Methods for Computing the Critical Temperature of the Electroweak Phase Transition

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The phase transition

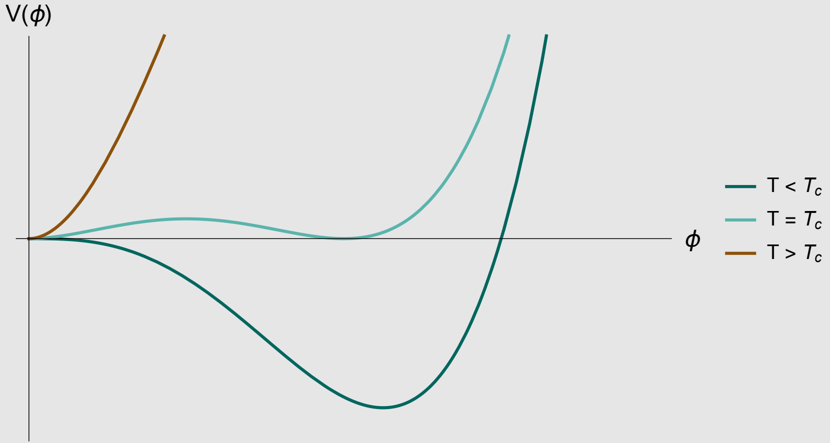

Figure 1: A first order phase transition.

The effective potential

- $V_{\text{eff}}(\phi) = V_0(\phi) + \hbar V_1(\phi) + \hbar^2 V_2(\phi) + \dots$
- The energy density in the presence of a background field $\langle \Phi(x) \rangle = \phi$.
- We can now solve $\frac{\partial V_{\text{eff}}}{\partial \phi}|_{\phi_m} = 0$ to minimize V_{eff} and find the "true" vacuum $\phi_m = \phi_0 + \hbar\phi_1 + \hbar^2\phi_2 + \dots$

The effective potential... at finite T

- $V_{\text{eff}}(\phi, T) = V_0(\phi) + \hbar V_1(\phi, T) + \hbar^2 V_2(\phi, T) + \ldots$
- The background energy density in the presence of a background field $\langle \Phi(x) \rangle_T = \phi(T)$, and a thermal medium with temperature T.
- We can now solve $\frac{\partial V_{\text{eff}}(\phi, T)}{\partial \phi}|_{\phi_m} = 0$ to minimize V_{eff} and find the "true" vacuum $\phi_m(T) = \phi_0 + \hbar\phi_1(T) + \hbar^2\phi_2(T) + \ldots$, in the presence of thermal fluctuations

Gauge dependence of V_{eff}

- In gauge theory, there are "unphysical" degrees of freedom: time-like polarizations of gauge bosons, ghosts and goldstone bosons.
- For observables, these d.o.f.'s must cancel among each other $\implies$ e.g. with R_ξ-gauge fixing, observables should be independent of ξ.
- But the effective potential depends on ξ:

$$\begin{aligned} \text{GB}: m^2_{GB}(\phi,\xi) &= \tilde{m}^2_{GB}(\phi) + \xi m^2_A(\phi) \\ \text{Ghost}: m^2_c(\phi,\xi) &= \xi m^2_A(\phi) \end{aligned}$$

Gauge dependence of V_{eff}

The energy density of the vacuum state *is* physical:

$$\frac{\partial V_{\text{eff}}}{\partial \phi}\Big|_{\phi_m} = 0 \implies \frac{\partial V_{\text{eff}}}{\partial \xi}\Big|_{\phi_m} = 0$$

Gauge dependence of V_{eff}

The energy density of the vacuum state *is* physical:

$$\frac{\partial V_{\text{eff}}}{\partial \phi}\Big|_{\phi_m} = 0 \implies \frac{\partial V_{\text{eff}}}{\partial \xi}\Big|_{\phi_m} = 0$$

- The **Traditional** way of finding the critical temperature T_{c} is to evaluate $V_{\text{eff}}(\phi)$ to one loop and minimize it numerically.
- This induces a gauge dependence in T_c, due to a mixing of orders of $\hbar$.

Gauge dependence of V_{eff}

Patel and Ramsey-Musolf [ArXiv:1101.4665]:
by appropriately truncating the power series in $\hbar$:

$$\phi_m = \phi_0 + \hbar\phi_1 + \ldots \implies$$
$$V_{\text{eff}}\left(\phi_m\right) = V_0\left(\phi_0\right) + \hbar V_1\left(\phi_0\right) + \hbar^2\left(V_2\left(\phi_0\right) - \frac{1}{2}\phi_1^2 \frac{\mathrm{d}^2 V_0}{\mathrm{d}\phi^2}\Big|_{\phi_0}\right) + \mathcal{O}\left(\hbar^3\right)$$

it is possible to determine T_c in a gauge invariant way
(**PRM** method).

PRM method

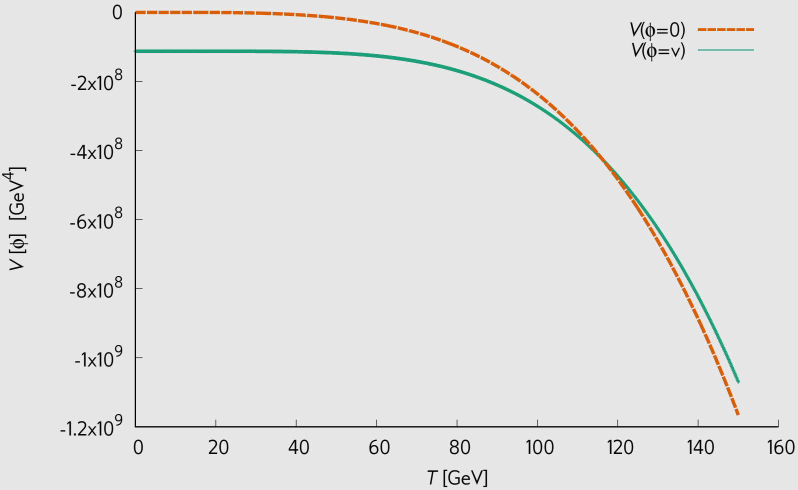

Figure 2: Determination of T_c in the SM, with $m_h = 125.09$ [GeV], using the **PRM** method.

Traditional

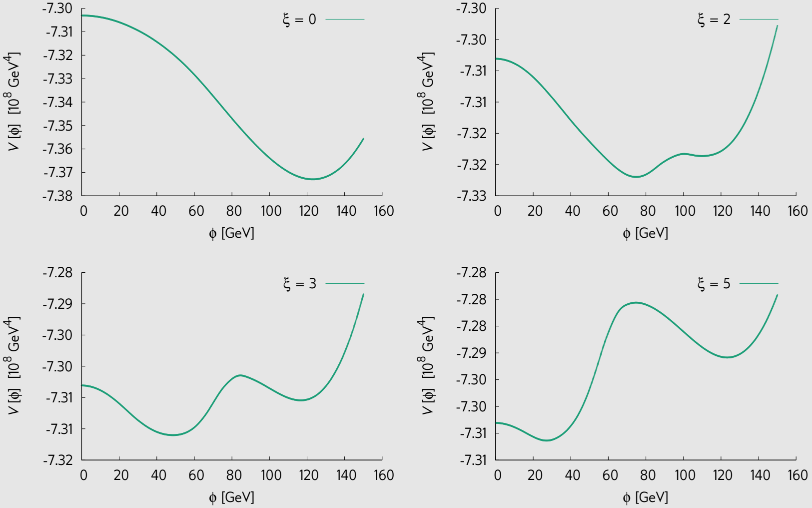

Figure 3: PRELIMINARY plots of $V_{\text{eff}}\left(\phi, T = 75 \text{ [GeV]}\right)$ in the SM, with $m_h = 65$ [GeV], for different values of ξ.

PRM & Traditional

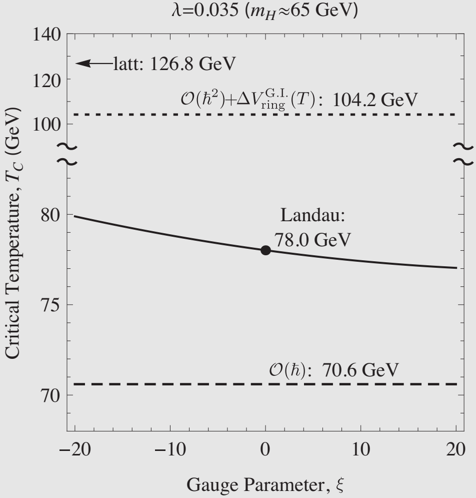

Figure 4: From Patel and Ramsey-Musolf's paper [ArXiv:1101.4665].

Purpose and goals of our C++ implementation

- Provide a testing ground for different models and methods
- Fast numerical evaluations
- Usability: straight-forward implementation of different models

The SM

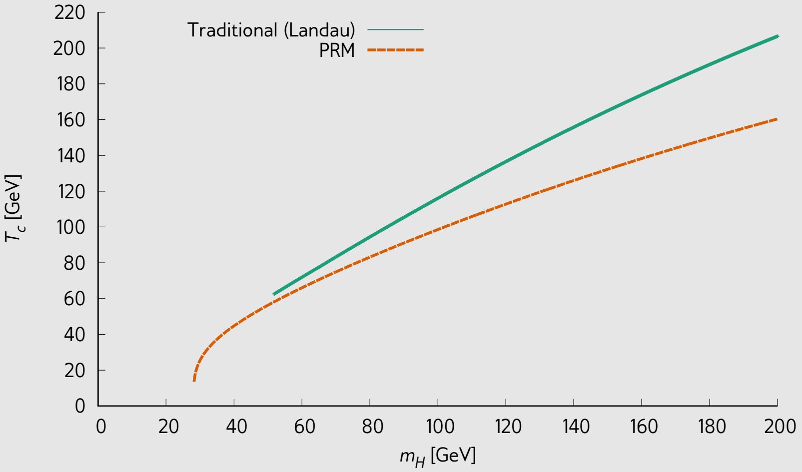

Figure 5: PRELIMINARY determination of T_c in the SM, for different values of the Higgs mass.

Summary

- The effective potential is ξ-dependent, but one can with care extract ξ-independent observables from it.
- Our goal: developing a C++ code to **test** if it is important to care about this ξ-dependence, for different models.

Questions?