = almost vanishing self-interaction to high scales
(see talks by Lindner, Haba, Nielsen)

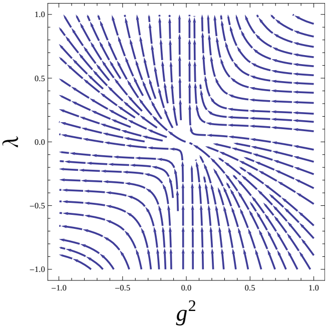


from: (Holthausen, Lim, Lindner '12)

"near-criticality"
(Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia '13)

Landau Pole at One-Loop





$$\beta_{g^2} = -b_0 g^4 \ , \quad \beta_\lambda = A\lambda^2 + B'\lambda g^2 + Cg^4$$

$$\lambda(g^2) = -\frac{g^2}{2A}\left\{B + \sqrt{\Delta}\tanh\left[\frac{\sqrt{\Delta}}{2b_0}\left(c - \log(g^2)\right)\right]\right\}$$

where

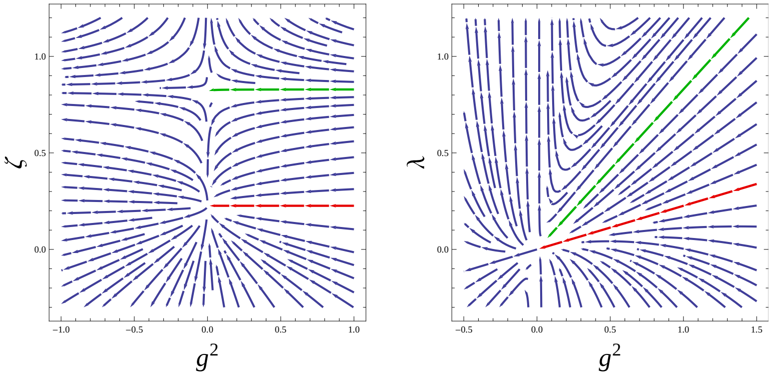
$$B = B' + b_0 \quad , \quad \Delta = B^2 - 4AC < 0$$

Total Asymptotic Freedom (TAF)

TAF condition (Higgs+Gauge):

if $\Delta > 0$, then UV asymptotics = FPs (Gross, Wilczek '73)

$$\beta_\zeta = g^2(A\zeta^2 + B\zeta + C) = 0, \quad \zeta = \frac{\lambda}{g^2}$$



Total Asymptotic Freedom (TAF)

TAF condition (Higgs+Gauge):

if $\Delta > 0$, then UV asymptotics = FPs (Gross, Wilczek '73)

$$\beta_\zeta = g^2(A\zeta^2 + B\zeta + C) = 0, \quad \zeta = \frac{\lambda}{g^2}$$

e.g.: holds for $SU(N \gg 1)$ and $b_0 \approx 0$ (many fermions)

TAF Models Classified: constrains on matter and symmetries

(Cheng, Eichten, Li '74) (Chang '74) (Fradkin, Kalashnikov '75)
(Chang, Perez-Mercader '78) (Bais, Weldon '78) (Callaway '88)
(Giudice, Isidori, Salvio, Strumia, '15) (Holdom, Ren, Zhang '15)

no TAF in the SM, guiding principle for BSM

Total Asymptotic Freedom (TAF)

How close can TAF be to the SM? Very close! Proposal:

TAF is possible already in the generic nonabelian Higgs model

How so? Where is the loophole?

TAF is realized OUT of the Deep Euclidean Limit

The vev v^2 scales like the RG scale k^2 in the UV

Functional RG (FRG) & Thresholds

v^2/k^2 never negligible, possibly diverging $\rightarrow$ strong threshold effects

Included by means of the FRG (mass-dependent scheme)
(Wilson '71, Wegner&Houghton '73, Polchinski '84, Wetterich '91)

Let us focus on SU(2)+fundamental scalar

$$\mu_W^2 = \frac{g^2\kappa}{2} \qquad \mu_{\mathrm{H}}^2 = 2\lambda\kappa$$

ϕ^4 1-loop

$$\beta_\lambda = \frac{1}{16\pi^2}\left\{12\lambda^2 - 9\lambda g^2 + \frac{9}{4}g^4\right\}$$

ϕ^4 FRG @NLO-derivative expansion

$$\beta_\lambda = \frac{1}{16\pi^2}\left\{12\lambda^2\left(\frac{1}{4} + \frac{3}{4(1+\mu_{\mathrm{H}}^2)^3}\right)\right.$$

$$\left. -9\lambda g^2\left(\frac{1}{2(1+\mu_W^2)} + \frac{1}{2(1+\mu_W^2)^2}\right) + \frac{9}{4}\frac{g^4}{(1+\mu_W^2)^3} + \cdots\right\}$$

Functional RG (FRG) & Higher-Dimensional Operators

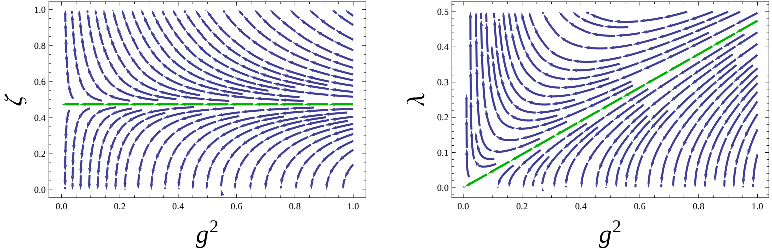
Local Effective Potential (full function) included

$$U = \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 + \frac{\lambda_3}{6k^2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^3 + \ldots .$$

to lowest order

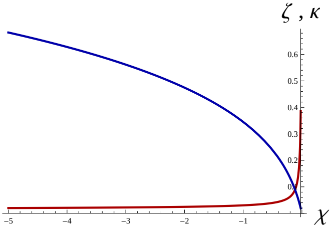
$$\beta_\zeta = g^2\left(A\zeta^2 + B\zeta + C\right) - \frac{1}{g^2}\left(\frac{\lambda_3}{16\pi^2} + \frac{9\lambda_3}{64\pi^2\zeta}\right) + \ldots$$

Positive-ζ FPs if $\lambda_3/g^4 = \chi$ is kept free and nonvanishing



Line of FPs

$\lambda/g^2 = \zeta, \quad \lambda_3/g^4 = \chi, \quad v^2/k^2 = 2\kappa$ finite and nonvanishing



Higher-dimensional operators suppressed by higher powers of g^2

Stable potential: $x = (gZ_\phi/k^2)\ \phi^\dagger\phi$

$$U \sim \frac{\zeta_*}{2}\ x^2 - \frac{3(3+4\zeta_*)}{128\pi^2} g\ x + O(g^2)$$

One free parameter (here ζ_*) = boundary condition of 1^{st}-order ODE

Generalization: $\lambda \sim g^{4P}$

Back to

$$\beta_\lambda = A\lambda^2 + B'\lambda g^2 + Cg^4 - \left(\frac{\lambda_3}{16\pi^2} + \frac{9\lambda_3 g^2}{64\pi^2\lambda}\right) + \ldots$$

look for scaling solutions: $\lambda = g^{4P}\zeta$

if $0 < P < 1/2$ and $\lambda_3 = g^{8P}\chi$ then

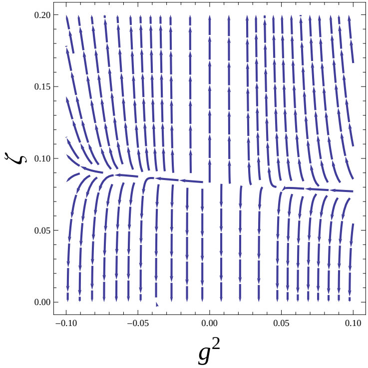
$$\beta_\zeta = \left(A\zeta^2 - \frac{\chi}{16\pi^2}\right) g^{4P} + O(g^2)$$

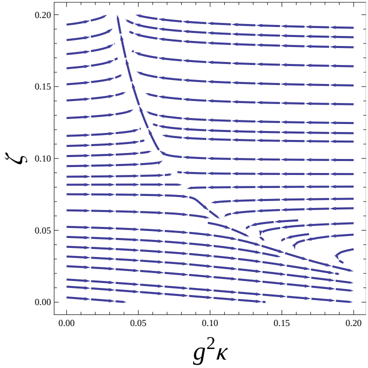
Positive-ζ FPs if χ is kept free and nonvanishing

Generalization: $\lambda \sim g^{4P}$

$P > 1/2$ is also possible but requires $v^2/k^2 = 2\kappa \to +\infty$ in the UV

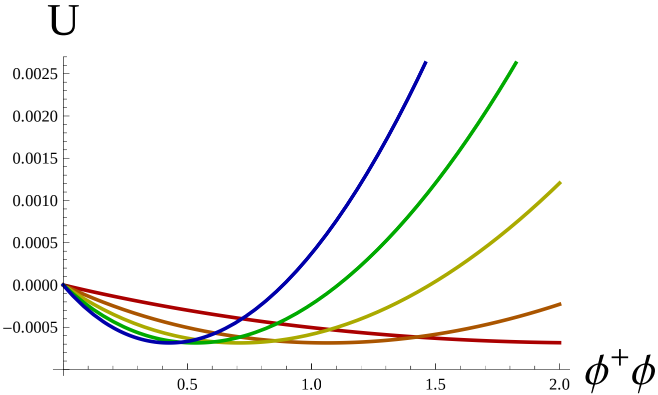
$$P = 1$$





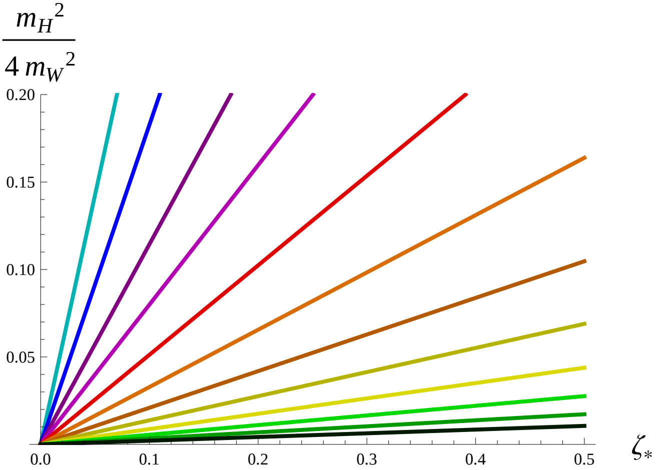
Generalization: $\lambda \sim g^{4P}$

$P > 1/2$ is also possible but requires $v^2/k^2 = 2\kappa \to +\infty$ in the UV



Higgs Phase and Masses

Every TAF scaling solution can be connected to the Higgs phase



Conclusions

- Nonabelian Higgs model allows for Total Asymptotic Freedom (TAF)
- TAF is compatible with any wanted Higgs/W mass ratio
- Small number of free UV parameters (g^2, v^2, λ, P)
- Infinitely many higher-dimensional operators predicted

Backup: Functional RG

1PI Functional RG (Wetterich '91,Morris '94)

1) Regularization: IR mass-like regulator $R_k(p^2)$

2) Average effective action Γ_k

$$k = 0 \xleftarrow{\quad\quad\quad\quad k \quad\quad\quad\quad} k = \Lambda$$

1PI effective action | bare action

3) Flow equation $\partial_t \equiv k\partial_k$

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_t R_k\right]$$

4) Truncation

$$\Gamma_k[\Phi] = \sum_i \bar{G}_i(k)\mathcal{O}_i[\Phi] \;\longrightarrow\; \partial_t \bar{G}_i(k) = \bar{\beta}_i(\bar{G}_j, k)$$

e.g. derivative expansion $\mathcal{L}_k = U_k(\Phi) + Z_\Phi \partial_\mu \Phi \partial^\mu \Phi + \cdots$

5) Rescalings

$$G_i(t) \equiv \frac{k^{-D_i}}{Z_\phi^{n_\phi/2} \cdots Z_W^{n_W/2}} \bar{G}_i(k) \;\longrightarrow\; \partial_t G_i = \beta_i(G_j)$$