Improved analysis of the CLFV process: $\mu^- e^- \rightarrow e^- e^-$ in muonic atom

Masato Yamanaka (Nagoya Univ., Japan)

M. Koike, Y. Kuno, J. Sato, M.Y., Phys. Rev. Lett. 105, 121601

Y. Kuno, J. Sato, T. Sato, Y. Uesaka, M.Y., arXiv:1601.XXXXX

Search for charged lepton flavor violation (cLFV)

- ☑ cLFV is an evidence of extension of scalar sector
- ☑ Various channels (e.g., $\mu \to e\gamma$, μ-e conversion, $\cdots$)
- ☑ Each reaction rate strongly depends on model structure

- ☑ cLFV search = One of the leading probe to new physics
- ☑ New cLFV reaction = big boost to understanding new physics

New process to search for cLFV

- ☑ New cLFV process: $\mu^- e^- \to e^- e^-$ in muonic atom

  [M. Koike, Y. Kuno, J. Sato, and M.Y., PRL105 (2010)]

- ☑ Sensitive to both photonic dipole cLFV and 4-Fermi contact cLFV
- ☑ Clean signal: back-to-back dielectron
- ☑ Useful to distinguish theoretical models by strong dependence on nucleus in the muonic atom
- ☑ One of the process searched for at COMET experiment (2016~)

- ☑ Important and worth investigating precisely

Aim of this work and outline of this talk

☑ Review of previous work

- New process: $\mu^- e^- \to e^- e^-$ in muonic atom
- Reaction rate

☑ Shortcoming and improvements

- Shortcomings for final electrons and initial leptons
- How to improve

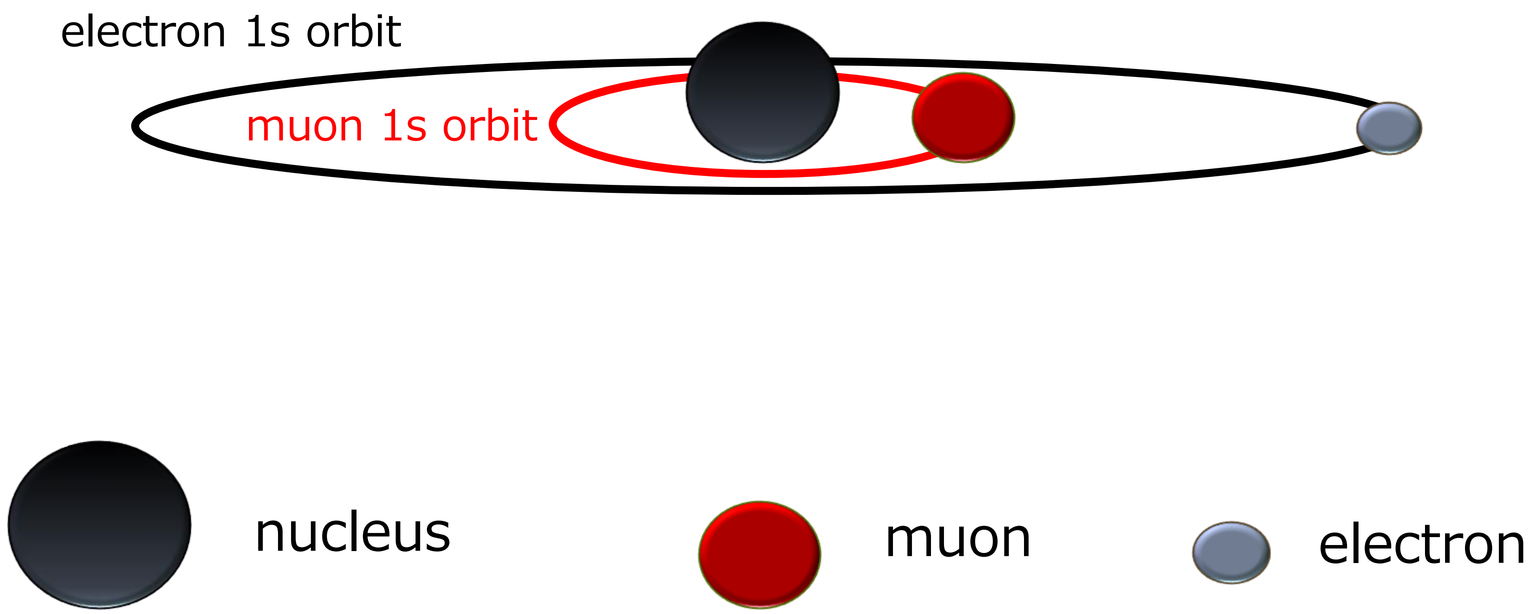
☑ Numerical results

- Comparison of new and previous results
- Large enhancement of the reaction rate

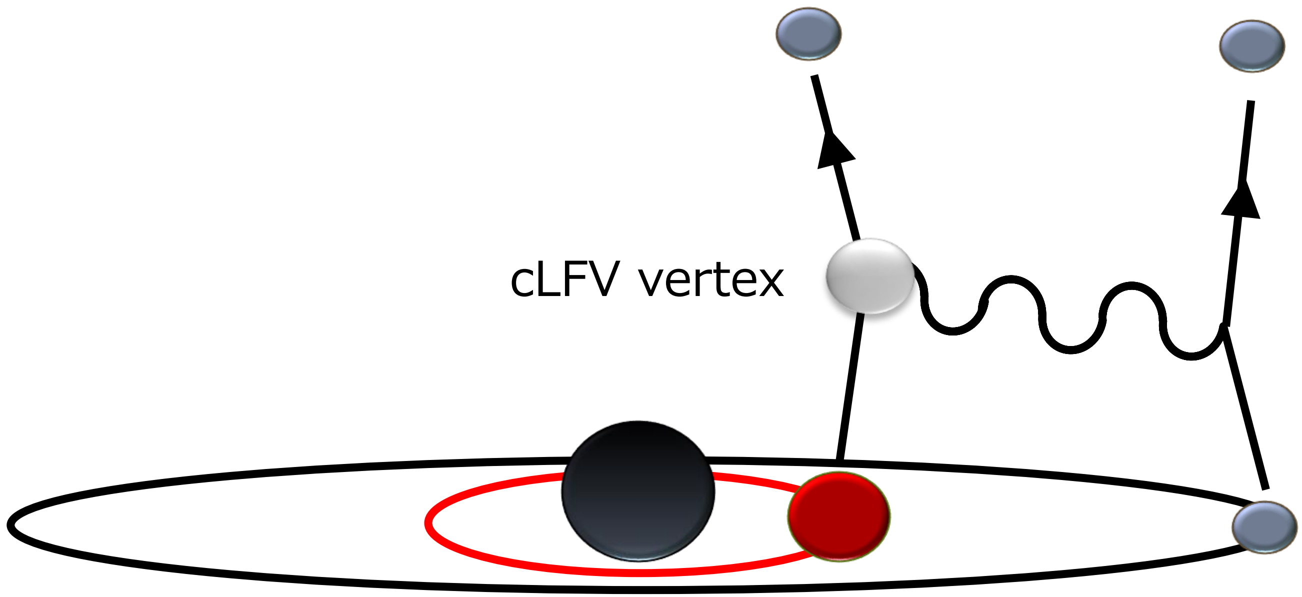
$\mu^- e^- \longrightarrow e^- e^-$ in muonic atom

Muonic atom

- ☑ Useful for various physics
  - Precision test of QED
  - M-e conversion search
  - Muon catalyzed fusion



$\mu^- e^- \to e^- e^-$ in muonic atom



Cross section of cLFV elemental process

Overlap of wave functions in initial state

☑ Reaction rate

$$\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{\rm rel} \left| \psi_{1S}^{(e)}(0; Z-1) \right|^2$$

Cross section of cLFV elemental process

- cLFV effective Lagrangian

$$\mathcal{L}_{\mu^-\mathrm{e}^-\to\mathrm{e}^-\mathrm{e}^-} = -\frac{4G_\mathrm{F}}{\sqrt{2}}\Big[m_\mu A_\mathrm{R}\,\overline{\mu_\mathrm{R}}\sigma^{\mu\nu}e_\mathrm{L}F_{\mu\nu} + m_\mu A_\mathrm{L}\,\overline{\mu_\mathrm{L}}\sigma^{\mu\nu}e_\mathrm{R}F_{\mu\nu}$$
$$+ g_1\left(\overline{\mu_\mathrm{R}}e_\mathrm{L}\right)\left(\overline{e_\mathrm{R}}e_\mathrm{L}\right) + g_2\left(\overline{\mu_\mathrm{L}}e_\mathrm{R}\right)\left(\overline{e_\mathrm{L}}e_\mathrm{R}\right)$$
$$+ g_3\left(\overline{\mu_\mathrm{R}}\gamma^\mu e_\mathrm{R}\right)\left(\overline{e_\mathrm{R}}\gamma_\mu e_\mathrm{R}\right) + g_4\left(\overline{\mu_\mathrm{L}}\gamma^\mu e_\mathrm{L}\right)\left(\overline{e_\mathrm{L}}\gamma_\mu e_\mathrm{L}\right)$$
$$+ g_5\left(\overline{\mu_\mathrm{R}}\gamma^\mu e_\mathrm{R}\right)\left(\overline{e_\mathrm{L}}\gamma_\mu e_\mathrm{L}\right) + g_6\left(\overline{\mu_\mathrm{L}}\gamma^\mu e_\mathrm{L}\right)\left(\overline{e_\mathrm{R}}\gamma_\mu e_\mathrm{R}\right) + (\mathrm{H.c.})\Big]$$

- Calculated in the standard way, i.e., by using plane wave

- Reaction rate

$$\Gamma(\mu^-\mathrm{e}^- \to \mathrm{e}^-\mathrm{e}^-; Z) = 2\sigma v_\mathrm{rel}\left|\psi_{1\mathrm{S}}^{(\mathrm{e})}(0; Z-1)\right|^2$$

Cross section of cLFV elemental process

- ☑ cLFV effective Lagrangian

$$\mathcal{L}_{\mu^-\mathrm{e}^-\to\mathrm{e}^-\mathrm{e}^-} = -\frac{4G_\mathrm{F}}{\sqrt{2}}\Big[m_\mu A_\mathrm{R}\,\overline{\mu_\mathrm{R}}\sigma^{\mu\nu}e_\mathrm{L}F_{\mu\nu} + m_\mu A_\mathrm{L}\,\overline{\mu_\mathrm{L}}\sigma^{\mu\nu}e_\mathrm{R}F_{\mu\nu}$$

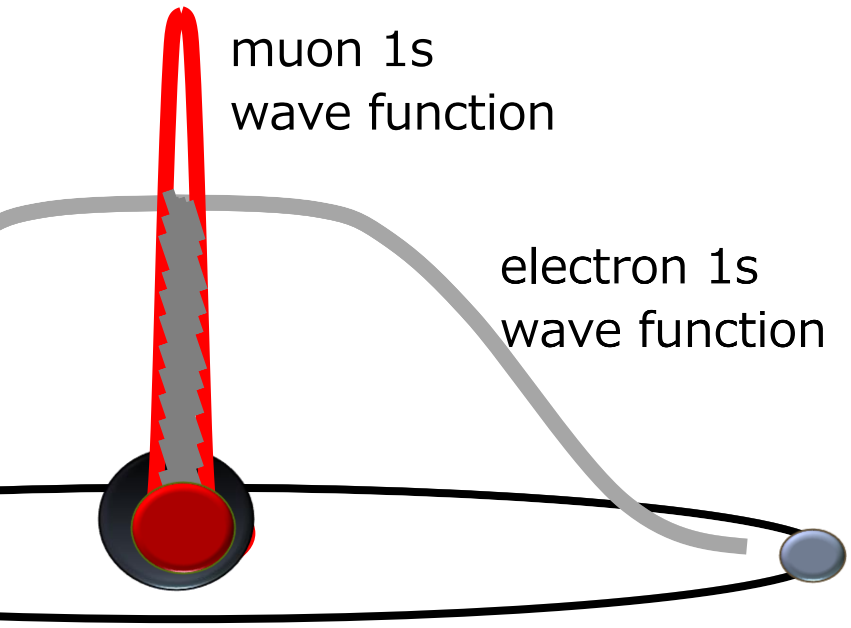
$$+ g_1\left(\overline{\mu_\mathrm{R}}e_\mathrm{L}\right)\left(\overline{e_\mathrm{R}}e_\mathrm{L}\right) + g_2\left(\overline{\mu_\mathrm{L}}e_\mathrm{R}\right)\left(\overline{e_\mathrm{L}}e_\mathrm{R}\right)$$

$$+ g_3\left(\overline{\mu_\mathrm{R}}\gamma^\mu e_\mathrm{R}\right)\left(\overline{e_\mathrm{R}}\gamma_\mu e_\mathrm{R}\right) + g_4\left(\overline{\mu_\mathrm{L}}\gamma^\mu e_\mathrm{L}\right)\left(\overline{e_\mathrm{L}}\gamma_\mu e_\mathrm{L}\right)$$

$$+ g_5\left(\overline{\mu_\mathrm{R}}\gamma^\mu e_\mathrm{R}\right)\left(\overline{e_\mathrm{L}}\gamma_\mu e_\mathrm{L}\right) + g_6\left(\overline{\mu_\mathrm{L}}\gamma^\mu e_\mathrm{L}\right)\left(\overline{e_\mathrm{R}}\gamma_\mu e_\mathrm{R}\right) + (\mathrm{H.c.})\Big]$$

Focused part in this talk

Overlap of wave functions



- ☑ Overlap = electron wave function at nucleus

  (∵ localization of muon at nucleus position)

- ☑ Electron wave function by solving Schrödinger Eq.

- ☑ Reaction rate

$$\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{\rm rel} \left| \psi_{1S}^{(e)}(0; Z-1) \right|^2$$

Branching ratio

☑ Branching ratio for cLFV 4-Fermi contact operator

$$\mathrm{BR}(\mu^- e^- \to e^- e^-) \equiv \frac{\text{reaction rate } \Gamma(\mu^- e^- \to e^- e^-)}{\text{muon capture rate } (1/\tilde{\tau}_\mu)}$$

$$= 3.31 \times 10^{-12} (Z-1)^3 \left(\tilde{\tau}_\mu / \tau_\mu\right) \left[G_{12} + 16 G_{34} + 4 G_{56} + 8 G'_{14} + 8 G'_{23} - 8 G'_{56}\right]$$

τ_μ : free muon lifetime

$$G_{ij} \equiv |g_i|^2 + |g_j|^2$$

$$G'_{ij} \equiv \mathrm{Re}\left(g_i^* g_j\right)$$

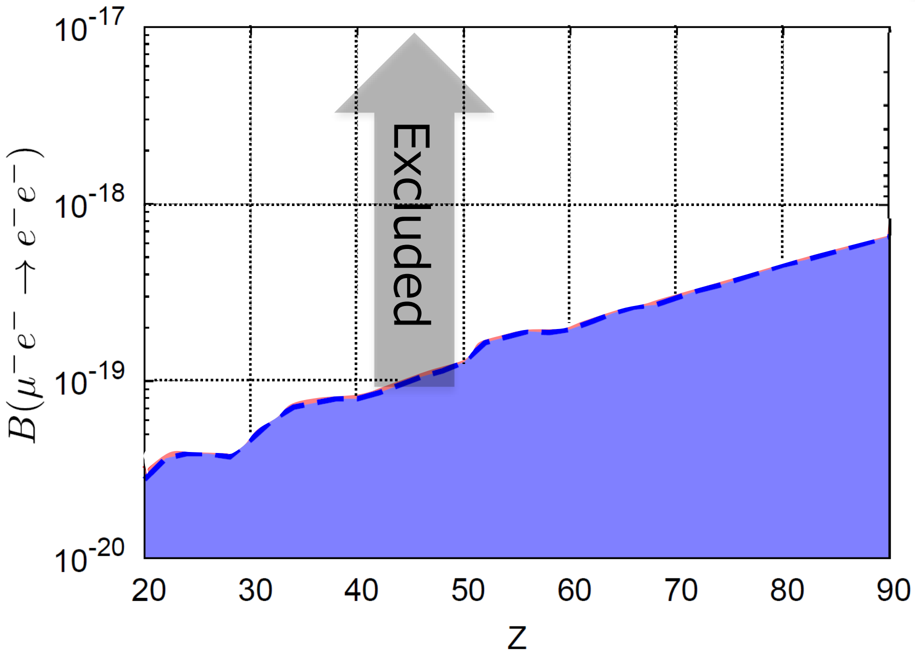
☑ Enhancement factor from overlap of wave functions

☑ Positive charge attracts electron toward the nucleus

Numerical result (previous work)

- ☑ The new process and $\mu \to 3e$ are described by same operator
- ☑ Relation between upper bounds of the new process and $\mu \to 3e$

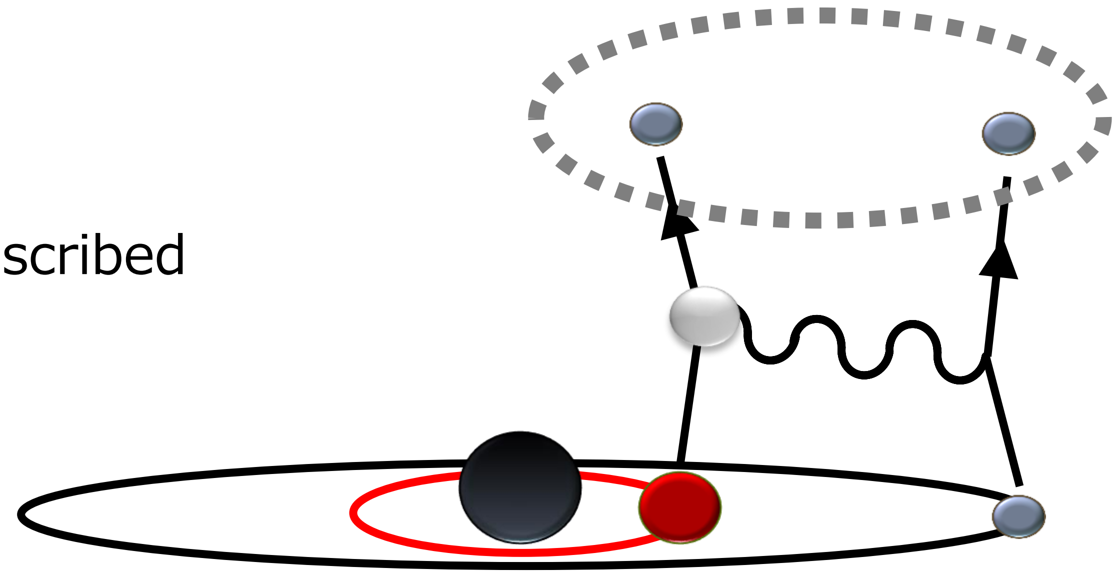
$$\frac{\mathrm{Br}(\mu^- e^- \to e^- e^-)}{\mathrm{Br}(\mu^+ \to e^+ e^+ e^-)} \lesssim 192\pi (Z-1)^3 \alpha^3 \left(\frac{m_e}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu}$$



- ☑ Large enhancement by large nucleus charge
- ☑ Example: Upper bound for Pb nucleus $\sim 5 \times 10^{-19}$

Shortcomings and improvements

Shortcoming and improvement (1)



- ☑ Shortcoming (1)

  Final electrons are described by plane wave

- ☑ Wave functions are distorted in nuclear Coulomb potential, especially for a large nucleus

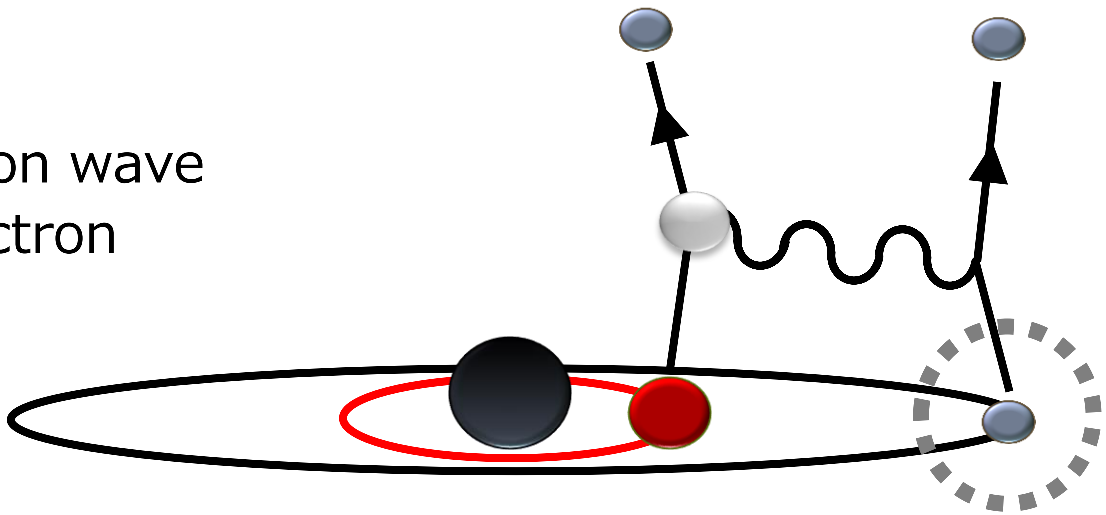
- ☑ Improvement (1)

  Plane wave → Coulomb scattering wave function of Dirac Eq.

Shortcoming and improvement (2)

☑ Shortcoming (2)

Non-relativistic electron wave function of bound electron



☑ The bound electron is relativistic Dirac particle

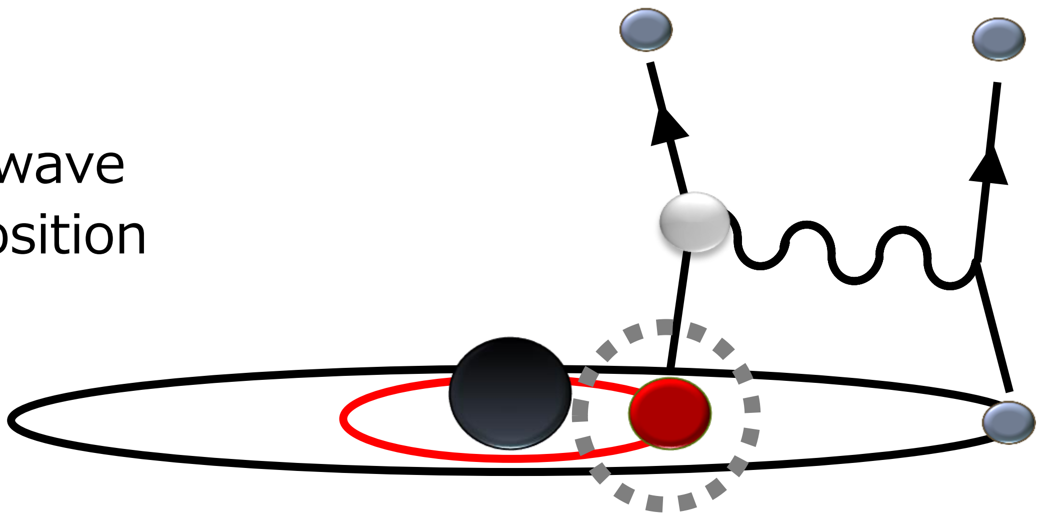
☑ Improvement (2)

NR wave function → Wave function of Dirac particle in point Coulomb potential

Shortcoming and improvement (3)

☑ Shortcoming (3)

Localization of muon wave function at nucleus position



☑ Muon is located around the surface of nucleus, not at center

☑ Improvement (3)

Localized muon wave function → Wave function of Dirac particle in Coulomb potential by finite size nucleus

Numerical results

Improved formulation of the reaction rate

- Reaction rate (improved version)

$$\Gamma = 2\pi \sum_f \sum_i \delta(E_f - E_i) \left| \langle \psi^e_{\vec{p}_1} \psi^e_{\vec{p}_2} | \mathcal{L}_I | \psi^\mu_B \psi^e_B \rangle \right|^2$$

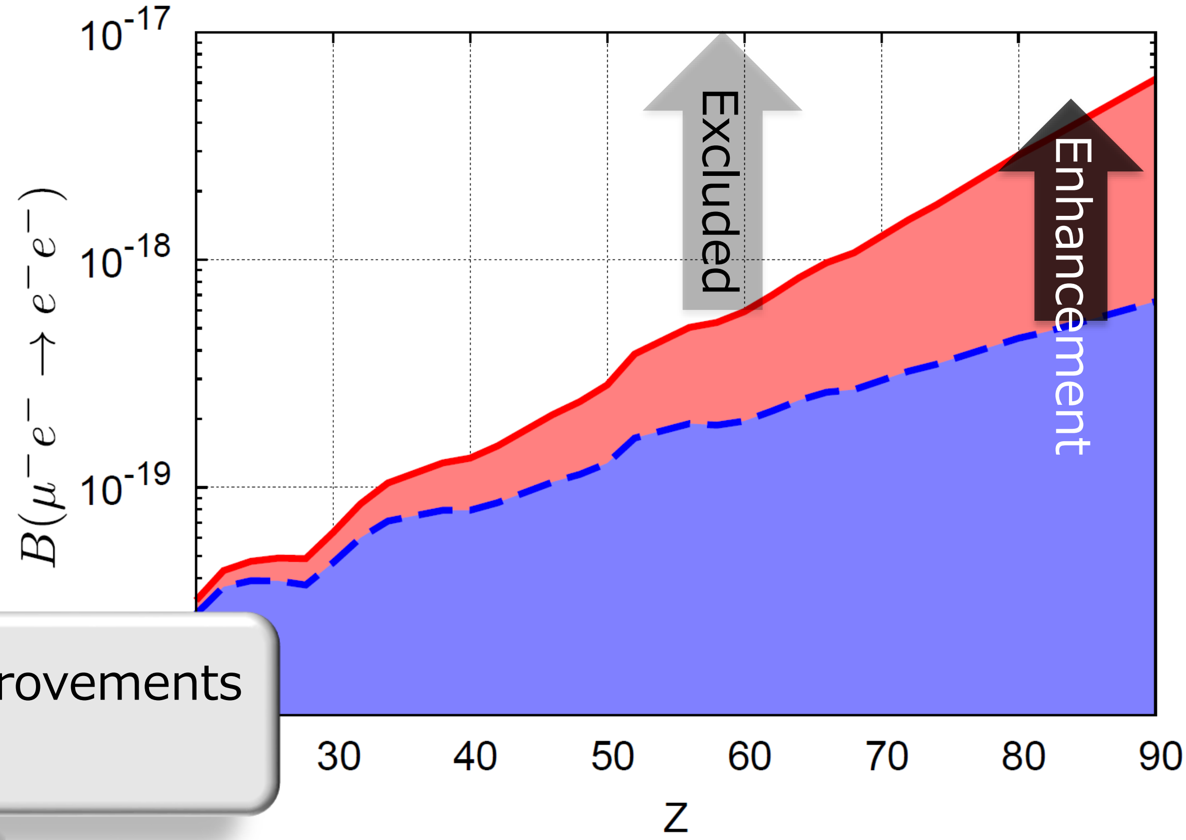
$$= \frac{G_F^2}{\pi^3} \int dE_{p_1} |\boldsymbol{p}_1||\boldsymbol{p}_2| \sum_{J,\kappa_1,\kappa_2} (2J+1)(2j_{\kappa_1}+1)(2j_{\kappa_2}+1)$$

$$\times \left| \sum_{i=1}^{6} g_i W_i(J, \kappa_1, \kappa_2, E_{p_1}) \right|^2$$

- A part of coupling of angular momentum

- cLFV interactions
- Racah coefficient (coupling of angular momentum)

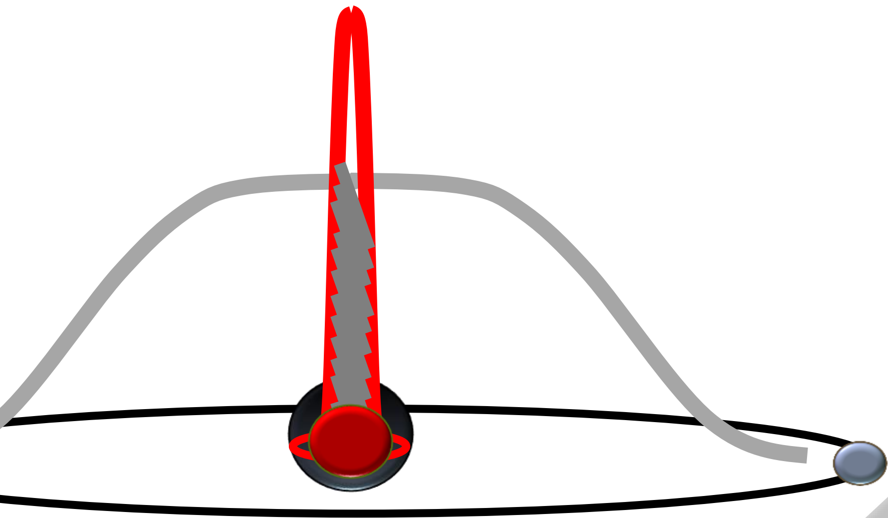
Numerical result



Upper bound by the improvements from $\mu \to 3e$ current limit

- ☑ Large enhancement of the reaction rate by the improvements
- ☑ Example: upper bound for pb nucleus $\sim 3.5 \times 10^{-18}$
- ☑ Close to muon intensity at experiments in near future

Reason of enhancement



- ☑ Wave function of initial electron approaches to nucleus

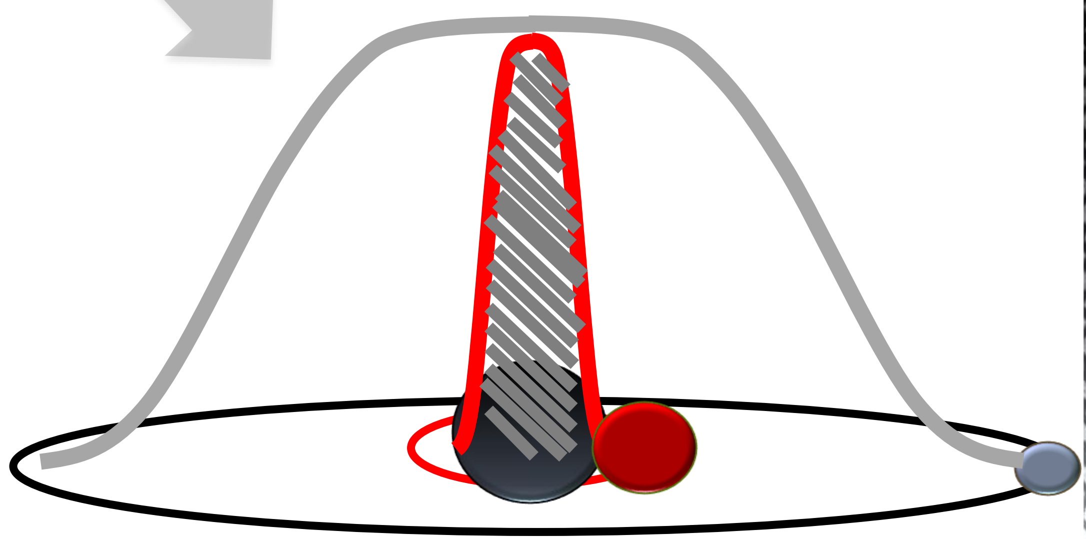
  Increase of overlap

- ☑ Wave function density of muon at nucleus becomes smaller

  Decrease of overlap

- ☑ Wave functions of final electron also approach to nucleus

  Increase of overlap

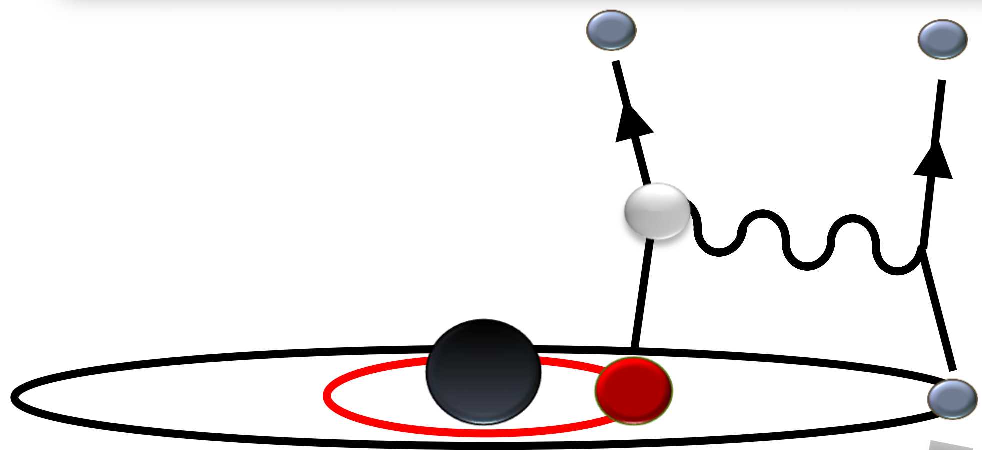


Summary

Summary

- ☑ New cLFV process:
  $\mu^- e^- \to e^- e^-$ in muonic atom
- ☑ Improvements of wave functions
- ☑ Large enhancement of the reaction rate by distorted electron wave functions
- ☑ First signal of cLFV may be discovered with the new process in near future

Plane wave → Coulomb scattering wave function of Dirac Eq.



NR wave function → Wave function of Dirac particle in point Coulomb

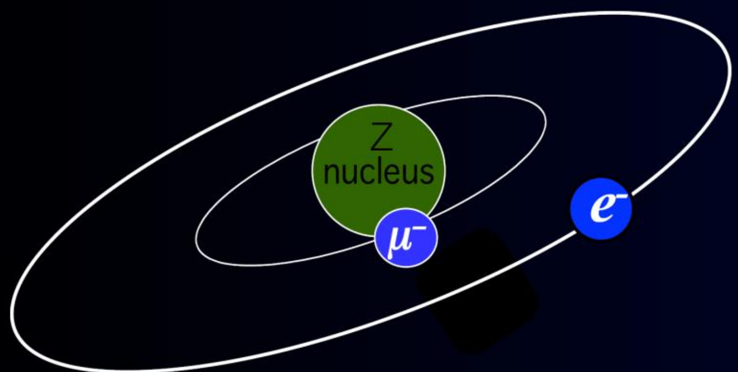
Localized muon wave function → Dirac wave function in Coulomb potential by finite size nucleus

Backup slides

Other CLFV Physics at COMET Phase-I

$\mu^- + N(Z) \rightarrow e^+ + N(Z-2)$

$\mu^- + e^- \rightarrow e^- + e^-$



- $\mu^- e^- \rightarrow e^- e^-$ has two-body final state, although $\mu^+ \rightarrow e^+ e^+ e^-$ is a 3-body decay.
- A muonium CLFV decay such as $\mu^+ e^- \rightarrow e^+ e^+$ is a 2-body decay having a larger phase space, but the overwrap of μ^+ and e^- is small.

The overwrap between μ^- and e^- is proportional to Z^3. For Z=82 (Pb), the overwrap increases by a factor of 5×10^5 over the muonium. The rate is 10^{-17} to 10^{-18}.

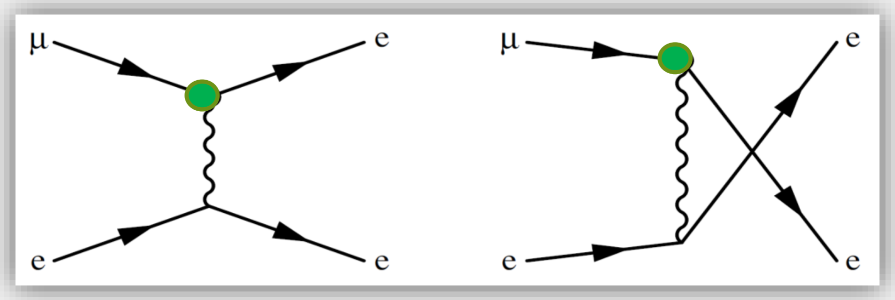
Discussion (photonic dipole Interaction)

Cross section of cLFV elemental process

- ☑ cLFV effective Lagrangian

$$\mathcal{L}_{\mu^-e^-\to e^-e^-} = -\frac{4G_F}{\sqrt{2}}\Big[m_\mu A_R\,\overline{\mu_R}\sigma^{\mu\nu}e_L F_{\mu\nu} + m_\mu A_L\,\overline{\mu_L}\sigma^{\mu\nu}e_R F_{\mu\nu}$$
$$+ g_1(\overline{\mu_R}e_L)(\overline{e_R}e_L) + g_2(\overline{\mu_L}e_R)(\overline{e_L}e_R)$$
$$+ g_3(\overline{\mu_R}\gamma^\mu e_R)(\overline{e_R}\gamma_\mu e_R) + g_4(\overline{\mu_L}\gamma^\mu e_L)(\overline{e_L}\gamma_\mu e_L)$$
$$+ g_5(\overline{\mu_R}\gamma^\mu e_R)(\overline{e_L}\gamma_\mu e_L) + g_6(\overline{\mu_L}\gamma^\mu e_L)(\overline{e_R}\gamma_\mu e_R) + (\mathrm{H.c.})\Big]$$

Dipole photonic interaction



Branching ratio (photonic dipole Int.)

- ☑ Branching ratio

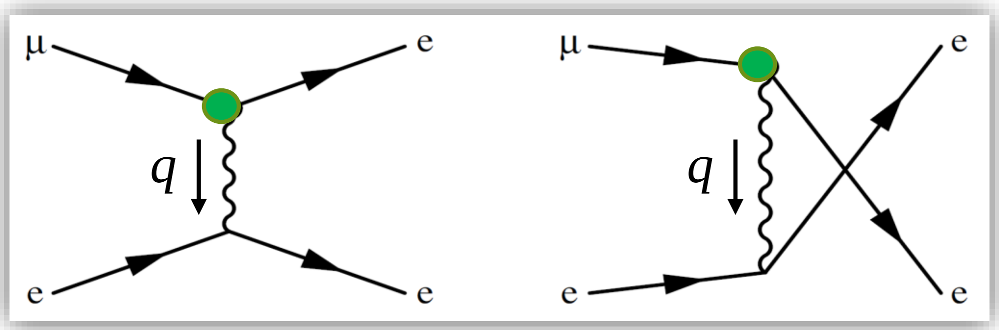
$$\mathrm{BR}(\mu^- e^- \to e^- e^-) = 2.08 \times 10^{-9} (Z-1)^3 \left(\tilde{\tau}_\mu / \tau_\mu\right) \left[|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2 \right]$$

- ☑ 1000 times larger than the 4-Fermi contact case
- ☑ Why? (next page)

- ☑ cLFV effective coupling

- ☑ Enhancement factor from overlap of wave functions
- ☑ Positive charge attracts electron toward the nucleus

Why the photonic BR >> the 4-Fermi BR?



☑ Photon propagator in non-relativistic limit $\frac{1}{q^2} \simeq \frac{1}{m_\mu m_e}$

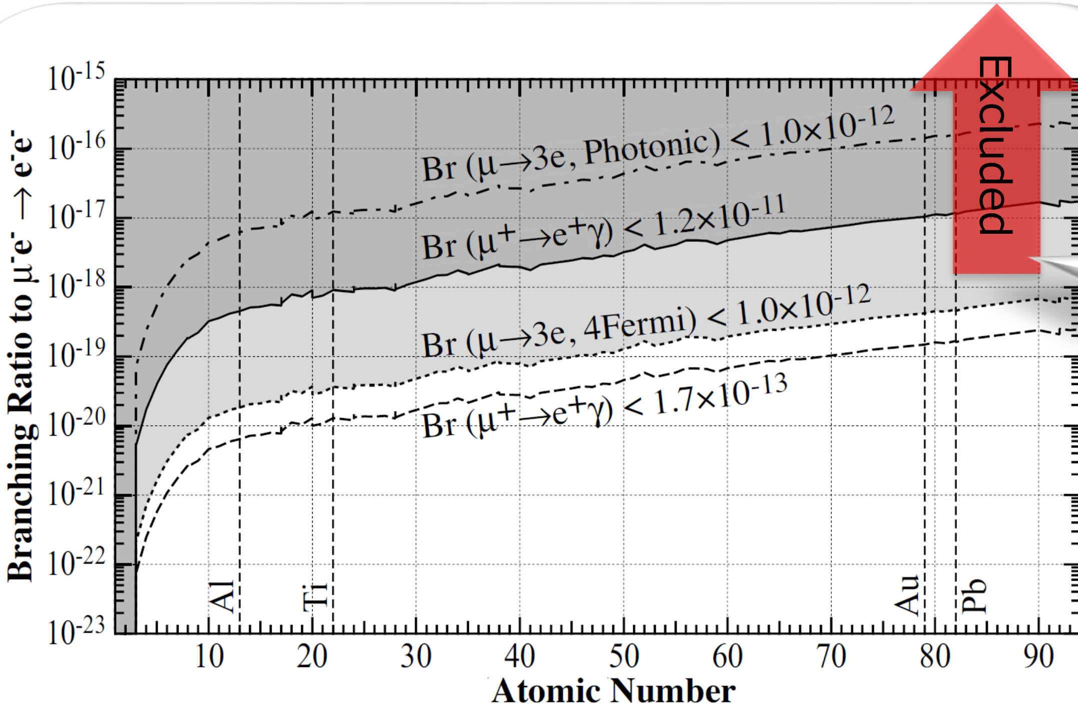
☑ Large enhancement factor

$$\mathrm{BR}(\mu^- e^- \to e^- e^-)\Big|_{\mathrm{photonic}} \propto \mathrm{BR}(\mu^- e^- \to e^- e^-)\Big|_{\text{4-Fermi}} \times \left(\frac{m_\mu^2}{m_e^2}\right)$$

Upper bound (photonic dipole Int.)

- ☑ The new process and $\mu \to e\gamma$ are described by same operator
- ☑ Relation between upper bounds of the new process and $\mu \to e\gamma$

$$\frac{\mathrm{Br}(\mu^- e^- \to e^- e^-)}{\mathrm{Br}(\mu^+ \to e^+ \gamma)} \lesssim 4(Z-1)^3 \alpha^4 \frac{m_e}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu}$$



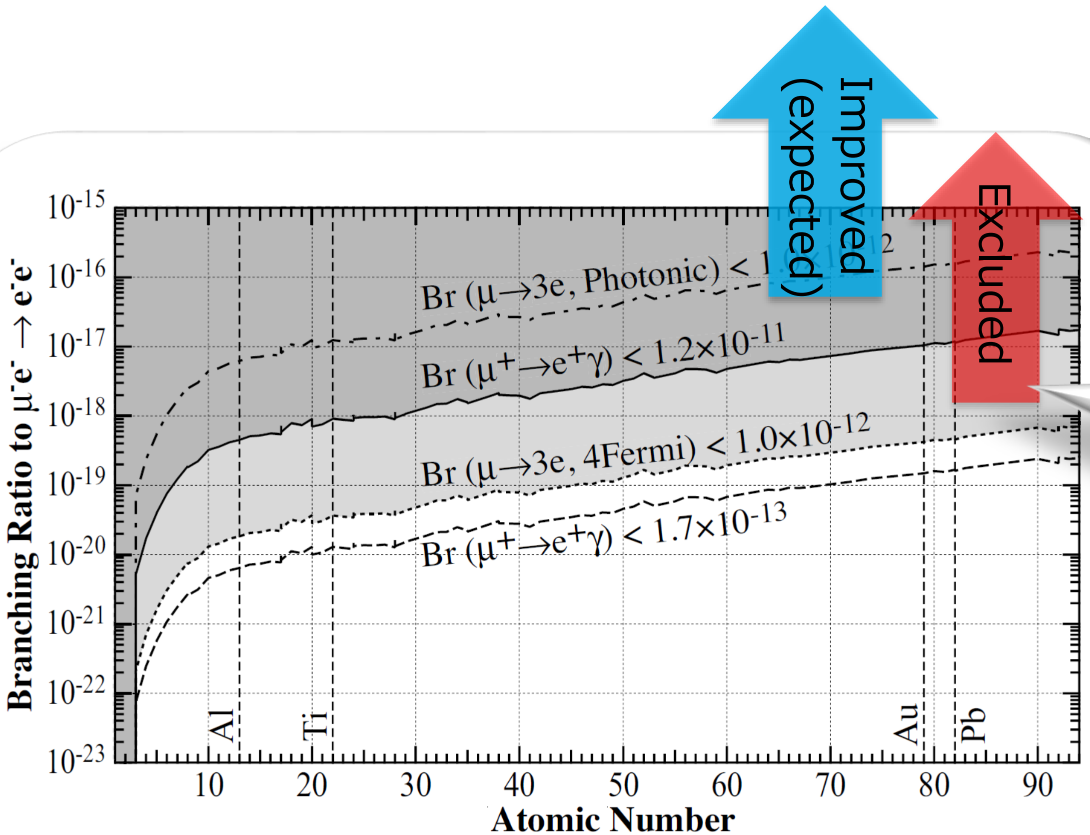
Latest limit:
$\mathrm{BR}(\mu \to e\gamma) < 5.7 \times 10^{-13}$

[MEG collaboration, PRL (2013)]

Upper bound (photonic dipole Int.)

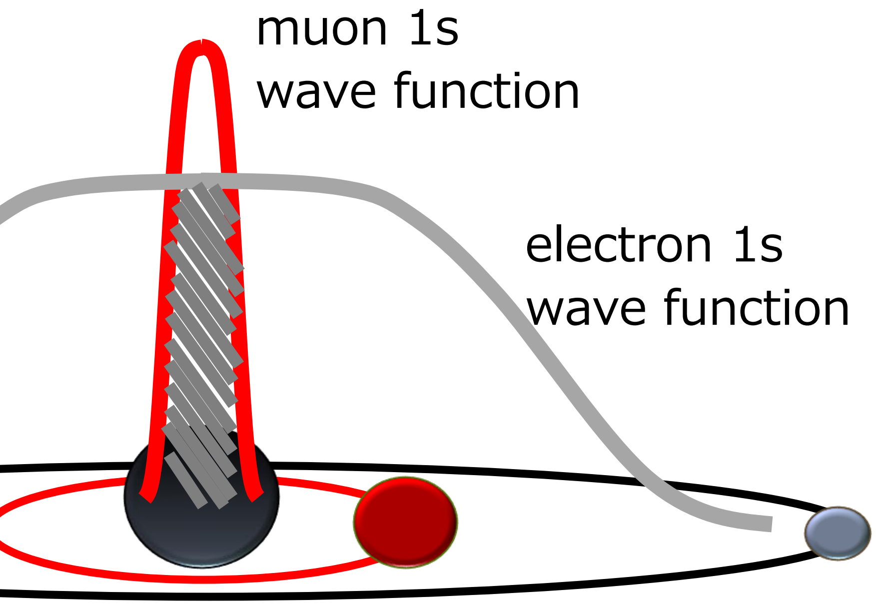
☑ Expectation: enhancement of the reaction rate by the improvements (work in progress)

☑ Could be discovered at next generation experiments



Latest limit:
$BR(\mu \to e\gamma) < 5.7 \times 10^{-13}$
[MEG collaboration, PRL (2013)]

Overlap of wave functions



☑ Overlap = electron wave function at nucleus

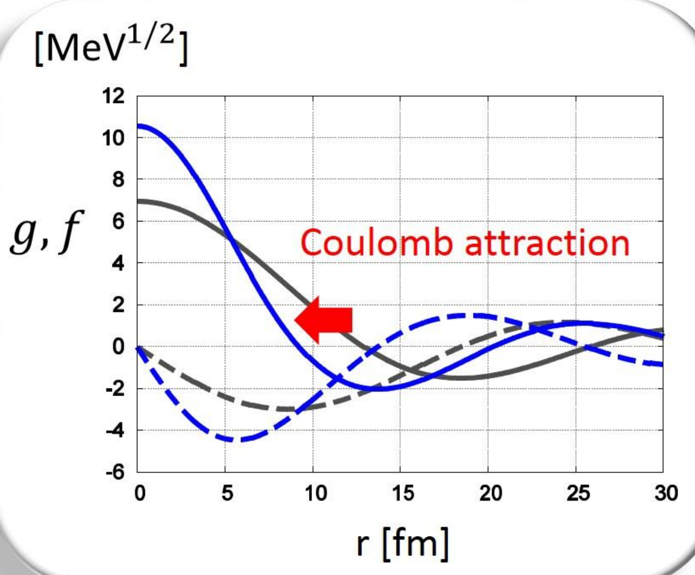
(∵ localization of muon at nucleus position)

☑ Electron wave function by solving Schrödinger Eq.

$$\psi_{1S}^{(e)}(r;Z) = \frac{(Z\alpha m_e)^{3/2}}{\sqrt{\pi}} \exp(-Z\alpha m_e r)$$

☑ This leads large enhancement factor

Shortcoming and improvement (1)



$$\psi_\kappa(\vec{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa(\hat{r}) \\ if_\kappa(r)\chi_{-\kappa}(\hat{r}) \end{pmatrix}$$

solid line : upper component, g

dotted line : lower component, f

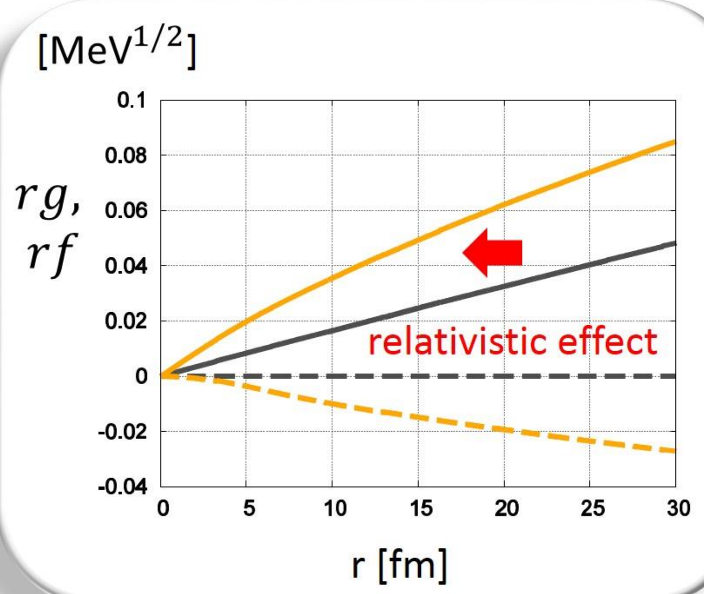
(black line : w.f. used in previous work)

☑ Density near the center position becomes larger, and leads to enhancement of the reaction rate

☑ Improvement (1)

Plane wave → Coulomb scattering wave function of Dirac Eq.

Shortcoming and improvement (2)



$$\psi_\kappa(\vec{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa(\hat{r}) \\ if_\kappa(r)\chi_{-\kappa}(\hat{r}) \end{pmatrix}$$

solid line : upper component, g

dotted line : lower component, f

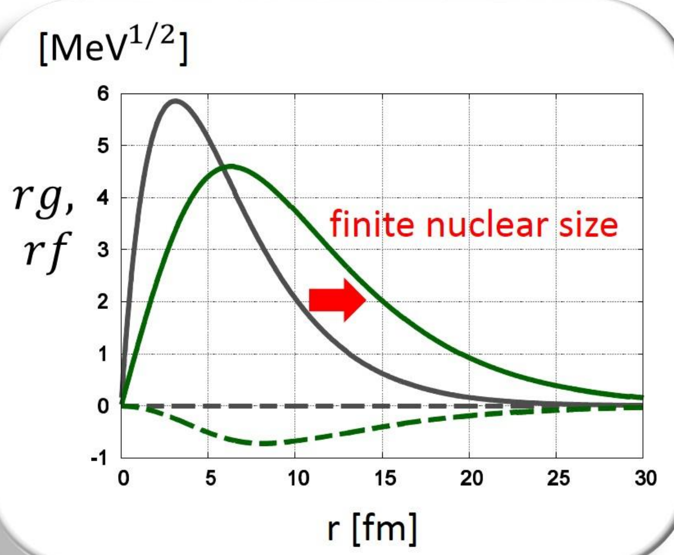
(black line : w.f. used in previous work)

☑ More attracted, and leads to enhancement of the overlap

☑ Improvement (2)

NR wave function → Wave function of Dirac particle in point Coulomb potential

Shortcoming and improvement (3)



$$\psi_\kappa(\vec{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa(\hat{r}) \\ i f_\kappa(r)\chi_{-\kappa}(\hat{r}) \end{pmatrix}$$

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☑ Density near the center position becomes smaller

☑ Improvement (3)

Localized muon wave function → Wave function of Dirac particle in Coulomb potential by finite size nucleus

Muon intensity at next generation Exp.

Muon intensity in working and future experiments

Collaboration	Searching for	Intensity
MEG	$\mu \to e\gamma$	$10^{7.5}\ \mu/s$
MUSIC	$\mu \to 3e$	$10^{8}\ \mu/s$
COMET	$\mu^- N \to e^- N$	$10^{11}\ \mu/s$
Mu2E (E973)	$\mu^- N \to e^- N$	$10^{11}\ \mu/s$
PRISM	$\mu^- N \to e^- N$	$10^{12}\ \mu/s$