Inflation and higher order corrections to Higgs/Starobinsky models

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(with Z. Lalak and M. Lewicki)

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- Plateau can be obtain in e.g. Starobinsky model, Brans-Dicke gravity, Higgs inflation, α attractors...
- Higher order terms may violate the existence of the plateau, for instance for $R + \frac{R^2}{6M^2} + \alpha_3 \frac{R^3}{M^3} + \ldots$ one obtains the sufficiently long plateau only for fine tuning of all α_i constants.
- Let's try to use higher order corrections to plateau-like inflationary models as a source of the saddle-point inflation!

Convention: $8\pi G = M_{pl}^{-2} = 1$, where $M_{pl} \sim 2 \times 10^{18}\, GeV$

$f(R)$ gravity

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Let us generalize this term into

$$\frac{1}{2} \int d^4x \sqrt{-g}\; R \;\to\; \frac{1}{2} \int d^4x \sqrt{-g}\; f(R) \,. \tag{1}$$

Then the modified Einstein equation looks as follows

$$F(R) R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} + \left[g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu \right] F(R) = T_{\mu\nu} \,, \tag{2}$$

where $F = f' = \frac{df}{dR}$ and $T_{\mu\nu}$ is the energy-momentum tensor.

$f(R)$ as a Brans-Dicke theory

The action of $f(R)$ can be rewritten as

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\varphi R - U(\varphi) + \mathcal{L}_{\rm m}\right] \ , \tag{3}$$

where

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On the other hand the action of Brans-Dicke theory is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\varphi R - \frac{\omega_{\mathrm{BD}}}{2\varphi}(\nabla\varphi)^2 - U(\varphi) + \mathcal{L}_{\mathrm{m}}\right] \tag{4}$$

$f(R)$ is a Brans-Dick theory with $\omega_{\mathrm{BD}} = 0$

From Brans-Dicke to Einstein frame

The gravitational part of the action may be canonical after transformation to Einstein frame

$$\tilde{g}_{\mu\nu} = \varphi g_{\mu\nu} \tag{5}$$

which gives the action of the form of

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2}\tilde{R} - \frac{\beta}{4}\left(\frac{\tilde{\nabla}\varphi}{\varphi}\right)^2 - \frac{U(\varphi)}{\varphi^2} \right] , \tag{6}$$

where $\beta = 2\omega_{BD} + 3$.

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where $V = U/\varphi^2 \, (\varphi = \varphi(\phi))$.

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where $V = U/\varphi^2 \, (\varphi = \varphi(\phi))$. BD = GR + scalar field

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- For Brans-Dicke theory higher powers of the Jordan frame potential, like e.g. $(\varphi - 1)^4$ in B-D generalisation of Starobinsky
- Plateau-like inflation can be also generated by the Higgs inflation, with Jordan frame potential $V = \lambda\varphi^4$ and non-minimal coupling term $\xi\varphi^2 R$. Then the higher order terms are non-renormalisible terms of the potential, like e.g. $\lambda_6\varphi^6$.

The Starobinsky model and saddle-point inflation

The oldest and one of the most successful inflationary models

$$f(R) = R + \frac{R^2}{6M^2} \quad \Rightarrow \quad V(\varphi) = \frac{3}{4}M^2\left(1 - \frac{1}{\varphi}\right)^2 \qquad (7)$$

The model is great because of small r and non-gaussianities, which are pefectly consistent with the data. Nevertheless higher order corrections of the form

$$\sum_{n=3}^{\infty} \alpha_n \frac{R^n}{M^{2(n-1)}} \qquad (8)$$

may spoil the plateau. How to get inflation without the R^2 domination? We need a small, but very flat part of the Einstein frame potential, i.e. we need the saddle-point inflation.

The Starobinsky model and saddle-point inflation

The saddle-point of the Einstein frame potential is defined by

$$V_\phi = V_{\phi\phi} = 0 \tag{9}$$

The other option - inflection-point inflation, for which

$$V_\phi \neq 0 \quad \text{and} \quad V_{\phi\phi} = 0 \tag{10}$$

We denote both of those points as ϕ_s. The $R = R_s$ is the Ricci scalar for the Einstein frame saddle-point.

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What is the problem? The saddle-point inflation predicts $n_s \simeq 0.92$, which is inconsistent with PLANCK data. This requires significant influence of the R^2 term in order to generate proper form of the power spectrum.

Saddle-point inflation with vanishing k derivatives

In general one can define the saddle point with first k derivatives vanishing. In that case $1 - n_s \simeq \frac{2k}{N_\star(k-1)}$ when freeze-out of primordial inhomogeneities happens close to the saddle point. Thus, for sufficiently big k one can fit the Planck data!

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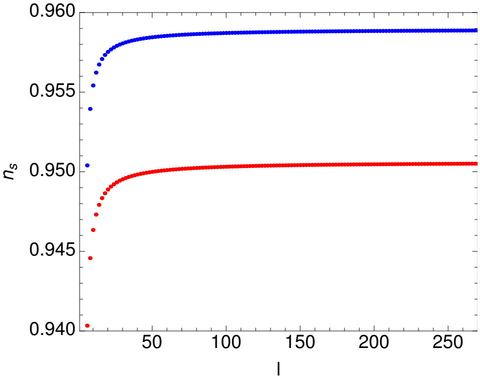
$$f(R) = R + \alpha_2 \frac{R^2}{M^2} + \sum_{n=3}^{l} \alpha_n \frac{R^n}{M^{2(n-1)}} , \qquad (11)$$

We want first $l-2$ derivatives to vanish, which gives

$$R_s = \sqrt{p}\, M^2 , \qquad \alpha_n = (-1)^{n-1} \frac{2(l-3)!}{(l-n)!(n-1)!} p^{\frac{3-n}{2}} \qquad (12)$$

where $p := \sqrt{(l-1)\,(l/2-1)}$. You can sum it up and obtain the analytical form of $f(R)$.

Saddle-point inflation with vanishing k derivatives



Numerical results for $N_\star = 50$ and $N_\star = 60$ (red and blue dots respectively). All values of r obtained in this analysis are consistent with PLANCK, but n_s fits the PLANCK data only for $N_\star \simeq 60$. No R^2 term needed

The $l \to \infty$ limit

For $l \to \infty$ one finds

$$f(R) = R\left(e^{-\frac{\sqrt{2}R}{M_o^2}} + \frac{\sqrt{2}+\alpha_2}{M_o^2}R\right). \tag{13}$$

The α_2 tell us about the contribution of R^2 to $f(R)$. Even for $\alpha_2 = 0$ this guy fits the data perfectly well. What is the problem?

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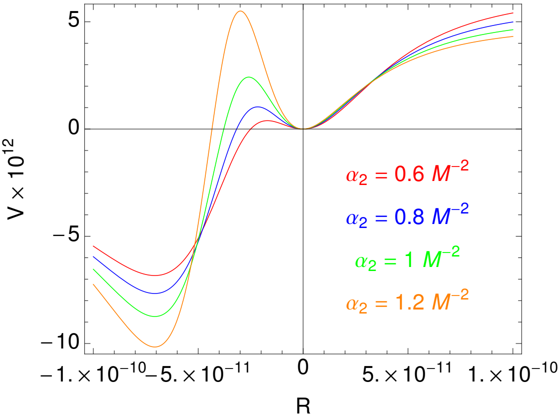
- The GR vacuum in not stable and some contribution of the Starobinsky term is needed in order to stabilise it.

Einstein frame potential

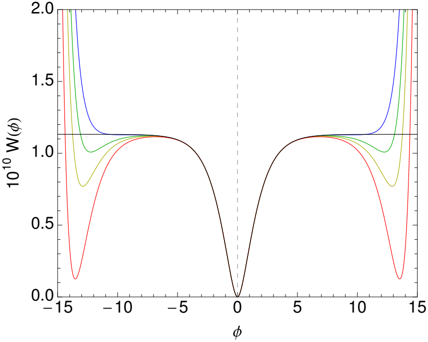


The GR minimum at $R = 0$ appears to be meta-stable, with a possibility of tunnelling to anti de Sitter vacuum. In order to avoid overshooting the minimum at $R = 0$ one requires $\alpha_2 \gtrsim 0.7$.

Higher order corrections in Higgs inflation

For the non-minimal coupling to gravity $\xi\varphi^2$ we introduce the Jordan frame potential

$$V = \frac{\lambda}{4}\varphi^4 + \frac{\lambda_6}{6}\varphi^6 + \frac{\lambda_8}{8}\varphi^8 + \ldots \tag{14}$$



W and ϕ are Einstein frame potential and field respectively

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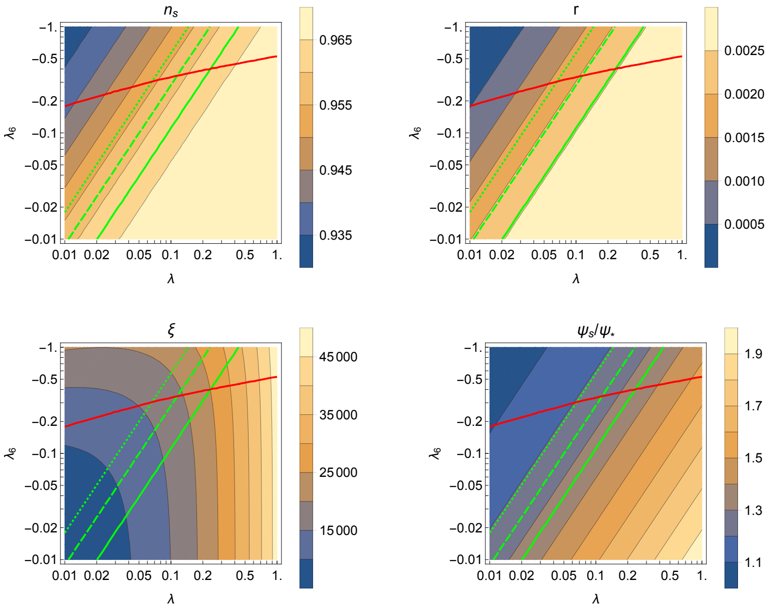
- Topological inflation at the local maxima
- De-Sitter expansion in local minima
- Possible cyclic universe if our vacuum is meta-stable
- Saddle-point or inflection-point inflation if $\lambda_6 \sim 3\left(\lambda\lambda_8/(4\xi)\right)^{1/3}$

Power spectra



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- They open brand new possibilities for the inflationary scenarios in plateau-like potentials
- If the long plateau is not possible one can seek for a short, but very flat part of the Einstein frame potential $\rightarrow$ saddle-point inflation
- Pure saddle point needs the help of the plateau, but for inflection-point inflation no Starobinsky is needed!