The Higgs, the top and the singlet scalar – gravity and the stability of the effective potential

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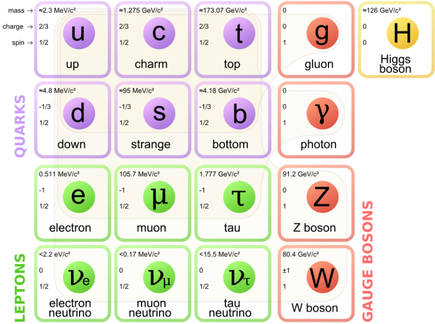
Outline

The Standard Model



'Standard Model of Elementary Particles'
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What is missing?

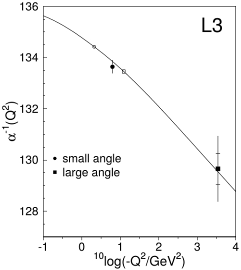
- dark matter
- inflaton
- dark energy
- gravity

The running of the constants

Are the coupling constants constant in quantum field theory?

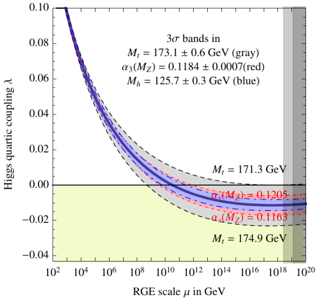
- To be meaningful quantum field theory requires renormalization.
- Renormalization introduces momentum/energy dependence to the renormalized constants.

The running of the fine structure constant



L3 Collaboration, Phys. Lett. B *476 (2000) 40*

The running of the Higgs quartic constant

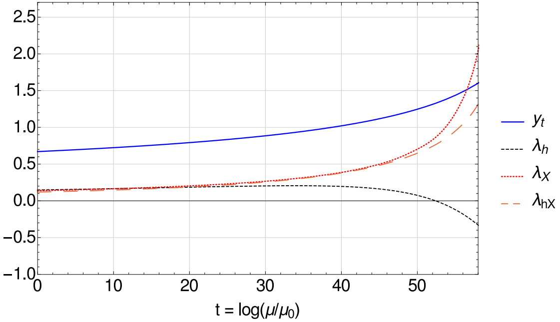


G. Degrassi et al. JHEP 08 (2012) 98

The scalar singlet extension of the Standard Model:

$$V_{HX} = m_H^2|H|^2 + \lambda_h|H|^4 + m_X^2 X^2 + \lambda_X X^4 + \lambda_{hX}|H|^2 X^2.$$

- A review of the properties of X and the flat spacetime stability of the extended SM in the context of LHC:
  T. Robens, T. Stefaniak, Europ. Phys. J. **C** 03 (2015) 75
- The running of the quartic and the top Yukawa couplings:

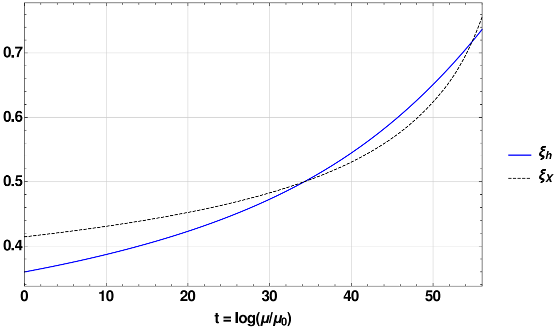


- $v_h = 246.2$ GeV,
  $v_X = v_h/\tan\beta$,
  $\tan\beta = 0.3$,
- $m_{H_-} = 125.5$ GeV,
  $m_{H_+} = 650$ GeV,
- $\sin(\alpha) = 0.15$,
- $y_t = 0.9359$

- Tree-level potential of scalars in the presence of gravity:

$$V_{HX} = m_H^2|H|^2 + \lambda_h|H|^4 - \xi_h|H|^2R + m_X^2X^2 + \lambda_X X^4 - \xi_X X^2 R + \lambda_{hX}|H|^2X^2.$$

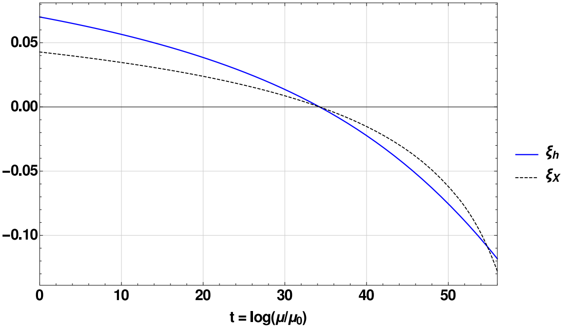
- The running of the non-minimal coupling of scalars to gravity
  – the $\xi_h = \xi_X = 0.5$ case



- Tree-level potential of scalars in the presence of gravity:

$$V_{HX} = m_H^2|H|^2 + \lambda_h|H|^4 - \xi_h|H|^2R + m_X^2X^2 + \lambda_XX^4 - \xi_XX^2R + \lambda_{hX}|H|^2X^2.$$

- The running of the non-minimal coupling of scalars to gravity
  – the $\xi_h = \xi_X = 0$ case



The one-loop effective potential for the Higgs-top-mediator sector:

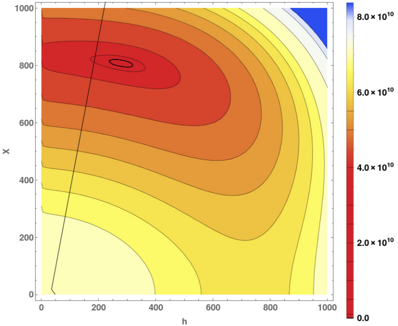
$$V^{(1)} = -\Big\{ -\frac{1}{2}\left[m_h^2 - \xi_h R\right] h^2 - \frac{\lambda_h}{4} h^4 - \frac{\lambda_{hX}}{4} h^2 X^2 - \frac{1}{2}\left[m_X^2 - \xi_X R\right] X^2 - \frac{\lambda_X}{4} X^4 +$$
$$+ \frac{\hbar}{64\pi^2}\Big[- a_+^2 \ln\left(\frac{a_+}{\mu^2}\right) - a_-^2 \ln\left(\frac{a_-}{\mu^2}\right) + \frac{3}{2}\left(a_+^2 + a_-^2\right) + 8b^2 \ln\left(\frac{b}{\mu^2}\right) - 12b^2 + \frac{1}{3} y_t^2 h^2 \ln\left(\frac{b}{\mu^2}\right) R - y_t^4 h^4 \ln\left(\frac{b}{\mu^2}\right) +$$
$$- \frac{4}{180}\left(-R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right)\left(\ln\left(\frac{a_+}{\mu^2}\right) + \ln\left(\frac{a_-}{\mu^2}\right) - 2\ln\left(\frac{b}{\mu^2}\right)\right) - \frac{4}{3} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} \ln\left(\frac{b}{\mu^2}\right)\Big]\Big\}.$$

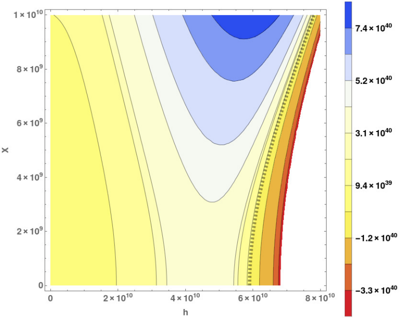
$$b = \frac{1}{2} y_t^2 h^2 - \frac{1}{12} R,$$
$$a_\pm = \frac{1}{2}\Big\{\Big[m_X^2 + m_h^2 - \left(\xi_X + \xi_h - \frac{2}{6}\right) R + \left(3\lambda_h + \frac{1}{2}\lambda_{hX}\right) h^2 + \left(3\lambda_X + \frac{1}{2}\lambda_{hX}\right) X^2\Big] +$$
$$\pm \sqrt{\left[m_X^2 - m_h^2 - \left(\xi_X - \xi_h\right) R + \left(\frac{1}{2}\lambda_{hX} - 3\lambda_h\right) h^2 + \left(3\lambda_X - \frac{1}{2}\lambda_{hX}\right) X^2\right]^2 + 4\left(\lambda_{hX} hX\right)^2}\Big\}.$$

In the radiation dominated Friedmann-Lemaître-Robertson-Walker universe we have:

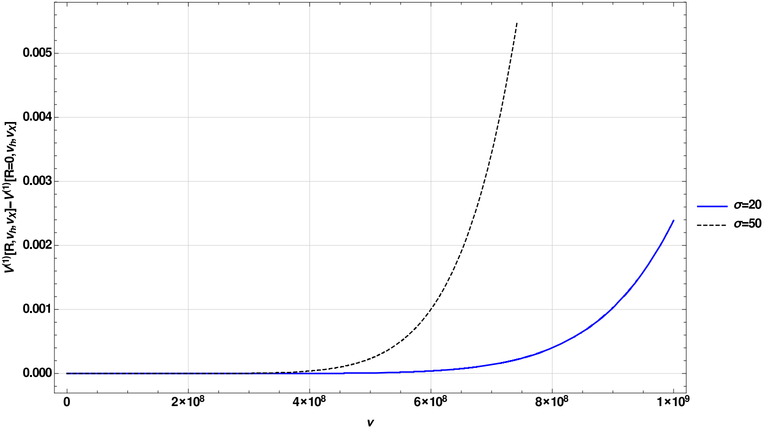
$$R = 0, \quad -R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{4}{3}\left(\bar{M}_P^{-2}\rho\right)^2, \quad R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{8}{3}\left(\bar{M}_P^{-2}\rho\right)^2.$$

The small and large field regimes of the one-loop effective potential; $\mu = \frac{y_t}{\sqrt{2}} h$, $\rho = \sigma\nu^4 + \mu^4$ and $\sigma = 50$, $\nu = 10^9$ GeV:





The influence of gravity in the small field region (around the electroweak minimum):



How big curvature do we need?

- $$V(h^2) = \left[\frac{1}{2}m_h^2 + \frac{1}{64\pi^2}\frac{4}{180}\frac{4}{3}\left(\bar{M}_P^{-2}\rho\right)^2\frac{\tilde{b}}{h^2}\right]h^2 = m_{eff}^2(h)h^2,$$
$$\rho = 4\pi v_h|m_h|\sqrt{\frac{135}{2\tilde{b}}}\bar{M}_P^2 \to \mu \sim 10^{10} \div 10^{11}\,GeV$$

- $$V(h^4) = \frac{1}{4}\left[\lambda_{eff}(h) + \frac{4}{64\pi^2}\frac{4}{3}\frac{8}{3}\left(\bar{M}_P^{-2}\rho\right)^2\frac{\tilde{c}}{h^4}\right]h^4 = \frac{1}{4}\bar{\lambda}_{eff}(h)h^4,$$
$$\rho = 4\pi h_0^2\bar{M}_P^2\sqrt{\frac{9|\lambda_{eff}|}{32\tilde{c}}} \to \mu \sim 10^{13} \div 10^{14}\,GeV$$

Summary

- Using the flat spacetime method for obtaining the effective action above the energy scale of 10^{10}GeV may lead to inaccuracies.
- Classical gravity induces new terms in the effective action.
- These new terms may have an impact on the problem of the stability of the Standard Model vacuum.

Thank you for your attention.

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