THE SUPERSYMMETRIC GEORGI-MACHACEK MODEL

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Outline

- The Georgi-Machacek (GM) Model
  - Motivations
  - Deficiencies of the model
- The Supersymmetric Extension of the GM Model (SCTM model)
  - Motivations
  - Details of the Model
- The GM model as limit of the SCTM
  - Deriving the GM Model from SCTM
  - SUSY constraints on GM parameters
- Some Phenomenology
  - How to distinguish SGM- from GM-model

The Georgi-Machacek (GM) Model

Georgia and Machacek: NPB262 (1985), many recent studies e.g. H. Logan, et. al.: 1709.01883, 1708.08753

Motivations: The Haber Principle and also:

- In the SM the ρ-parameter is exactly 1 at tree level, this is a consequence of a custodial SU(2) symmetry:

$$\Phi = \begin{pmatrix} h^{o*} & h^{+} \\ h^{-*} & h^{o} \end{pmatrix} \qquad V = \mu Tr(\Phi^{\dagger}\Phi) + \lambda\left(Tr(\Phi^{\dagger}\Phi)\right)^{2}$$

$$\Phi \xrightarrow{SU(2)_L \times SU(2)_R} U_L \Phi U_R^{\dagger}$$

Manifestly invariant under: $SU(2)_L \times SU(2)_R$

After SSB: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_c$

Under this residual symmetry, in the absence of hypercharge the W's transform as a triplet and would be degenerate.

THE GEORGI-MACHACEK (GM) MODEL

In addition to the doublet add triplet reps. However, for $\rho = 1$ we want to preserve the custodial symmetry, we must add at least two triplets, one with hypercharge 0 and the other with hypercharge 1.

$$\mathbf{X} = \begin{pmatrix} \psi_o^* & \phi_+ & \psi_{++} \\ \psi_- & \phi_o & \psi_+ \\ \psi_{--} & \phi_- & \psi_o \end{pmatrix} \qquad \mathbf{X} \xrightarrow{SU(2)_L \times SU(2)_R} U_L \mathbf{X} U_R^\dagger$$

$$V_{GM} = \frac{\mu_2^2}{2} Tr\left[\Phi^\dagger \Phi\right] + \frac{\mu_3^2}{2} Tr\left[X^\dagger X\right] + \lambda_1 Tr\left[\Phi^\dagger \Phi\right]^2 + \lambda_2 Tr\left[\Phi^\dagger \Phi\right] Tr\left[X^\dagger X\right]$$
$$+\lambda_3 Tr\left[X^\dagger X X^\dagger X\right] + \lambda_4 Tr\left[X^\dagger X\right]^2 - \lambda_5 Tr\left[\Phi^\dagger \tau^a \Phi \tau^b\right] Tr\left[X^\dagger t^a X t^b\right]$$
$$-M_1 Tr\left[\Phi^\dagger \tau^a \Phi \tau^b\right](UXU^\dagger)_{ab} - M_2 Tr\left[X^\dagger t^a X t^b\right](U\bar{X}U^\dagger)_{ab},$$

SSB in the Georgi Model

$$<\Phi>_o = \begin{pmatrix} v_H & 0 \\ 0 & v_H \end{pmatrix} \qquad <\mathbf{X}>_o = \begin{pmatrix} v_\Delta & 0 & 0 \\ 0 & v_\Delta & 0 \\ 0 & 0 & v_\Delta \end{pmatrix}$$

$$\Phi \xrightarrow{SU(2)_L \times SU(2)_R} U_L \Phi U_R^\dagger \qquad \mathbf{X}_c \xrightarrow{SU(2)_L \times SU(2)_R} U_R \mathbf{X} U_L^\dagger$$

Then vacuum is invariant only if $U_R = U_L$:

$$\Rightarrow SU(2)_L \times SU(2)_R \to SU(2)_c$$

$$m_W^2 = m_Z^2 \cos\theta_w = \frac{g^2 v^2}{4}$$

With: $v^2 = v_H^2 + 8v_\Delta^2$

The Custodial Multiplets in the GM Model

The custodial symmetry $SU(2)_c$ preserves hermiticity and the trace. Since we can always write X in the form:

$$\chi = \left[\frac{1}{2}(X + X^{\dagger}) - Tr[X]\right] + \frac{1}{2}(X - X^{\dagger}) + Tr[X]$$

Then each term transforms separately under $SU(2)_c$ and each forms a different rep. of $SU(2)_c$

$$H_5^{++} = \psi_{++} \quad H_5^{\pm} = \frac{(\psi_{\pm} - \phi_{\pm})}{\sqrt{2}} \quad H_5^{0} = \frac{(\sqrt{2}\psi_{or} - 2\phi_{o})}{\sqrt{6}}$$

$$\zeta_{+} = \frac{(\psi_{+} + \phi_{+})}{\sqrt{2}} \quad \zeta_{o} = \psi_{oi} \quad \zeta_{-} = \frac{(\psi_{-} + \phi_{-})}{\sqrt{2}} \qquad H_1^{o'} = \frac{\sqrt{2}\psi_{oR} + \phi_{o}}{\sqrt{3}}$$

Likewise for the doublet fields: $h^{+}, h^{-}, and\ h_i^{o}$ form a triplet under $SU(2)_c$; and h_r^{o} form a singlet. The fields in yellow mix to give the Goldstone bosons and a physical pseudo-scalar triplet. The singlets mix to give a SM-like scalar and a second (generally higher mas) physical scalar

The Georgi-Machacek (GM) Model

The good:

- Rich phenomenology with doubly charged scalars and HWZ couplings
- Arises naturally in Higgs composite models and Little Higgs models
- Predicts $\rho = 1$ at tree level. This allows sizeable contributions to EWSB from the triplet sector.

The Bad:

- The problem with the GM model is that one loop corrections to ρ depend quadratically on the cut-off scale. This is expected since the custodial symmetry is violated by hypercharge. To cancel the quadratic divergence you have to add custodial breaking counter terms to the potential. The coefficient of these terms is then tuned to cancel the quadratic contributions to $\delta\rho$. (Gunion, Wudka, RV: PRD43 (1991))

- Therefore, the GM model requires even more tuning than the SM. Furthermore in the GM model the ρ parameter is not a predicted value, but rather a parameter that must be fixed by experiment.

- Similarly to the SM an extension to SUSY model cures this quadratic divergence

The Supersymmetric GM Model (SCTM)

(L. Cort, M. Garcia, M. Quiros: 1308.4025, M. Garcia, S. Gori, M. Quiros, RV, R. Vega-Morales, T. Yu: 1409.5737

$$\Phi = \begin{pmatrix} h_1^{o*} & h_2^{+} \\ h_1^{-} & h_2^{o} \end{pmatrix} \qquad \mathbf{X} = \begin{pmatrix} \xi_o & \phi_+ & \psi_{++} \\ \xi_- & \phi_o & \psi_+ \\ \xi_{--} & \phi_- & \psi_o \end{pmatrix}$$

$$X_c = C X^T C \qquad \Phi_c = \sigma_2 \Phi^T \sigma_2 \qquad C = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Like G-parity

$$\mathbf{X}_c \xrightarrow{SU(2)_L \times SU(2)_R} U_R \mathbf{X} U_L^\dagger$$

- The manifestly $SU(2)_L X SU(2)_R$ invariant super potential takes the form:

$$W_0 = \frac{\mu}{2} Tr\left(\Phi_c \Phi\right) + 2 * \lambda Tr\left(\Phi_c \sigma_i \Phi \sigma_j\right)(U\mathbf{X}U^\dagger)_{ij} + \frac{\mu_\Delta}{2} Tr\left(\mathbf{X}_c \mathbf{X}\right) + \frac{\lambda_3}{6} Tr\left(\mathbf{X}_c T_i \mathbf{X} T_j\right)(U\mathbf{X}U^\dagger)_{ij}$$

- After EWSB there remains a custodial $SU(2)_c$ symmetry in the scalar potential and states can be classified into custodial multiplets

- Note that now we have two complex doublets, two complex $Y = +1, -1$ triplets, and one complex $Y = 0$ triplet,
- The scalar spectrum is doubled

The Supersymmetric GM Model

- As in the GM model after SSB the states can be classified into custodial multiplets by observing that If we define:

$$X_{\pm} = X \pm X_{c} \Rightarrow C X_{\pm}^{T} C = \pm X_{\pm}$$

- A transformation under $SU(2)_c$ preserves this property and X_+ and X_- transform independently. Thus if we write,

$$X = \left(\frac{X_{+} + X_{-}}{2} - Tr[X] \right) + \left(\frac{X_{+} - X_{-}}{2} \right) + Tr[X]$$

- Since the Y=0 triplet is complex and since the $Y = \pm 1$ triplets are independent dof, there are two of each of the reps: 2 fiveplets, 2 triplets, and 2 singlets
- We repeat the similar procedure for the doublet fields resulting in a 2 triplets and 2 singlets.
- The fiveplets will mix, the triplets will mix and the singlets will mix resulting in the physical states.
- In the notation of Quiros the physical fields are then: a real-scalar fiveplet, $\left(F_s^{\{\pm\pm\}}, F_s^{\pm}, F_s^{o}\right)$; a pseudo-scalar fiveplet, $\left(F_p^{\{\pm\pm\}}, F_p^{\pm}, F_p^{o}\right)$; two real scalar triplets, $\left(T_{(1,2)}^{\pm}, T_{(1,2)}^{o}\right)$; one pseudo-scalar triplet, $\left(A^{\pm}, A^{o}\right)$; two real-scalar singlets, (S_1^o, S_2^o); *and* two pseudo-scalar singlets, (P_1^o, P_2^o), plus the Goldstone bosons. The guys in yellow would correspond to the GM-like scalars.

The SCTM Scalar Potential:

RV, R. Vega-Morales (University of Granada) and Keping Xie (SMU) 1711.05329

$$
\begin{aligned}
V_{\mathrm{F}} = {} & \mu^2 Tr[\Phi^\dagger\Phi] + \mu_\Delta^2 Tr[X^\dagger X] + \lambda^2 \Big(Tr[\Phi^\dagger\Phi]^2 - \frac{1}{4} Tr[\Phi_c\Phi] Tr[\Phi_c^\dagger\Phi^\dagger] + Tr[X^\dagger T_i X T_j] Tr[\Phi^\dagger\sigma_i\Phi\sigma_j] \\
& + Tr[X^\dagger X] Tr[\Phi^\dagger\Phi] - Tr[X^\dagger T_i X] Tr[\Phi^\dagger\sigma_i\Phi] - Tr[X_c^\dagger T_i X_c] Tr[\Phi_c^\dagger\sigma_i\Phi_c] \Big) \\
& - \frac{\lambda_3^2}{2} \left(Tr[X^\dagger X X^\dagger X] - Tr[X^\dagger X]^2 \right) - \frac{\lambda_3\lambda}{4} \left(Tr[X^\dagger T_i X_c^\dagger T_j] Tr[\Phi\sigma_j\Phi_c\sigma_i] + c.c. \right) \\
& - \frac{\lambda_3\mu_\Delta}{2} \left(Tr[X^\dagger T_i X T_j][UXU^\dagger]_{i,j} + c.c. \right) - \lambda\mu \left(Tr[\Phi^\dagger\sigma_i\Phi\sigma_j] \left[UXU^\dagger \right]_{i,j} + c.c. \right) \\
& + \frac{\lambda\mu_\Delta}{2} \left(Tr[\Phi_c\sigma_i\Phi\sigma_j][UX^\dagger U^\dagger]_{i,j}] + c.c. \right)
\end{aligned}
$$

$$
\begin{aligned}
V_{\mathrm{soft}} = {} & m_H^2 Tr[\Phi^\dagger\Phi] + m_\Delta^2 Tr[X^\dagger X] + \Big(\frac{B_\Delta}{2} Tr[X_c X] - \frac{B}{2} Tr[\Phi_c\Phi] + \frac{A_\lambda}{2} Tr[\Phi_c\sigma_i\Phi\sigma_j](UXU^\dagger)_{ij} \\
& - \frac{A_\Delta}{6} Tr[X_c t_i X t_j](UXU^\dagger)_{ij} + c.c. \Big)
\end{aligned}
$$

The SCTM Mass Spectrum in the limit of small v_Δ:

GM-like scalars

$$m_{S_1}^2 = 3\lambda^2$$

$$m_{S_2}^2 = x + \lambda(3\lambda - \lambda_3)$$

$$m_{F_s}^2 = x + \frac{1}{2}\lambda\lambda_3$$

$$m_A^2 = x + \frac{1}{2}\lambda\left(4\lambda - \lambda_3\right)$$

Mirror scalars

$$m_{F_p}^2 = z - \frac{1}{2}\lambda\lambda_3$$

$$m_{T_1}^2 = 2(\xi^2 + \chi^2)$$

$$m_{T_2}^2 = z$$

$$m_{p_1}^2 = 2(\xi^2 + \chi^2)$$

$$m_{p_2}^2 = z$$

where:

$$x = \frac{B_\Delta + m_\Delta^2 + \mu_\Delta^2}{v^2}$$

$$z = \frac{m_\Delta^2 + \mu_\Delta^2 - B_\Delta}{v^2}$$

$$\chi = \frac{\mu_h}{v}$$

$$\xi = \frac{m_h}{v}$$

The mirror scalars can be decoupled by taking the parameters (z, χ, ξ) large. We would then recover the GM Model spectrum. The effective potential should then correspond to that of the GM potential.

The SUSY constraints on the GM Model

If we impose the contraints: $X_c = X^t$ and $\Phi_c = \Phi^t$, the expression for $V_F + V_{\{soft\}}$ reduces to the same form as the potential for the GM model:

$$
\begin{aligned}
V_{\mathrm{F}} = {} & \mu^2 Tr[\Phi^\dagger\Phi] + \mu_\Delta^2 Tr[X^\dagger X] + \lambda^2\Big(Tr[\Phi^\dagger\Phi]^2 - \frac{1}{4}Tr[\Phi_c\Phi]Tr[\mathrm{X}_c^\dagger\Phi^\dagger] + Tr[X^\dagger T_i X T_j]Tr[\Phi^\dagger\sigma_i\Phi\sigma_j] + Tr[X^\dagger X]Tr[\Phi^\dagger\Phi] \\
& -Tr[X^\dagger T_i X]Tr[\Phi^\dagger\sigma_i\Phi] - Tr[X_c^\dagger T_i X_c]Tr[\Phi_c^\dagger\sigma_i\Phi_c]\Big) + \frac{\lambda\mu_\Delta}{2}\Big(Tr[\Phi_c\sigma_i\Phi\sigma_j][UX^\dagger U^\dagger]_{i,j}] + c.c.\Big) - \frac{\lambda_3^2}{2}\Big(Tr[X^\dagger X X^\dagger X] - Tr[X^\dagger X]^2\Big) \\
& - \frac{\lambda_3\lambda}{4}\Big(Tr[X^\dagger T_i X_c^\dagger T_j]Tr[\Phi\sigma_j\Phi_c\sigma_i] + c.c.\Big) - \frac{\lambda_3\mu_\Delta}{2}\Big(Tr[X^\dagger T_i X T_j][UXU^\dagger]_{i,j} + c.c.\Big) - \lambda\mu\Big(Tr[\Phi^\dagger\sigma_i\Phi\sigma_j]\big[UXU^\dagger\big]_{i,j} + c.c.\Big)
\end{aligned}
$$

$$
V_{\mathrm{soft}} = m_H^2 Tr[\Phi^\dagger\Phi] + m_\Delta^2 Tr[X^\dagger X] + \Big(\frac{B_\Delta}{2}Tr[X_c X] - \frac{B}{2}Tr[\Phi_c\Phi] + \frac{A_\lambda}{2}Tr[\Phi_c\sigma_i\Phi\sigma_j](UXU^\dagger)_{ij} - \frac{A_\Delta}{6}Tr[X_c t_i X t_j](UXU^\dagger)_{ij} + c.c.\Big)
$$

$$
\begin{aligned}
V_{GM} = {} & \frac{\mu_2^2}{2}Tr\left[\Phi^\dagger\Phi\right] + \frac{\mu_3^2}{2}Tr\left[X^\dagger X\right] + \lambda_1 Tr\left[\Phi^\dagger\Phi\right]^2 + \lambda_2 Tr\left[\Phi^\dagger\Phi\right]Tr\left[X^\dagger X\right] + \lambda_3 Tr\left[X^\dagger X X^\dagger X\right] + \lambda_4 Tr\left[X^\dagger X\right]^2 \\
& -\lambda_5 Tr\left[\Phi^\dagger\tau^a\Phi\tau^b\right]Tr\left[X^\dagger t^a X t^b\right] - M_1 Tr\left[\Phi^\dagger\tau^a\Phi\tau^b\right](UXU^\dagger)_{ab} - M_2 Tr\left[X^\dagger t^a X t^b\right](U\bar{X}U^\dagger)_{ab},
\end{aligned}
$$

By comparing these expression with the substitutions above we obtain a mapping between the parameters.

The Matching Conditions:

$$\lambda_1 = \frac{3}{4}\lambda^2,\ \lambda_2 = \lambda^2,\ \lambda_3 = -\frac{1}{2}\lambda_\Delta^2,$$
$$\lambda_4 = \frac{1}{2}\lambda_\Delta^2,\ \lambda_5 = 2\lambda(\lambda_\Delta - 2\lambda),$$
$$M_1 = 4\big[\lambda(2\mu - \mu_\Delta) - A_\lambda\big],\ M_2 = \frac{1}{3}(3\lambda_\Delta\mu_\Delta + A_\Delta),$$
$$\mu_2^2 = 2(\mu^2 + m_H^2) + B,\ \mu_3^2 = 2(\mu_\Delta^2 + m_\Delta^2) + B_\Delta$$

These imply constraints on the GM parameters:

$$\lambda_1 = \frac{3}{4}\lambda_2,\ \lambda_3 = -\lambda_4,$$
$$\lambda_5 = -4\lambda_2 + 2\sqrt{2\lambda_2\lambda_4}$$

- Note that the five $\lambda's$ can be written in terms of just two, also $\lambda_{\{1,2,4\}}$ are positive definite, and λ_3 is negative.

How do we distinguish a GM Model from its SUSY extension in the decoupling limit (aca SGM)?

- The matching relations guarantee that in the scalar sector the masses and coupling in both models will be the same.
- We can measure observables and check whether or not the SUSY conditions are satisfied. However, it may be that by chance the GM model happens to satisfy those conditions without the need of a SUSY origin.
- The smoking gun will have to come from the fermionic sector of the SUSY model.

The neutral mass matrix: $\Psi^o = (\tilde{h}_1, \tilde{\delta}_1, \tilde{\gamma}, \tilde{Z}, \tilde{h}_3, \tilde{\delta}_3, \tilde{\delta}_5)$

$$M_F^0 = \begin{pmatrix} \frac{3}{\sqrt{2}}\lambda v_\Delta - \mu & \sqrt{3}\lambda v_H & 0 & 0 & 0 & 0 & 0 \\ \sqrt{3}\lambda v_H & -\sqrt{2}\lambda_\Delta v_\Delta + \mu_\Delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{g^2 M_{\tilde{B}} + g'^2 M_{\tilde{W}}}{g^2+g'^2} & \frac{gg'(M_{\tilde{W}} - M_{\tilde{B}})}{g^2+g'^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{gg'(M_{\tilde{W}} - M_{\tilde{B}})}{g^2+g'^2} & \frac{g'^2 M_{\tilde{B}} + g^2 M_{\tilde{W}}}{g^2+g'^2} & \sqrt{\frac{1}{2}(g^2+g'^2)}\, v_H & \sqrt{2(g^2+g'^2)}\, v_\Delta & 0 \\ 0 & 0 & 0 & \sqrt{\frac{1}{2}(g^2+g'^2)}\, v_H & \frac{1}{\sqrt{2}}\lambda v_\Delta + \mu & -\sqrt{2}\lambda v_H & 0 \\ 0 & 0 & 0 & \sqrt{2(g^2+g'^2)}\, v_\Delta & -\sqrt{2}\lambda v_H & \frac{1}{\sqrt{2}}\lambda_\Delta v_\Delta - \mu_\Delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}}\lambda_\Delta v_\Delta + \mu_\Delta \end{pmatrix}$$

Lightest neutralino is LSP and makes viable DM candidate

(A. Delgado, M. Garcia-Pepin, B. Ostdiek, M. Quiros: 1504.02486)

SGM versus GM model at the LHC

- We study under what circumstances the GM model can mimic the SGM model and when they should be easily distinguishable
- To reduce our parameter space, we assume a gauge mediated SUSY breaking mechanism to set $A_\lambda = A_\Delta = 0$ (A. Delgado, M. Garcia-Pepin, M. Quiros: 1505.07469)
- Gives 1-to-1 mapping between SGM and GM parameters:

$$(\lambda, \lambda_\Delta, \mu, \mu_\Delta, v_H, v_\Delta) \Leftrightarrow (\lambda_2, \lambda_4, M_1, M_2, v_\phi, v_\chi)$$

- Use Higgs mass and EW scale measurements to eliminate vev's:

v=256 GeV and $m_h = 125$ GeV

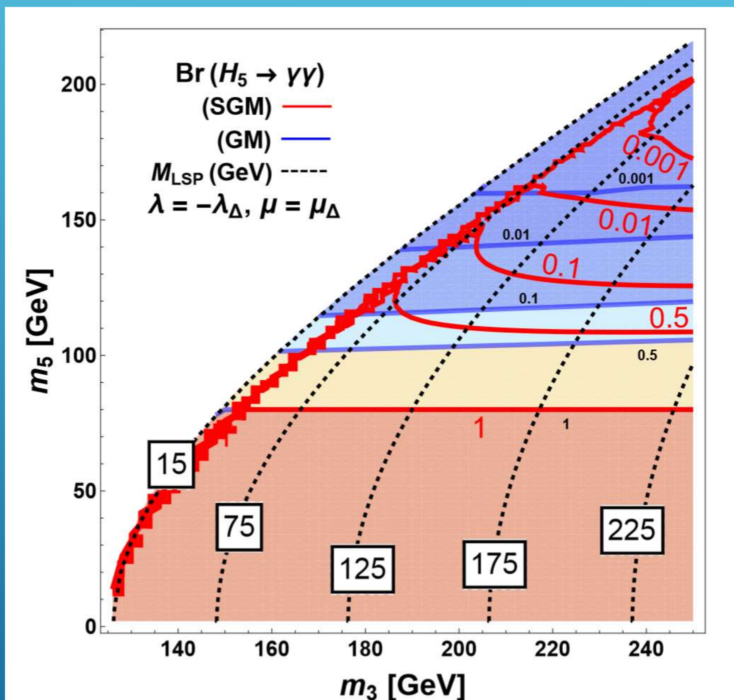
- This reduces the number of parameters to four we further reduce these to two by focusing on two representative scenarios:

Point 1: $\lambda = -\lambda_\Delta$ and $\mu = \mu_\Delta$ Point 2: $\lambda = \lambda_\Delta$ and $\mu = \mu_\Delta$

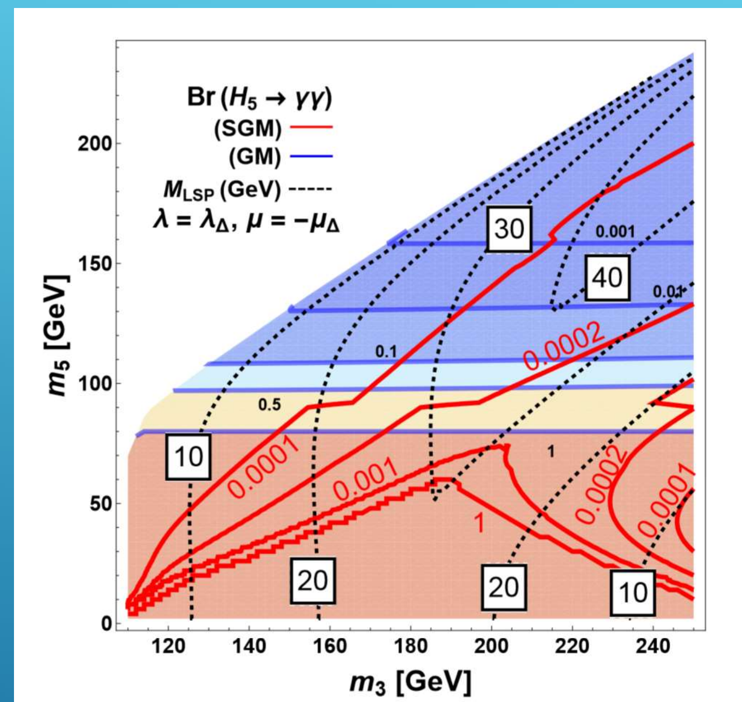
- We trade these two parameters for m_3 and m_5 the masses of the triplets and fiveplets respectively and scan over the range $100 < m_{3,5} < 250\ GeV$

SGM versus GM with diphotons

Heavy LSP



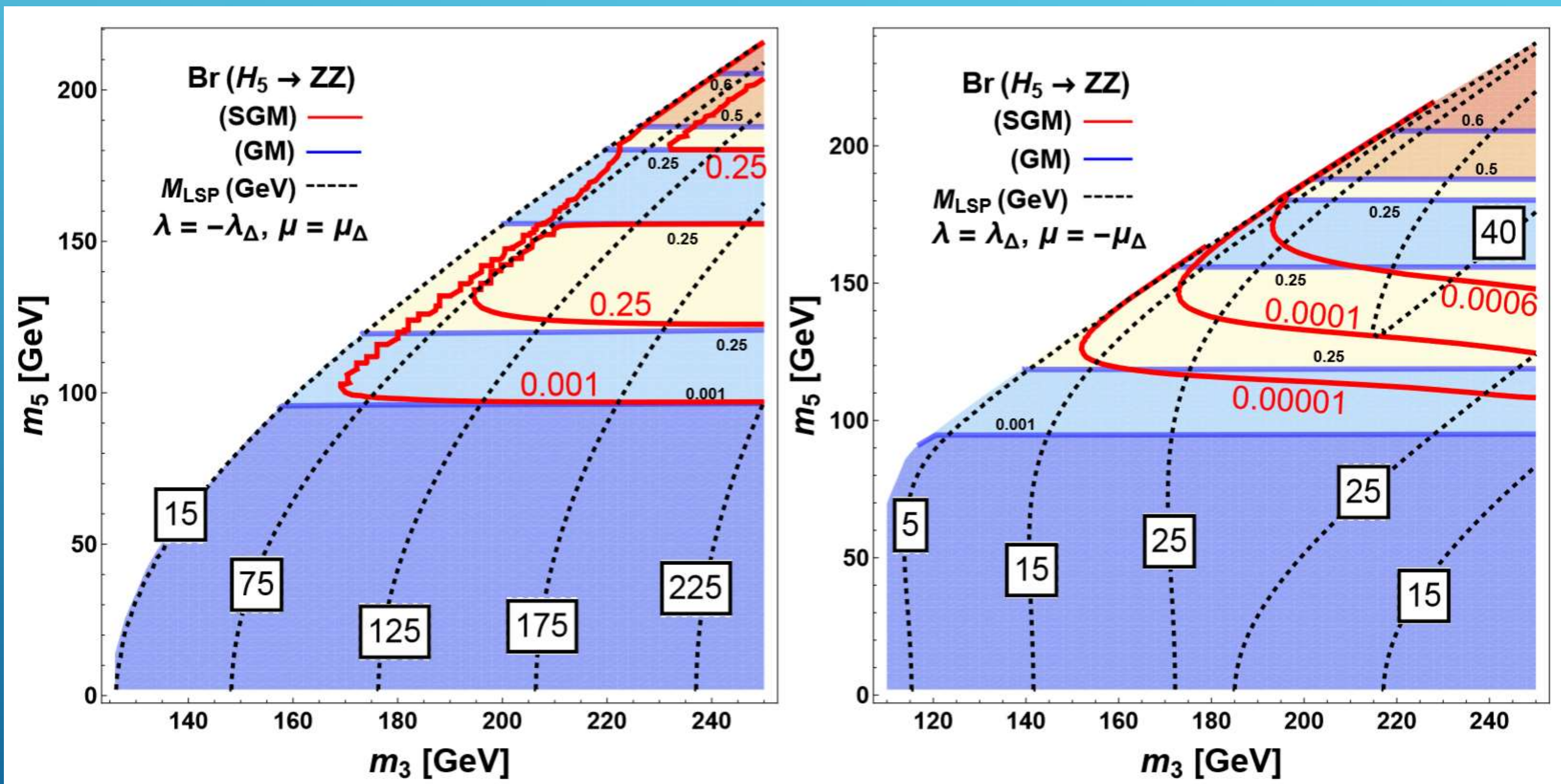
Light LSP



As we go to larger LSP masses GM and SGM converge

Light LSP opens up invisible decay width and reduces BR to photons in SGM V allows for possibility of evading constraints

We see similar qualitative behavior for H5 → ZZ decays

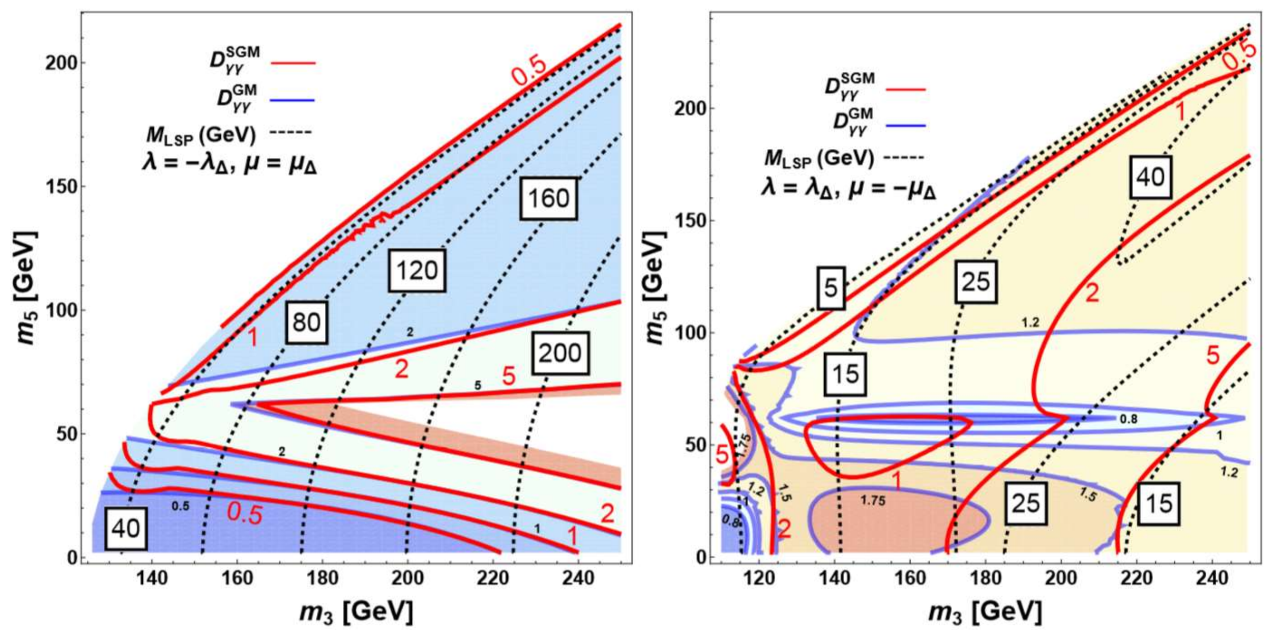


A suppressed ZZ branching ratio may be sign of SUSY origin

SGM versus GM with 'Higgs Golden Ratio'

- Can also affect decays of 125 GeV SM-like Higgs boson
- Higgs golden ratio very precisely measured (eventually O(1%))
- (A. Djouadi, J. Quevillon, RVM: 1509.03913)

$$\mathcal{D}_{\gamma\gamma}^{SGM(GM)} \equiv \frac{Br_{h\to\gamma\gamma}^{GM(SGM)}/Br_{h\to ZZ}^{GM(SGM)}}{Br_{h\to\gamma\gamma}^{SM}/Br_{h\to ZZ}^{SM}}$$



Ongoing Work and Conclusions

Ongoing and future work:

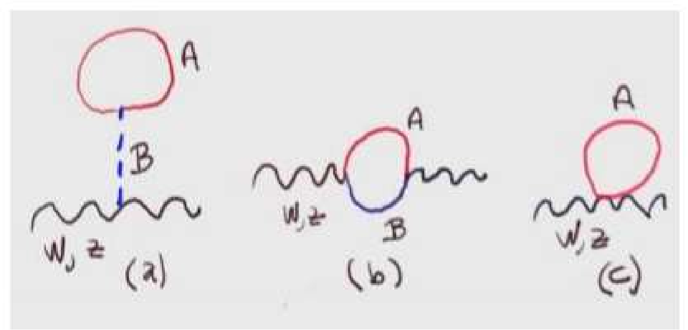
- Dedicated study of low mass diphoton signals
- Including NLO and RG evolution effects
- More general scan to find allowed parameter space
- Differential studies in $4l$ and $2l$ + γ channels

Summary and Conclusions:

- GM model was obtained as a limit of SCTM, we have dubbed this limit the Supersymmetric GM model
- SUSY implies constraints on the parameters of the GM potential
- SGM contains decays to DM V escape constraints from γγ
- Many other potentially striking signals at the LHC

Extra Slides

ρ at one loop:



$$\Delta\rho\,|_{loop} = \frac{g'^2 s_H^2}{4\pi M_{H_5}^2}\Lambda^2$$

Adding custodial breaking counter terms leads to misalignment of the vacuum:

$$<\phi_o> = <\psi_o>(1+\delta) = v_\Delta(1+\delta)$$

$$m_W^2 = \frac{g^2 v^2}{4}(1 + s_H^2\delta)$$

While m_Z^2 is unafected

$$\Rightarrow \Delta\rho|_\delta = s_H^2\delta$$

Therefore pick: $\delta = \frac{g'^2}{4\pi M_{H_5}^2}\Lambda^2$

Gunion, Wudka, RV: PRD43 (1991)

The Custodial Fields:

$$h_1^0 = \frac{1}{\sqrt{2}}(H_1^0 + H_2^0)$$
$$h_3^+ = H_2^+, \quad h_3^0 = \frac{1}{\sqrt{2}}(H_1^0 - H_2^0), \quad h_3^- = H_1^-$$

$$\delta_1^0 = \frac{\phi^0 + \chi^0 + \psi^0}{\sqrt{3}}$$
$$\delta_3^+ = \frac{\psi^+ - \phi^+}{\sqrt{2}}, \ \delta_3^0 = \frac{\chi^0 - \psi^0}{\sqrt{2}}, \ \delta_3^- = \frac{\phi^- - \chi^-}{\sqrt{2}}$$
$$\delta_5^{++} = \psi^{++}, \ \delta_5^+ = \frac{\phi^+ + \psi^+}{\sqrt{2}}, \ \delta_5^0 = \frac{-2\phi^0 + \psi^0 + \chi^0}{\sqrt{6}}, \ \delta_5^- = \frac{\phi^- + \chi^-}{\sqrt{2}}, \ \delta_5^{--} = \chi^{--}$$

The Physical Fields:

$$G^0 = \cos\theta\, h_{3I}^0 + \sin\theta\, \delta_{3I}^0$$
$$G^{\mp} = \cos\theta\, \frac{h_3^{\pm *} - h_3^{\mp}}{\sqrt{2}} + \sin\theta\, \frac{\delta_3^{\pm *} - \delta_3^{\mp}}{\sqrt{2}}$$
$$A^0 = -\sin\theta\, h_{3I}^0 + \cos\theta\, \delta_{3I}^0$$
$$A^{\mp} = -\sin\theta\, \frac{h_3^{\pm *} - h_3^{\mp}}{\sqrt{2}} + \cos\theta\, \frac{\delta_3^{\pm *} - \delta_3^{\mp}}{\sqrt{2}}$$

$$(h_{1R}^0, \delta_{1R}^0)$$

$$(h_{1I}^0, \delta_{1I}^0)$$

$$T_H = \begin{pmatrix} \frac{1}{\sqrt{2}}(h_3^+ + h_3^{-*}) \\ h_{3R}^0 \\ \frac{1}{\sqrt{2}}(h_3^- + h_3^{+*}) \end{pmatrix}, \quad T_\Delta = \begin{pmatrix} \frac{1}{\sqrt{2}}(\delta_3^+ + \delta_3^{-*}) \\ \delta_{3R}^0 \\ \frac{1}{\sqrt{2}}(\delta_3^- + \delta_3^{+*}) \end{pmatrix}$$

$$F_S = \begin{pmatrix} \frac{1}{\sqrt{2}}(\delta_5^{++} + \delta_5^{--*}) \\ \frac{1}{\sqrt{2}}(\delta_5^+ + \delta_5^{-*}) \\ \delta_{5R}^0 \\ \frac{1}{\sqrt{2}}(\delta_5^- + \delta_5^{+*}) \\ \frac{1}{\sqrt{2}}(\delta_5^{--} + \delta_5^{++*}) \end{pmatrix}, \quad F_P = \begin{pmatrix} \frac{1}{\sqrt{2}}(\delta_5^{--*} - \delta_5^{++}) \\ \frac{1}{\sqrt{2}}(\delta_5^{-*} - \delta_5^+) \\ \delta_{5I}^0 \\ \frac{1}{\sqrt{2}}(\delta_5^{+*} - \delta_5^-) \\ \frac{1}{\sqrt{2}}(\delta_5^{++*} - \delta_5^{--}) \end{pmatrix}$$

The SCTM Mass Spectrum in the limit of small v_Δ:

$$m_{F_s}^2 = x + \frac{\lambda\lambda_3}{2} + A_{3s}\tau - 4\lambda\lambda_3\tau^2$$

$$m_{F_p}^2 = z - \frac{\lambda\lambda_3}{2} - A_{3s}\tau + 4\zeta\lambda_3\tau + \left(2\lambda_3^2 + 4\lambda\lambda_3\right)\tau^2.$$

$$m_A^2 = \frac{2x + 4\lambda^2 - \lambda_3\lambda - 2A_{3s}\tau + 4\left(-8\lambda^2 + 2\lambda_3\lambda + \lambda_3^2\right)\tau^2}{2 - 16\tau^2}$$

$$\mathrm{x} + 3\,\lambda^2 - 2\,\mathrm{A3s}\,\tau - \lambda\,\lambda_3 + \frac{24\,\mathrm{A3s}\,\mathrm{x}\,\lambda\,\tau^3\,\lambda_3}{(\mathrm{x} - \lambda\,\lambda_3)^2} + \tau^2\left(-24\,\lambda^2 + 8\,\lambda\,\lambda_3 + 6\,\lambda_3^2 + \frac{12\,\mathrm{x}^2}{\mathrm{x} - \lambda\,\lambda_3}\right)$$

$$3\,\lambda^2 - \frac{24\,\mathrm{A3s}\,\mathrm{x}\,\lambda\,\tau^3\,\lambda_3}{(\mathrm{x} - \lambda\,\lambda_3)^2} + \tau^2\left(-24\,\lambda^2 - \frac{12\,\mathrm{x}^2}{\mathrm{x} - \lambda\,\lambda_3}\right)$$

The Lower Bound Constraints on the Potential (Logan et.al. 1404.2640)

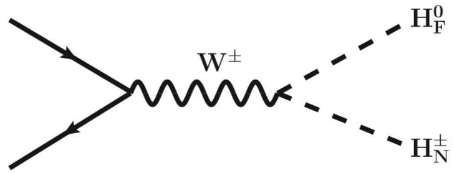
$$V^{(4)}(r, \tan\gamma, \zeta, \omega) = \frac{r^4}{(1+\tan^2\gamma)^2}\left[\lambda_1 + (\lambda_2 - \omega\lambda_5)\tan^2\gamma + (\zeta\lambda_3 + \lambda_4)\tan^4\gamma\right].$$

$$\lambda_1 > 0, \qquad \zeta\lambda_3 + \lambda_4 > 0, \qquad \text{and} \quad \lambda_2 - \omega\lambda_5 + 2\sqrt{\lambda_1(\zeta\lambda_3 + \lambda_4)} > 0.$$

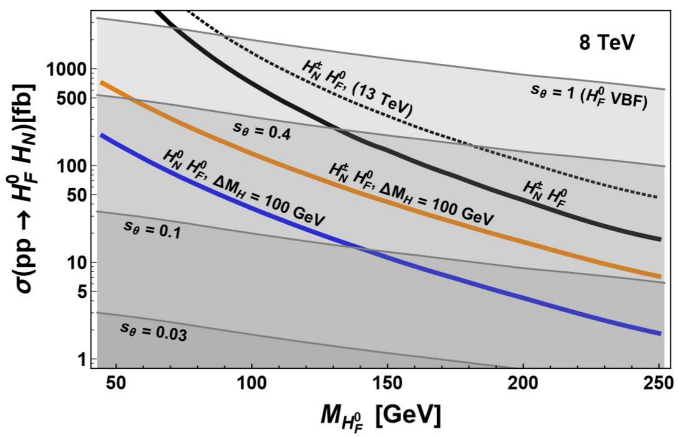
In the SGM $\lambda_4 = \lambda_3$, therefore the middle condition is satisfied as long as $\zeta < 1$. A problem can arise at $\zeta = 1$. However, in that case the quartic term drops out and V^4 Is reduce to a quadratic which is positive definite for the parameters as defined in the matching conditions. It can also be shown that the last condition is also satisfied for the allowed values of ω.

Fermiophobic Higgs Production

- Unlike in SM there is a DY Higgs pair production mechanism

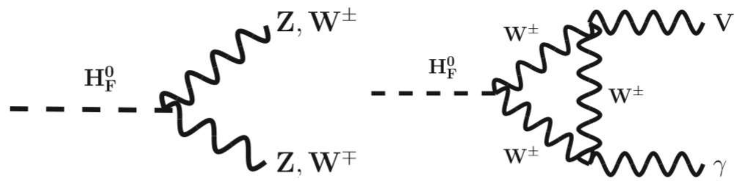


- For small VEV it is the dominant production mechanism



Fermiophobic Higgs Decays

- No $b\bar{b}$ (or gg) decays $\Rrightarrow$ large BR into $\gamma\gamma$, VV possible



- At low masses $\gamma\gamma$ dominates while at high masses WW/ZZ