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Gauge coupling unification without new particle thresholds

Mikhail Shaposhnikov

Work with Georgios Karananas, 1703.02964

Motivation: the hierarchy problem
41 years soon, dob June 15, 1976

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Gauge-symmetry hierarchies*

Eldad Gildener
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 15 June 1976)

It is shown that one cannot artifically establish a gauge hierarchy of any desired magnitude by arbitrarily adjusting the scalar-field parameters in the Lagrangian and using the tree approximation to the potential; radiative corrections will set an upper bound on such a hierarchy. If the gauge coupling constant is approximately equal to the electromagnetic coupling constant, the upper bound on the ratio of vector-meson masses is of the order of $\alpha^{-1/2}$, independent of the sclar-field masses and their self-couplings. In particular, the usual assumption that large scalar-field mass ratios in the Lagrangian can induce large vector-meson mass ratios is false. A thus far unsuccessful search for natural gauge hierarchies is briefly discussed. It is shown that if such a hierarchy occurred, it would have an upper bound of the order of $\alpha^{-1/2}$.

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Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

Hierarchy of Interactions in Unified Gauge Theories*

H. Georgi,† H. R. Quinn, and S. Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 15 May 1974)

We present a general formalism for calculating the renormalization effects which make strong interactions strong in simple gauge theories of strong, electromagnetic, and weak interactions. In an SU(5) model the superheavy gauge bosons arising in the spontaneous breakdown to observed interactions have mass perhaps as large as 10^{17} GeV, almost the Planck mass. Mixing-angle predictions are substantially modified.

GUTs

Proposal, going back to 70ties: Strong, weak and electromagnetic interactions are part of the same gauge force and are unified at high energies:

$$SU(3) \times SU(2) \times U(1) \in G$$

- 1973 - Pati, Salam: $G = SU(4) \times SU(2) \times SU(2)$. Lepton number as 4th colour, left-right symmetry
- 1974 - Georgi, Glashow $G = SU(5)$
- 1975 - Fritzsch, Minkowski $G = SO(10)$. All fermions of one generation are in one representation $\underline{16}$!

GUTs

Generic features of GUTs:

- charge quantisation is automatic
- quantum numbers of SM fermions can be understood
- $\sin^2\theta_W$ can be predicted: gauge coupling unification.
- some relations between quark and lepton masses (e.g. bottom quark and τ lepton) can appear
- common prediction: instability of matter, proton decay

Looks great!

Main trouble: hierarchy problem

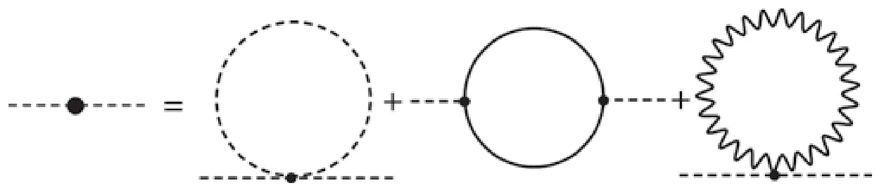
Extra particles beyond the SM – leptoquarks (vector and scalar) must be very heavy, $M_X > 10^{15}$ GeV

- this is required by the gauge coupling unification
- this is needed for stability of matter, proton lifetime $\tau_p > 10^{34}$ years

Hierarchy: $(\frac{M_X}{M_W})^2 \simeq 10^{28}$

Two faces of hierarchy

- Ad hoc tuning between the parameters (masses and couplings of different multiplets) at the tree level with an accuracy of 26 orders of magnitude
- Stability of the Higgs mass against radiative corrections Gildener, '76



$$\delta m_H^2 \simeq \alpha_{GUT}^n M_X^2$$

Tuning is needed up to 14th order of perturbation theory!

Proposed solutions

Stability of EW scale: requirement of “naturalness”:

- Low energy SUSY: compensation of bosonic loops by fermionic loops
- Composite Higgs boson - new strong interactions
- Large extra dimensions

All require new physics right above the Fermi scale, which was expected to show up at the LHC

However, the LHC has discovered something quite unexpected : the Higgs boson and nothing else, confirming the Standard Model.

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For $\mathbf{125}$ GeV Higgs mass the Standard Model is a self-consistent weakly coupled effective field theory for all energies up to the quantum gravity scale $\boldsymbol{M_P \sim 10^{19}}$ GeV

Should we abandon Grand Unification?

Should we abandon Grand Unification?

Should we accept fine tunings in many orders of perturbation theory?

Main problem of the stability of the Higgs mass against radiative corrections: existence of superheavy particles, $\delta m_H^2 \propto M_X^2$.

Do we need lepto-quarks for GUTs?

Yes, if the Nature we know at EW scale repeats itself at the gauge coupling unification scale!

Physics at EW scale $\equiv$ dynamical Higgs mechanism $\equiv$ true Higgs boson

Perhaps, the physical meaning of the GUT scale is different from that of EW scale?

A lesson from gravity

Gauging of Poincaré group Einstein-Cartan-Sciama-Kibble theory
Covariant field strengths

$$\text{torsion (shifts):}\quad T^A_{\mu\nu} = \partial_\mu e^A_\nu - \partial_\nu e^A_\mu - \omega^A_{\mu B} e^B_\nu + \omega^A_{\nu B} e^B_\mu \ ,$$
$$\text{curvature (Lorentz):}\quad \omega^{AB}_{\mu\nu} = \partial_\mu \omega^{AB}_\nu - \partial_\nu \omega^{AB}_\mu - \omega^{AC}_\mu \omega^B_{\nu C} + \omega^{AC}_\nu \omega^B_{\mu C}$$

20 physical degrees of freedom = graviton + heavy states + ghosts (for generic action).
Gauge covariant constraint: vanishing of torsion

$$T^A_{\mu\nu} = 0 \ ,$$

We end up with the relation between the connection and the metric $\omega = \bar{\omega} \sim \partial e$ and

Einstein gravity with 2 degrees of freedom!

$\Rightarrow$ Poincaré group can be gauged with the vielbein only if (gauge invariant) constraints are added!

Gauge coupling unification without new particle thresholds

Idea: Take some GUT and remove all heavy degrees of freedom by imposing gauge-invariant constraints.
How does it work? SU(5) example.

- Scalar leptoquarks in $\underline{24}$, Σ .

Consider eigenvalues σ_i of Σ^2. They are gauge invariant - any condition on them does not break gauge symmetry

$$\sigma_1 = \sigma_2 = \sigma_3 = v_{GUT}^2 \,, \quad \sigma_4 = \sigma_5 = \frac{9}{4} v_{GUT}^2 \,,$$

From geometrical point of view, this operation confines the theory on a specific manifold in the field-space. When this is done, a generic $\boldsymbol{\Sigma}$ field can be expressed as

$$\boldsymbol{\Sigma^2 = U \begin{pmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5 \end{pmatrix} U^\dagger \,,}$$

with $\boldsymbol{U \in G}$. The above spans the twelve-dimensional space of Goldstones.

- Scalar leptoquarks in $\underline{5}$, H.

$$H^\dagger \Sigma^2 H - \frac{3}{10}\mathrm{Tr}(\Sigma^2) H^\dagger H = 0 \, .$$

This requirement eliminates the colour triplet contained in H, but leaves intact the remaining two components which are identified with the SM Higgs field

- Vector leptoquarks in 24

$$\text{Tr}\left([\Sigma, D_\mu \Sigma]^2\right) = 0 \ ,$$

All the heavy vector leptoquarks are set to zero, together with corresponding Goldstones. The twelve SM gauge fields are not affected.

Resulting theory: Renormalisable Standard Model which inherits from SU(5)

- fermion quantum numbers
- relations between the gauge couplings
- relations between the Yukawa couplings

Small Higgs mass requirement:

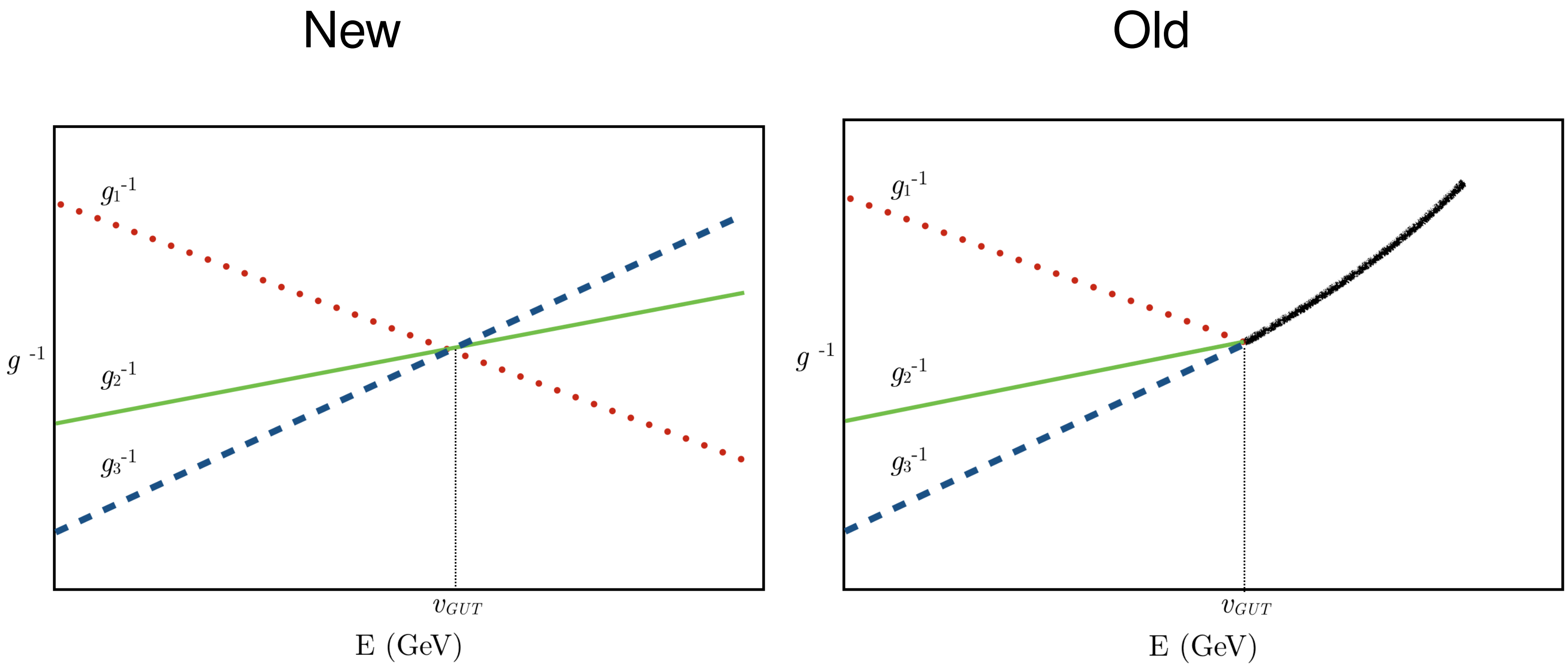
$$m_H^2 - \frac{1}{2}(\lambda_{HH} v_{EW}^2 + (\lambda_{\Sigma H} + \lambda'_{\Sigma H}) v_{GUT}^2) \sim \mathcal{O}(10^4)\ \text{GeV}^4\ .$$

This relation constitutes a fine-tuning that is not explained. It is, however a technically natural condition due to absence of superheavy particles. No proton decay!

Notations in potential:

$$V = -\frac{1}{2} m_\Sigma^2 \text{Tr}(\Sigma^2) - \frac{1}{2} m_H^2 H^\dagger H + \frac{1}{4}\lambda_{\Sigma\Sigma}\left(\text{Tr}(\Sigma^2)\right)^2 + \frac{15}{14}\lambda'_{\Sigma\Sigma}\text{Tr}(\Sigma^4)$$
$$+ \frac{1}{4}\lambda_{HH}\left(H^\dagger H\right)^2 + \frac{1}{2}\lambda_{\Sigma H}\text{Tr}(\Sigma^2) H^\dagger H + \frac{5}{3}\lambda'_{\Sigma H} H^\dagger \Sigma^2 H\ .$$

Gauge coupling unification



As in the Minimal SU(5):

- $v_{GUT} \simeq 10^{14}$ GeV, but no problem with the proton decay
- $\sin^2\theta_W \simeq 0.2$ – too small

How to correct $\sin^2\theta_W$? Proposal goes back to Hill; Shafi and Wetterich: add higher-dimensional operators suppressed by the Planck scale,

$$\mathcal{O}_{4+n} = \text{Tr}\left[F_{\mu\nu}\Sigma^k F^{\mu\nu}\Sigma^{n-k}\right]\ ,\quad 0 \leq k < n\ ,\quad n > 0\ ,$$

With our constraint on Σ, these terms modify the relation $g_1 = g_2 = g_3$ at the GUT scale, change the prediction of $\sin^2\theta_W$, and modifying v_{GUT}. The theory is still renormalisable and no new degrees of freedom are introduced!

A viable possibility: $v_{GUT} \simeq M_P$ – unity of all forces at the Planck scale?

Scale and conformal invariance or how to get “naturally” small Higgs mass

Scale and conformal invariance
or how to get “naturally” small Higgs mass

Why scale invariance?

Scale and conformal invariance or how to get "naturally" small Higgs mass

Why scale invariance?

If the mass of the Higgs boson is put to zero in the SM, the Lagrangian has a wider symmetry: it is scale and conformally invariant.

Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \rightarrow \sigma^n \Psi(\sigma x) ,$$

$n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

It is tempting to use this symmetry for solution of the hierarchy problem

Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} ,$$

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The way out: scale independent subtraction of divergences Englert, Truffin ’76; Wetterich ’88; MS, Zenhausern, ’08

Scale-invariant SU(5) construction

Extra field - dilaton χ. Also appears as a normalisation point in renormalisation procedure. Constraint for Σ should be replaced by

$$\sigma_1 = \sigma_2 = \sigma_3 = \alpha\chi^2 \,, \quad \sigma_4 = \sigma_5 = \frac{9\alpha}{4}\chi^2 \,,$$

where α is a dimensionless constant. The remaining two conditions for vectors and H remain the same.

The scale-invariant potential for the theory: add a quartic self-interaction for the dilaton, $\Lambda'\chi^4$, and replace the mass terms for Σ and H by the dilaton couplings:

$$m_\Sigma^2 = \frac{15\nu\alpha}{4}\chi^2 \,, \quad m_H^2 = \frac{15\mu\alpha}{2}\chi^2 \,,$$

Resulting potential for the Higgs field h:

$$V = \lambda \left(h^\dagger h - \frac{\beta}{2\lambda}\chi^2 \right)^2 + (\Lambda + \Lambda')\chi^4 \, ,$$

and λ, β, Λ are related to the constants appearing in V as

$$\lambda = \frac{\lambda_{HH}}{4} \, , \;\; \beta = \frac{15\alpha}{4}(\mu - \lambda_{\Sigma H} - \lambda'_{\Sigma H}) \, ,$$

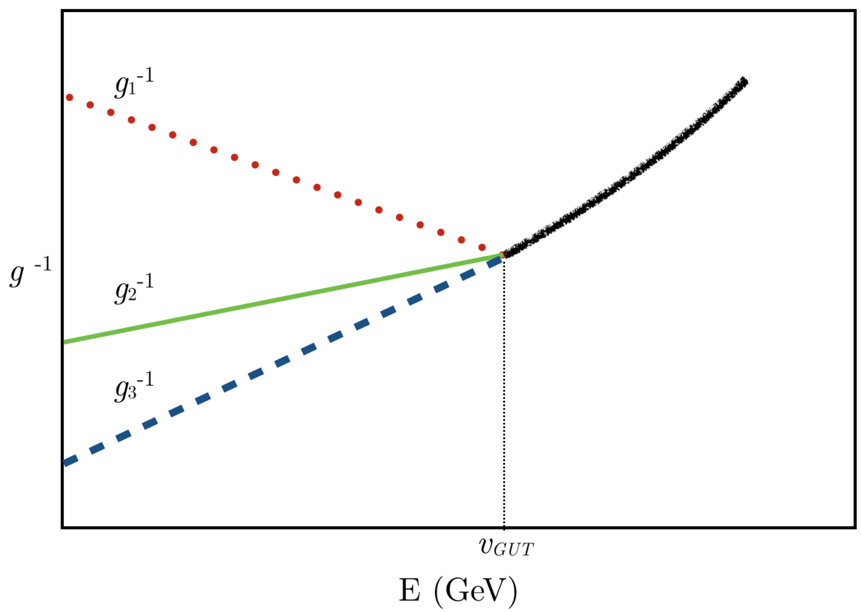
$$\Lambda = \left(\frac{15\alpha}{4}\right)^2 \left(\lambda_{\Sigma\Sigma} + \lambda'_{\Sigma\Sigma} - \nu - \lambda^{-1}_{HH}(\mu - \lambda_{\Sigma H} - \lambda'_{\Sigma H})^2 \right) \, .$$

Existence of flat direction (absence of cosmological constant) - unexplained fine-tuning, $\Lambda + \Lambda' = 0$. Gauge hierarchy condition $\frac{\beta}{\alpha} \ll 1$, is a technically natural requirement, since the dilaton has an approximate shift symmetry in the limit $\beta \to 0, \; \Lambda + \Lambda' \to 0$.

UV limit?

High energy limit, $E \gg v_{GUT}$: equivalent to $\chi \to 0$? $\Sigma = 0$ as a solution to all constraints? If true, the UV degrees of freedom are SU(5) gauge bosons, fermions, dilaton and the Higgs 5-plet.

Asymptotically free behaviour?



Inclusion of gravity

Planck scale: through non-minimal coupling of the dilaton to the Ricci scalar.

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + \xi_h h^2\right) \frac{R}{2} \,,$$

This term, for $\xi_\chi \sim 1$, does break the shift symmetry. However, this is a coefficient in front of graviton kinetic term. Since the graviton stays massless in any constant scalar background, the perturbative computations of gravitational corrections to the Higgs mass in scale-invariant regularisation are suppressed by M_P. There are no corrections proportional to M_P!

Consequences

- Theory is “natural” in perturbative sense: Higgs mass is stable against radiative corrections

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- The dilaton is massless in all orders of perturbation theory
- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential: it makes scale transformations to be internal symmetry!)
- Fifth force or Brans-Dicke constraints are not applicable to it

In our approach we have no new particles up to the gravitational Planck scale.

Where is new physics?

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Where is new physics?

- Observations of neutrino oscillations (in the SM neutrinos are massless and do not oscillate)
- Evidence for Dark Matter (SM does not have particle physics candidate for DM).
- No antimatter in the Universe in amounts comparable with matter (baryon asymmetry of the Universe is too small in the SM)
- Cosmological inflation is absent in canonical variant of the SM
- Accelerated expansion of the Universe (?) - though can be "explained" by a cosmological constant.

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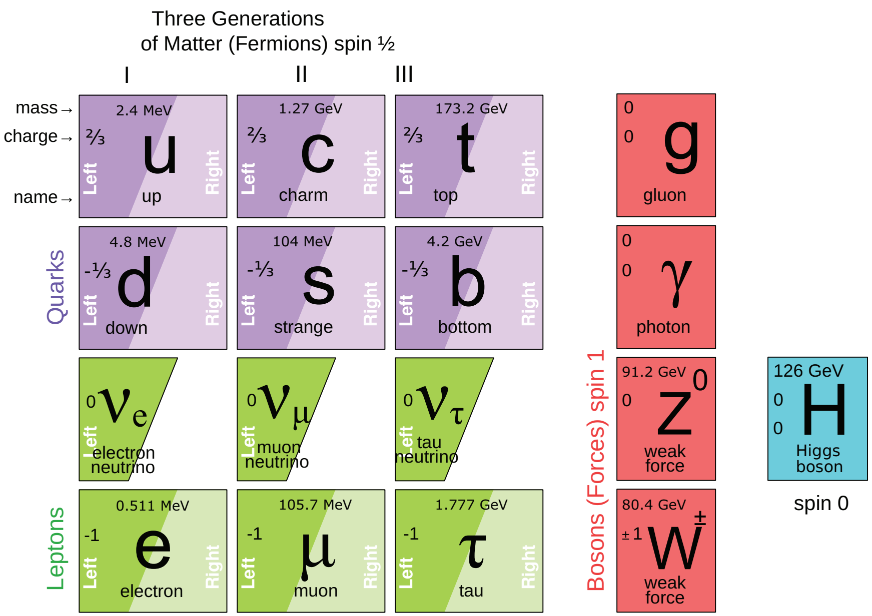
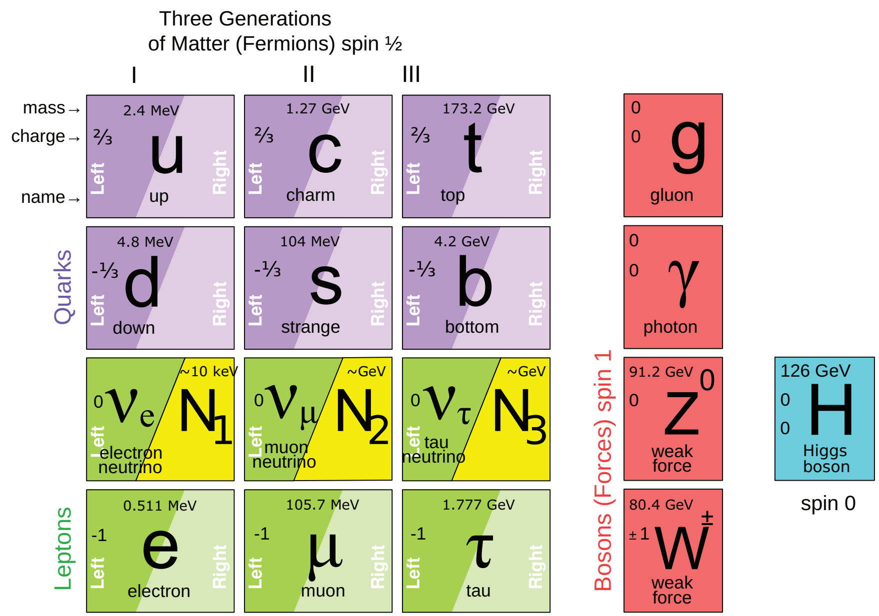
Below the Fermi scale

New physics below the Fermi scale: the νMSM





Role of the Higgs: EW symmetry breaking, inflation

Role of N_1 with mass in keV region: dark matter.

Role of $N_2, \; N_3$ with mass in 100 MeV – GeV region: “give” masses to neutrinos and produce baryon asymmetry of the Universe.

All fermions can be embedded in SO(10)

Conclusions

- The gauge coupling unification scale may be not related to the mass of any particle
- “Constrained GUTs" provide a specific example of unified theories without leptoquarks
- In these theories the EW scale is stable against radiative corrections

Problems and weak points

- The choice of GUT symmetry is arbitrary
- The choice of scalar multiplets is arbitrary
- Why 3 generations?
- Origin of constraints - why the one leading to the SM is the best one?