Implications of Symmetries in the Scalar Sector

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Scalars 2019

Warsaw, 11 September 2019

Work done in collaboration with D. Emmanuel-Costa, Howard E. Haber, Anton Kuncinas, O. M. Ogreid and P. Osland,
arXiv:1601.04654, DOI: 10.1007/JHEP08(2016)169, 10.1007/JHEP02(2016)154,
arXiv:1701.0476, DOI: 10.1007/JHEP08(2017)005,
arXiv:1808.08629 DOI: 10.1007/JHEP01(2019)042
Kuncinas, Master Thesis, U. Bergen 2019:
http://bora.uib.no/handle/1956/20467

Work Partially supported by:

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

europa.eu
European Union

QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL
PORTUGAL 2007.2013

COMPETE
PROGRAMA OPERACIONAL FACTORES DE COMPETITIVIDADE

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Symmetries play an important rôle in multi-Higgs models

- reduction of the number of free parameters
- experimental predictions
- symmetries help to control HFCNC

Connections can be established between Symmetries and:

- mass degeneracies in the scalar sector
- existence of massless scalars
- CP violation in the scalar sector

Symmetries of the 2 Higgs Doublet Model

$$\mathcal{V} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)$$
$$+\lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} . \qquad (2.1)$$

11 independent parameters

If all the parameters are real CP is explicitly conserved:

most general CP transformation $\quad \Phi_i \xrightarrow{\text{CP}} U_{ij} \Phi_j^*$

with U a unitary matrix which we can choose as the identity matrix when all parameters are real

However, there is still the possibility of Spontaneous Symmetry Breaking

T. D. Lee 1973

The above equation together with the assumption that the vacuum is CP invariant leads to

$U_{ij} \langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle \qquad \mathcal{L}(U\phi) = \mathcal{L}(\phi) \qquad \text{CP}|0\rangle = |0\rangle$

G. C. Branco, J. M. Gerard and W. Grimus 1984

CP is violated spontaneously by vevs of the form $(\rho_1 e^{i\theta}, \rho_2)$, in the region of parameters of the potential where ρ_1 and ρ_2 are different from zero and $e^{i\theta} \neq 1$

List of all possible Symmetries of the 2HDM

The complete list of such symmetries is known:

symmetry	transformation law		
$\mathbb{Z}_2$	$\Phi_1 \to \Phi_1$	$\Phi_2 \to -\Phi_2$	
$\mathrm{U}(1)$	$\Phi_1 \to \Phi_1$	$\Phi_2 \to e^{2i\theta}\Phi_2$	
$\mathrm{SO}(3)$	$\Phi_a \to U_{ab}\Phi_b$	$U \in \mathrm{U}(2)/\mathrm{U}(1)_\mathrm{Y}$	(for $a,\, b = 1, 2$)
GCP1	$\Phi_1 \to \Phi_1^*$	$\Phi_2 \to \Phi_2^*$	
GCP2	$\Phi_1 \to \Phi_2^*$	$\Phi_2 \to -\Phi_1^*$	
GCP3	$\Phi_1 \to \Phi_1^* \cos\theta + \Phi_2^* \sin\theta$	$\Phi_2 \to -\Phi_1^* \sin\theta + \Phi_2^* \cos\theta$	(for $0 < \theta < \frac{1}{2}\pi$)
Π_2	$\Phi_1 \to \Phi_2$	$\Phi_2 \to \Phi_1$	

Deshpande and Ma 1978, Ivanov 2007, Ferreira, Haber and Silva 2009, Ferreira, Haber, Maniatis, Nachtmann and Silva 2011, Battye, Brawn, Pilaftsis 2011, Pilaftsis 2011

There are three possible Higgs family symmetries (first three rows) and three classes of CP symmetries with different U matrices (next three rows)

There are seven additional accidental symmetries of the 2HDM scalar potential

Battye, Brawn, Pilaftsis 2011, Pilaftsis 2012

which are not exact symmetries since they are violated by the U(1) gauge kinetic term of the scalar potential, as well as by the Yukawa couplings, therefore, not considered here.

List of all possible Symmetries of the 2HDM (cont.)

Starting from a generic scalar potential given by Eq. (2.1) if the scalar potential respects one of the symmetries listed in Table 1, the coefficients of the scalar potential are constrained according to Table 2, in the basis where the symmetry is manifest

symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	$\mathrm{Re}\,\lambda_5$	$\mathrm{Im}\,\lambda_5$	λ_6	λ_7
$\mathbb{Z}_2$	-	-	0	-	-	-	-	-	-	0	0
U(1)	-	-	0	-	-	-	-	0	0	0	0
SO(3)	-	m_{11}^2	0	-	λ_1	-	$\lambda_1 - \lambda_3$	0	0	0	0
GCP1	-	-	real	-	-	-	-	-	0	real	real
GCP2	-	m_{11}^2	0	-	λ_1	-	-	-	-	-	$-\lambda_6$
GCP3	-	m_{11}^2	0	-	λ_1	-	-	$\lambda_1 - \lambda_3 - \lambda_4$	0	0	0
Π_2	-	m_{11}^2	real	-	λ_1	-	-	-	0	-	λ_6^*
$\mathbb{Z}_2 \oplus \Pi_2$	-	m_{11}^2	0	-	λ_1	-	-	-	0	0	0
$\mathrm{U}(1) \oplus \Pi_2$	-	m_{11}^2	0	-	λ_1	-	-	0	0	0	0

In all these cases the imposed symmetry leads to explicit CP conservation

In all cases GCP1, and also 2 and 3 there is invariance under hermitian conjugation

Ferreira, Haber and Silva 2009

Possibility of spontaneous CP violation with Z_2 softly broken

Branco and Rebelo 1985

Natural 2HDM mass degeneracies

Analysis of explicit expressions of the neutral scalar masses

or

Consider all possible symmetries of the 2HDM

Mass degenerate neutral scalars can only arise naturally in the 2HDM in the case of the IDM with Z_5 = 0

$$\mathcal{V} = Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \left[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \right] H_1^\dagger H_2 + \text{h.c.} \right\},$$

exact $\mathbb{Z}_2$ symmetry $H_1 \to +H_1$ and $H_2 \to -H_2$

$Y_3 = Z_6 = Z_7 = 0$ preserved by the vacuum

Physical scalar mass spectrum

$$m_h^2 = Z_1 v^2 , \qquad m_{H\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 v^2 ,$$

$$m_A^2 = m_{H\pm}^2 + \tfrac{1}{2}(Z_4 - Z_5) v^2 , \qquad m_H^2 = m_A^2 + Z_5 v^2 .$$

$$m_H = m_A, \text{ due to } Z_5 = 0.$$

Natural 2HDM mass degeneracies (cont.)

$Y_3 = Z_6 = Z_7 = 0$. **together with** $Z_5 = 0$

exact continuous unbroken U(1) symmetry $H_1 \to H_1 \qquad H_2 \to e^{i\theta} H_2$

It is this symmetry that is responsible for the mass degenerate states H and A

One can now define eigenstates of U(1) charge:

$$\phi^{\pm} = \frac{1}{\sqrt{2}}[H \pm iA]$$

Physical mass spectrum of the mass degenerate IDM:

$$m_h^2 = Z_1 v^2 \,,$$

$$m_{H^\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 v^2 \,,$$

$$m_{\phi^\pm}^2 = Y_2 + \tfrac{1}{2}(Z_3 + Z_4) v^2$$

Natural 2HDM mass degeneracies (cont.)

Although $\phi^\pm$ are mass degenerate states, they can be physically distinguished on an event by event basis.

The relevant interaction terms of $\phi^\pm$ are

$$\mathscr{L}_{\rm int} = \left[\tfrac{1}{2}g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{4c_W^2} Z_\mu Z^\mu\right]\phi^+\phi^- + \frac{ig}{2c_W} Z^\mu \phi^- \overleftrightarrow{\partial}_\mu\, \phi^+ - \frac{g}{\sqrt{2}}\left[iW_\mu^+ H^- \overleftrightarrow{\partial}^\mu\, \phi^+ + \text{h.c.}\right]$$
$$+\frac{eg}{\sqrt{2}}\left(A^\mu W_\mu^+ H^-\phi^+ + A^\mu W_\mu^- H^+\phi^-\right) - \frac{g^2 s_W^2}{\sqrt{2}c_W}\left(Z^\mu W_\mu^+ H^-\phi^+ + Z^\mu W_\mu^- H^+\phi^-\right)$$
$$-v(Z_3+Z_4)h\phi^+\phi^- - \tfrac{1}{2}\left[Z_2(\phi^+\phi^-)^2 + (Z_3+Z_4)h^2\phi^+\phi^-\right] - Z_2 H^+ H^-\phi^+\phi^- \,.$$

For example, Drell-Yan production via a virtual s-channel W^+ exchange can produce H^+ in association with ϕ^-, whereas virtual s-channel W^- exchange can produce H^- in association with ϕ^+. Thus, the sign of the charged Higgs boson reveals the U(1)-charge of the produced neutral scalar. The origin of this correlation lies in the fact that, by construction, H^+ and ϕ^+ both reside in H_2, whereas H^- and ϕ^- both reside in $H_2^\dagger$.

Models with three Higgs doublets

There is not yet a full study of all possible symmetries

e.g. Ivanov et al

In what follows we consider

Three Higgs doublet models with S_3 Symmetry

An Interesting model:

A CP-conserving multi-Higgs Model with irremovable complex coefficients

I.P. Ivanov and J.P. Silva, Phys. Rev. D **93**, 095014 (2016) [arXiv:1512.09276],

Was analysed by H. Haber, O. M. Ogreid, P. Osland, MNR, 2018

The Scalar potential

S_3 is the permutation group involving three objects, ϕ_1, ϕ_2, ϕ_3

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \frac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \text{hc}]$$

$$V_4 = A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{ C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \text{hc}] \}$$
$$+ \frac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \text{hc}] + \sum_{i \neq j \neq k \neq i, j<k} \{ \frac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \text{hc}]$$
$$+ \frac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \text{hc}] + \frac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \text{hc}] \}$$

Derman, 1979

here all fields appear on equal footing

this representation is not irreducible, for instance, the combination

$$\phi_1 + \phi_2 + \phi_3$$

remains invariant, it splits into two irreducible representations,

doublet and singlet: $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad h_S$

Decomposition into these two irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

This definition does not treat equally

$$\phi_1, \phi_2, \phi_3$$

they could be interchanged

Notice similarity with tribimaximal mixing:

Harrison, Perkins and Scott, 1999

$$(F =) \quad \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

The scalar potential in terms of fields from irreducible representations

$$
\begin{aligned}
V_2 &= \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \\
V_4 &= \lambda_8 (h_S^\dagger h_S)^2 + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) + \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 \\
&\quad + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\
&\quad + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] \\
&\quad + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \mathrm{h.c.}] \\
&\quad + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \mathrm{h.c.}]
\end{aligned}
$$

Das and Dey

no symmetry under the interchange of h_1 and h_2

however there is symmetry for $h_1 \to -h_1$

equivalent doublet representation $\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$

now there is symmetry for $\chi_1 \leftrightarrow \chi_2$

In the special case $\lambda_4 = 0$ **the potential has SO(2) symmetry:**

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Danger: massless scalar!

Constraining the potential by the vevs

Possibility of SCPV - real parameters

Let us start with real vacua (no CP violation)

Three minimisation conditions:

can be solved to give μ_0^2 and μ_1^2 in terms of the quartic coefficients:

$$\mu_0^2 = \frac{1}{2w_S}\left[\lambda_4(w_2^2 - 3w_1^2)w_2 - (\lambda_5 + \lambda_6 + 2\lambda_7)(w_1^2 + w_2^2)w_S - 2\lambda_8 w_S^3\right], \tag{4.2a}$$

$$\mu_1^2 = -\frac{1}{2}\left[2(\lambda_1 + \lambda_3)(w_1^2 + w_2^2) + 6\lambda_4 w_2 w_S + (\lambda_5 + \lambda_6 + 2\lambda_7)w_S^2\right], \tag{4.2b}$$

$$\mu_1^2 = -\frac{1}{2}\left[2(\lambda_1 + \lambda_3)(w_1^2 + w_2^2) - 3\lambda_4(w_2^2 - w_1^2)\frac{w_S}{w_2} + (\lambda_5 + \lambda_6 + 2\lambda_7)w_S^2\right]. \tag{4.2c}$$

Eqs (4.2b) and (4.2c) obtained dividing by w_1 and w_2 respectively

$$\lambda_4 = 4A - 2(C + \bar{C} + D) - E_1 - E_2 + E_4 = 0$$

Consistency requires:

- for $w_1 = 0$ the corresponding derivative is zero - no clash
- or else $\lambda_4(3w_2^2 - w_1^2)w_S = 0$ i. e., $\lambda_4 = 0$ or $w_1 = \pm\sqrt{3}w_2$ or $w_S = 0.$
- for $w_S = 0.$ special condition: $\lambda_4 w_2(3w_1^2 - w_2^2) = 0,$ i. e., in addition:

$$\lambda_4 = 0 \text{ or } w_2 = \pm\sqrt{3}w_1, \text{ or else } w_2 = 0.$$

SSB, real vacua, residual symmetries

Derman, Tsao Phys. Rev. D20 (1979) 1207:

(x, x, x) S_3 ; (x, x, y) S_2 ; (x, y, z) = (x, -x, 0) S_2 $\lambda_4 \neq 0$

Translation into doublet singlet notation

(x, x, x) $\rightarrow$ (0,0, ω_S) $w_1 = 0$ (also verifies $w_1 = \pm\sqrt{3}w_2$)

(x, -x, 0) $\rightarrow$ $(\omega_1, 0, 0)$ $w_S = 0$ together with $w_2 = 0.$

(x, 0, -x) $\rightarrow$ $(\omega_1, \omega_2, 0)$ $w_S = 0$ together $w_2 = \sqrt{3}w_1$

(0,x, -x) $\rightarrow$ $(\omega_1, \omega_2, 0)$ $w_S = 0$ together with $w_2 = -\sqrt{3}w_1$

(x, x, y) translates into $(0, w_2, w_S)$; consistency condition: $w_1 = 0.$

(x, y, x) translates into $(w_1, -\frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = -\sqrt{3}w_2$

(y, x, x) translates into $(w_1, \frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = \sqrt{3}w_2$

For $\lambda_4 = 0$ SO(2) symmetry implies (x, y, z) possible solution

Vacuum	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	Comment
R-0	$0, 0, 0$	$0, 0, 0$	Not interesting
R-I-1	x, x, x	$0, 0, w_S$	$\mu_0^2 = -\lambda_8 w_S^2$
R-I-2a	$x, -x, 0$	$w, 0, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_1^2$
R-I-2b	$x, 0, -x$	$w, \sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-I-2c	$0, x, -x$	$w, -\sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-II-1a	x, x, y	$0, w, w_S$	$\mu_0^2 = \frac{1}{2}\lambda_4 \frac{w_2^3}{w_S} - \frac{1}{2}\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2 + \frac{3}{2}\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1b	x, y, x	$w, -w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1c	y, x, x	$w, w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-2	$x, x, -2x$	$0, w, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2,\ \lambda_4 = 0$
R-II-3	$x, y, -x-y$	$w_1, w_2, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right), \lambda_4 = 0$
R-III	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	$\mu_0^2 = -\frac{1}{2}\lambda_a(w_1^2 + w_2^2) - \lambda_8 w_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right) - \frac{1}{2}\lambda_a w_S^2,$ $\lambda_4 = 0$

$$\lambda_a = \lambda_5 + \lambda_6 + 2\lambda_7,$$

$$\lambda_b = \lambda_5 + \lambda_6 - 2\lambda_7.$$

Complex vacua

Table 2: Complex vacua. Notation: $\epsilon = 1$ and -1 for C-III-d and C-III-e, respectively; $\xi = \sqrt{-3\sin 2\rho_1/\sin 2\rho_2}$, $\psi = \sqrt{[3+3\cos(\rho_2 - 2\rho_1)]/(2\cos\rho_2)}$. With the constraints of Table 4 the vacua labelled with an asterisk (*) are in fact real.

	IRF (Irreducible Rep.)	RRF (Reducible Rep.)
	w_1, w_2, w_S	ρ_1, ρ_2, ρ_3
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x+iy, x-iy, x$
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho} - \frac{y}{2}, -xe^{i\rho} - \frac{y}{2}, y$
C-III-d,e	$\pm i\hat{w}_1, \epsilon\hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho} \pm ix, re^{i\rho} \mp ix, \frac{3}{2}re^{-i\rho} - \frac{1}{2}re^{i\rho}$
C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho} \pm ix, re^{-i\rho} \mp ix, \frac{3}{2}re^{i\rho} - \frac{1}{2}re^{-i\rho}$
C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2\sigma_1)}{1+9\tan^2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\pm\hat{w}_2 e^{-i\arctan(3\tan\sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho} + x, -re^{i\rho} + x, x$
C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho} + x, -re^{-i\rho} + x, -re^{i\rho} + re^{-i\rho} + x$
C-IV-c	$\sqrt{1+2\cos^2\sigma_2}\hat{w}_2$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho} + r\sqrt{3(1+2\cos^2\rho)} + x$, $re^{i\rho} - r\sqrt{3(1+2\cos^2\rho)} + x, -2re^{i\rho} + x$
C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm\hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho} + x, (r_2 - r_1)e^{i\rho} + x, -r_2 e^{i\rho} + x$
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2} + re^{i\rho_1}\xi + x, re^{i\rho_2} - re^{i\rho_1}\xi + x$, $-2re^{i\rho_2} + x$
C-IV-f	$\sqrt{2 + \frac{\cos(\sigma_1 - 2\sigma_2)}{\cos\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1} + re^{i\rho_2}\psi + x$, $re^{i\rho_1} - re^{i\rho_2}\psi + x, -2re^{i\rho_1} + x$
C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

Constraints

Vacuum	Constraints
C-I-a	$\mu_1^2 = -2\left(\lambda_1 - \lambda_2\right)\hat{w}_1^2$
C-III-a	$\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \frac{4\cos\sigma_2\hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-b	$\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_1^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\lambda_b \hat{w}_S^2,$ $\lambda_4 = 0$
C-III-c	$\mu_1^2 = -(\lambda_1 + \lambda_3)(\hat{w}_1^2 + \hat{w}_2^2),$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0$
C-III-d,e	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{(\hat{w}_1^2 - \hat{w}_2^2)^2}{\hat{w}_S^2} - \epsilon\lambda_4 \frac{(\hat{w}_1^2 - \hat{w}_2^2)(\hat{w}_1^2 - 3\hat{w}_2^2)}{4\hat{w}_2\hat{w}_S}$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \epsilon\lambda_4 \frac{\hat{w}_S(\hat{w}_1^2 - \hat{w}_2^2)}{4\hat{w}_2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_7 = \frac{\hat{w}_1^2 - \hat{w}_2^2}{\hat{w}_S^2}(\lambda_2 + \lambda_3) - \epsilon\frac{(\hat{w}_1^2 - 5\hat{w}_2^2)}{4\hat{w}_2\hat{w}_S}\lambda_4$
C-III-f,g	$\mu_0^2 = -\frac{1}{2}\lambda_b\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\lambda_b\hat{w}_S^2, \lambda_4 = 0$
C-III-h	$\mu_0^2 = -2\lambda_b\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \mp\frac{2\cos\sigma_2\hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-i	$\mu_0^2 = \frac{16\left(1-3\tan^2\sigma_1\right)^2}{(1+9\tan^2\sigma_1)^2}(\lambda_2 + \lambda_3)\frac{\hat{w}_2^4}{\hat{w}_S^2} \pm \frac{6\left(1-\tan^2\sigma_1\right)(1-3\tan^2\sigma_1)}{(1+9\tan^2\sigma_1)^{\frac{3}{2}}}\lambda_4\frac{\hat{w}_2^3}{\hat{w}_S}$ $-\frac{2(1+3\tan^2\sigma_1)}{1+9\tan^2\sigma_1}(\lambda_5 + \lambda_6)\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\frac{4(1+3\tan^2\sigma_1)}{1+9\tan^2\sigma_1}(\lambda_1 - \lambda_2)\hat{w}_2^2 \mp \frac{\left(1-3\tan^2\sigma_1\right)}{2\sqrt{1+9\tan^2\sigma_1}}\lambda_4\hat{w}_2\hat{w}_S$ $-\frac{1}{2}(\lambda_5 + \lambda_6)\hat{w}_S^2,$ $\lambda_7 = -\frac{4\left(1-3\tan^2\sigma_1\right)\hat{w}_2^2}{(1+9\tan^2\sigma_1)\hat{w}_S^2}(\lambda_2 + \lambda_3) \mp \frac{\left(5-3\tan^2\sigma_1\right)\hat{w}_2}{2\sqrt{1+9\tan^2\sigma_1}\hat{w}_S}\lambda_4$

Vacuum	Constraints
C-IV-a*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_1^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = 0$
C-IV-b	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)^2}{\hat{w}_S^2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = -\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-c	$\mu_0^2 = 2\cos^2\sigma_2\left(1 + \cos^2\sigma_2\right)\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$ $-\left(1 + \cos^2\sigma_2\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left[2\left(1 + \cos^2\sigma_2\right)\lambda_1 - \left(2 + 3\cos^2\sigma_2\right)\lambda_2 - \cos^2\sigma_2\lambda_3\right]\hat{w}_2^2$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = -\frac{2\cos\sigma_2\hat{w}_2}{\hat{w}_S}\left(\lambda_2 + \lambda_3\right), \lambda_7 = \frac{\cos^2\sigma_2\hat{w}_2^2}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-d*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = 0$
C-IV-e	$\mu_0^2 = \frac{\sin^2(2(\sigma_1 - \sigma_2))}{\sin^2(2\sigma_1)}\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$ $-\frac{1}{2}\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_1 - \lambda_2\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = -\frac{\sin(2(\sigma_1 - \sigma_2))\hat{w}_2^2}{\sin 2\sigma_1\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-f	$\mu_0^2 = -\frac{(\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1)\cos(\sigma_2 - \sigma_1)}{2\cos^2\sigma_1}\lambda_4\frac{\hat{w}_2^3}{\hat{w}_S}$ $-\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{2\cos\sigma_1}\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{\cos\sigma_1}\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2$ $-\frac{3\cos 2\sigma_1 + 2\cos(2(\sigma_1 - \sigma_2)) + \cos 2\sigma_2 + 4}{4\cos(\sigma_1 - \sigma_2)\cos\sigma_1}\lambda_4\hat{w}_2\hat{w}_S - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_2 + \lambda_3 = -\frac{\cos\sigma_1\hat{w}_S}{2\cos(\sigma_2 - \sigma_1)\hat{w}_2}\lambda_4, \lambda_7 = -\frac{\cos(\sigma_2 - \sigma_1)\hat{w}_2}{2\cos\sigma_1\hat{w}_S}\lambda_4$
C-V*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0, \lambda_7 = 0$

Complex vacua, Spontaneous CP Violation

Table 1: Spontaneous CP violation

Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV
C-I-a	X	no	C-III-f,g	0	no	C-IV-c	X	yes
C-III-a	X	yes	C-III-h	X	yes	C-IV-d	0	no
C-III-b	0	no	C-III-i	X	no	C-IV-e	0	no
C-III-c	0	no	C-IV-a	0	no	C-IV-f	X	yes
C-III-d,e	X	no	C-IV-b	0	no	C-V	0	no

No spontaneous CP violation in any of the cases with

$$\lambda_4 = 0$$

The case of $\lambda_4 = 0$

Potential has additional continuous SO(2) symmetry

$$\lambda_4 = 4A - 2(C + \bar{C} + D) - E_1 - E_2 + E_4 = 0$$

Derman (1979), "unnatural"

Spontaneous breaking of this SO(2) symmetry leads to massless particles

Possible solution: break the symmetry softly.
The most general quadratic potential can be written:

$$V = V_2 + V_2' + V_4$$

$$V_2' = \mu_2^2 \left(h_1^\dagger h_1 - h_2^\dagger h_2\right) + \frac{1}{2}\nu_{12}^2 \left(h_1^\dagger h_2 + \text{h.c.}\right) + \frac{1}{2}\nu_{01}^2 \left(h_S^\dagger h_1 + \text{h.c.}\right) + \frac{1}{2}\nu_{02}^2 \left(h_S^\dagger h_2 + \text{h.c.}\right).$$

Table 1: Complex vacua, for the unbroken S_3 case, with massless states and degeneracies indicated. The first entry in the parenthesis refers to the charged sector, the second one to the neutral sector. In the footnotes below, L indicates that a linear expression in its arguments vanishes.

Vacuum	name	λ_4	symmetry	# massless states	degeneracies
C_{0xy}	C-III-a	$\surd$		none	none
C_{x0y}	C-III-b	0	SO(2)	(none,1)	none
C_{x0y}	C-IV-a	0^{α}	$\mathrm{SO}(2) \otimes \mathrm{U}(1)_s$	(none,2)	(none,2)
C_{xy0}	C-I-a	$\surd$		none	(none,2)
C_{xy0}	C-III-c	0^{β}	SO(2)	(none,2)	(none,2)
C_{xyz}	C-III-d,e	$\surd$		none	none
C_{xyz}	C-III-f,g	0	SO(2)	(none,1)	none
C_{xyz}	C-III-h,i	$\surd$		none	none
C_{xyz}	C-IV-b	0	SO(2)	(none,1)	none
C_{xyz}	C-IV-c	$^{\gamma}$	x	(none,1)	none
C_{xyz}	C-IV-d	0^{α}	$\mathrm{SO}(2) \otimes \mathrm{U}(1)_s$	(none,2)	(none,2)
C_{xyz}	C-IV-e	0	SO(2)	(none,1)	none
C_{xyz}	C-IV-f	$^{\gamma}$	x	(none,1)	none
C_{xyz}	C-V	$0^{\alpha,\beta}$	$\mathrm{SO}(2) \otimes \mathrm{U}(1)_1 \otimes \mathrm{U}(1)_2 \otimes \mathrm{U}(1)_s$	(none,3)	(none,3)

$^{\alpha}$ Also $\lambda_7 = 0$. $^{\beta}$ Also $\lambda_2 + \lambda_3 = 0$. $^{\gamma}$ $L(\lambda_2 + \lambda_3, \lambda_4)$, $L(\lambda_2 + \lambda_3, \lambda_7)$.

$\lambda_4 = 0$ potential acquires an additional SO(2) symmetry between the two members of the S3 doublet

$\lambda_4 = 0$ together with $\lambda_7 = 0$ the potential acquires an additional SO(2) together with a U(1) symmetry

$\lambda_4 = 0$ $\lambda_7 = 0$ and $\lambda_2 + \lambda_3 = 0$ SO(2) symmetry plus symmetry under independent rephasing of each doublet

A particularly interesting complex vacuum configuration

Vacuum C-III-c

$$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$$

Constraints:

$$\mu_1^2 = -(\lambda_1 + \lambda_3)(\hat{w}_1^2 + \hat{w}_2^2),$$
$$\lambda_2 + \lambda_3 = 0, \lambda_4 = 0$$

(acquires SO(2) symmetry)

Does not violate CP spontaneously

O. M. Ogreid, P. Osland, M. N. R. , 2017

Can be rewritten as: $(w_1, w_2, w_S) = (\hat{w}e^{i\sigma/2}, \hat{w}e^{-i\sigma/2}, 0) = (\hat{w}e^{i\sigma}, \hat{w}, 0)$

It is the only complex vacuum in the full list with a nontrivial phase that is not constrained by the minimisation conditions

Two massless neutral scalars, one of them is a Goldstone boson associated to the breaking of SO(2)

The C-III-c model without soft breaking terms

phase σ is not determined by the potential.

Two massless states in the neutral sector apart from the would-be Goldstone boson

S_3 doublet and the S_3 singlet do not mix in the mass terms

In the neutral sector of the S_3-doublet, there is only one massive (CP-even) state

The S_3-singlet sector has two massive states (S_1 and S_2)

$$m_{S_1}^2 = \mu_0^2 + \tfrac{1}{2}(\lambda_5 + \lambda_6)v^2 - \lambda_7 \cos\sigma v^2$$
$$m_{S_2}^2 = \mu_0^2 + \tfrac{1}{2}(\lambda_5 + \lambda_6)v^2 + \lambda_7 \cos\sigma v^2$$

The phase sigma which is left undetermined by the potential is related to the mass splitting of these neutral scalars and also parametrises some of the triliniar couplings involving the scalar fields

The C-III-c model with soft S_3-breaking

Table 6: Summary of softly-broken C-III-c-like vacua. Here, "SBT" stands for "Soft-breaking terms". When the two moduli are equal, we denote it $\hat{w}$. In the last column we listed the symmetry responsible for no spontaneous CP violation.

Case	Constraints	Allowed SBT	Vacuum	CP
1	$\lambda_4=0,\ \lambda_2+\lambda_3=0$ C-III-c	none	$(\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0)$ $\equiv (\hat{w}e^{i\sigma/2}, \hat{w}e^{-i\sigma/2}, 0)$	conserving SO(2)
2	$\lambda_4=0,\ \lambda_2+\lambda_3\neq 0$ $\cos(\sigma_2-\sigma_1)=0,\ \hat{w}_1\neq\hat{w}_2$	μ_2^2	$(\pm i\hat{w}_1, \hat{w}_2, 0)$	conserving $h_1\to -h_1$
3	$\lambda_4=0,\ \lambda_2+\lambda_3\neq 0$ $\cos(\sigma_2-\sigma_1)\neq 0,\ \hat{w}_1=\hat{w}_2$	ν_{12}^2	$(\hat{w}e^{i\sigma_1}, \hat{w}e^{i\sigma_2}, 0)$ $\equiv (\hat{w}e^{i\sigma/2}, \hat{w}e^{-i\sigma/2}, 0)$	conserving $h_1\leftrightarrow h_2$
4	$\lambda_4=0,\ \lambda_2+\lambda_3\neq 0$ no other conditions	$\mu_2^2,\ \nu_{12}^2$	$(\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0)$	violating
5	$\lambda_4\neq 0,\ \lambda_2+\lambda_3=0$ $\cos(\sigma_2-\sigma_1)=0,\ \hat{w}_1=\hat{w}_2$ C-I-a	none	$(\pm i\hat{w}, \hat{w}, 0)$	conserving $h_1\to -h_1$
6	$\lambda_4\neq 0,\ \lambda_2+\lambda_3=0$ $\cos(\sigma_2-\sigma_1)=0,\ \hat{w}_1\neq\hat{w}_2$	ν_{02}^2	$(\pm i\hat{w}_1, \hat{w}_2, 0)$	conserving $h_1\to -h_1$
7	$\lambda_4\neq 0,\ \lambda_2+\lambda_3=0$ $\cos(\sigma_2-\sigma_1)\neq 0,\ \hat{w}_1=\hat{w}_2$	ν_{01}^2	$(\hat{w}e^{i\sigma}, \hat{w}, 0)$	violating
8	$\lambda_4\neq 0,\ \lambda_2+\lambda_3=0$ no other conditions	$\nu_{01}^2,\ \nu_{02}^2$	$(\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0)$	violating
9	$\lambda_4\neq 0,\ \lambda_2+\lambda_3\neq 0$ $\cos(\sigma_2-\sigma_1)=0,\ \hat{w}_1\neq\hat{w}_2$	$\mu_2^2,\ \nu_{02}^2$	$(\pm i\hat{w}_1, \hat{w}_2, 0)$	conserving $h_1\to -h_1$
10	$\lambda_4\neq 0,\ \lambda_2+\lambda_3\neq 0$ $\cos(\sigma_2-\sigma_1)\neq 0,\ \hat{w}_1=\hat{w}_2$	$\nu_{12}^2,\ \nu_{01}^2$	$(\hat{w}e^{i\sigma}, \hat{w}, 0)$	violating
11	$\lambda_4\neq 0,\ \lambda_2+\lambda_3\neq 0$ $\sigma_2-\sigma_1\neq 0,\ \hat{w}_1\neq\hat{w}_2$	all	$(\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0)$	violating

$$V_2' = \mu_2^2\left(h_1^\dagger h_1 - h_2^\dagger h_2\right) + \frac{1}{2}\nu_{12}^2\left(h_1^\dagger h_2 + \text{h.c.}\right) + \frac{1}{2}\nu_{01}^2\left(h_S^\dagger h_1 + \text{h.c.}\right) + \frac{1}{2}\nu_{02}^2\left(h_S^\dagger h_2 + \text{h.c.}\right).$$

CONCLUSIONS

Symmetries play a crucial rôle in multi-Higgs models

Multi-Higgs models provide interesting scenarios for Dark Matter

Symmetries are needed to stabilise Dark Matter

The question of whether CP is violated spontaneously or explicitly is still open

Multi-Higgs Models have a rich phenomenology

Discoveries at the LHC are eagerly awaited