

# A smoking gun signature of 3HDM

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In collaboration with

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work in progress and arXiv: 2309.XXXX

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# The Standard Model and its shortcomings

- A Higgs boson discovered
- No significant deviation from the SM
- No signs of new physics

But no explanation for

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- Fermion mass hierarchy
- Vacuum stability
- Dark Matter & ...

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**The recently discovered 125-GeV scalar can be a portal to the dark sector.**

**problem:** Current direct and indirect detection as well as relic density bound strongly constrain the simplistic possibilities.

# BSMs to the rescue

Scalar extensions with a  $Z_2$  symmetry: 3HDM: SM + 2 scalar doublets

## CP-conserving I(2+1)HDM

$$\phi_1, \phi_2, \phi_3$$

$$g_{Z_2} = \text{diag}(-1, -1, +1)$$

$$VEV = (0, 0, v)$$

[JHEP1401(2014)052], [Phys.Rev.D90, 075015(2014)], [arXiv : 1907.12522]

# The scalar potential with explicit CPC

$$V_{3HDM} = V_0 + V_{Z_2}$$

$$V_0 = \sum_i^3 \left[ -\mu_i^2 (\phi_i^\dagger \phi_i) + \lambda_{ii} (\phi_i^\dagger \phi_i)^2 \right] \\ + \sum_{i,j}^3 \left[ \lambda_{ij} (\phi_i^\dagger \phi_i) (\phi_j^\dagger \phi_j) + \lambda'_{ij} (\phi_i^\dagger \phi_j) (\phi_j^\dagger \phi_i) \right]$$

$$V_{Z_2} = -\mu_{12}^2 (\phi_1^\dagger \phi_2) + \lambda_1 (\phi_1^\dagger \phi_2)^2 + \lambda_2 (\phi_2^\dagger \phi_3)^2 + \lambda_3 (\phi_3^\dagger \phi_1)^2 + h.c.$$

The  $Z_2$  symmetry

$$\phi_1 \rightarrow -\phi_1, \quad \phi_2 \rightarrow -\phi_2, \quad \phi_3 \rightarrow \phi_3, \quad \text{SM fields} \rightarrow \text{SM fields}$$

[Phys.Lett.B695(2011)459 – 462]

# Parameters of the model

- All parameters of the potential to be real
- “dark” parameters  $\lambda_1, \lambda_{11}, \lambda_{22}, \lambda_{12}, \lambda'_{12}$  (values have been fixed in agreement with the theoretical constraints.)
- $\mu_1^2 = n\mu_2^2, \quad \lambda_3 = n\lambda_2, \quad \lambda_{31} = n\lambda_{23}, \quad \lambda'_{31} = n\lambda'_{23}$
- fixed by the Higgs mass  $\mu_3^2 = v^2\lambda_{33} = m_h^2/2$

## 6 important parameters

- Mass splittings  $\mu_{12}^2, \lambda_2$
- Higgs-DM coupling  $\lambda_2, \lambda_{23}, \lambda'_{23}$
- Mass scale of inert particles  $\mu_2^2$

[*Eur. Phys. J. C*80(2020)2, 135]

# The mass eigenstates

The doublet compositions

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{H_1^0 + iA_1^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{H_2^0 + iA_2^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} G^+ \\ \frac{\nu + G^0}{\sqrt{2}} \end{pmatrix}$$

The mass eigenstates

$$\begin{aligned} H_1 &= \cos \theta_h H_1^0 + \sin \theta_h H_2^0, & A_1 &= \cos \theta_a A_1^0 + \sin \theta_a A_2^0 \\ H_2 &= \cos \theta_h H_1^0 - \sin \theta_h H_2^0, & A_2 &= \cos \theta_a A_2^0 - \sin \theta_a A_1^0 \\ H_1^\pm &= \cos \theta_c \phi_1^\pm + \sin \theta_c \phi_2^\pm, & H_2^\pm &= \cos \theta_c \phi_2^\pm - \sin \theta_c \phi_1^\pm \end{aligned}$$

$H_1$  is assumed to be the DM candidate

## • Input parameters:

DM mass  $m_{H_1}$ , Mass of second CP-even scalar  $m_{H_2}$ ,  
Higgs-DM coupling  $g_{H_1 H_1 h}$ , angles  $\theta_c$ ,  $\theta_a$  and  $n$ .

# Constraints

- **Vacuum stability:** scalar potential  $V$  bounded from below
- **Perturbative unitarity:** eigenvalues  $\Lambda_i$  of the high-energy scattering matrix fulfill the condition  $|\Lambda_i| < 8\pi$
- **Collider:** bounds on masses of the scalars
  - Limits from gauge bosons width:  
$$m_{H_i} + m_{H_j^\pm} \geq m_W, \quad m_{A_i} + m_{H_j} \geq m_Z, \quad 2 m_{H_{1,2}^\pm} \geq m_Z$$
  - Limits on charged scalar mass and lifetime:  
$$m_{H_i^\pm} \geq 70 \text{ GeV}, \quad \tau \leq 10^{-7} \text{ s} \rightarrow \Gamma_{\text{tot}} \geq 10^{-18} \text{ GeV}$$
  - Allowed by Higgs invisible branching ratio,  $Br(h \rightarrow \text{inv.}) < 19\%$
  - Allowed by Higgs total decay width,  $\mu^{\text{tot}}(h)$  as well as Higgs signal strength data.
- **DM constraints:** Relic density, Direct and indirect detection bounds.

# Relevant DM scenario

In the low mass region ( $m_{H_1} < m_Z$ )

We can have multiple scenarios but we are interested in

- **Scenario I: coannihilation** with  $H_2, A_{1,2}$ :

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} < M_{H_1^\pm, H_2^\pm}$$

[JHEP09(2018)059]

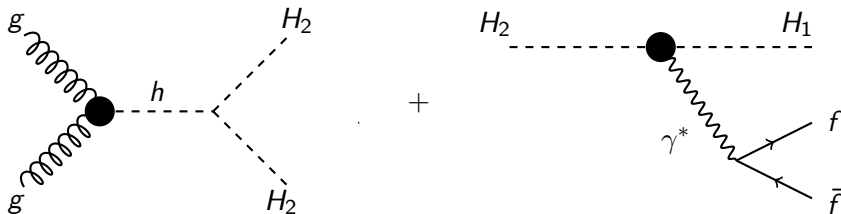
## CPC DM at the LHC

Looking for a **smoking-gun** signal of the 3HDM which is not allowed in the 2HDM with one inert doublet.

The  $\cancel{E}_T + 4l$  signature at the LHC

# Smoking gun Signal

- We focused on,

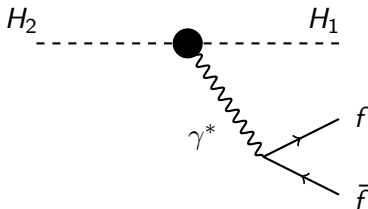


In the CPC I(2+1)HDM, a process contributing to the  $\cancel{E}_T l^+ l^- l^+ l^-$  signature is

$$gg \rightarrow h \rightarrow H_2 H_2 \rightarrow H_1 H_1 \gamma^* \rightarrow H_1 H_1 l^+ l^- l^+ l^-,$$

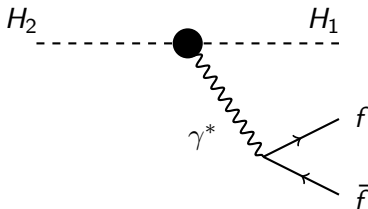
where the off-shell  $\gamma^*$  splits into  $l^+ l^-$  and the  $H_1$  states escape detection and will give  $\cancel{E}_T$ .

# Smoking gun Signal



**Figure:** Radiative decay of the heavy neutral particle  $H_2 \rightarrow H_1 \gamma^* \rightarrow H_1 l^+ l^-$ .

# Smoking gun Signal

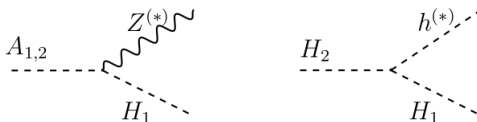


**Figure:** Radiative decay of the heavy neutral particle  $H_2 \rightarrow H_1 \gamma^* \rightarrow H_1 l^+ l^-$ .

- $m_{H_2} - m_{H_1}$  is very small
- $H_2$ , into the lightest inert state,  $H_1$ , and a virtual photon, which then would split into a light  $l\bar{l}$  pair.

# Inert cascade decays at the LHC

When there is a **large mass splitting** between DM and other inert particles:

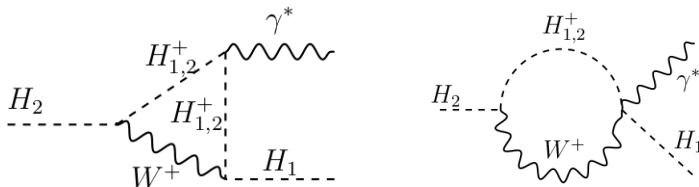


It can give the **tree level** process  $E_{miss}^T + l^+ l^- l^+ l^-$ :

$$pp \rightarrow H_2 H_2 / A_{1,2} A_{1,2} \rightarrow H_1 H_1 Z^* Z^* \rightarrow H_1 H_1 l^+ l^- l^+ l^-$$

# Inert cascade decays at the LHC

When there is a **small mass splitting** between DM and other inert particles (winning scenarios):

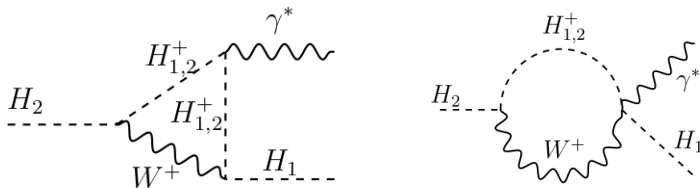


It can give the **loop level** process  $E_{miss}^T + l^+ l^- l^+ l^-$ :

$$pp \rightarrow H_2 H_2 / A_{1,2} A_{1,2} \rightarrow H_1 H_1 \gamma^* \gamma^* \rightarrow H_1 H_1 l^+ l^- l^+ l^-$$

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It can give the **loop level** process  $E_{miss}^T + l^+ l^- l^+ l^-$ :

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The **smoking gun** channel

- We are looking for Benchmarks with small mass gap ( $\Delta m$ ) between  $H_2$  and  $H_1$

BPs	$m_{H_1}$	$m_{H_2}$	$\Delta m$	$n$	$g_{H_1 H_1 h}$	$\theta_h$	$\sigma(pp \rightarrow H_1 H_1 2\mu^+ 2\mu^-)$
$BP1 : I_5^{50}$	50	55	5	0.83	0.01	0.105	6.923 fb
$BP2 : I_{10}^{50}$	50	60	10	0.70	0.01	0.103	4.0 fb

Table: Parameter choices of our Benchmark points (BPs)

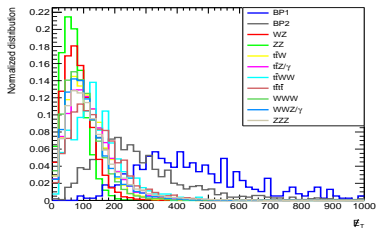
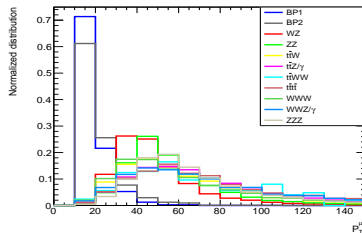
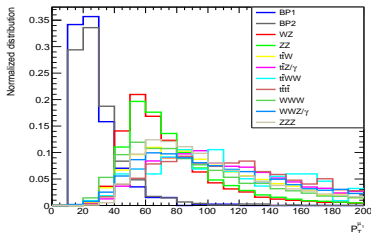
# Signal and backgrounds

- **Signal:** At least 3—muon with at least one pair of Opposite sign  $\mu + \not{E}_T$ .

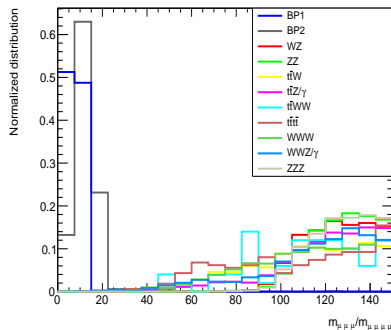
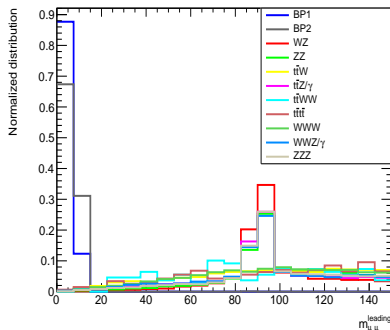
# Signal and backgrounds

- **Signal:** At least 3—muon with at least one pair of Opposite sign  $\mu + \not{E}_T$ .
- **Backgrounds:**
  - 1) **Di-boson**,  $VV(V : W, Z, \gamma)$ : Mainly  $WZ/\gamma$  and  $ZZ$  have large contribution where both  $V$  can decay leptonically.
  - 2) **Tri-boson**,  $VVV(V : W, Z, \gamma)$ : Mainly consider  $WWZ/\gamma$ ,  $WWW$  and  $ZZZ$ . All vector bosons are supposed to decay leptonically.
  - 3)  $t\bar{t}X$ , ( $X : W, Z, \gamma, WW, t\bar{t}$ ): The fully leptonic decay mode of  $t\bar{t}X$  can give us atleast three lepton with at least one pair of  $\mu$  with opposite charge.

# Distributions



# Distributions



**Figure:** Normalized distribution of invariant mass of two leading muons and invariant mass of all muons for signal BPs and backgrounds.

# Results

## Cuts:

1) **Pre-selection Cut:** We are looking for events where we can have at least three or four muons in final state with no  $b - jet$ .

2) **Cut-A:**

- $m_{\mu\mu}^{leading}$  and  $m_{\mu\mu}^{\Delta R_{min}}$  has to be less than 50 GeV.
- $m_{\mu\mu\mu}/m_{\mu\mu\mu\mu}$  has to be less than 70 GeV.

3) **Cut-B:**

- $m_{\mu\mu}^{leading}$  and  $m_{\mu\mu}^{\Delta R_{min}}$  has to be less than 20 GeV.
- $m_{\mu\mu\mu}/m_{\mu\mu\mu\mu}$  has to be less than 30 GeV.
- $\Delta R_{\mu\mu}^{leading, sub-leading} < 1.0$  and  $\Delta R_{\mu\mu}^{leading, sub-sub-leading} < 1.2$ .
- $\Delta\eta_{\mu\mu}^{leading, sub-leading} < 1.0$  and  $\Delta\eta_{\mu\mu}^{leading, sub-sub-leading} < 1.0$ .

# Results

Datasets	Cross-section ( $fb$ )	Pre-selection Cut	Cut – A	Cut – B
$BP1$	6.961	17	16	16
$BP2$	3.733	59	58	58
$WZ$	163.4068	97691	9	0
$ZZ$	16.554	22614	2	0
$WWW$	0.248862	185	3	0
$WWZ/\gamma$	0.04978	96	1	0
$ZZZ$	$9.3516 \times 10^{-3}$	16	0	0
$t\bar{t}W$	0.606	114	2	0
$t\bar{t}Z/\gamma$	0.3045	136	1	0
$t\bar{t}WW$	$1.279 \times 10^{-3}$	0	0	0
$t\bar{t}t\bar{t}$	$1.51359 \times 10^{-3}$	0	0	0

**Table:** Signal and background events cross-section at and Number of Events after cuts at  $\sqrt{s} = 14$  TeV and  $\mathcal{L} = 3000fb^{-1}$  for  $\geq 3\text{-}\mu + E_T$  final state.

# Significance

- we calculated the projected significance ( $\mathcal{S}$ ) in the  $3\mu + \cancel{E}_T$  channel for each benchmark point, for **14 TeV LHC** with **3000 fb<sup>-1</sup>**. The significance  $\mathcal{S}$  is defined as follows:

$$\mathcal{S} = \sqrt{2[(S + B)\text{Log}(1 + \frac{S}{B}) - S]}$$

*The 'AsimovPaper' (Cowan, Cranmer, Gross, Vitells, EPJC71(2011)1 – 19)*

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BP	$\mathcal{S}(\text{Pre} - \text{selection})$	$\mathcal{S}(\text{Cut} - A)$
BP1	0.05 $\sigma$	3.77 $\sigma$
BP2	0.17 $\sigma$	13.67 $\sigma$

- with **Cut – B** we will end up with signal only events
- $L = 300 \text{ fb}^{-1}$  also can give us fully background eliminated signals after **Cut – A** itself.

# Summary

- Inert Doublet Model
  - a good DM model with rich phenomenology, however, **very constrained**.
- CP-Conserving in **I(2+1)HDM**
  - SM-like active sector:  $H_3 \equiv h^{SM}$
  - The inert sector:  $H_{1,2}, A_{1,2}, H_{1,2}^\pm$ ,  $H_1 \rightarrow \text{DM}$
  - less constrained DM sector with low mass DM particle
  - New **Smoking-gun** signature at the LHC:  $m_{H_2}$  and  $m_{H_1}$  are close
  - Good signal significance in  $3\mu + \cancel{E}_T$  channel over backgrounds at HL-LHC.

*Thank you for your attaintion....*

# Back-up slides

## BSMs to the rescue

**Solution:** Scalar extensions with a  $Z_2$  symmetry:

- 2HDM: SM + scalar doublet
  - Type-I, Type-II, ...:  $\phi_1, \phi_2 \Rightarrow \text{CPV}, \text{DM}$
  - IDM - I(1+1)HDM:  $\phi_1, \phi_2 \Rightarrow \text{DM}, \text{CPV}$
- 3HDM: SM + 2 scalar doublets
  - Weinberg model:  $\phi_1, \phi_2, \phi_3 \Rightarrow \text{CPV}, \text{DM}$
  - I(1+2)HDM:  $\phi_1, \phi_2, \phi_3 \Rightarrow \text{DM}, \text{CPV}$
  - I(2+1)HDM:  $\phi_1, \phi_2, \phi_3 \Rightarrow \text{CPV}, \text{DM}$

# Dark Matter (DM)

around 25 % of the Universe is:

- cold
- non-baryonic
- neutral
- very weakly interacting

⇒ **Weakly Interacting Massive Particle**

- stable due to the discrete symmetry

$$\underbrace{\text{DM DM} \rightarrow \text{SM SM}}_{\text{pair annihilation}}, \quad \underbrace{\text{DM} \not\rightarrow \text{SM}, \dots}_{\text{stable}}$$

# Higgs-portal DM

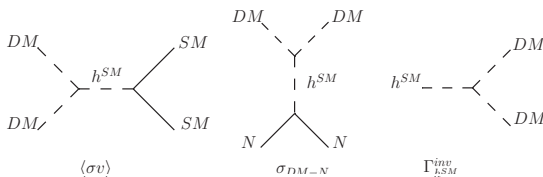
Simplest realisation: the SM with  $\Phi_{SM} + Z_2$ -odd scalar  $S$ :

$$S \rightarrow -S, \quad \text{SM fields} \rightarrow \text{SM fields}$$

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial S)^2 - \frac{1}{2}m_{DM}^2 S^2 - \lambda_{DM} S^4 - \lambda_{hDM} \Phi_{SM}^2 S^2$$

Higgs-portal interaction:

SM sector  $\xleftrightarrow{\text{Higgs}}$  DM sector



given by the same coupling

## 2HDM with CP-violation ( $\oplus M$ )

The general scalar potential

$$\begin{aligned}
 V = & \mu_1^2(\phi_1^\dagger\phi_1) + \mu_2^2(\phi_2^\dagger\phi_2) - \left[ \mu_3^2(\phi_1^\dagger\phi_2) + h.c. \right] \\
 & + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
 & + \left[ \frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + h.c. \right].
 \end{aligned}$$

$$Z_2 \text{ symmetry} \Rightarrow \lambda_6 = \lambda_7 = 0$$

The doublets composition with  $\tan\beta = v_2/v_1$

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + h_1^0 + ia_1^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + h_2^0 + ia_2^0}{\sqrt{2}} \end{pmatrix}$$

## CP-mixed mass eigenstates

- $2 \times 2$  charged mass-squared matrix

$$\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} \Rightarrow \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

- $4 \times 4$  neutral mass-squared matrix

$$\begin{pmatrix} a_1^0 \\ h_1^0 \\ a_2^0 \\ h_2^0 \end{pmatrix} \Rightarrow \begin{pmatrix} G^0 \\ H_1 \\ H_2 \\ H_3 \end{pmatrix}$$

CPV severely constrained from SM data

## The Inert Doublet Model (CPV)

Scalar potential  $V$  invariant under a  $Z_2$ -transformation:

$$Z_2: \quad \phi_1 \rightarrow \phi_1, \quad \phi_2 \rightarrow -\phi_2, \quad \text{SM fields} \rightarrow \text{SM fields}$$

$$\begin{aligned}
V &= -\frac{1}{2} \left[ m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 \right] + \frac{1}{2} \left[ \lambda_1 \left( \phi_1^\dagger \phi_1 \right)^2 + \lambda_2 \left( \phi_2^\dagger \phi_2 \right)^2 \right] \\
&+ \lambda_3 \left( \phi_1^\dagger \phi_1 \right) \left( \phi_2^\dagger \phi_2 \right) + \lambda_4 \left( \phi_1^\dagger \phi_2 \right) \left( \phi_2^\dagger \phi_1 \right) + \frac{1}{2} \lambda_5 \left[ \left( \phi_1^\dagger \phi_2 \right)^2 + \left( \phi_2^\dagger \phi_1 \right)^2 \right]
\end{aligned}$$

- All parameters are real  $\rightarrow$  no CP violation
- Only  $\phi_1$  couples to fermions
- The whole Lagrangian is explicitly  $Z_2$ -symmetric

## DM in the IDM

## The Inert minimum

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- $Z_2$ -symmetry survives the EWSB

$$g_{Z_2} = \text{diag}(+1, -1)$$

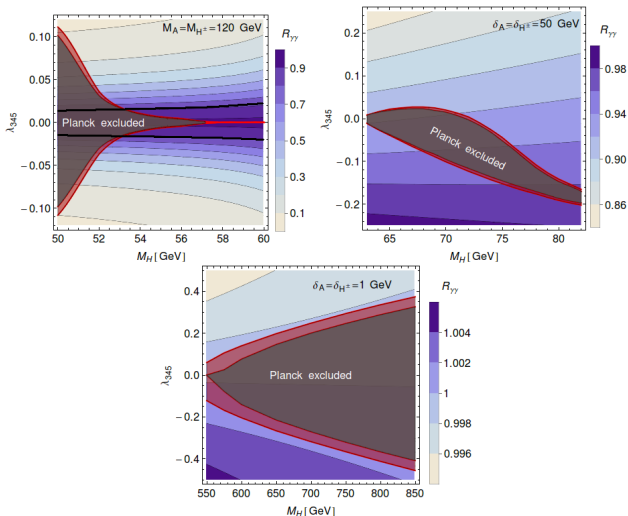
$$VEV = (v, 0)$$

- $\phi_1$  is active (plays the role of the SM-Higgs)
- $\phi_2$  is “dark” or inert (with 4 dark scalars  $H, A, H^\pm$ )

→ the lightest scalar is a candidate for the DM

# $h \rightarrow \gamma\gamma$ signal strength

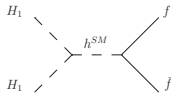
(JHEP 09 (2013) 055)



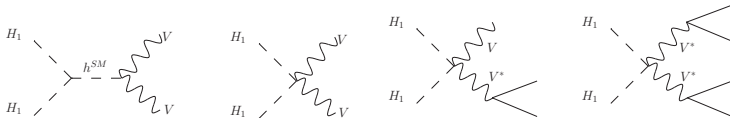
# CP-conserving $I(2+1)$ HDM

# Dark Matter Annihilation

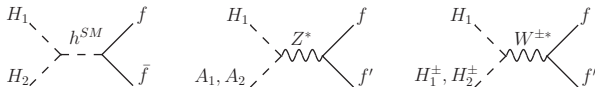
- annihilation through Higgs into fermions; dominant channel for  $M_{DM} < M_h/2$



- annihilation to gauge bosons; crucial for heavy masses



- coannihilation; when particles have similar masses



## DM Annihilation Scenarios

(A) no coannihilation effects:

$$M_{H_1} < M_{H_2, A_1, A_2, H_1^\pm, H_2^\pm}$$

(I) **coannihilation** with  $H_2, A_{1,2}$ :

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} < M_{H_1^\pm, H_2^\pm}$$

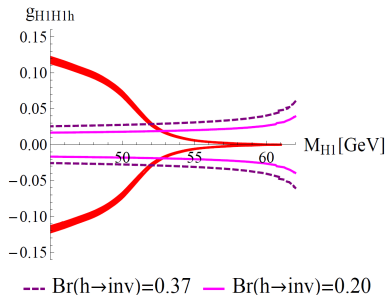
(G) coannihilation with  $H_2, A_{1,2}, H_{1,2}^\pm$ :

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} \approx M_{H_1^\pm, H_2^\pm}$$

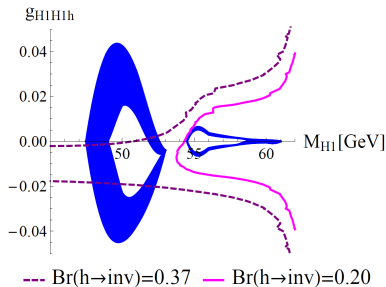
(H) coannihilation with  $A_1, H_1^\pm$ :

$$M_{H_1} \approx M_{A_1} \approx, H_1^\pm < M_{H_2, A_2, H_2^\pm}$$

# LHC vs Planck $M_{DM} < M_h/2$



case A



case I

- $Br(h \rightarrow inv) < 37\%$  &  $\Omega_{DM} h^2 \Rightarrow$

- Case A:  $M_{DM} \gtrsim 53$  GeV    • Case I: most masses are OK

# Masses and mixing angles

- The CP-even neutral inert fields

The pair of inert neutral scalar gauge eigenstates,  $H_1^0, H_2^0$ , are rotated by

$$R_{\theta_h} = \begin{pmatrix} \cos \theta_h & \sin \theta_h \\ -\sin \theta_h & \cos \theta_h \end{pmatrix}, \text{ with } \tan 2\theta_h = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda_{\phi_1} - \mu_2^2 + \Lambda_{\phi_2}}$$

into the mass eigenstates,  $H_1, H_2$ , with squared masses

$$m_{H_1}^2 = (-\mu_1^2 + \Lambda_{\phi_1}) \cos^2 \theta_h + (-\mu_2^2 + \Lambda_{\phi_2}) \sin^2 \theta_h - 2\mu_{12}^2 \sin \theta_h \cos \theta_h,$$

$$m_{H_2}^2 = (-\mu_1^2 + \Lambda_{\phi_1}) \sin^2 \theta_h + (-\mu_2^2 + \Lambda_{\phi_2}) \cos^2 \theta_h + 2\mu_{12}^2 \sin \theta_h \cos \theta_h,$$

where  $\Lambda_{\phi_1} = \frac{1}{2}(\lambda_{31} + \lambda'_{31} + 2\lambda_3)v^2$ ,  $\Lambda_{\phi_2} = \frac{1}{2}(\lambda_{23} + \lambda'_{23} + 2\lambda_2)v^2$ .

# Masses and mixing angles

- **The charged inert fields**

The pair of inert charged gauge eigenstates,  $\phi_1^\pm, \phi_2^\pm$ , are rotated by

$$R_{\theta_c} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}, \text{ with } \tan 2\theta_c = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda'_{\phi_1} - \mu_2^2 + \Lambda'_{\phi_2}}$$

into the mass eigenstates,  $H_1^\pm, H_2^\pm$ , with squared masses

$$m_{H_1^\pm}^2 = (-\mu_1^2 + \Lambda'_{\phi_1}) \cos^2 \theta_c + (-\mu_2^2 + \Lambda'_{\phi_2}) \sin^2 \theta_c - 2\mu_{12}^2 \sin \theta_c \cos \theta_c,$$

$$m_{H_2^\pm}^2 = (-\mu_1^2 + \Lambda'_{\phi_1}) \sin^2 \theta_c + (-\mu_2^2 + \Lambda'_{\phi_2}) \cos^2 \theta_c + 2\mu_{12}^2 \sin \theta_c \cos \theta_c,$$

where  $\Lambda'_{\phi_1} = \frac{1}{2}(\lambda_{31})v^2$ ,  $\Lambda'_{\phi_2} = \frac{1}{2}(\lambda_{23})v^2$ .

# Masses and mixing angles

- The CP-odd neutral inert fields**

The pair of inert pseudo-scalar gauge eigenstates,  $A_1^0, A_2^0$ , are rotated by

$$R_{\theta_a} = \begin{pmatrix} \cos \theta_a & \sin \theta_a \\ -\sin \theta_a & \cos \theta_a \end{pmatrix}, \text{ with } \tan 2\theta_a = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda''_{\phi_1} - \mu_2^2 + \Lambda''_{\phi_2}},$$

into the mass eigenstates,  $A_1, A_2$ , with squared masses

$$m_{A_1}^2 = (-\mu_1^2 + \Lambda''_{\phi_1}) \cos^2 \theta_a + (-\mu_2^2 + \Lambda''_{\phi_2}) \sin^2 \theta_a - 2\mu_{12}^2 \sin \theta_a \cos \theta_a,$$

$$m_{A_2}^2 = (-\mu_1^2 + \Lambda''_{\phi_1}) \sin^2 \theta_a + (-\mu_2^2 + \Lambda''_{\phi_2}) \cos^2 \theta_a + 2\mu_{12}^2 \sin \theta_a \cos \theta_a,$$

$$\text{where } \Lambda''_{\phi_1} = \frac{1}{2}(\lambda_{31} + \lambda'_{31} - 2\lambda_3)v^2, \quad \Lambda''_{\phi_2} = \frac{1}{2}(\lambda_{23} + \lambda'_{23} - 2\lambda_2)v^2.$$

# Dependent parameters in terms of input parameters

$$\Lambda_{\phi_2} = \frac{v^2 g_{H_1 H_1 h}}{4(\sin^2 \theta_h + n \cos^2 \theta_h)},$$

$$\Lambda'_{\phi_2} = \frac{2\mu_{12}^2}{(1-n)\tan 2\theta_c} + \mu_2^2,$$

$$\Lambda''_{\phi_2} = \frac{2\mu_{12}^2}{(1-n)\tan 2\theta_a} + \mu_2^2,$$

$$\mu_2^2 = \Lambda_{\phi_2} - \frac{m_{H_1}^2 + m_{H_2}^2}{1+n},$$

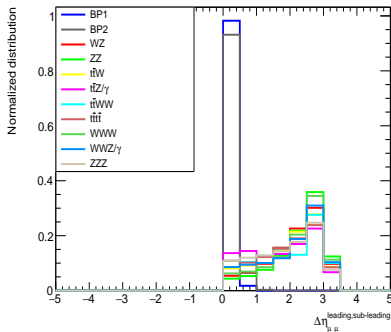
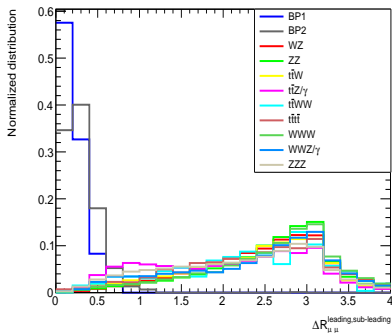
$$\mu_{12}^2 = \frac{1}{2} \sqrt{(m_{H_1}^2 - m_{H_2}^2)^2 - (-1+n)^2(\Lambda_{\phi_2} - \mu_2^2)^2},$$

$$\lambda_2 = \frac{1}{2v^2}(\Lambda_{\phi_2} - \Lambda''_{\phi_2}),$$

$$\lambda_{23} = \frac{2}{v^2} \Lambda'_{\phi_2},$$

$$\lambda'_{23} = \frac{1}{v^2}(\Lambda_{\phi_2} + \Lambda''_{\phi_2} - 2\Lambda'_{\phi_2})$$

# Distributions



**Figure:** Normalized Distribution of  $\Delta R$  and  $\Delta\eta$  of leading and sub-leading muon for signal BPs and backgrounds.

# Asimov estimate for discovery significance in counting experiment

## Discovery significance for $n \sim \text{Poisson}(s + b)$

Consider the case where we observe  $n$  events, model as following Poisson distribution with mean  $s + b$ .

Here assume  $b$  is known.

- 1) For an observed  $n$ , what is the significance  $Z_0$  with which we would reject the  $s = 0$  hypothesis?
- 2) What is the expected (or more precisely, median)  $Z_0$  if the true value of the signal rate is  $s$ ?

Taken from the slides 'Asimov estimate for discovery significance in counting experiment' by Glen Cowan) 1 – 19

## Gaussian approximation for Poisson significance

For large  $s + b$ ,  $n \rightarrow x \sim \text{Gaussian}(\mu, \sigma)$ ,  $\mu = s + b$ ,  $\sigma = \sqrt{s + b}$ .

For observed value  $x_{\text{obs}}$ ,  $p$ -value of  $s = 0$  is  $\text{Prob}(x > x_{\text{obs}} | s = 0)$ ,:

$$p_0 = 1 - \Phi\left(\frac{x_{\text{obs}} - b}{\sqrt{b}}\right)$$

Significance for rejecting  $s = 0$  is therefore

$$Z_0 = \Phi^{-1}(1 - p_0) = \frac{x_{\text{obs}} - b}{\sqrt{b}}$$

Expected (median) significance assuming signal rate  $s$  is

$$\text{median}[Z_0 | s + b] = \frac{s}{\sqrt{b}}$$

*Taken from the slides' Asimov estimate for discovery significance in counting experiment' by Glen Cowan) 1 - 19*

## Better approximation for Poisson significance

Likelihood function for parameter  $s$  is

$$L(s) = \frac{(s+b)^n}{n!} e^{-(s+b)}$$

or equivalently the log-likelihood is

$$\ln L(s) = n \ln(s + b) - (s + b) - \ln n!$$

Find the maximum by setting  $\frac{\partial \ln L}{\partial s} = 0$

gives the estimator for  $s$ :  $\hat{s} = n - b$

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## Approximate Poisson significance (continued)

The likelihood ratio statistic for testing  $s = 0$  is

$$q_0 = -2 \ln \frac{L(0)}{L(\hat{s})} = 2 \left( n \ln \frac{n}{b} + b - n \right) \quad \text{for } n > b, \text{ 0 otherwise}$$

For sufficiently large  $s + b$ , (use Wilks' theorem),

$$Z_0 \approx \sqrt{q_0} = \sqrt{2 \left( n \ln \frac{n}{b} + b - n \right)} \quad \text{for } n > b, \text{ 0 otherwise}$$

To find median $[Z_0|s+b]$ , let  $n \rightarrow s + b$  (i.e., the Asimov data set):

$$\text{median}[Z_0|s + b] \approx \sqrt{2 \left( (s + b) \ln(1 + s/b) - s \right)}$$

This reduces to  $s/\sqrt{b}$  for  $s \ll b$ .

*Taken from the slides 'Asimov estimate for discovery significance in counting experiment' by Glen Cowan* 1 – 19