

# Conformal Extensions of the Standard Model

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# Hierarchy Problems

- 1) why are scales vastly different
- 2) why do scales remain vastly different under quantum corrections

## SM + embedding at $\Lambda$

$$\delta M_H^2 = \frac{\Lambda^2}{32\pi^2 V^2} (6M_W^2 + 3M_Z^2 + 3M_H^2 - 12M_t^2) \sim \Lambda^2 \gg M_H^2$$

**SM + Dirac neutrinos:** no problem – just like SM

**SM + Majorana neutrinos:**

- more than one scale: VEV and the Majorana mass(es)  $M$

→ generates a HP problem for large  $M$  even if  $y_\nu$  is tiny

$$\delta m_H^2 \simeq \frac{y_\nu^2}{16\pi^2} M^2 \quad y_\nu^2 = M m_\nu / v^2$$

→  $M \lesssim 10^7 - 10^8 \text{ GeV}$  ↔ see-saw, leptogenesis, ...



# Look again carefully at the SM as a QFT

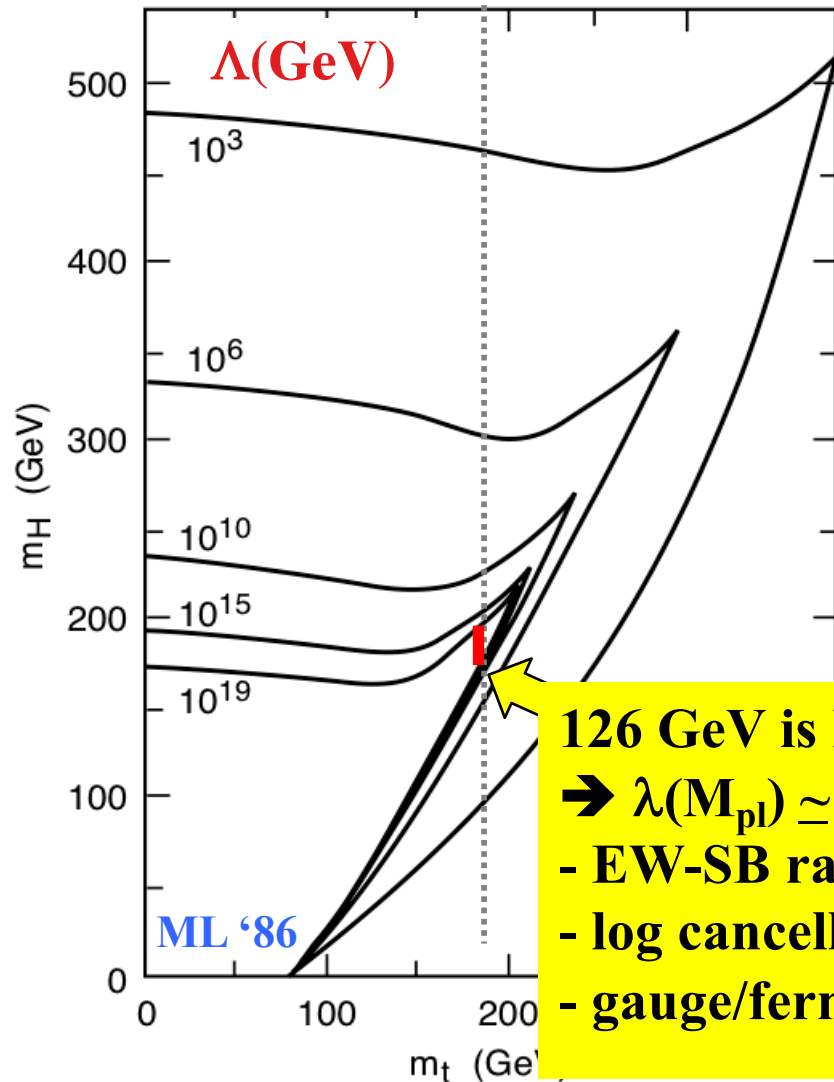
- The SM itself (without embedding) is a 4d QFT like QED
    - infinities, renormalization  $\leftrightarrow \delta^2 \rightarrow$  only differences are calculable
    - SM itself is perfectly OK  $\rightarrow$  many things unexplained...
  - Has (like QED) a **triviality problem** (Landau poles  $\leftrightarrow$  infinite  $\lambda$ )
    - triviality = inconsistency  $\rightarrow$  requires some scale  $\Lambda$  where the SM is embedded
    - running  $U(1)_Y$  coupling: pole well beyond Planck scale... - like in QED
    - running Higgs / top coupling  $\rightarrow$  upper bounds on  $m_H$  and  $m_t$
  - Another potential problem is **vacuum instability** ( $\leftrightarrow$  negative  $\lambda$ )
    - does occur in SM for large top mass  $> 79$  GeV  $\rightarrow$  lower bounds on  $m_H$
- $\rightarrow$  important detail: SM has only one single scale  $v=246$  GeV**

The SM as QFT (without an embedding) works perfectly:

- a hard cutoff  $\Lambda$  and the sensitivity towards  $\Lambda$  has no meaning
- renormalizable, calculable ... - just like QED
- BUT: an embedding is required  $\leftrightarrow$  triviality...

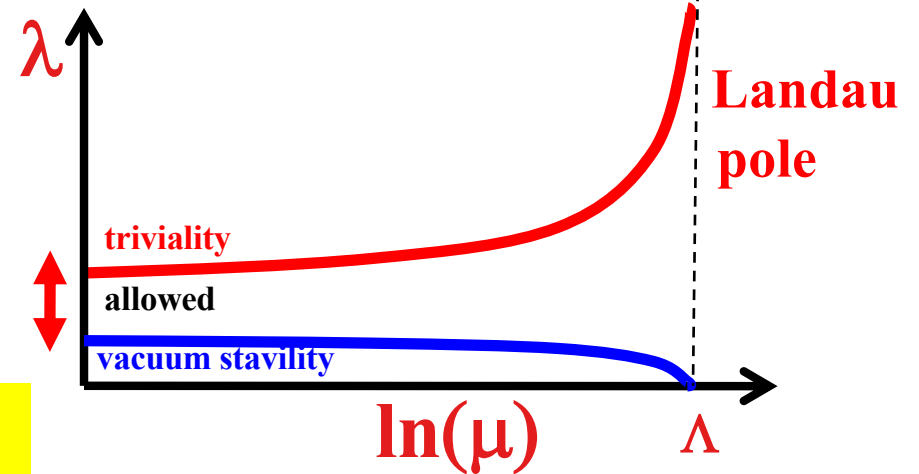
# A remarkable Coincidence

- SM is a renormalizable QFT like QED w/o hierarchy problem
- Cutoff “ $\Lambda$ ” has no meaning → **triviality, vacuum stability**



$$126 \text{ GeV} < m_H < 174 \text{ GeV}$$

SM does not exist w/o embedding  
- U(1) coupling, Higgs self-coupling



- RGE arguments seem to work
- but we need some embedding

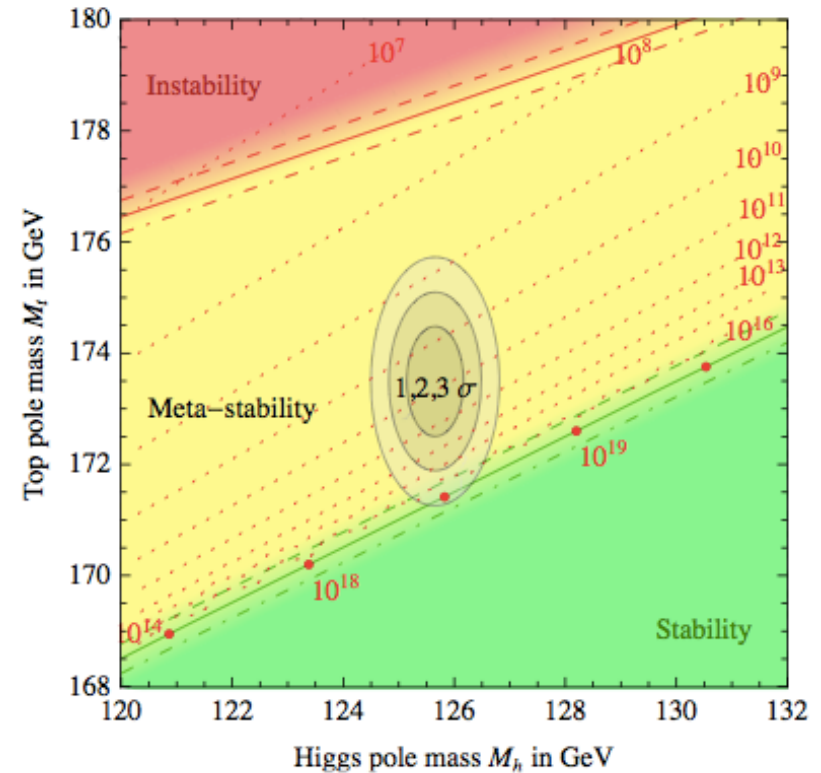
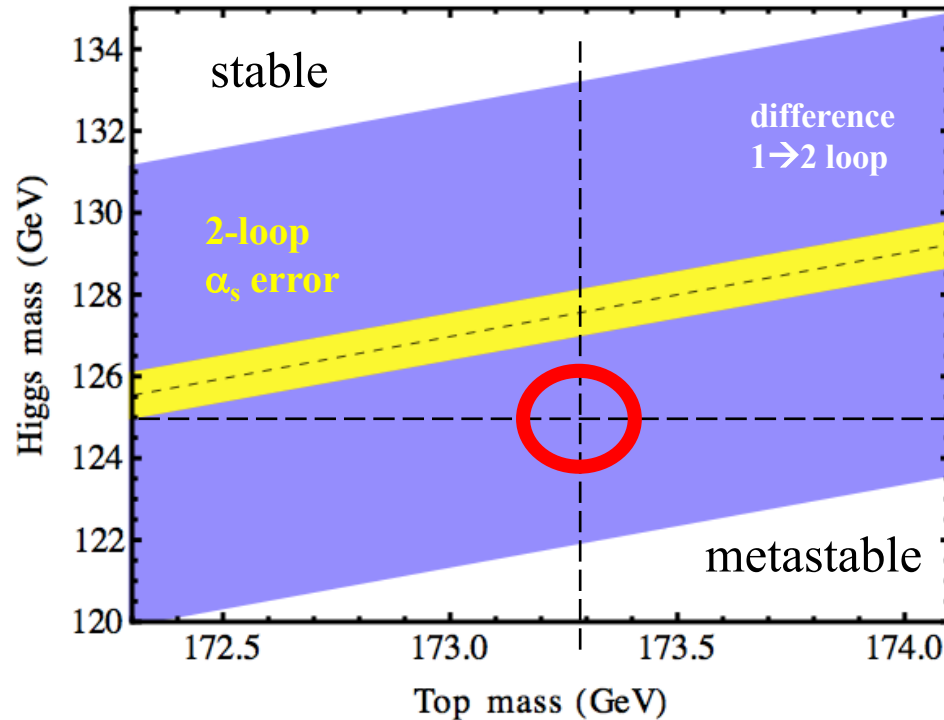
# Is the Higgs Potential at $M_{\text{Planck}}$ flat?

Holthausen, ML, Lim

12 Dec 2011

Elias-Miro, Espinosa, Giudice, Isidori, Riotto, Strumia

13 Dec 2011



**Experimental values indicate metastability. Is it fully established?**

→ we need to include DM, neutrino masses, ...? are all errors (EX+TH) fully included?

→ be cautious about claiming that metastability is established

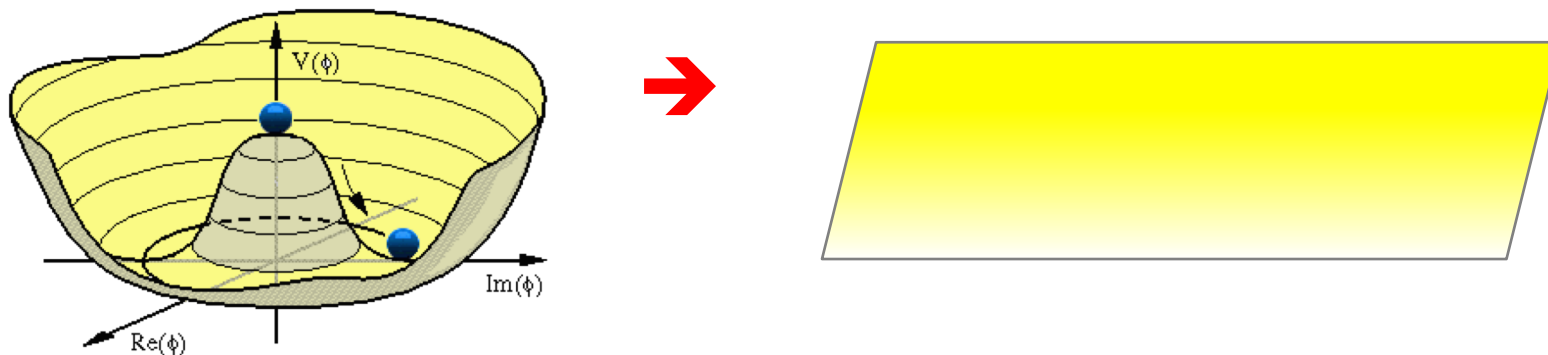
→ **An important observation:**

- remarkable relation between weak scale,  $m_t$ , couplings and  $M_{\text{Planck}}$   $\leftrightarrow$  precision

- remarkable interplay between gauge, Higgs and top loops (log divergences – not  $\Lambda^2$ )

# Is there a Message?

- $\lambda(M_{\text{Planck}}) \simeq 0$ ?  $\rightarrow$  remarkable log cancellations  $\leftrightarrow$  CA  $\sim$   $\beta$ -fcts.  
 $M_{\text{planck}}$ ,  $M_{\text{weak}}$ , gauge, Higgs & Yukawa couplings are unrelated
- remember:  $\mu$  is the only single scale of the SM  $\rightarrow$  special role
  - $\rightarrow$  if in addition  $\mu^2 = 0 \rightarrow V(M_{\text{Planck}}) \simeq 0$
  - $\rightarrow$  flat Mexican hat (<1%) at the Planck scale!



- $\rightarrow$  conformal (or shift) symmetry as solution to the HP
- $\rightarrow$  combined conformal & EW symmetry breaking
  - conceptual issues
  - minimal realizations  $\leftrightarrow$  SM seems to know about high scales  $\rightarrow$  bottom-up
    - $\leftrightarrow$  many new d.o.f. (fields, big reps.)  $\sim$  UV-instabilities

# The Problem: EXPLICIT Scales

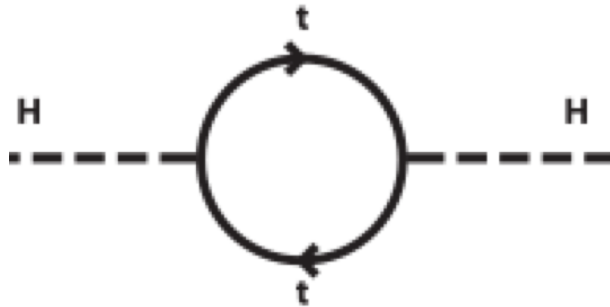
- Renormalizable QFT with two scalars  $\varphi, \Phi$  with masses  $m, M$  and a hierarchy  $m \ll M$
  - These scalars must interact since  $\varphi^\dagger\varphi$  and  $\Phi^\dagger\Phi$  are singlets  
→  $\lambda_{\text{mix}}(\varphi^\dagger\varphi)(\Phi^\dagger\Phi)$  must exist (= portal) in addition to  $\varphi^4$  and  $\Phi^4$
  - Quantum corrections  $\sim M^2$  drives both masses to the (heavy) scale  
→ vastly different explicit scalar scales are generically unstable
- 
- Since SM Higgs exists → problem: embedding with a 2<sup>nd</sup> scalar
    - gauge extensions: LR, PS, GUTs → must be broken...
    - even for SUSY GUTS → doublet-triplet splitting...
    - also for fashinable Higgs-portal scenarios...

## Ways out:

- no 2<sup>nd</sup> Higgs → just the SM → triviality → requires a new scale...
- symmetry: SUSY, ... → conformal symmetry = no explicit scales!  
→ all scales emerge from no-scale theories



# Conformal Symmetry and the Hierarchy Problem



**Theories without any scale in  $\mathcal{L} \rightarrow$  CS**

**Non-linear realizations of CS:**

- $\rightarrow$  symmetry is classically preserved**
- $\rightarrow$  naïve power counting invalid**
- $\rightarrow$  classically: no  $\Lambda^2$ ,  $\log(\Lambda)$  divergences**

**Conformal Anomaly (CA = breaking by loops)**

**anomaly  $\sim$  trace of energy momentum tensor**

**$\leftrightarrow \beta$ -functions  $\leftrightarrow \log(\Lambda)$**

**$\rightarrow$  CA does not fully restore naive power counting:  
 $\log(\Lambda)$ , but no  $\Lambda^2$**

**$\rightarrow$  avoids hierarchy problem**

**$\rightarrow$  dimensional transmutation of conformal theories  
by log running of couplings like in chiral QCD**

# Conformal Symmetry and SM Extensions

## Main idea:

- Do not introduce any fundamental (explicit) scales  
→ theories with conformal or shift symmetry
- Dynamical breaking of CS → Coleman Weinberg  $V_{\text{eff}}$   
→ all scale(s) by dimensional transmutation  
→ non-linear realization of CS:
  - naïve power counting ( $\sim \Lambda^2$ ) misleading
  - similar to gauge symmetry and vector boson masses
- An UV complete theory should have UV fixedpoints...

**The SM parameters may point in that direction!**

# Generic Questions

- Isn't the Planck-scale spoiling things (explicit scale, cut-off, ...)?
  - non-linear realization of conformal symmetry...
  - **conformal gravity...**
  - protected by conformal symmetry up to conformal anomaly
  - **generate  $M_{\text{Planck}}$  by dimensional transmutation**
  - for now assumption:  $M_{\text{Planck}}$  somehow generated in a conformal setting
- Are  $M_{\text{planck}}$  and  $M_{\text{weak}}$  connected?
  - 1<sup>st</sup> part: assumed to be independently generated scales
  - later more...
- UV: ultimate solution should be asymptotically safe → **UV-FPs...**
- Significant conceptual change for scale setting:  
Until now a rollover of scale generation:  $\text{SM} \rightarrow \text{BSM} \rightarrow \text{GUT} \rightarrow \text{gravit} @ M_{\text{Planck}}$   
Requires a new concept @  $M_{\text{planck}}$  → strings, ...  
**CS: Absolute scales meaningless, relative scales are calculable quantum effects**  
**Fully consistent realization → now new concept for scale setting required**

# Realizing the Idea

# Why the minimalistic SM does not work

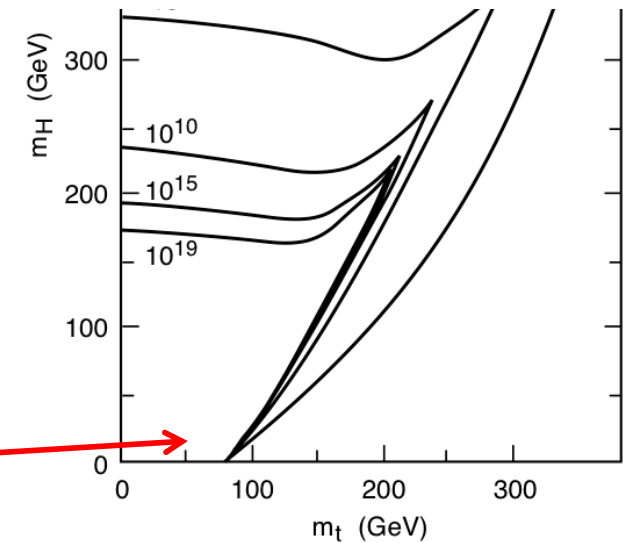
Minimalistic version:  $\rightarrow$  “SM-”

SM + with  $\mu=0 \leftrightarrow$  CS

Coleman Weinberg: effective potential

$\rightarrow$  CS breaking (**dimensional transmutation**)

$\rightarrow$  induces for  $m_t < 79$  GeV  
a Higgs mass  $m_H = 8.9$  GeV

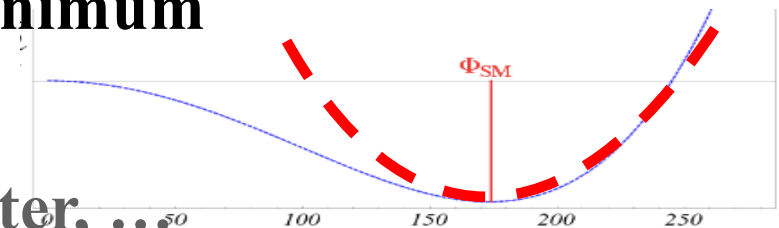


- Success: **no-scale SM  $\rightarrow$  SM** but: Higgs and top too do not fit

- DSB for weak coupling  $\leftrightarrow$  CS = phase boundary  
 $\rightarrow$  scale set by running couplings  $\rightarrow$  gap eqn: hierarchical!

- Reason for  $m_H \ll v$ :  $V_{\text{eff}}$  flat around minimum  
 $\leftrightarrow m_H \sim \text{loop factor} \sim 1/16\pi^2$

AND: We need neutrino masses, dark matter, ...



# Realizing the Idea via Higgs Portals

- SM scalar  $\Phi$  plus some new scalar  $\varphi$  (or more scalars)
- CS  $\rightarrow$  no scalar mass terms
- the scalar portal  $\lambda_{\text{mix}}(\varphi^+\varphi)(\Phi^+\Phi)$  must exist

$\rightarrow$  a condensate of  $\langle\varphi^+\varphi\rangle$  produces  $\lambda_{\text{mix}}\langle\varphi^+\varphi\rangle(\Phi^+\Phi) = \mu^2(\Phi^+\Phi)$   
 $\rightarrow$  effective mass term for  $\Phi$

- CS anomalous ...  $\rightarrow$  breaking  $\rightarrow$  only  $\ln(\Lambda)$   
 $\rightarrow$  implies a TeV-ish condensate for  $\varphi$  to obtain  $\langle\Phi\rangle = 246$  GeV
- Model building possibilities / phenomenological aspects:
  - $\varphi$  could be an effective field of some hidden sector DSB
  - further particles could exist in hidden sector; e.g. confining...
  - extra hidden U(1) potentially problematic  $\leftrightarrow$  U(1) mixing
  - avoid Yukawas which couple visible and hidden sector $\rightarrow$  phenomenology safe due to Higgs portal, but there is TeV-ish new physics!

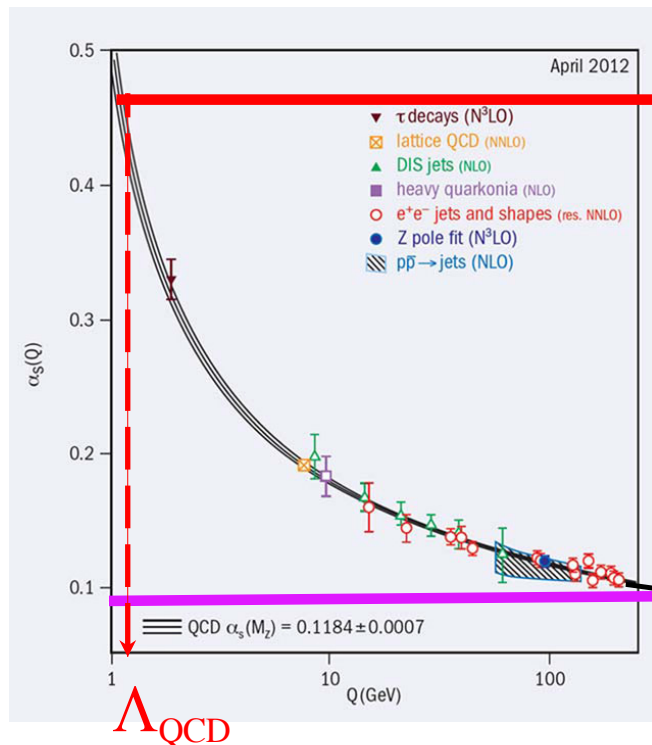
# Rather minimalistic: SM + QCD Scalar S

J. Kubo, K.S. Lim, ML New scalar representation S  $\rightarrow$  QCD gap equation:

$$\text{---}\bullet\text{---} = \text{---}\text{---} + \text{---}\bullet\text{---} + \dots \rightarrow C_2(S)\alpha(\Lambda) \gtrsim X$$

$C_2(\Lambda)$  increases with larger representations

$\leftrightarrow$  condensation for smaller values of running  $\alpha$



$$q=3 \quad \mathcal{L} = \mathcal{L}_{\text{SM}, m^2 \rightarrow 0} + (D_{\mu, ij} S_j)^\dagger (D_{ik}^\mu S_k) + \lambda_{HS} H^\dagger H S^\dagger S - \lambda_{1_i} [\bar{S} \times S \times \bar{S} \times S]_{1_i}$$

$$\lambda_{HS} \langle S^\dagger S \rangle H^\dagger H \rightarrow \lambda_{HS} \Lambda^2 H^\dagger H$$

$$m_h^2 = 2\lambda_{HS} \Lambda^2 \quad \frac{\lambda_h}{\lambda_{HS}} = \frac{\Lambda^2}{v^2}$$

# SM $\otimes$ hidden SU(3)<sub>H</sub> Gauge Sector

Holthausen, Kubo, Lim, ML

- hidden SU(3)<sub>H</sub>:

$$\mathcal{L}_H = -\frac{1}{2}\text{Tr } F^2 + \text{Tr } \bar{\psi}(i\gamma^\mu D_\mu - yS)\psi$$

gauge fields ;  $\psi = 3_H$  with SU(3)<sub>F</sub> ; **S = real singlet scalar**

- SM coupled by S via a Higgs portal:

$$V_{\text{SM}+S} = \lambda_H (H^\dagger H)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{HS} S^2 (H^\dagger H)$$

- no scalar mass terms
- use similarity to QCD, use NJL approximation, ...
- $\chi$ -ral symmetry breaking in hidden sector:  
SU(3)<sub>L</sub> × SU(3)<sub>R</sub> → SU(3)<sub>V</sub> → **generation of TeV scale**  
→ **transferred into the SM sector through the singlet S**  
→ **dark pions are PGBs: naturally stable → DM**



# Realizing the Idea: Specific Models

SM + extra singlet or doublet:  $\Phi, \varphi$

Nicolai, Meissner Farzinnia, He, Ren, Foot, Kobakhidze, Volkas, Hill, ...

Minimal B-L extension of SM:  $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

Iso, Okada, Orikasa

Minimal LR-model:  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  Holthausen, ML, Schmidt

SM  $\otimes$   $SU(N)_H$  with new N-plet in a hidden sector

Ko, Carone, Ramos, Holthausen, Kubo, Lim, ML, Hambye, Strumia, ...

SM + QCD colored scalar which condenses at TeV scale Kubo, Lim, ML

SM  $\otimes$  [ $SU(2)_X \otimes U(1)_X$ ]

Altmannshofer, Bardeen, Bauer, Carena, Lykken

...

**Since the SM-only version does not work  $\rightarrow$  observable effects:**

- Higgs coupling to other scalars (singlet, hidden sector, ...)
- dark matter candidates  $\leftrightarrow$  hidden sectors & Higgs portals
- consequences for neutrino masses

# Conformal Symmetry & Neutrino Masses

ML, S. Schmidt and J. Smirnov

- No explicit scale  $\rightarrow$  no explicit (Dirac or Majorana) mass term  
 $\rightarrow$  only Yukawa couplings  $\otimes$  generic scales
- Enlarge the Standard Model field spectrum  
like in 0706.1829 - R. Foot, A. Kobakhidze, K.L. McDonald, R. Volkas
- Consider direct product groups: SM  $\otimes$  HS
- Two scales: **CS breaking scale at O(TeV) + induced EW scale**

**Important consequence for fermion mass terms:**

- $\rightarrow$  spectrum of Yukawa couplings  $\otimes$  TeV or EW scale
- $\rightarrow$  interesting consequences  $\leftrightarrow$  Majorana mass terms are no longer expected at the generic L-breaking scale  $\rightarrow$  anywhere

# Examples

$$\mathcal{M} = \begin{pmatrix} 0 & y_D \langle H \rangle \\ y_D^T \langle H \rangle & y_M \langle \phi \rangle \end{pmatrix}$$

→ generically expect a TeV seesaw

BUT:  $y_M$  can be tiny

→ wide range of sterile masses → including pseudo-Dirac case

→ suppressed  $0\nu\beta\beta$

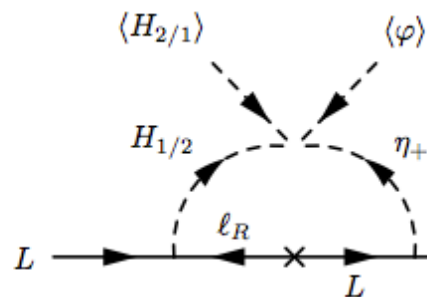
## Yukawa seesaw:

SM +  $\nu_R$  + singlet

$$\langle \phi \rangle \approx \text{TeV}$$

$$\langle H \rangle \approx 1/4 \text{ TeV}$$

## Radiative masses



$$\mathcal{M} = m_L \quad \text{or}$$

$$\mathcal{M} = \begin{pmatrix} \mu_1 & y_D \langle H \rangle \\ y_D^T \langle H \rangle & \mu_2 \end{pmatrix}$$

→ pseudo-Dirac case

## The punch line:

all usual neutrino mass terms can be generated

→ suitable scalars required

→ no explicit masses:

**all via Yukawa couplings**

→ different numerical expectations  $\leftrightarrow$  could easily explain keV masses

# Phenomenological Impact

Conventional see-saw:

$$\begin{pmatrix} 0 & \mathbf{m}_D \\ \mathbf{m}_D & \mathbf{M}_R \end{pmatrix}$$

→ ultra heavy

$$\simeq M_R$$

→ ultra light

$$\simeq \frac{m_D^2}{M_R} \ll m_D$$

- Explains nicely known active neutrino masses
- But what if intermediate sterile neutrino states were found?
  - keV sterile  $\nu$ 's as warm dark matter
  - evidences for eV sterile
  - TeV-ish sterile neutrinos and improved EW precision fits (e.g. 1302.1872)

→ hard to explain in conventional see-saw and variants

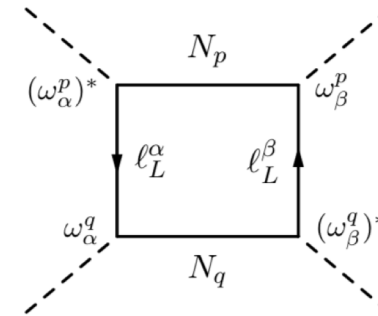
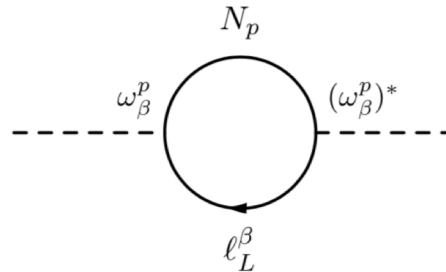
→ easy in conformal neutrino scenarios  $\longleftrightarrow M_R = g_Y * \langle \text{VEV} \rangle$

# The Neutrino Option

An interesting possibility: Connection between EWSB and neutrinos

**Neutrino option:** Brivio

→  $V_{\text{eff}}$  from neutrino loops



**Conformal Realization of the Neutrino Option:** Brdar, Emonds, Helmboldt, ML

→ conformal symmetry +  $V_{\text{eff}}$  from neutrino loops (not from Higgs portal)

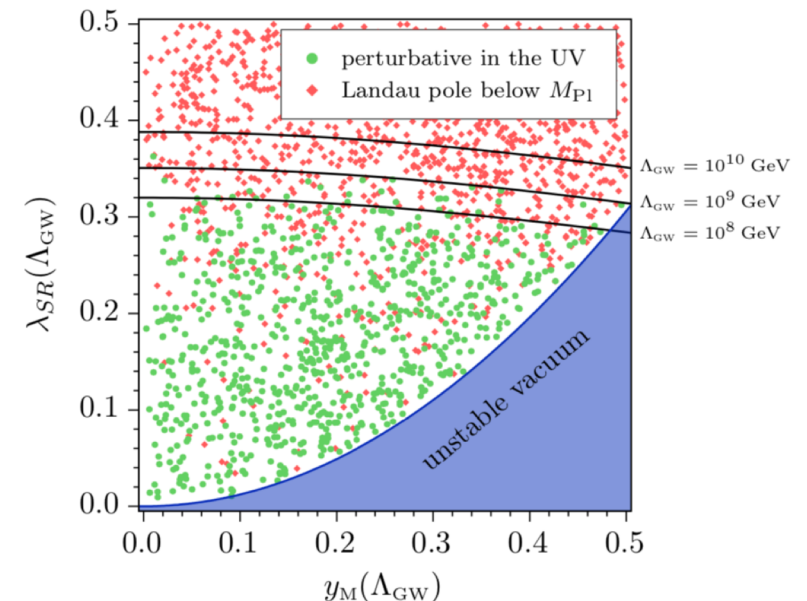
SM particle content

3x NR

2x scalar SM singlets: S, R

$$\mathcal{L} \supseteq \frac{1}{2} \partial_\mu S \partial^\mu S + \frac{1}{2} \partial_\mu R \partial^\mu R + i \bar{N}_R \not{\partial} N_R - V(H, S, R) - \left( \frac{1}{2} y_M S \bar{N}_R N_R^c + y_\nu \bar{L} \tilde{H} N_R + \text{h.c.} \right)$$

→ consistent UV-complete realization of the idea



# Conformal Symmetry & Dark Matter

## Different natural and viable options:

- 1) eV, **keV = DM**, TeV, ... sterile neutrino mass easily possible  $\leftrightarrow$  not so easy in standard see-saw's
  - 2) New particles which are fundamental or composite DM candidates:
    - hidden sector pseudo-Goldstone-bosons
    - stable color neutral bound states from new QCD representations
- some look like WIMPs
- others are extremely weakly coupled (via Higgs portal)
- or even coupled to QCD (threshold suppressed...)

# Including the Planck Scale

# The Planck Scale from CS Breaking

## Conformal Gravity (CG):

- more symmetry  $\rightarrow$  CG claimed to be power counting renormalizable
- CG may have a ghost...  $\rightarrow$  see later

Idea: Generate  $M_{\text{Planck}}$  in conformal gravity  $\otimes$  SU(N)

$\rightarrow$  gauge assisted condensate via SU(N) field

$\rightarrow M_{\text{Planck}}$  becomes an effective scale

Kubo, ML, Schmitz, Yamada      similar ideas: Donoghue, Menezes, ...

$$S_C = \int d^4x \sqrt{-g} \left[ -\hat{\beta} S^\dagger S R + \hat{\gamma} R^2 - \frac{1}{2} \text{Tr} F^2 + \right. \\ \left. + g^{\mu\nu} (D_\mu S)^\dagger D_\nu S - \hat{\lambda} (S^\dagger S)^2 + a R_{\mu\nu} R^{\mu\nu} + b R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right]$$

R = Ricci curvature scalar,  $R_{\mu\nu}$  = Ricci tensor,  $R_{\mu\nu\alpha\beta}$  = Riemann tensor

F = field-strength tensor of the SU( $N_c$ ) gauge theory ; S = complex scalar in fund. rep.  $\rightarrow N_c$

$\rightarrow$  most general diffeomorphism invariance, gauge invariance, and global scale invariance



## Condensation in $SU(N_c)$ gauge sector

→ **dimensional transmutation:**  $\langle S^+ S \rangle \rightarrow$  effective Planck mass

$$M_{\text{planck}} = \sqrt{2 \beta f_0} = \frac{N_c \beta}{16\pi^2} (2 \lambda f_0) \left( 1 + 2 \ln \frac{2 \lambda f_0}{\Lambda^2} \right) \quad \text{with } f_0 = \langle S^+ S \rangle$$

→ Effectively normal GR

What about the ghost problem of CG?

→ go into broken phase after condensation:

- normal gravity

- + hidden CS – nonlinearly realized + CA =  $\beta$ -functions

- renormalizability should be preserved

- ghost: should be absent like in normal GR  $\leftrightarrow M_{\text{planck}}$  is effective

→ May lead to viable QFT with dynamical CS breaking

# Dilaton-Scalaron Inflation

Effective Jordan-frame Lagrangian:

$$\frac{\mathcal{L}_{\text{eff}}^J}{\sqrt{-g_J}} = -\frac{1}{2} B(\chi) M_{\text{Pl}}^2 R_J + G(\chi) R_J^2 + \frac{1}{2} g_J^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \rightarrow \text{auxiliary field } \Psi \rightarrow$$

$$\frac{\mathcal{L}_{\text{eff}}^J}{\sqrt{-g_J}} = -\left[\frac{1}{2} B(\chi) M_{\text{Pl}}^2 - 2G(\chi)\psi\right] R_J + \frac{1}{2} g_J^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) - G(\chi)\psi^2$$

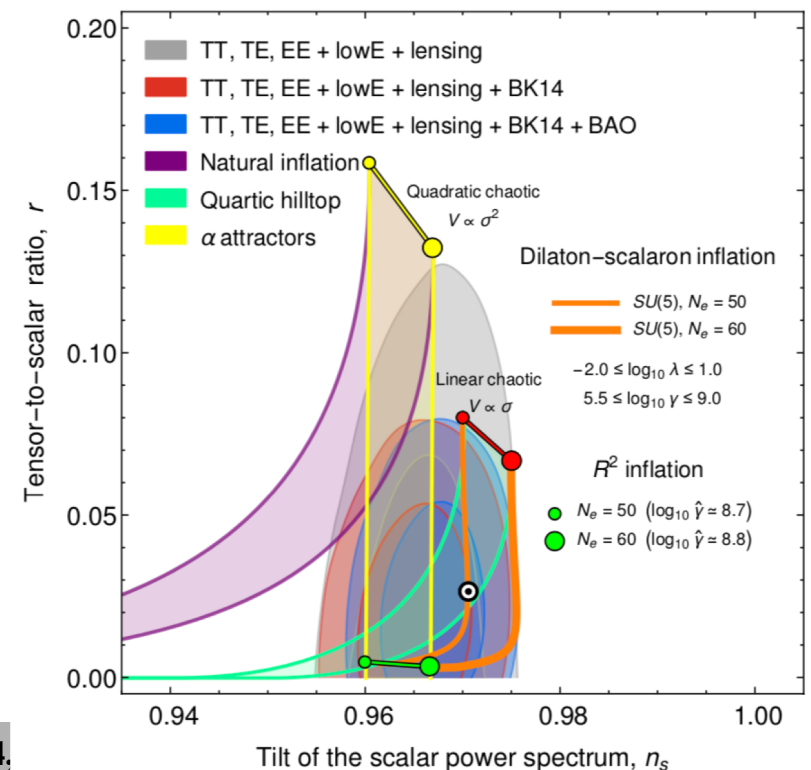
Weyl rescaling:  $g_{\mu\nu} = \Omega^2 g_{\mu\nu}^J$   $\Omega^2 = e^{\Phi(\phi)}$ ,  $\Phi(\phi) = \frac{\sqrt{2}\phi}{\sqrt{3}M_{\text{Pl}}}$

Einstein-frame scalar potential:

$$V(\chi, \phi) = e^{-2\Phi(\phi)} \left[ U(\chi) + \frac{M_{\text{Pl}}^4}{16G(\chi)} \left( B(\chi) - e^{\Phi(\phi)} \right)^2 \right]$$

→ Slow role inflation

→ fits data very well!



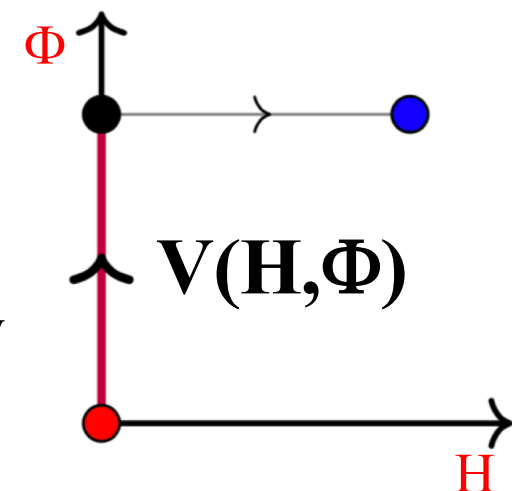
# Scale Dependence: EW vs. Planck Scale

- Assume:
    - SM scale generated by some TeV-ish conformal extension
    - Planck scale generation by conformal gravity  $\otimes$  gauge sector
- Do we understand the hierarchy between EW and Planck scale?

$$V = \lambda_1(\mathbf{H}^\dagger \mathbf{H})^2 + \underbrace{\lambda_2(\mathbf{H}^\dagger \mathbf{H})(\Phi^\dagger \Phi)}_{\text{portal coupling}} + \lambda_3(\Phi^\dagger \Phi)^2$$

→ Does  $\lambda_2$  portal lead to the usual hierarchy problem? → ideas

- sequential breaking by RG running → ‘CW tumbling’  
 $m^2 = 0$  is boundary broken/unbroken
  - SSB for tiny attractive force
  - if  $\langle \Phi^\dagger \Phi \rangle$  condenses first (stronger coupling)
  - portal can induce  $m^2 > 0$  for H → shifts SSB boundary
  - 2<sup>nd</sup> SSB by log running of couplings



# Summary

- **SM works (so far) perfectly**
  - be a bit more patient: new physics may be around the corner...
  - or maybe it is time to re-consider some things...
- **The old hierarchy problem(s)...? No new physics observed**
  - $\lambda(M_{\text{Planck}}) = 0$  ?  $\leftrightarrow$  precise value for  $m_t$  **→ is there a message?**
  - SM embeddings into QFTs with conformal symmetry**
    - combined conformal & electro-weak symmetry breaking
    - implications for BSM phenomenology
    - implications for Higgs couplings, neutrino physics, dark matter, ...
  - testable consequences: @LHC, dark matter, neutrinos**
- **Planck scale generation by gauge induced breaking of conformal GR**
  - very nice phenomenology: inflation...
  - consistent quantum gravity: renormalizability?, ghost?
    - $\leftrightarrow$  normal GR from a theory with more symmetry
  - stabilizing large scale hierarchies...
  - trans-Planck: just be a different phase - no new concept required