

Primordial Black Holes as a probe of EW phase transition

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Scalars2023, Sep 13-16, 2023

Higgs sector remains unknown

Non-minimal Higgs sectors

- Multi-plet structure
- Symmetries
- Strength of interaction

Important bound from data

vev, M_h ,
SM-like (nearly aligned)

...

Phase transition

CP Violation

DM candidate

Models for tiny neutrino masses

Inflatons

...

Rich Phenomenology



Example: 2HDM with softly broken Z_2

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \frac{m_3^2}{2} (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \\ + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + (\text{h.c.}) \right]$$

Φ_1 and $\Phi_2 \Rightarrow h, H, A^0, H^\pm \oplus$ Goldstone bosons
 $\overset{\text{CPeven}}{\uparrow} \quad \overset{\text{CPodd}}{\uparrow} \quad \overset{\text{charged}}{\uparrow}$

CPeven CPodd

masses {

$$m_h^2 = v^2 \left(\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \frac{\lambda}{2} \sin^2 2\beta \right) + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_H^2 = M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda) \sin^2 \beta \cos^2 \beta + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2,$$

$$m_A^2 = M_{\text{soft}}^2 - \lambda_5 v^2.$$

M_{soft} : soft breaking scale

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i a_i) \end{bmatrix} \quad (i = 1, 2)$$

Diagonalization

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \quad \begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix} \\ \begin{bmatrix} w_1^\pm \\ w_2^\pm \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

$$\frac{v_2}{v_1} \equiv \tan \beta$$

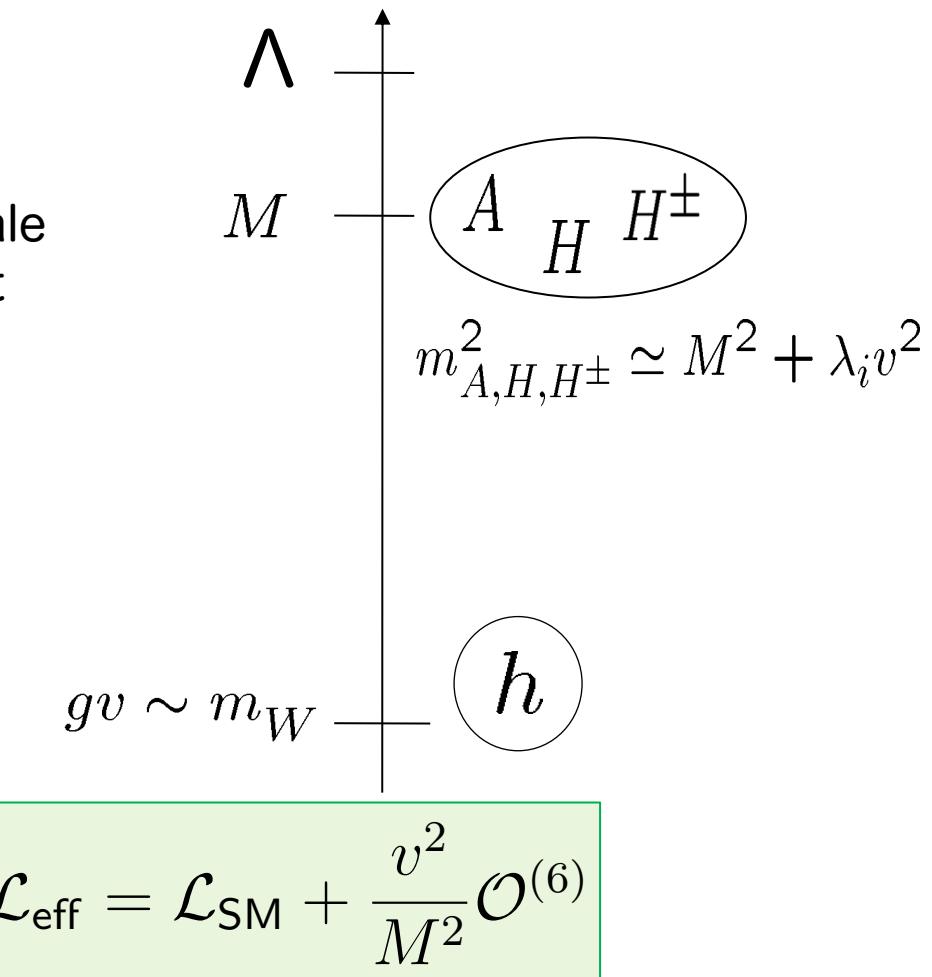
$$M_{\text{soft}} \quad (= \frac{m_3}{\sqrt{\cos \beta \sin \beta}}):$$

soft-breaking scale
of the discrete symm.

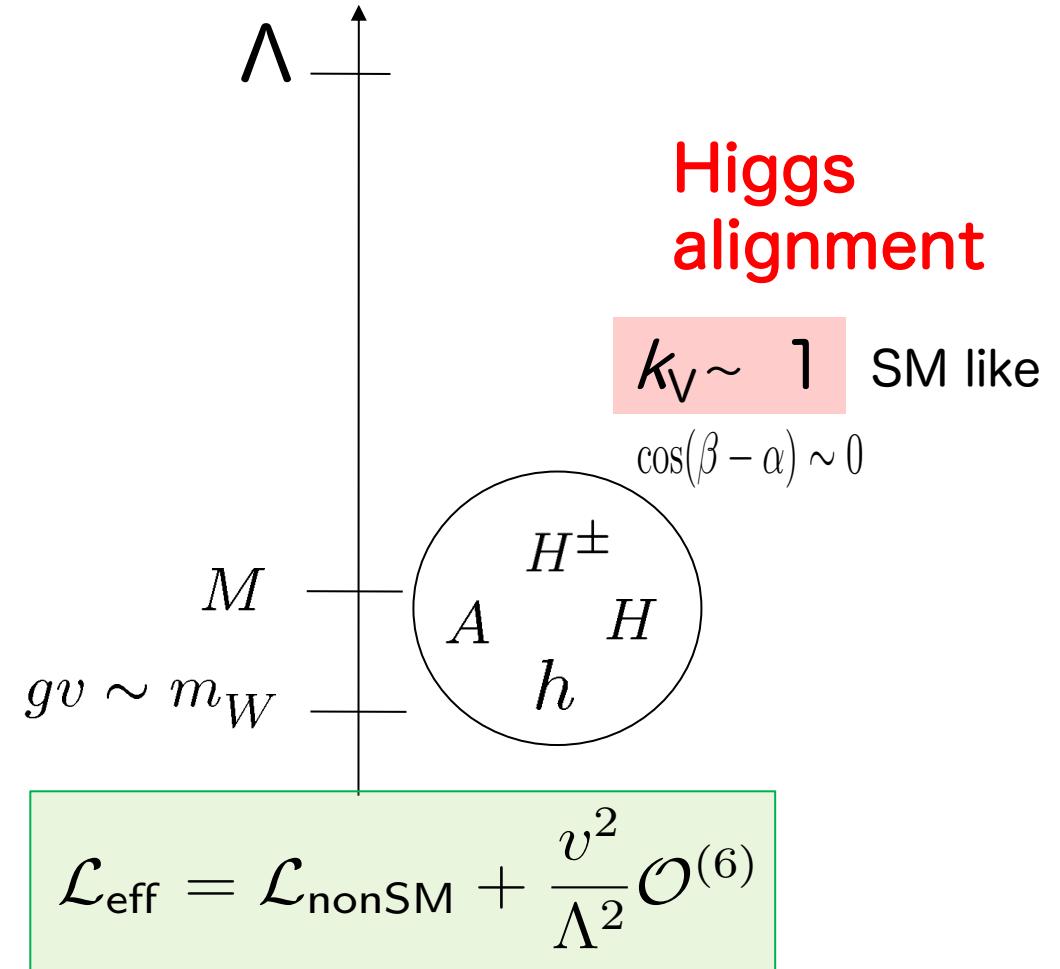
How SM-like is realized?

Λ : Cutoff

M : Mass scale
irrelevant
to VEV



Effective Theory is the SM
Decoupling



Effective Theory is an extended Higgs sector
Alignment and Non-Decoupling

Alignment and non-decoupling case

- To study physics of this case is interesting
- New physics appears at TeV scale
- Testable at experiments

EWSB (Higgs physics)

WIMP

Hierarchy problem

EW Phase Transition

...

g-2, B-anomalies,

...

- 1st OPT for EW baryogenesis is a concrete example
- Today's subject

EW phase transition

EWSB

→ EWPT exists in thermal history of Universe

Aspect of PT is crucial
for EW Baryogenesis

EW Baryogenesis

Sakharov Conditions

Kuzmin, Ruvakov, Shaposhnikov (1985)

- 1) B non-conservation \rightarrow Sphaleron transition at high T
- 2) C and CP violation \rightarrow C violation (SM is a chiral theory)
CP in BSM sectors
- 3) Departure from thermal equilibrium \rightarrow EWPT is strongly 1st OPT

Extension of the Higgs sector is required

Condition of Strongly First OPT

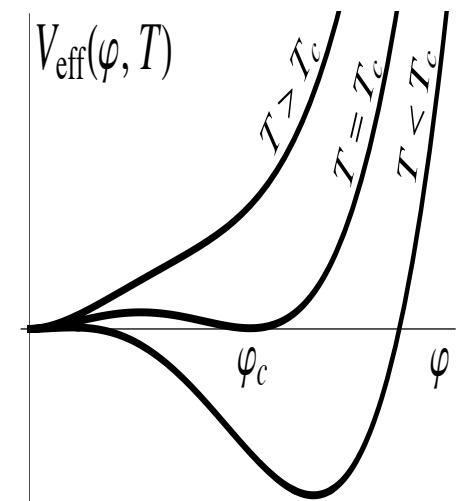
In the broken phase, sphaleron should quickly decouple to avoid wash out

$$\Gamma_{\text{sph}} < H$$



$$\frac{\varphi_c}{T_c} \gtrsim 1$$

Physics of Higgs potential



Extended Higgs can satisfy the condition

Effective Potential
at finite T (HTE)

$$V_{\text{eff}}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - \underline{ET}\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + \dots$$

SM

The condition
cannot be satisfied

$$\frac{\varphi_c}{T_c} = \frac{2E}{\lambda_T} \simeq \frac{6m_W^3 + 3m_Z^3 + \dots}{3\pi v m_h^2} \ll 1$$

Extended Higgs: Strong 1st OPT possible due to quantum effect

$$\frac{\varphi_C}{T_C} \simeq \frac{1}{3\pi v m_h^2} \left\{ 6m_W^3 + 3m_Z^3 + \sum_{\Phi} m_{\Phi}^3 \left(1 - \frac{M^2}{m_{\Phi}^2} \right)^3 \left(1 + \frac{3M^2}{2m_{\Phi}^2} \right) \right\} > 1$$

Quantum effects of Φ ($= H, A, H^+, \dots$)

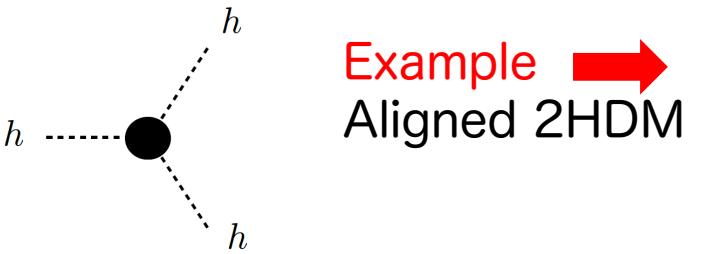
Prediction: Large deviation in the hhh coupling

$$\lambda_{hhh} \simeq \frac{3m_h^2}{v^2} \left\{ 1 - \frac{m_t^4}{\pi^2 v^2 m_h^2} + \sum_{\Phi} \frac{m_{\Phi}^4}{12\pi^2 v^2 m_h^2} \left(1 - \frac{M^2}{m_{\Phi}^2} \right)^3 \right\} > \lambda_{hhh}^{\text{SM}}$$

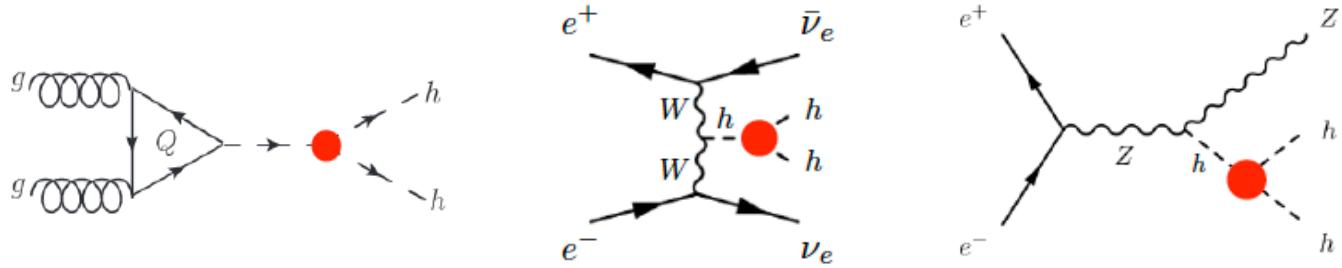
Test of strongly 1st OPT

Strongly 1st OPT
 → A large deviation in the hhh coupling

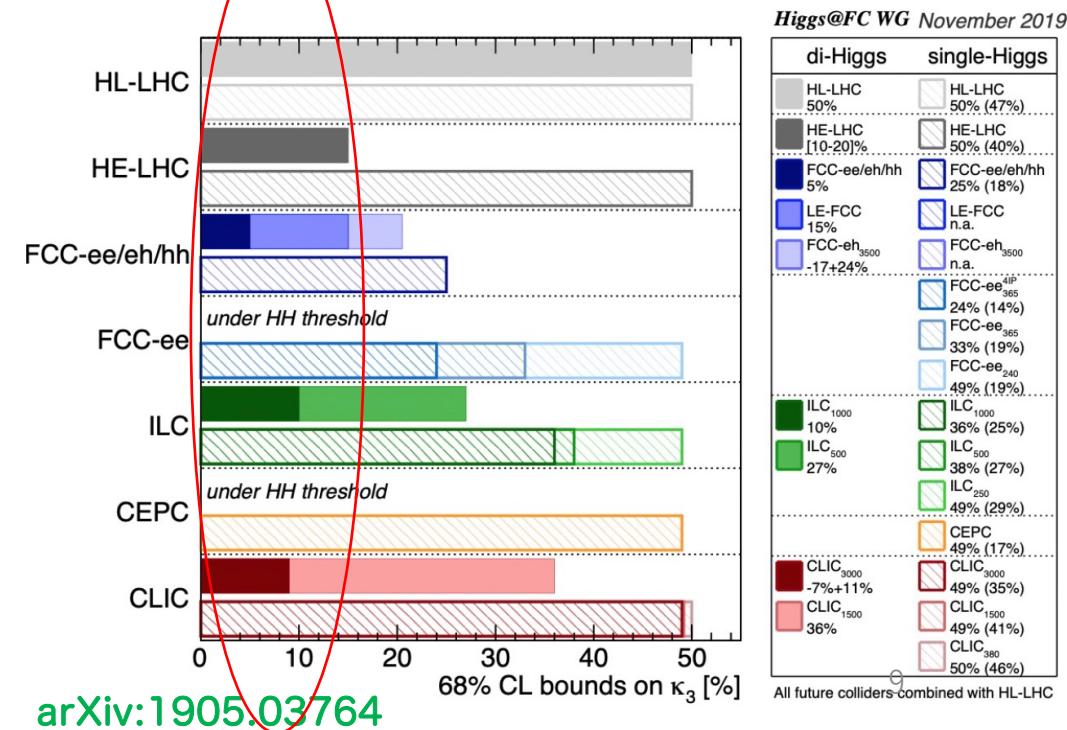
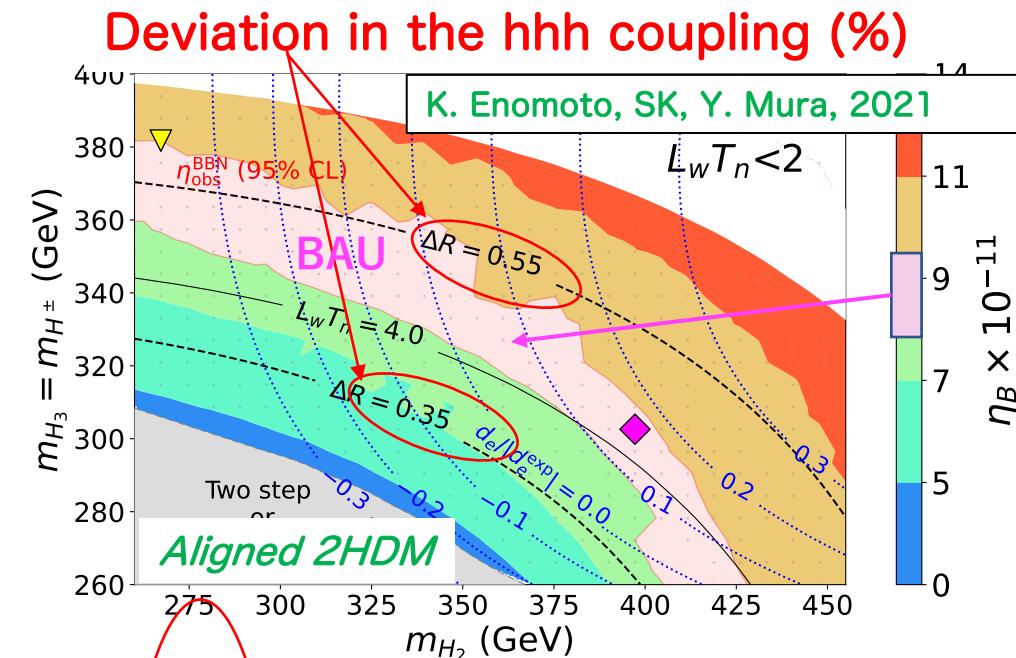
SK, Y. Okada, E. Senaha, 2005



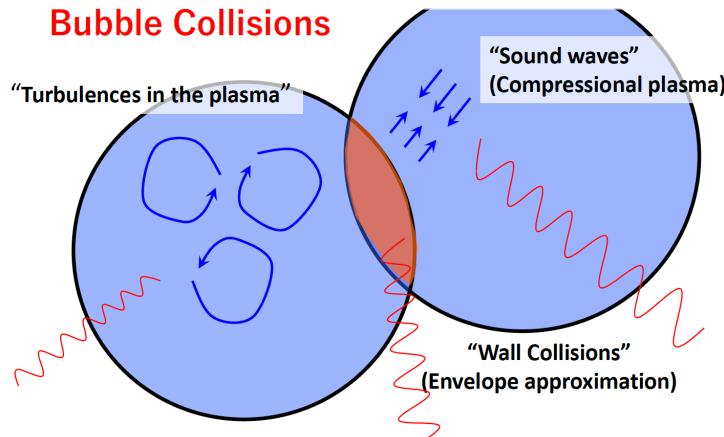
The hhh coupling can be measured at HL-LHC, or future e^+e^- colliders



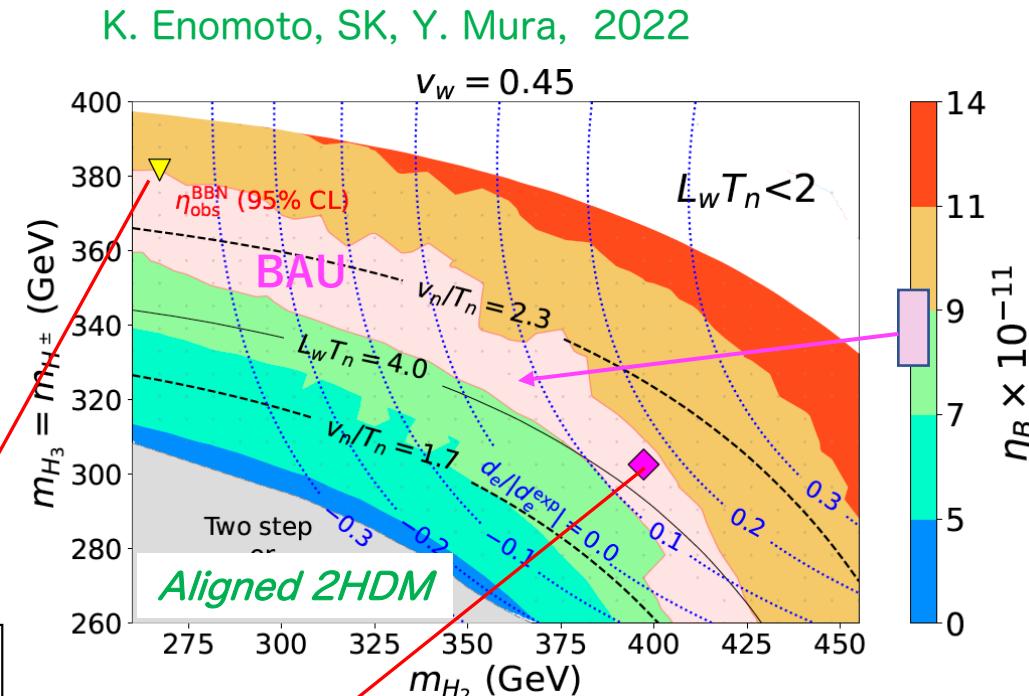
EW Baryogenesis can be directly tested by the hhh measurement



GW from 1stOPT

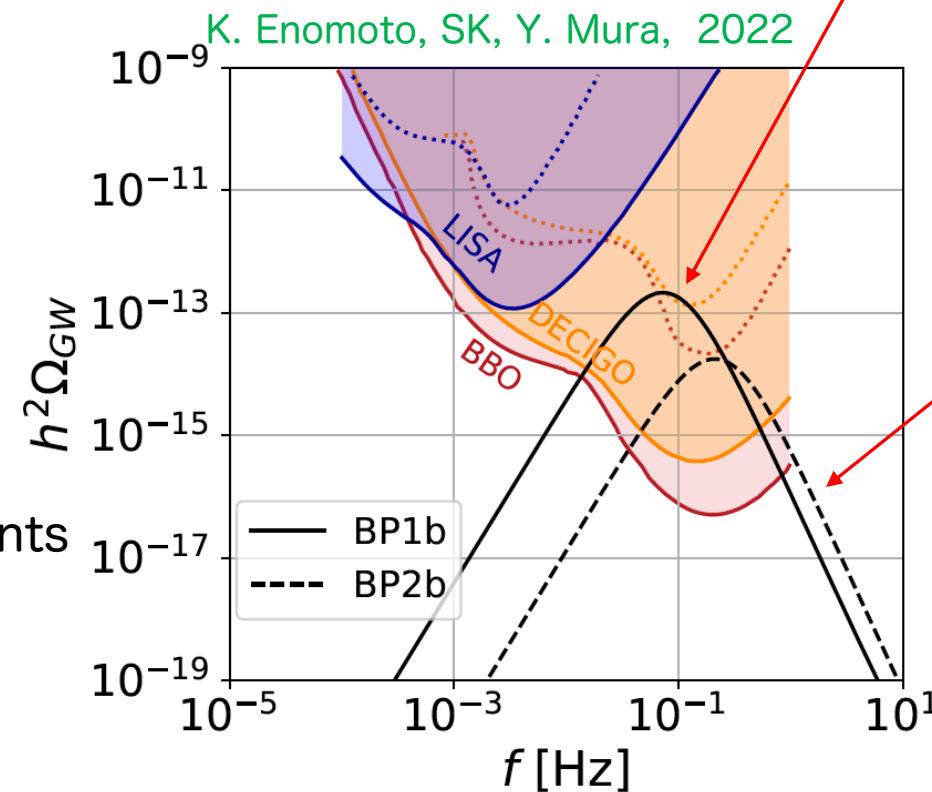


Example
Aligned 2HDM



GWs for benchmark
points of BAU

They may be tested
by future GW experiments



Dotted curves: Sensitivity Curve
M. Breitbach et al., arXiv: 1811.11175

Solid curves: $h^2 \Omega_{PISC}$ [SNR criterion]
J. Cline et al., arXiv: 2102.12490

naHEFT (for describing non-decoupling property)

(nearly aligned)

SK, R. Nagai (2021)

$$\mathcal{L}_{\text{naHEFT}} = \mathcal{L}_{\text{SM}} - \frac{\kappa_0}{64\pi^2} [\mathcal{M}^2(\varphi)]^2 \ln \frac{\mathcal{M}^2(\varphi)}{\mu^2}$$

$$\begin{aligned}\mathcal{M}^2(h) &= M^2 + \frac{\kappa_p}{2} \varphi^2 \\ &= M^2 + \frac{\kappa_p}{2} (h + v)^2\end{aligned}$$

In the decoupling region ($M^2 \gg \kappa_p v^2$),

$$V_{\text{BSM}}(\varphi) \simeq \frac{\lambda_\Phi^3}{64\pi^2 \textcolor{blue}{M^2}} \varphi^6 = \frac{1}{\Lambda^2} \varphi^6 \Rightarrow \text{SMEFT is a good approximation}$$

SMEFT is not good in the non-decoupling region ($M^2 < \kappa_p v^2$)

[Falkowski, Rattazzi, JHEP 10 (2019), Cohen et. al, JHEP 03 (2021)]

Three free parameters Λ , κ_0 , r

$$\Lambda = \sqrt{M^2 + \frac{\kappa_p}{2} v^2}, \quad \kappa_0, \quad r = \frac{\kappa_p v^2}{\Lambda^2} \quad \textcolor{red}{r} : \text{non-decouplingness}$$

Mass of new particles d.o.f of new particles

$r \sim 0 \Rightarrow M^2 \gg \frac{\kappa_p}{2} v^2$ Decoupling

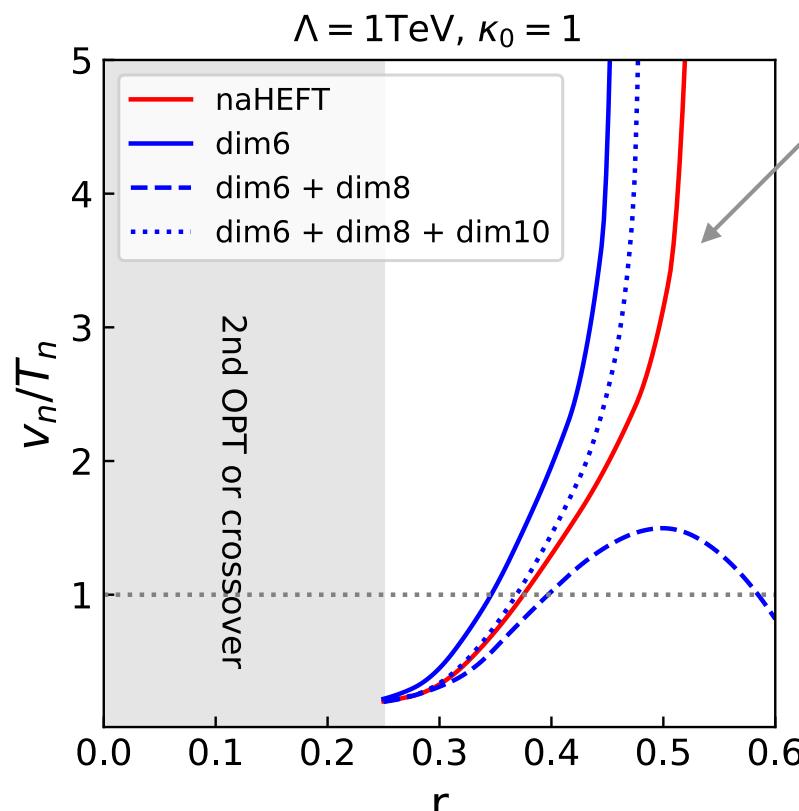
$r \sim 1 \Rightarrow M^2 \ll \frac{\kappa_p}{2} v^2$ Non-decoupling

- The naHEFT at finite temperatures

SK, R. Nagai, M. Tanaka (2022)

$$V_{\text{EFT}} = V_{\text{SM}} + \frac{\kappa_0}{64\pi^2} [\mathcal{M}^2(\phi)]^2 \ln \frac{\mathcal{M}^2(\phi)}{\mu^2} + \frac{\kappa_0}{2\pi^2} T^4 J_{\text{BSM}} \left(\frac{\mathcal{M}^2(\phi)}{T^2} \right)$$

$$J_{\text{BSM}}(a^2) = \int_0^\infty dk^2 k^2 \ln [1 - \text{sign}(\kappa_0) e^{-\sqrt{k^2+a^2}}] \quad \mathcal{M}^2(\phi) = M^2 + \frac{\kappa_p}{2} \phi^2$$



Consistent with results in the SM with a singlet
M. Kakizaki, SK, T.Matsui (2015)

Large deviation in v_n/T_n exists b/w the SMEFT
and naHEFT



SMEFT may not be appropriate when we
discuss the strongly first order EWPT

$$r = \frac{\frac{\kappa_p v^2}{2}}{\Lambda^2}$$

$r \sim 0 \Rightarrow M^2 \gg \frac{\kappa_p}{2} v^2$ Decoupling

$r \sim 1 \Rightarrow M^2 \ll \frac{\kappa_p}{2} v^2$ Non-decoupling

GW from 1st order EWPT in naHEFT

Nucleation rate
Linde 1983

$$\Gamma(T) \sim A(T) \exp\left(-\frac{S_3}{T}\right)$$

$$S_3(T) = \int d^3x \left[\frac{1}{2}(\vec{\nabla}\varphi^b)^2 + V_{\text{EFT}}(\varphi^b, T; \Lambda, \kappa_0, r) \right]$$

Nucleation completion condition (\rightarrow transition temp T_t)

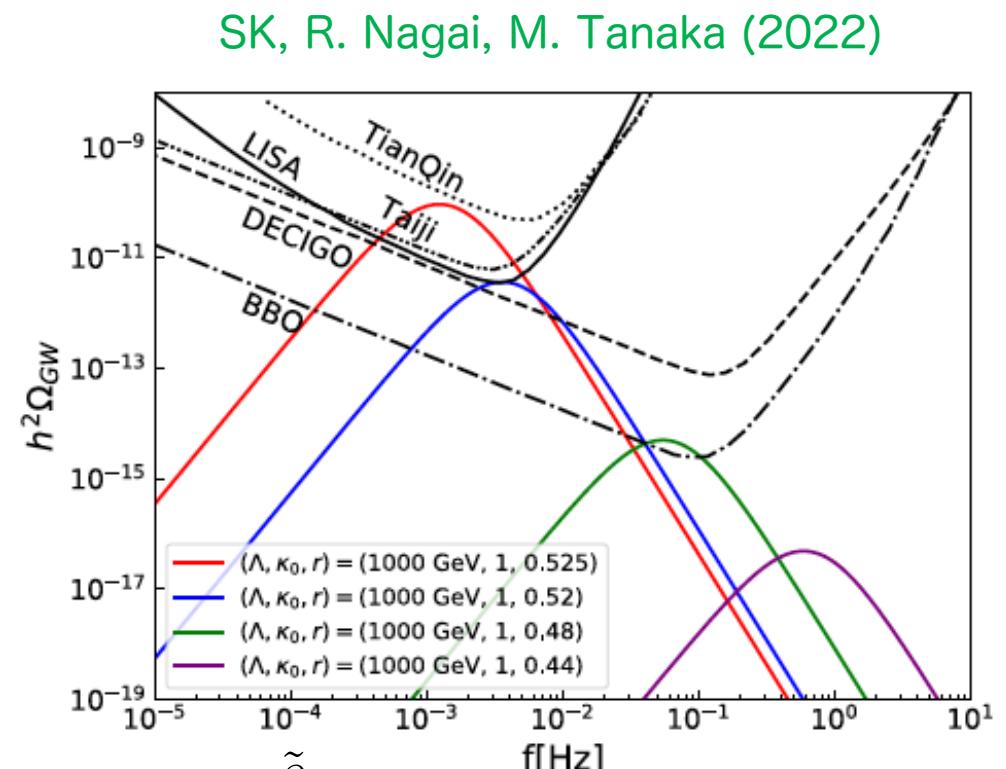
$$\left. \frac{\Gamma}{H^4} \right|_{T=T_t} \simeq 1$$

$$\frac{S_3(T_t)}{T_t} = 4 \ln(T_t/H_t) \simeq 140$$

- α Latent heat (released E of false vacuum)
- β Inverse of duration of phase transition

GW Spectrum C.Caprini et al., arXiv:1512.06239

$$\tilde{\Omega}_{\text{sw}} h^2 \simeq 2.65 \times 10^{-6} \frac{v_b}{\tilde{\beta}} \left(\frac{\kappa(v_b, \alpha)\alpha}{1+\alpha} \right)^2 \quad \tilde{f}_{\text{sw}} \simeq 1.9 \times 10^{-5} \text{Hz} \frac{\tilde{\beta}}{v_b}$$



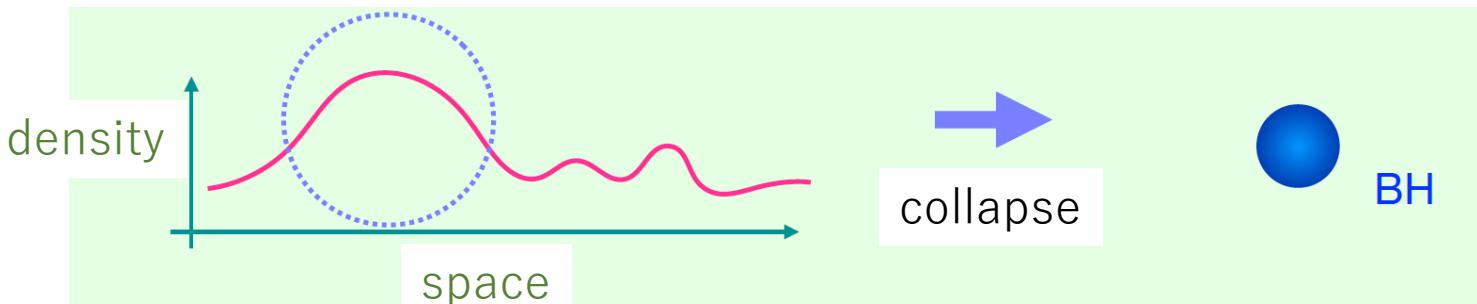
Test of EWPT

- Colliders (hhh, ⋯) HL-LHC 2028? ~
 ILC(500) 2040s??~
- Space based GW interferometer LISA 2037? ~
 DECIGO 2040s??~

A new possibility of testing EWPT by earlier experiment ?

Primordial Black Holes?

PBH Formation



Primordial black holes (PBH) : BHs formed before the star formation

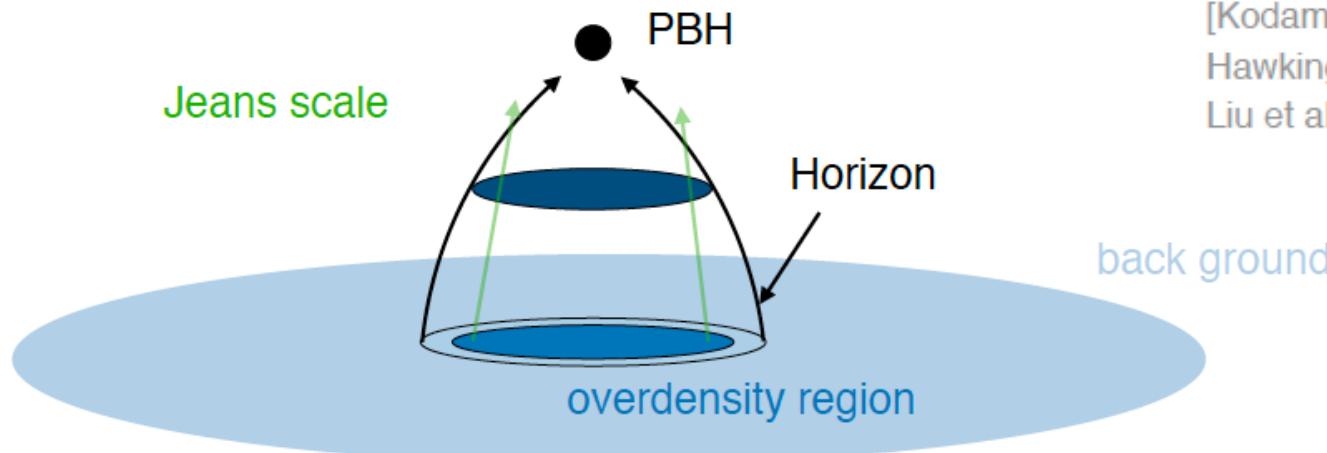
Condition for the PBH formation

$$\delta = \frac{\rho_{\text{over}} - \rho_{\text{back}}}{\rho_{\text{back}}} > \delta_C$$

[Hawking, Mon. Not. Roy. Astron. Soc. 152 (1971),
Hawking and Carr, Mon. Not. Roy. Astron. Soc. 168 (1974),
Harada, Yoo and Kohri, PRD 88 (2013)]

$\delta > \delta_C$ can be satisfied when the FOPT occurs

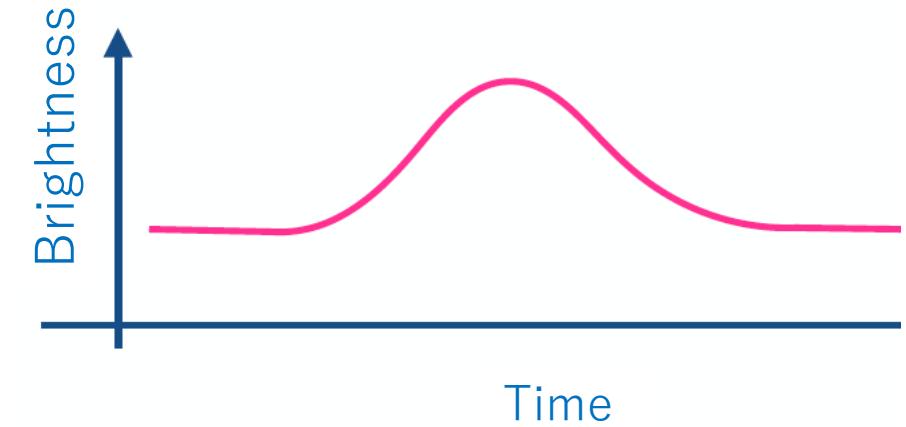
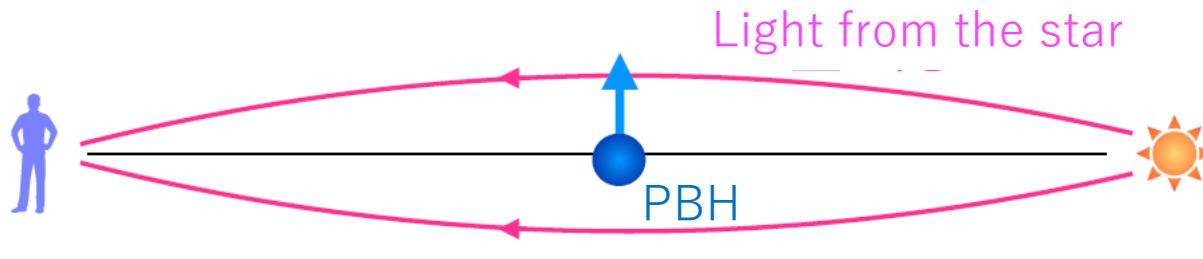
→ PBHs might be produced by the FOPT



[Kodama, Sasaki and Sato, PTP 68 (1982);
Hawking, Moss and Stewart, PRD 26 (1982)
Liu et al., PRD105 (2022)]

PBH search

- Gravitational microlensing effect
- Brightness is up by passing PBH



Non-observation → constraint on the PBH abundance

Subaru HSC, OGLE (Optical Gravitational Lensing Experiment)

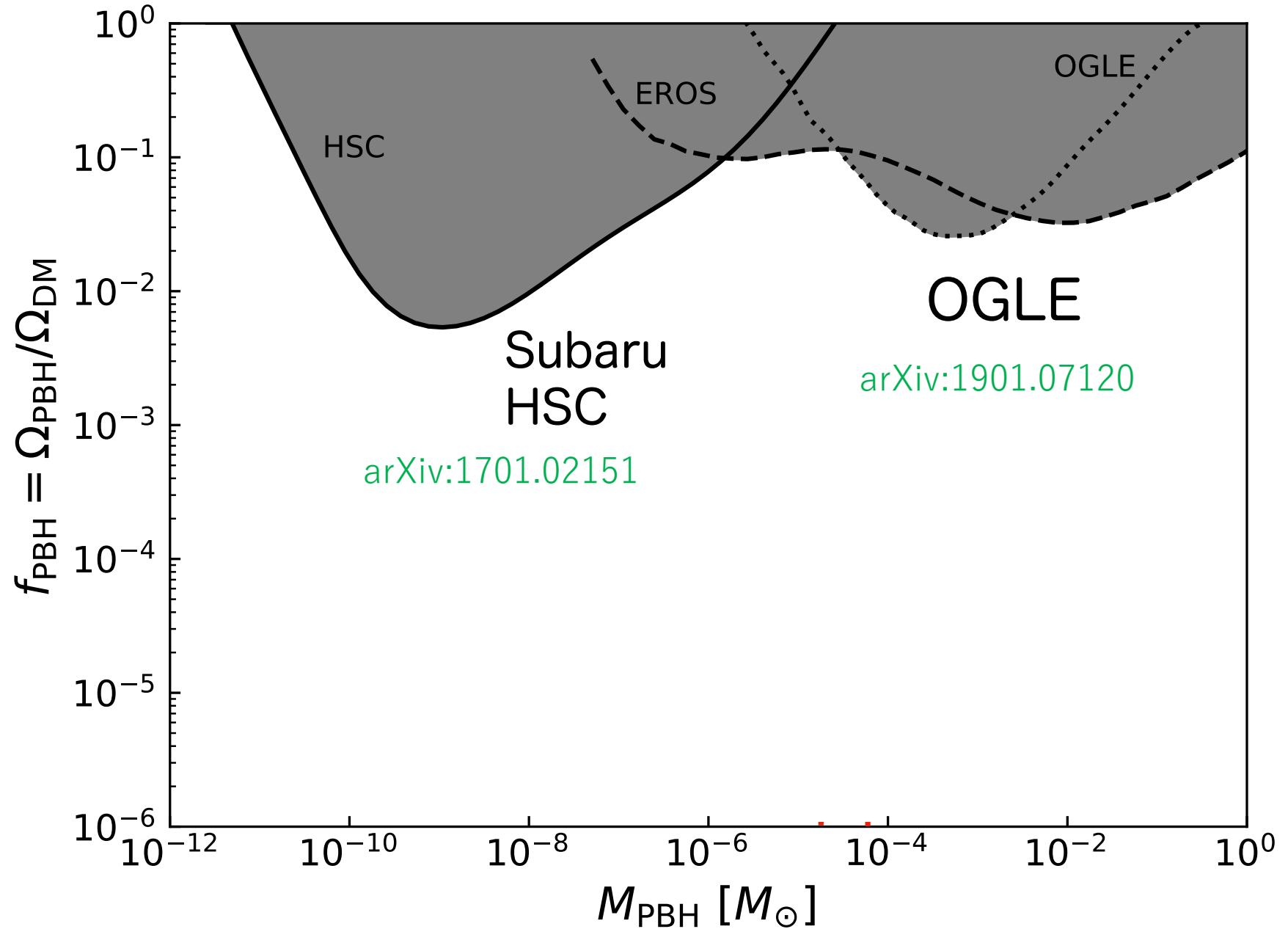
astro-ph/0607207

Current Data have
already constrained
some mass regions

Microlensing effect
of visible light

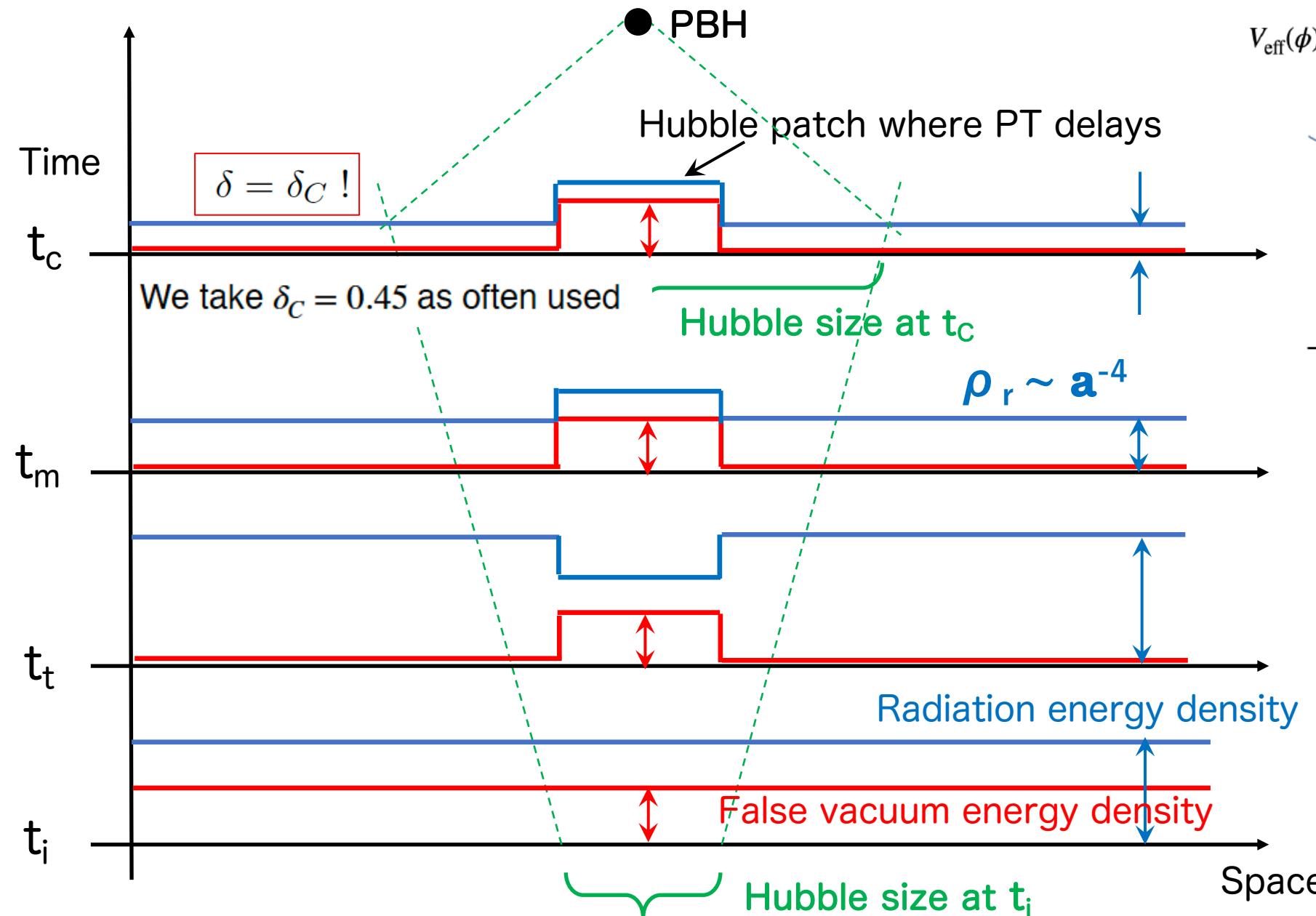
Improve in near future

2023~ PRIME telescope
2026~ Roman telescope

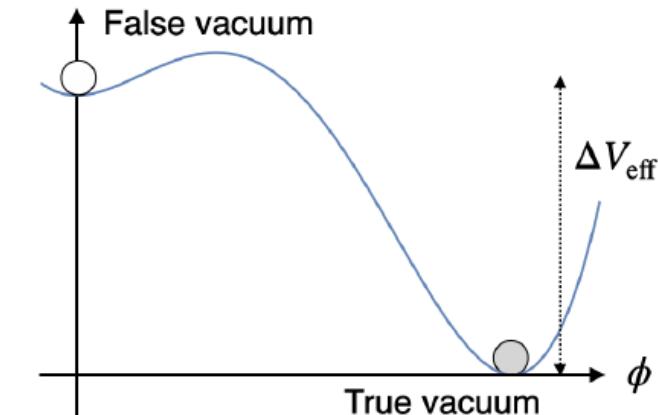


PBH formation from the contrast

$$\delta = \frac{\rho_{\text{over}} - \rho_{\text{back}}}{\rho_{\text{back}}} > \delta_C$$



$$\delta_C = 0.45$$



False vacuum
energy density

$\rho_v \sim \text{const}$
 $\sim F(t) \Delta V$

Radiation
energy density

$\rho_r \sim a^{-4}$

Rate of staying at the symmetric phase in a Hubble volume

$$F(t) = \exp \left[-\frac{4\pi}{3} \int_{t_i}^t dt' \Gamma(t') a^3(t') r^3(t, t') \right] \quad r(t, t') \equiv \int_{t'}^t a^{-1}(\tau) d\tau$$

Radiation Dominant $p = \rho/3$

$$\rho_v = F(t) \Delta V \quad \begin{array}{l} \text{vacuum energy density} \\ \text{is constant} \end{array}$$

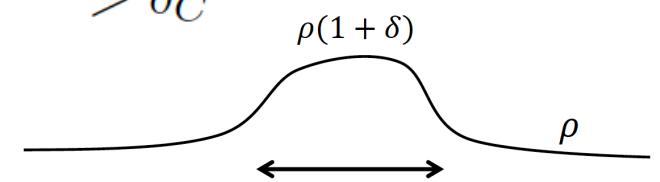
Friedmann eq $H^2 = \frac{\rho_v + \rho_r + \rho_w}{3}$ ρ_r Radiation (wall) energy density $\propto a^{-4}(t)$

Conservation $\frac{d(\rho_r + \rho_w)}{dt} + 4H(\rho_r + \rho_w) = \left(-\frac{d\rho_v}{dt} \right) \rho_w \quad (v_w = 1)$

$\rho_r(t)$, $H(t)$ are determined

Then, from the criterion $\delta_c = 0.45$,
 T_{PBH} is determined

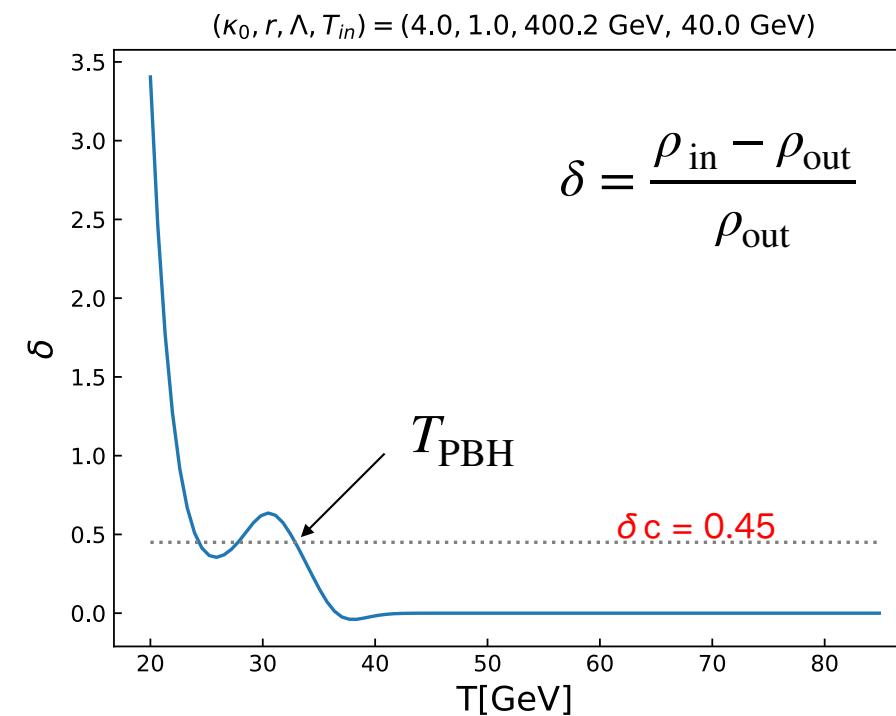
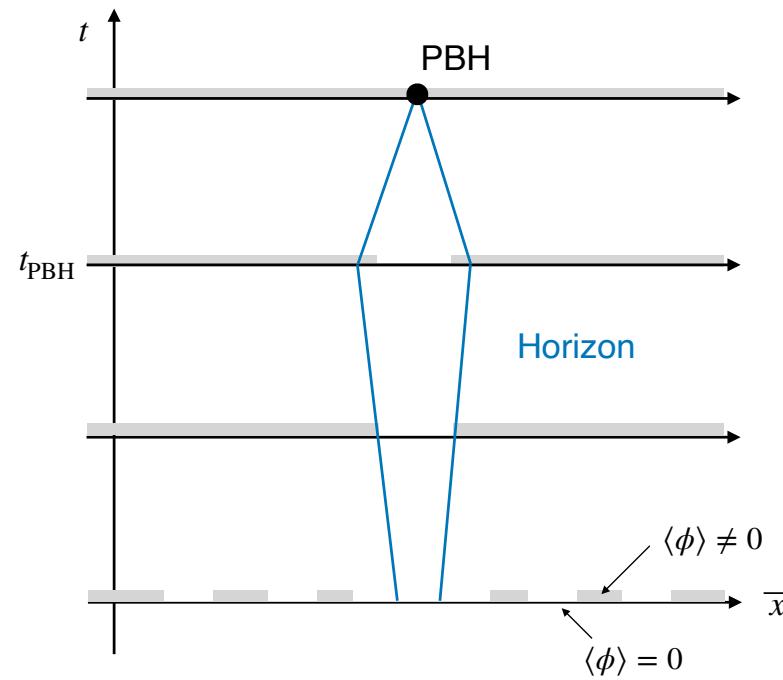
$$\delta = \frac{\rho_{\text{over}} - \rho_{\text{back}}}{\rho_{\text{back}}} > \delta_c$$



$\delta > 0.45$ gravitational collapse!

How to calculate the fraction of PBH

1. Evaluate the possibility that the symmetry is not broken in a Hubble volume at t_{PBH}
2. Calculate how many Hubble patches at t_{PBH} are included in the present Hubble volume



$$f_{\text{PBH}}^{\text{EW}} \equiv \frac{\Omega_{\text{PBH}}^{\text{EW}}}{\Omega_{\text{CDM}}} \sim 1.49 \times 10^{11} \left(\frac{0.25}{\Omega_{\text{CDM}}} \right) \left(\frac{T_{\text{PBH}}}{100 \text{ GeV}} \right) P(t_{\text{PBH}})$$

$$P(t_n) = \exp \left[-\frac{4\pi}{3} \int_{t_i}^{t_n} \frac{a^3(t)}{a^3(t_{\text{PBH}})} H^{-3}(t_{\text{PBH}}) \Gamma(t) dt \right]$$

PBH from 1st OEWPT

K. Hashino, SK, T. Takahashi, 2021

K. Hashino, SK, T. Takahashi, M. Tanaka 2023

Mass of PBH from EWPT is determined by t_{PBH}

$$M_{\text{PBH}} \approx \frac{4\pi}{3} H^{-3}(t_{\text{PBH}}) \rho_c = 4\pi H^{-1}(t_{\text{PBH}})$$

$$M_{\text{PBH}} \sim 10^{-5} M_{\odot}$$

Microlensing observations

Subaru HSC <https://hsc.mtk.nao.ac.jp/ssp>

OGLE <http://ogle.astrow.u.edu.pl/>

Future observations

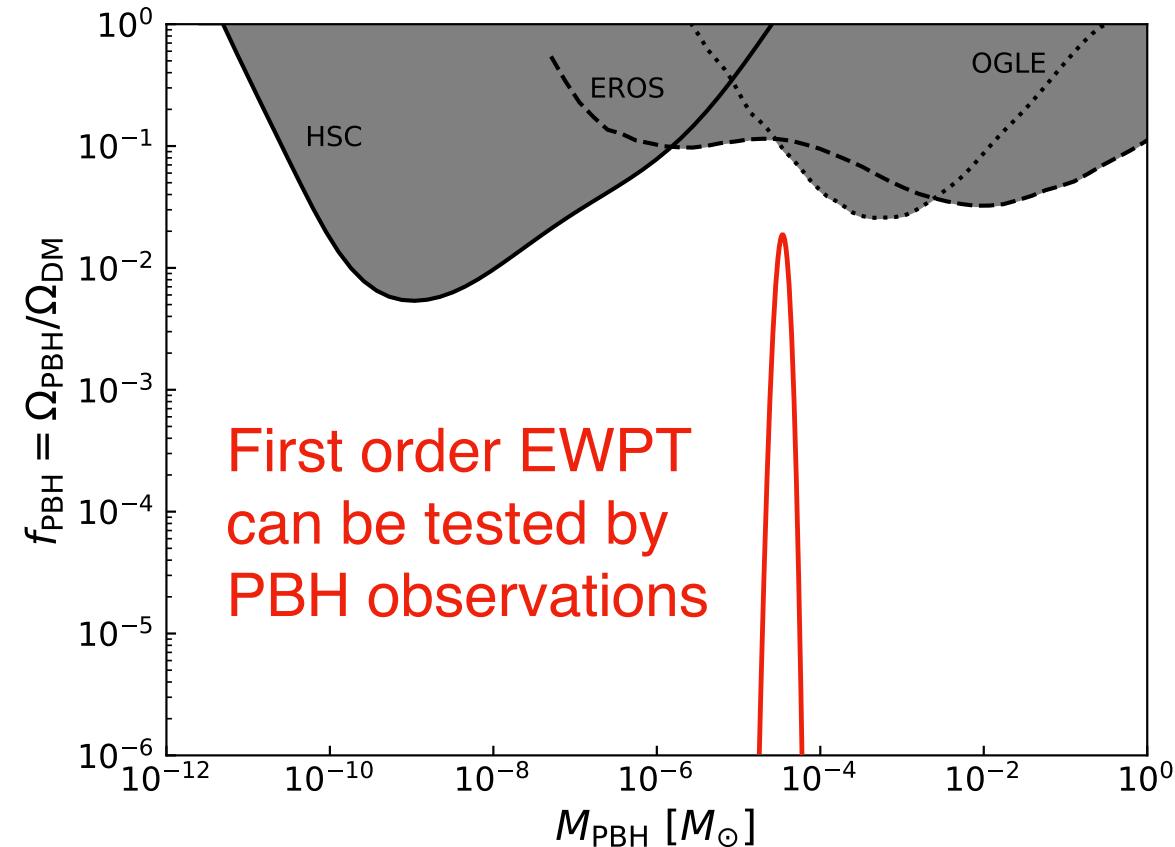
PRIME 2023~

<http://www-ir.ess.sci.osaka-u.ac.jp/prime/index.html>

Roman 2026~

<https://roman.gsfc.nasa.gov>

f_{PBH} is constrained by 10^{-4}



First order EWPT
can be tested by
PBH observations

Using far infrared rays:
sensitive to the microlensing
from center galaxy

Theory prediction on α - β plane

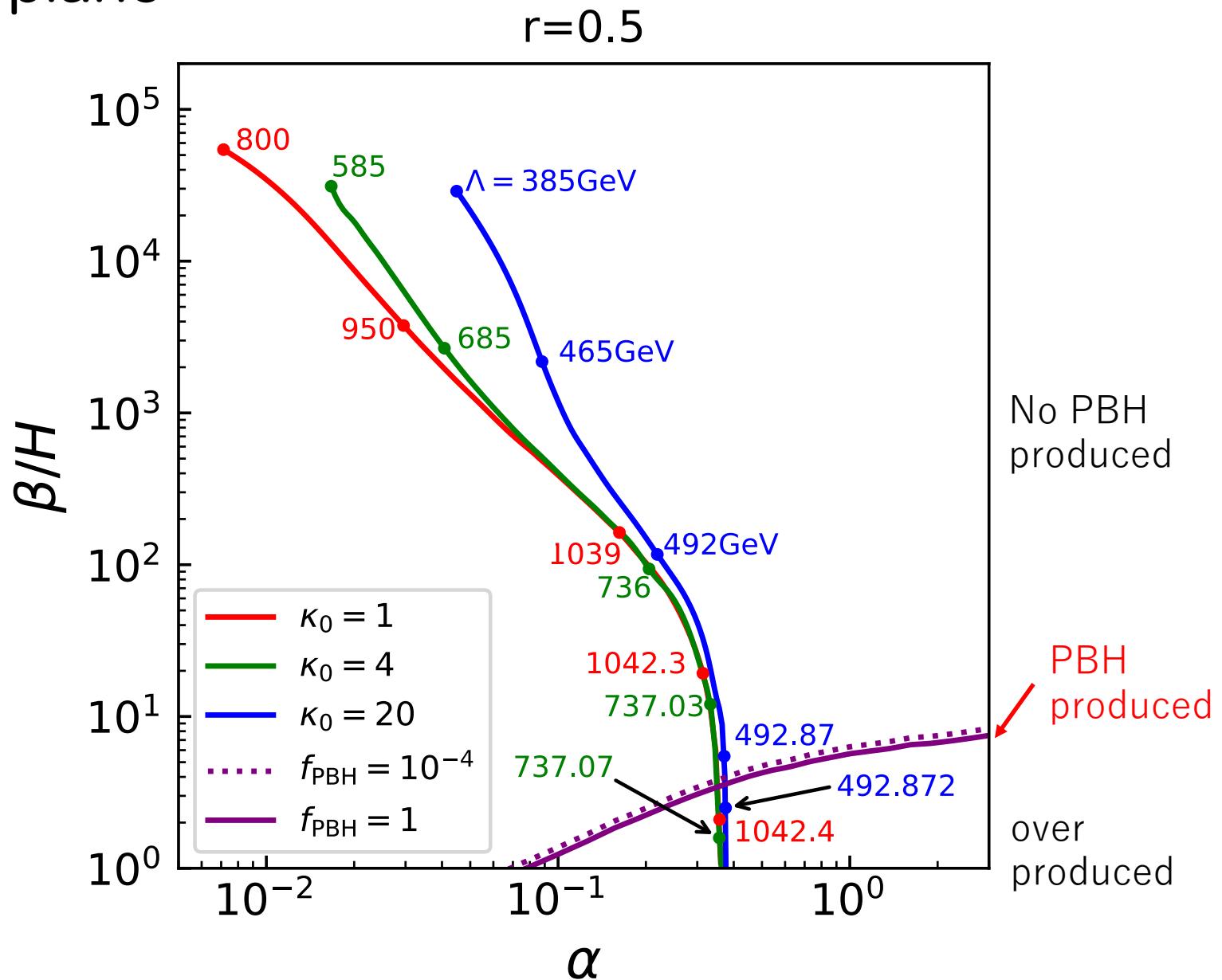
Non-dec $r = 0.5$

DOF $\kappa_0 = 1, 4, 20$
for various Λ

New scale

With contours of f_{PBH}

$10^{-4} < f_{\text{PBH}} < 1$



Strongly 1st OPT

Parameter region

Sphaleron decoupling

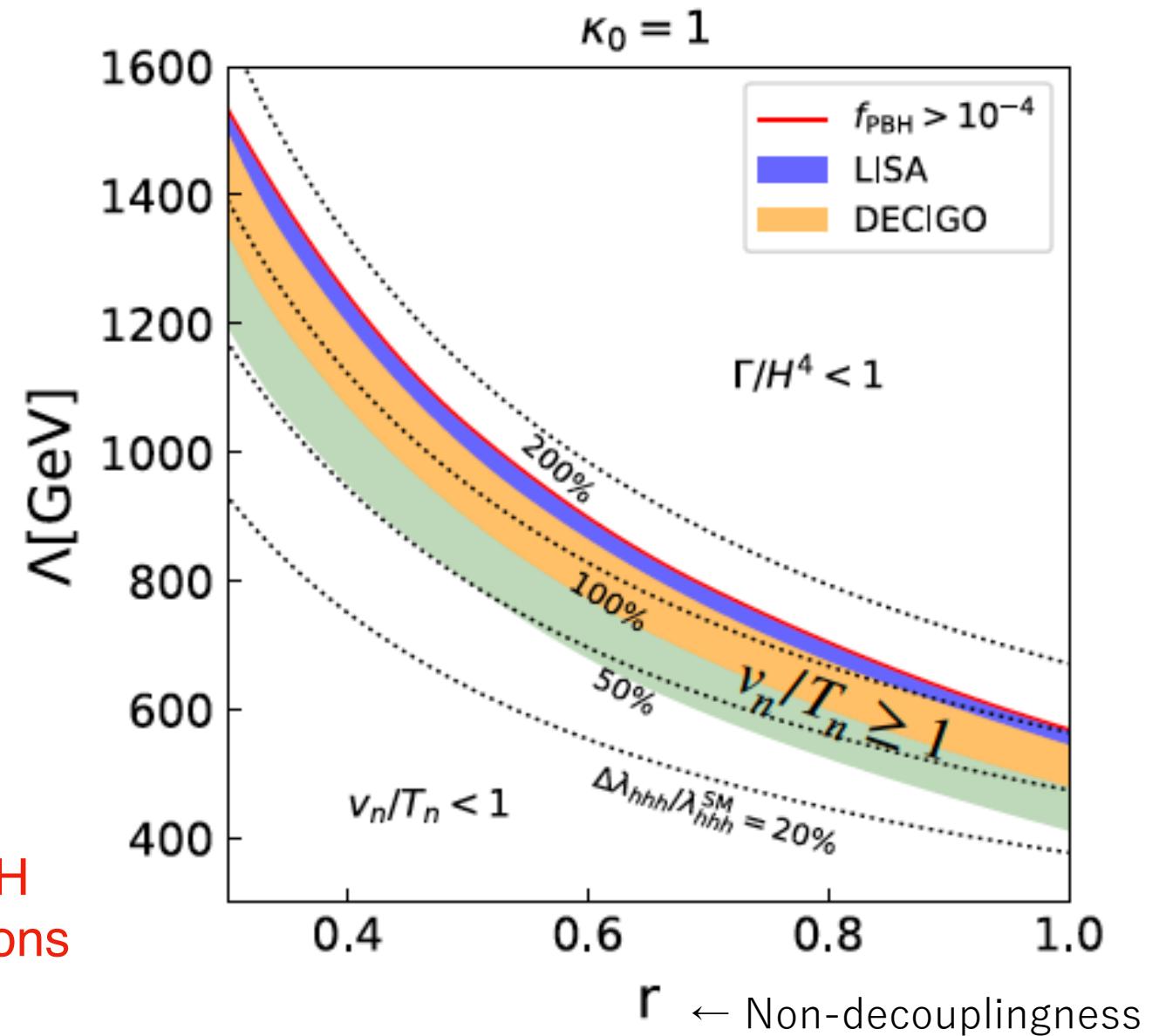
$$\frac{\varphi_c}{T_c} \gtrsim 1$$

Bubble nucleation completion

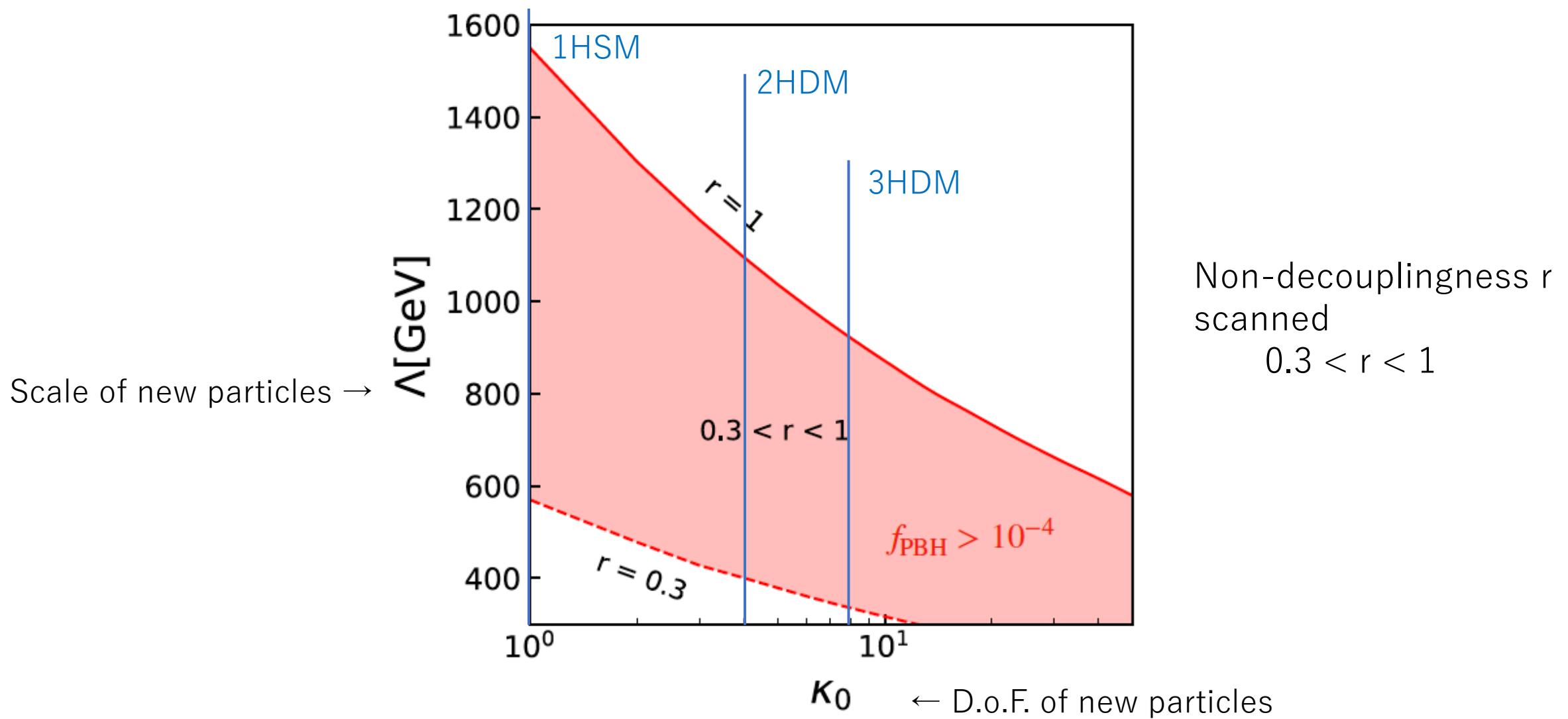
$$\left. \frac{\Gamma}{H^4} \right|_{T=T_t} \simeq 1$$

- PBH (red)
- GW (LISA)
- GW (DECIGO)
- Only $\Delta \lambda_{hhh}$ (HL-LHC, ILC, ...)

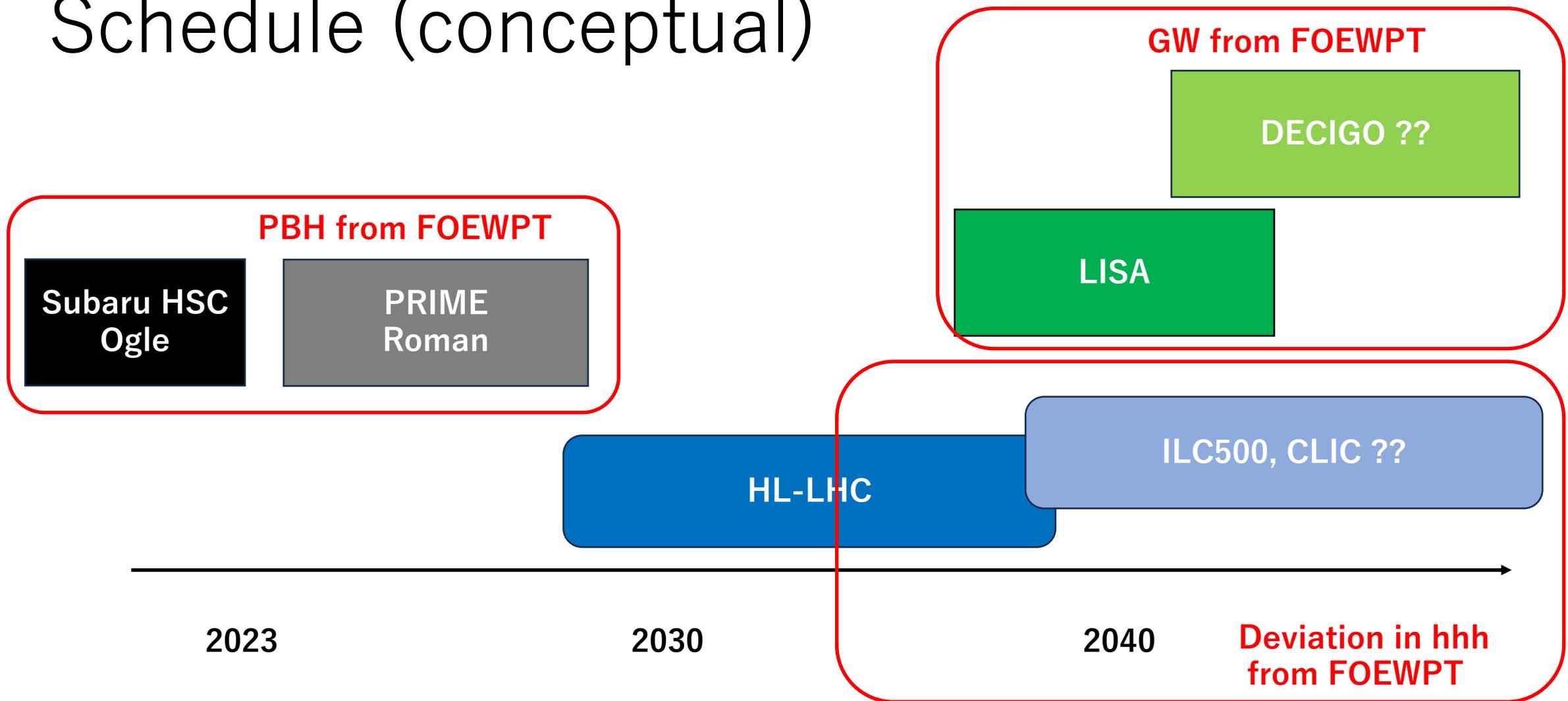
First order EWPT can be explored by PBH observations in addition to GW observations and collider experiments



Parameter Regions testable by PBH



Schedule (conceptual)



Complementary

Summary

- EWPT is the next target
 - If non-decoupling property, EWPT can be strongly first order

Thank you!

From bubble dynamics to GW spectrum

Nucleation rate per
time and volume
Linde 1983

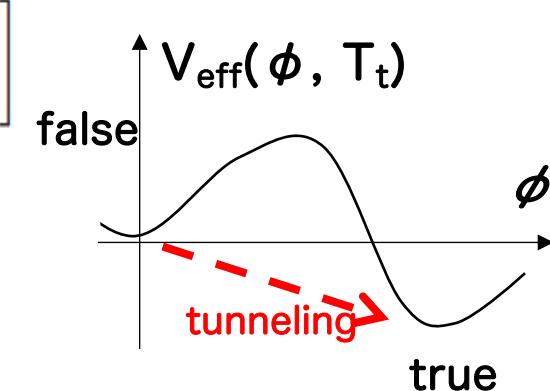
$$\Gamma(T) = \Gamma_0 \exp(-S_3/T)$$

$$S_3 = \int d^3r \left[\frac{1}{2}(\vec{\nabla}\varphi_b)^2 + V_{\text{eff}}(\varphi_b, T) \right]$$

T_t transition temp
is determined by

$$\left. \frac{\Gamma}{H^4} \right|_{T=T_t} \simeq 1$$

$$\frac{S_3(T_t)}{T_t} = 4 \ln(T_t/H_t) \simeq 140$$



$$\left. \begin{array}{ll} \alpha & \text{Latent heat (released E of False vacuum)} \\ & \epsilon(T) = -V_{\text{eff}}(\varphi_B(T), T) + T \frac{\partial V_{\text{eff}}(\varphi_B(T), T)}{\partial T} \\ \beta & \text{Inverse of duration of phase transition} \\ & \beta = - \left. \frac{dS_E}{dt} \right|_{t=t_t} \simeq \left. \frac{1}{\Gamma} \frac{d\Gamma}{dt} \right|_{t=t_t} \quad \tilde{\beta} = \frac{\beta}{H_t} \end{array} \right\}$$

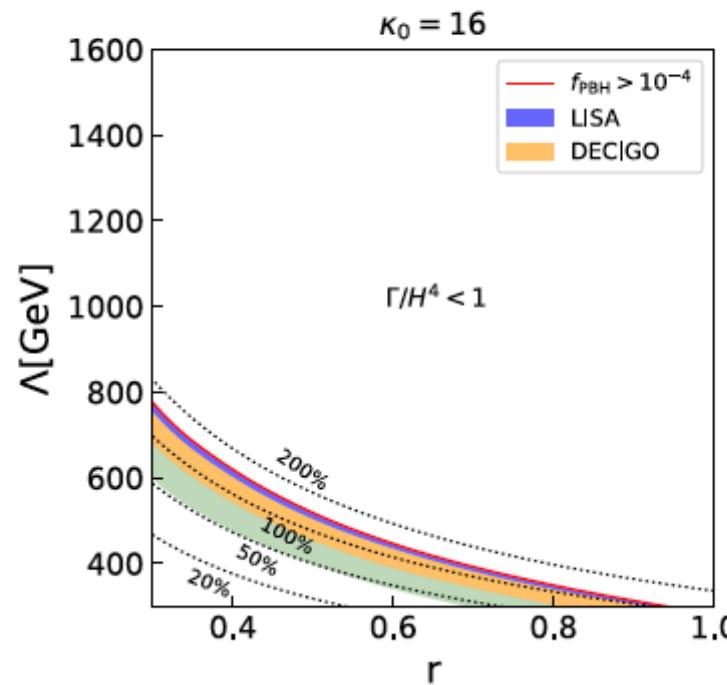
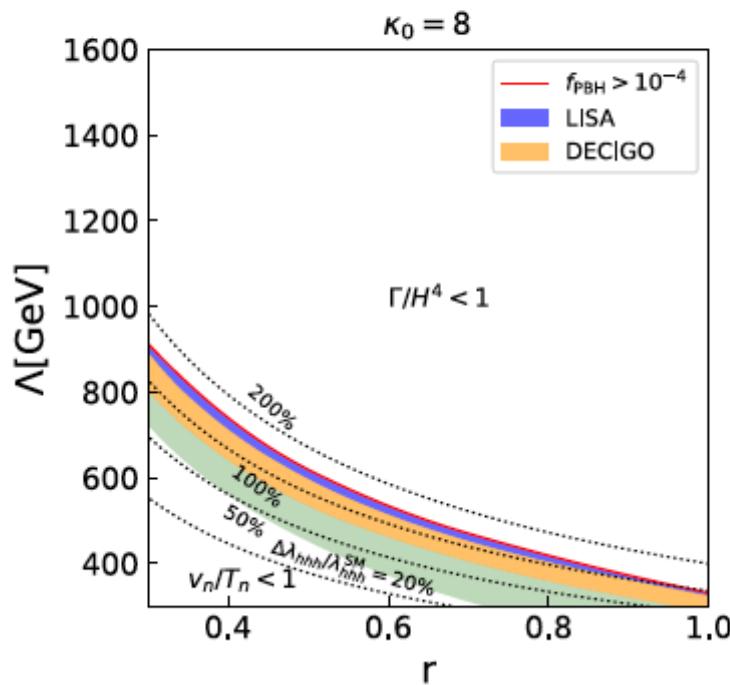
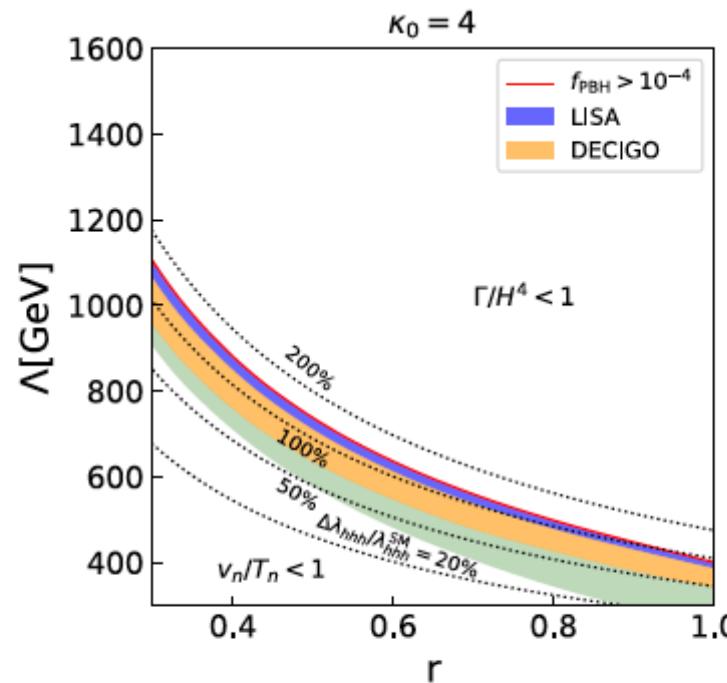
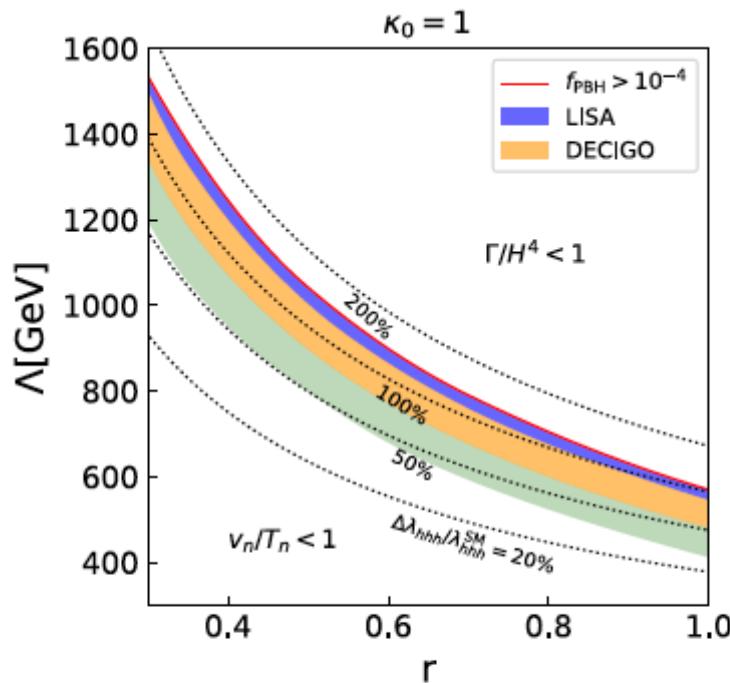
GW Spectrum is determined by T_t, α, β, v_b

C.Caprini et al., arXiv:1512.06239

ex) GW strength and peak frequency from sound waves (Fitting function)

$$\tilde{\Omega}_{\text{sw}} h^2 \simeq 2.65 \times 10^{-6} \frac{v_b}{\tilde{\beta}} \left(\frac{\kappa(v_b, \alpha) \alpha}{1 + \alpha} \right)^2 \quad \tilde{f}_{\text{sw}} \simeq 1.9 \times 10^{-5} \text{Hz} \frac{\tilde{\beta}}{v_b}$$

v_b : wall velocity



Nearly aligned Higgs EFT

Higgs EFT

Fergulio (1993),
Giudice et al (2007), ...

SK, R Nagai (2021)

$$\mathcal{L}_{\text{naHEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}},$$

$$\mathcal{L}_{\text{BSM}} = \xi \left[-\frac{\kappa_0}{4} [\mathcal{M}^2(h)]^2 \ln \frac{\mathcal{M}^2(h)}{\mu^2} \right]$$

$$\xi = \frac{1}{16\pi^2} \quad U = \exp \left(\frac{i}{v} \pi^a \tau^a \right)$$
$$\mathcal{M}^2(h), \mathcal{F}(h), \mathcal{K}(h), \mathcal{Y}_\psi^{ij}(h), \hat{\mathcal{Y}}_\psi^{ij}(h)$$

arbitrary polinominals

$$+ \frac{v^2}{2} \mathcal{F}(h) \text{Tr} [D_\mu U^\dagger D^\mu U] + \frac{1}{2} \mathcal{K}(h) (\partial_\mu h) (\partial^\mu h)$$
$$- v \left(\bar{q}_L^i U \left[\mathcal{Y}_q^{ij}(h) + \hat{\mathcal{Y}}_q^{ij}(h) \tau^3 \right] q_R^j + h.c. \right) - v \left(\bar{l}_L^i U \left[\mathcal{Y}_l^{ij}(h) + \hat{\mathcal{Y}}_l^{ij}(h) \tau^3 \right] l_R^j + h.c. \right)$$

To describe non-decoupling effects
we put a CW type structure (1-loop)

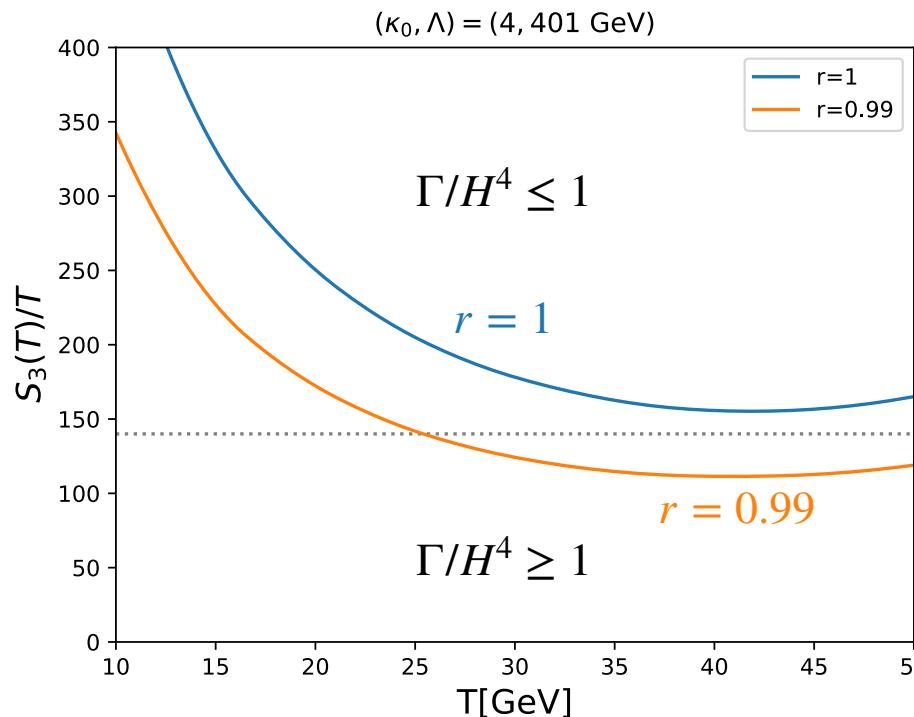
Further assume the form

$$\mathcal{M}^2(h) = M^2 + \frac{\kappa_p}{2} (h + v)^2$$

Bubble nucleation

- Nucleation rate of vacuum bubbles [Linde; Nucl. Phys. B216 (1983)]

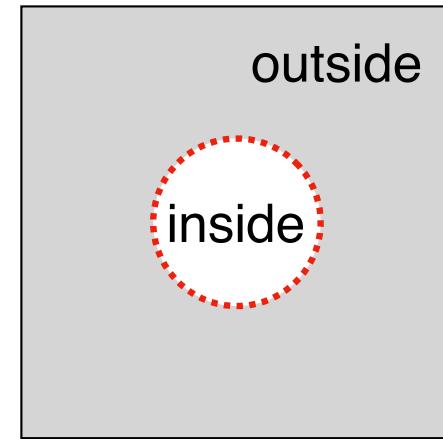
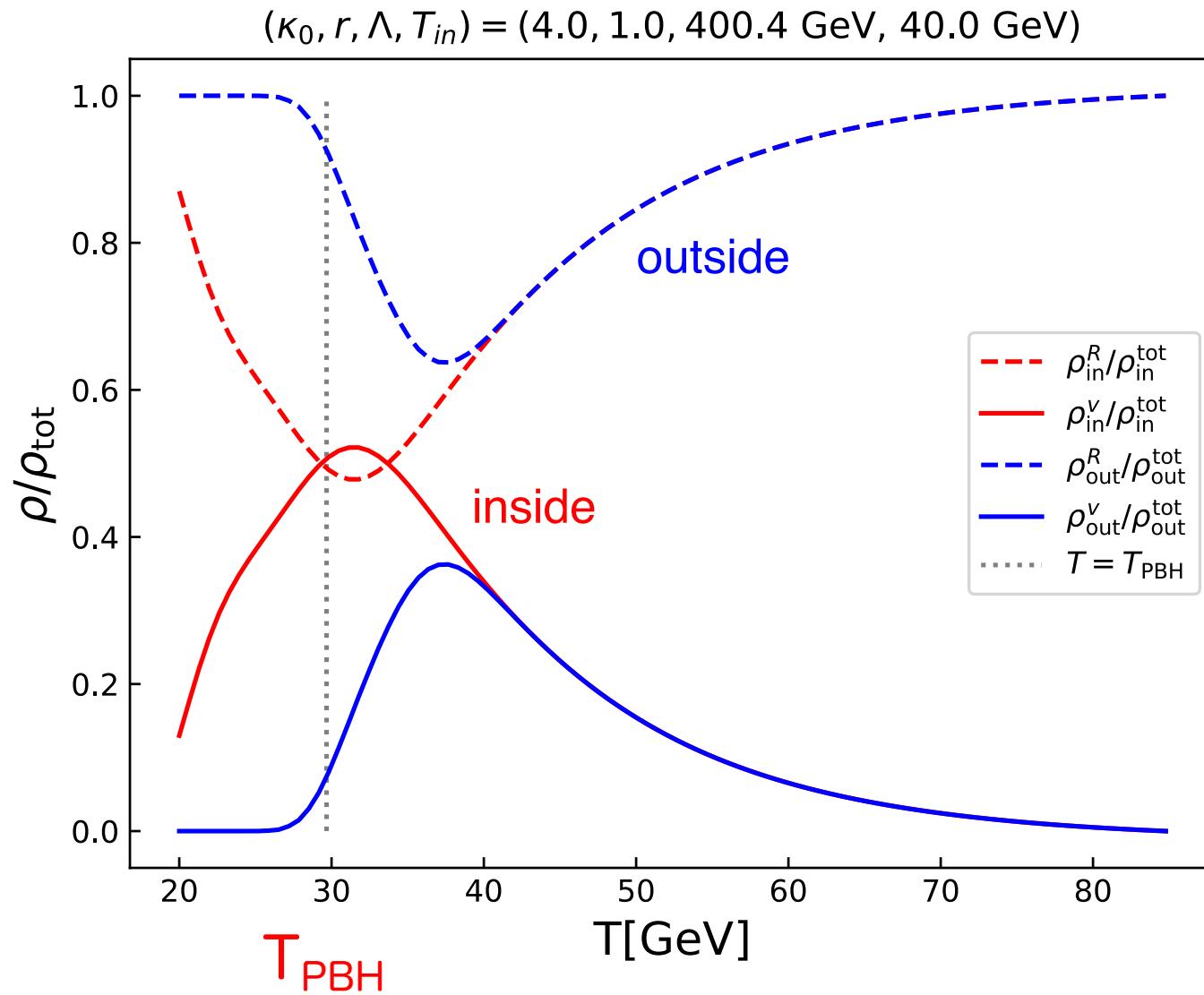
$$\Gamma_{\text{bubble}} \simeq A(T) \exp \left[-\frac{S_3(T)}{T} \right], \quad S_3(T) = \int d^3x \left[\frac{1}{2} (\nabla \varphi^b)^2 + V_{\text{eff}}(\varphi^b, T) \right]$$



Non-decoupling effects are required
to realize the delay of first-order EWPT

$$\Gamma/H^4 = 1 \Leftrightarrow S_3/T \sim 140$$

Ratio of energy density

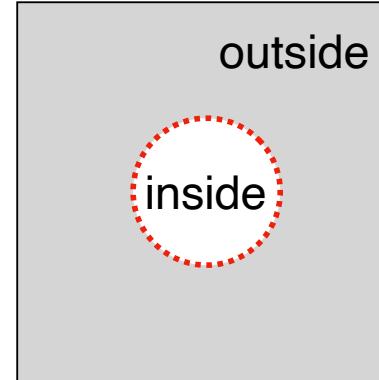
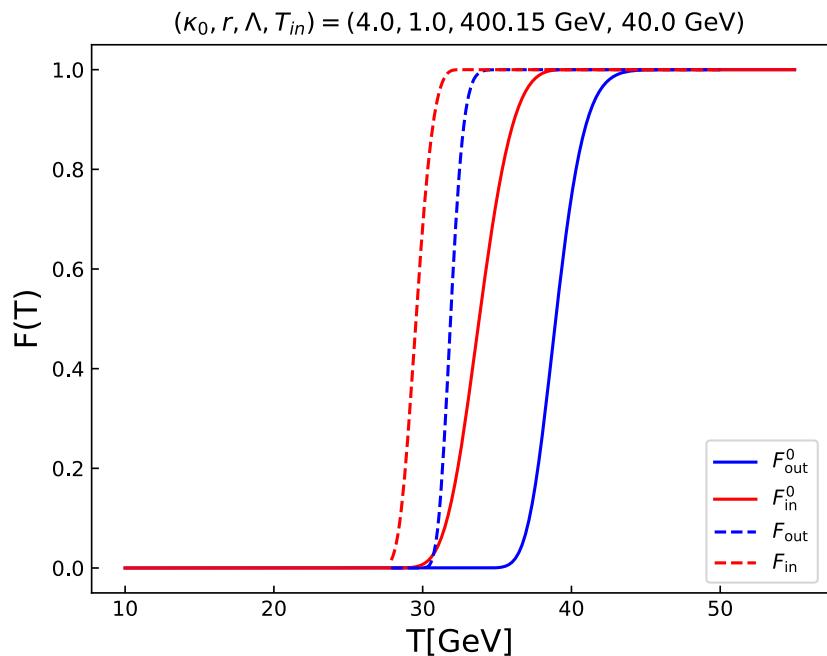


$$\rho_{\text{tot}} = \rho_{\text{rad}} + \rho_{\text{vac}}$$

$$\delta = \frac{\rho_{\text{over}} - \rho_{\text{back}}}{\rho_{\text{back}}} > \delta_C$$

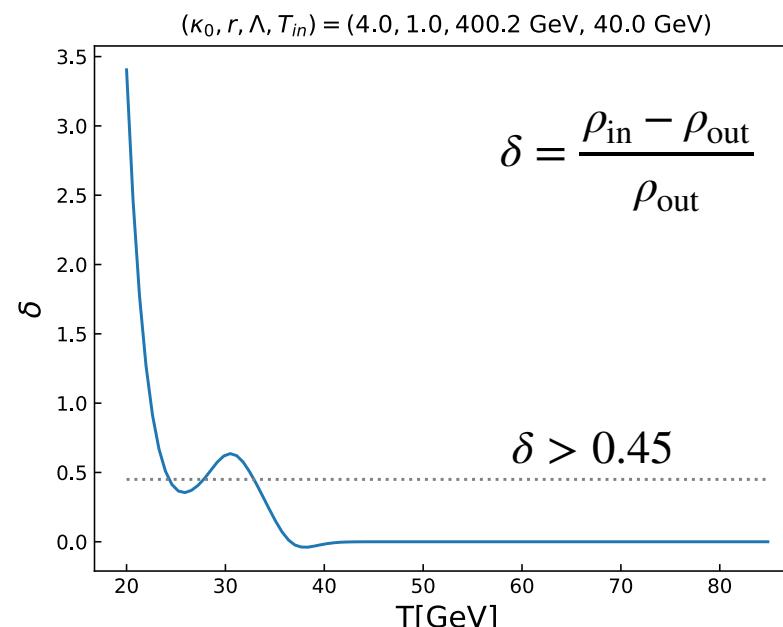
$$\delta C = 0.45$$

Fraction of the false vacuum



$$F(t) = \exp \left[-\frac{4\pi}{3} \int_{t_i}^t dt' \Gamma(t') a^3(t) r^3(t, t') \right]$$

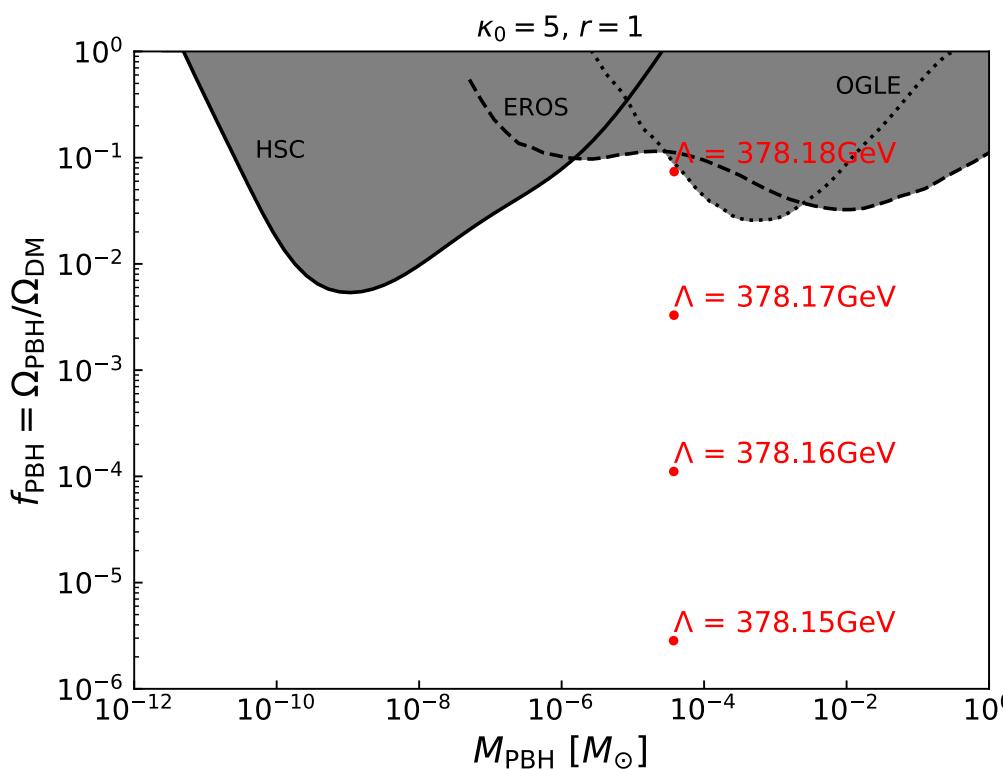
$$r(t, t') \equiv \int_{t'}^t \frac{v_w}{a(\tilde{t})} d\tilde{t}$$



Fraction of PBH

$$f_{\text{PBH}}^{\text{EW}} \equiv \frac{\Omega_{\text{PBH}}^{\text{EW}}}{\Omega_{\text{CDM}}} \sim 1.49 \times 10^{11} \left(\frac{0.25}{\Omega_{\text{CDM}}} \right) \left(\frac{T_{\text{PBH}}}{100 \text{ GeV}} \right) P(t_{\text{PBH}}),$$

$$P(t_n) = \exp \left[-\frac{4\pi}{3} \int_{t_i}^{t_n} \frac{a^3(t)}{a^3(t_{\text{PBH}})} \frac{1}{H^3(t_{\text{PBH}})} \Gamma(t) dt \right], \quad \Gamma_{\text{bubble}}(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} \exp \left[-\frac{S_3(T)}{T} \right],$$



General 2HDM

Most general Higgs potential

$$\begin{aligned} V = & -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \left(\mu_3^2(\Phi_1^\dagger \Phi_2) + h.c. \right) \\ & + \frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) \\ & + \left\{ \left(\frac{1}{2}\lambda_5\Phi_1^\dagger \Phi_2 + \lambda_6\Phi_1^\dagger \Phi_1 + \lambda_7\Phi_2^\dagger \Phi_2 \right) \Phi_1^\dagger \Phi_2 + h.c. \right\}, \quad (\mu_3, \lambda_5, \lambda_6, \lambda_7 \in \mathbb{C}) \end{aligned}$$

Higgs basis

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_1 + iG^0) \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2 + ih_3) \end{pmatrix}$$

Most general Yukawa Interation

$$\begin{aligned} \mathcal{L}_y \supset & \sum_{ij} \left\{ \sum_{k=1}^3 -\overline{f_{iL}} g_{k,ij}^f f_{jR} H_k \right. \\ & \left. + \left\{ \overline{u_{iR}} (\rho^u{}^\dagger V_{CKM})_{ij} d_{jL} - \overline{u_{iL}} (V_{CKM} \rho^d)_{ij} d_{jR} - \overline{\nu_{iL}} \rho_{ij}^e e_{jR} \right\} H^+ \right\} + h.c., \end{aligned}$$

How we narrow down?
No principle

Use experimental data!
and basic requirements

Higgs alignment

Higgs potenshal

$$\begin{aligned}
 V = & -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - (\mu_3^2(\Phi_1^\dagger \Phi_2) + h.c.) \\
 & + \frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) \\
 & + \left\{ \left(\frac{1}{2}\lambda_5\Phi_1^\dagger \Phi_2 + \lambda_6\Phi_1^\dagger \Phi_1 + \lambda_7\Phi_2^\dagger \Phi_2 \right) \Phi_1^\dagger \Phi_2 + h.c. \right\}, \quad (\mu_3, \lambda_5, \lambda_6, \lambda_7 \in \mathbb{C})
 \end{aligned}$$

Higgs basis

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_1 + iG^0) \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2 + ih_3) \end{pmatrix}$$

$$m_{H^\pm}^2 = M^2 + \frac{1}{2}\lambda_3 v^2$$

To satisfy LHC data, need to avoid mixing between h and heavy Higgs bosons: $\lambda_6 \sim 0$

Higgs
Alignment
(h : SM-like)

Mass matrix
of neutral scalar
bosons

$$\mathcal{M}^2 = v^2 \begin{pmatrix} \lambda_1 & \text{Re}[\lambda_6] & -\text{Im}[\lambda_6] \\ \text{Re}[\lambda_6] & \frac{M^2}{v^2} + \frac{1}{2}(\lambda_3 + \lambda_4 + \text{Re}[\lambda_5]) & -\frac{1}{2}\text{Im}[\lambda_5] \\ -\text{Im}[\lambda_6] & -\frac{1}{2}\text{Im}[\lambda_5] & \frac{M^2}{v^2} + \frac{1}{2}(\lambda_3 + \lambda_4 - \text{Re}[\lambda_5]) \end{pmatrix} = \begin{pmatrix} m_h^2 & 0 & 0 \\ 0 & m_{H_2}^2 & 0 \\ 0 & 0 & m_{H_3}^2 \end{pmatrix}$$

rephasing

Physical Phase in the Higgs potential : $\arg[\lambda_7] \equiv \theta_7$

We work on this Higgs alignment scenario in the following discussion

Simply $\lambda_6 = 0$

2HDM (scenario 1)

Higgs potential

$$\begin{aligned} V = & -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - (\mu_3^2(\Phi_1^\dagger \Phi_2) + h.c.) \\ & + \frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) \\ & + \left\{ \left(\frac{1}{2}\lambda_5 \Phi_1^\dagger \Phi_2 + \lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2 \right) \Phi_1^\dagger \Phi_2 + h.c. \right\}, \quad (\mu_3, \lambda_5, \lambda_6, \lambda_7 \in \mathbb{C}) \end{aligned}$$

Higgs basis

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_1 + iG^0) \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2 + ih_3) \end{pmatrix}$$

$$m_{H^\pm}^2 = M^2 + \frac{1}{2}\lambda_3 v^2$$

To satisfy LHC data, avoid mixing between h and heavy Higgs bosons: $\lambda_6 \sim 0$

Mass matrix of neutral scalar bosons

$$\mathcal{M}^2 = v^2 \begin{pmatrix} \lambda_1 & \text{Re}[\lambda_6] & -\text{Im}[\lambda_6] \\ \text{Re}[\lambda_6] & \frac{M^2}{v^2} + \frac{1}{2}(\lambda_3 + \lambda_4 + \text{Re}[\lambda_5]) & -\frac{1}{2}\text{Im}[\lambda_5] \\ -\text{Im}[\lambda_6] & -\frac{1}{2}\text{Im}[\lambda_5] & \frac{M^2}{v^2} + \frac{1}{2}(\lambda_3 + \lambda_4 - \text{Re}[\lambda_5]) \end{pmatrix} = \begin{pmatrix} m_h^2 & 0 & 0 \\ 0 & m_{H_2}^2 & 0 \\ 0 & 0 & m_{H_3}^2 \end{pmatrix}$$

Higgs alignment

$\arg[\lambda_7] \equiv \theta_7$

rephasing

Avoiding FCNC: Yukawa alignment is imposed by hand $y_f^2 = \zeta_f y_f^1$ ($f = u, d, e$)

$$\mathcal{L}_y = -\overline{Q}_L \frac{\sqrt{2}M_u}{v} (\tilde{\Phi}_1 + \zeta_u^* \tilde{\Phi}_2) u_R - \overline{Q}_L \frac{\sqrt{2}M_d}{v} (\Phi_1 + \zeta_d \Phi_2) d_R - \overline{L}_L \frac{\sqrt{2}M_e}{v} (\Phi_1 + \zeta_e \Phi_2) e_R + h.c.$$

Yukawa alignment

Pich and Tuzon (2009)

Multiple CPV phases

Higgs potential $\arg[\lambda_7] \equiv \theta_7$
Yukawa couplings $\arg[\zeta_u] \equiv \theta_u, \arg[\zeta_d] \equiv \theta_d, \arg[\zeta_e] \equiv \theta_e$