An Ultralight Axion in Supersymmetry and Strings and Cosmology at Small Scales [1]

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[1] Based on work with James Halverson and Cody Long, arXiv:1703.07779 (hep-ph).

Cosmology at small scales

Cosmological models with cold dark matter such as ΛCDM have been pretty successful in explaining the large-scale structure of the universe.

However, at smaller scales at the level of galaxies ~ 10 kpc or less ΛCDM does not do well, as there are phenomena which cannot be explained with ΛCDM alone.

Two main problems emerge [2]

1. "Cusp-Core" problem: N-body simulations show that CDM collapse leads to cuspy dark matter halos. The observed galaxy rotation curves are better fit by constant dark matter density cores

$$\textbf{NFW} \qquad\qquad \textbf{Berkert}$$

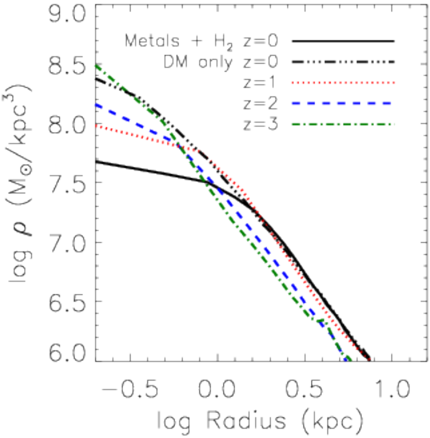
$$\rho(r) = \frac{\rho_0 \delta}{\frac{r}{r_s}(1+\frac{r}{r_s})^2}, \quad \rho(r) = \frac{\rho_0 r_0^3}{(r+r_0)(r^2+r_0^2)} \qquad (1)$$

2. "Missing satellite" problem: CDM predicts too many dwarf galaxies[3] There are nine dwarf satellite galaxies known around the Milky way with in ~ 250 kpc. However, CDM simulations give a factor of $\sim 5-20$ more satellites.

[2] Weinberg D. H., Bullock J. S., Governato F., Kuzio de Naray R., Peter A. H. G., 2015, Proceedings of the National Academy of Science, 112, 12249.

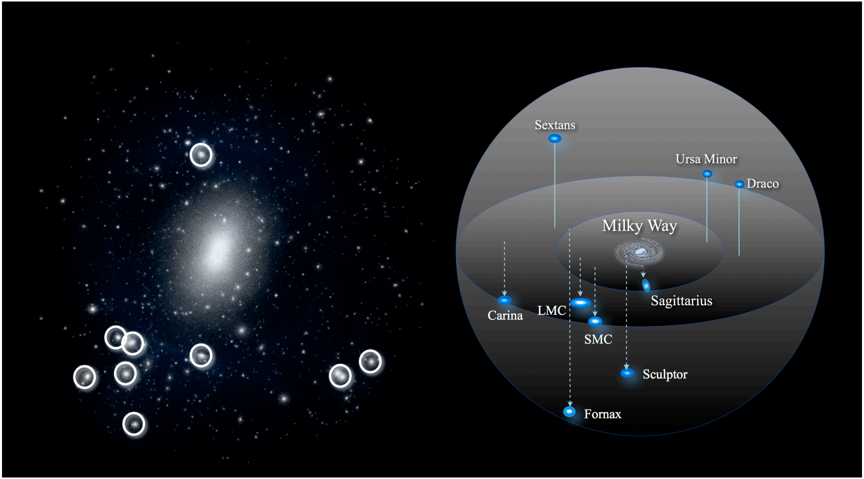
[3] Dwarf galaxies have $(10^8 - 10^9)$ stars, compared to Milky Way's $(2-4)\times 10^{11}$ stars.

Cusp-Core problem: Perhaps this problem could be solved by complex dynamics and baryonic physics.



Governato, F., Zolotov, A., Pontzen, A., Christensen, C., Oh, S. H., Brooks, A. M., Quinn, T., Shen, S., and Wadsley, 'Cuspy no more: how outflows affect the central dark matter and baryon distribution in Λ cold dark matter galaxies", J. 2012, MNRAS, 422(May), 1231-1240.

Missing satellite galaxy problem: N-body simulations with CDM show many more satellite galaxies than observed [4].



[4] Weinberg D. H., Bullock J. S., Governato F., Kuzio de Naray R., Peter A. H. G., 2015, "Cold dark matter: controversies on small scales", "Proceedings of the National Academy of Science, 112, 12249.

Aside from complex dynamics and baryonic matter, another possibility involves particle physics models such as (repulsive) self-interactions dark matter or a particle of de Broglie wavelength of $\sim \mathbf{1}$ kpc and a mass which lies in the range [5], [6].

$$\boldsymbol{m = (1 - 10) \times 10^{-22}\,\mathrm{eV}}\cdot$$

Dark matter of this type has been discussed under various names

- Fuzzy DM (Hu et al. 2000)
- Wave DM (Bray 2012; Schive et al. 2014)
- Ultra-light Axion (ULA) DM (Hlozek et al 2015)
- Bose -Einstein Condensate (BEC) DM (Boehmer & Harko 2007)

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[5] L. Hui, J. P. Ostriker, S. Tremaine and E. Witten, Phys. Rev. D **95**, no. 4, 043541 (2017) [arXiv:1610.08297 [astro-ph.CO]].

[6] D. J. E. Marsh, Phys. Rept. **643**, 1 (2016) [arXiv:1510.07633 [astro-ph.CO]].

$O(10^{-22})\,\mathrm{eV}$ is an extremely small mass scale (cf: $m_\nu \sim 0.1 - 10^{-5}\,\mathrm{eV}$). How could such a scale arise in particle physics?

The simplest possibility is an axion arising from the breaking of a shift symmetry

$$I = \int d^4x \sqrt{g} \left[-\frac{1}{2} F^2 g^{\mu\nu} \partial_\mu a \, \partial_\nu a - \mu^4 (1 - \cos a) \right].$$

F is axion decay constant and $m_a = \frac{\mu^2}{F}$.

QCD[7]: $\mu = \Lambda_{QCD} \sim 200\ \mathrm{MeV}$, $F \leq 10^{12}\,\mathrm{GeV}$. Here an axion mass of 10^{-22} eV cannot be generated.

[7] Peccei, Quinn, Weinberg, Wilczek, Kim, Nilles, Shifman, Vainstein, Zkharov, Dine, Fischler, Srednicki, $\cdots$

The axion we consider here is another type of axion possibly a string axion [8] not related to QCD.

We assume that in this more general context the effective μ is set by non-perturbative effects, such as string instantons, and so m_a depends on two parameters.

In a variety of string models[9], the axion decay constant is much larger than $\mathbf{10^{12}\,GeV}$ typically lying in the range of $\mathbf{(10^{16} - 10^{18})\,GeV}$.

This allows for greater freedom in the axion mass and relic abundance.

[8] P. Svrcek and E. Witten, JHEP **0606**, 051 (2006).

Hui et al. analysis[9]

$$10^{18}\,\mathrm{GeV} \gtrsim F \gtrsim 10^{16}\,\mathrm{GeV}.$$

μ is generated by nonperturbative instanton effects and is model dependent. A rough formula is

$$\mu^4 \sim M_{\mathrm{Pl}}^2 \Lambda^2 e^{-S},$$

where S is the instanton action and Λ is a model dependent factor which can vary over a very wide range:

$$10^{18}\,\mathrm{GeV} \gtrsim \Lambda \gtrsim 10^{4}\,\mathrm{GeV}.$$

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[9] L. Hui, J. P. Ostriker, S. Tremaine and E. Witten, Phys. Rev. D **95**, no. 4, 043541 (2017) [arXiv:1610.08297 [astro-ph.CO]].

Ultralight axion miracle analogous to WIMP miracle

With $F = 10^{17}\,\mathrm{GeV}$, $m = 10^{-22}$ eV can be gotten if [10],

$$S = \begin{cases} 165 & \text{if } \Lambda = 10^4\,\mathrm{GeV} \\ 198 & \text{if } \Lambda = 10^{11}\,\mathrm{GeV} \\ 230 & \text{if } \Lambda = 10^{18}\,\mathrm{GeV}. \end{cases}$$

If assume $S \sim S_0 = 2\pi/\alpha_G$ then

$$S_0 = \begin{cases} 126 & \text{for } \alpha_G = 1/20 \\ 157 & \text{for } \alpha_G = 1/25 \\ 188 & \text{for } \alpha_G = 1/30. \end{cases}$$

Here one finds that there is an overlap of S_0 and S for a range of values. Thus instantons can generate the desired mass. Using misalignment mechanism one also finds that the relic density is given by

$$\Omega_{\mathrm{axion}} \sim 0.1 \left(\frac{F}{10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{1/2}.$$

[10] L. Hui et al., Phys. Rev. D **95**, no. 4, 043541 (2017) [arXiv:1610.08297 [astro-ph.CO]].

Ultralight axion mass and the electroweak scale [11]

The effective scalar potential V_a that gives the axion its mass must respect all of the symmetries of the theory.

$$V_a = \tilde{A}\, \mathcal{O}_H\, \mathcal{O}_V\, \cos(a/F)\,,$$

where $\mathcal{O}_H$ is a hidden sector operator, and $\mathcal{O}_V$ is a visible sector operator that contains Higgs and possibly a singlet s.

To generate axion mass $\mathcal{O}_H\, \mathcal{O}_V$ must have non-vanishing VEVs. Defining $A := \tilde{A}\langle\mathcal{O}_H\rangle$ the relevant part of V_a is

$$V_a = A\, \frac{s^{2m}(h^\dagger h)^{2k}}{\Lambda^{4k+2m-4}}\, \cos(a/F)\,.$$

[11] J. Halverson, C. Long, P.N., "An Ultralight Axion in Supersymmetry and Strings and Cosmology at Small Scales" arXiv:1703.07779 (hep-ph).

This leads to

$$m_a \simeq \Lambda_{EW} \left(\frac{\Lambda_{EW}}{\Lambda}\right)^{n-1} .$$

where $n = 2k + m$ and Λ is some ultraviolet cutoff and assuming $F = \Lambda$, and $\langle s \rangle \sim \langle h \rangle \sim \Lambda_{\mathrm{EW}}$.

For high scale cutoff $\Lambda \simeq 10^{18}\ \mathrm{GeV}$ and $\Lambda_{EW} \simeq 10^{2}\ \mathrm{GeV}$, this gives

$$\begin{aligned} m_a &\simeq 10^{11}\ \mathrm{eV} && \text{for } n = 1\ (\mathrm{EW})\,, \\ m_a &\simeq 10^{-5}\ \mathrm{eV} && \text{for } n = 2,\ (\nu)\,, \\ m_a &\simeq 10^{-21}\ \mathrm{eV} && \text{for } n = 3\ \ (\text{UL})\,. \end{aligned}$$

For $n = 3$ a mass scale of $10^{-21}\,\mathrm{eV}$ arises naturally from a combination of the electroweak scale and the Planck scale. The same mass then gives the right relic density. One may call it the $n = 3$ miracle.

SUSY Models

We consider two chiral axionic fields which are oppositely charged under the shift symmetry

$$S_1 \to e^{i\lambda} S_1,\ \ S_2 \to e^{-i\lambda} S_2\,,$$
$$W_s = \mu_0 S_1 S_2 + \frac{\lambda_s}{2M}(S_1 S_2)^2\,.$$

SSB gives

$$S_i = (F + \rho_i) e^{i a_i / F}$$
$$V = 4F^2 \mu_0^2 \left[1 - \cos\left(\frac{\sqrt{2} a_+}{F}\right)\right]\,.$$

where $a_\pm = \frac{1}{\sqrt{2}}(a_1 \pm a_2)$. The axion a_+ becomes massive with a mass $2\sqrt{2}\mu_0$ while a_- remains massless. In fact, the entire plus multiplet becomes massive while the minus multiplet remains massless.

Mass for a_-

To generate a mass for a_- include a non-perturbative term W_n

$$\begin{aligned} W &= W_s + W_n \,, \\ W_n &= A(e^{-\alpha S_1} + e^{-\alpha S_2}) \,, \end{aligned}$$

W_n violates the shift symmetry and leads to a potential for a_- :

$$V = V_0[1 - \cos(\alpha F \sin(a_-/\sqrt{2}F))] \,,$$

$$V_0 \equiv 2\alpha^2 A^2 e^{-2\alpha F} e^{-\alpha F \cos(a_-/\sqrt{2}F)}$$

$$m_{a_-} \simeq \alpha^2 A e^{-\alpha F} \,.$$

Using numerics of Hui et al., i.e., $F = 10^{17}\,\mathrm{GeV}$, $\alpha^2 A = 10^{12}$, GeV, $\alpha F = 99$, one finds $m_{a_-} = 10^{-21}\,\mathrm{eV}$. A similar analysis holds for the axino ξ_-.

Ultralight axion using EW scale and the Planck scale [12]

$$
\begin{aligned}
W &= W_s + \frac{c}{M_{\mathrm{Pl}}^{2k-2}}(S_1 + S_2)(H_1 H_2)^k \,. \\
V(a_-) &= V_0(1 - \cos(\frac{a_-}{\sqrt{2}F})) \\
V_0 &\equiv (\frac{cF}{M_{\mathrm{Pl}}^{2k-2}}(v_1 v_2)^{k-1})^2 (v_1^2 + v_2^2) \\
M_{a_-} &= \Lambda_{\mathrm{EW}} \left(\frac{\Lambda_{\mathrm{EW}}}{\Lambda}\right)^{n-1} \qquad (2)
\end{aligned}
$$

where $\Lambda_{EW} \simeq v_1 \sim v_2$ and $\Lambda \simeq F \simeq M_{Pl}$. $n = 2k - 1$ and $n = 3$, $\Lambda \sim 10^{18}\,\mathrm{GeV}$ gives the ultralight axion. We note that after soft terms are taken into account we need $n = 5$.

[12] J. Halverson, C. Long and PN, arXiv:1703.07779 [hep-ph].

Ultralight axion using simplified string models

4d effective SUGRA theory constructed from a string compactification has scalar fields known as moduli. These fields arise from the reduction of the metric and various p-form gauge fields. The ultralight axion could be one of the these fields.

The moduli come in various forms: the dilaton, Kahler, complex. We will focus on Kahler moduli and consider the possibility of an ultralight particle in KKLT[13]

$$K = -3\log(T+\bar{T}),$$
$$W = W_0 + Ae^{-qT},$$

where A and W_0 are independent of T:

$$T = \tau + i\theta.$$

[13] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).

Using the $\boldsymbol{N=1}$ SUGRA potential

$$V = e^{K}(K^{i\bar{j}}\mathcal{D}_i\mathcal{D}_{\bar{j}} - 3|W|^2)\,,$$
$$\mathcal{D}_i = W_{,i} + K_{,i}W\,,$$

stabilizing τ so that $<\tau> = \tau_0 + \tau'$ canonical variables are

$$(\tau', \theta) \rightarrow (\rho, a)$$

$$V(a) = \delta(1 - \cos(\gamma a))\,,$$
$$\delta = -\frac{qAW_0}{2\tau_0^2}e^{-q\tau_0}, \gamma = \frac{\sqrt{2}q\tau_0}{\sqrt{3}}\,.$$

m_a at the AdS minimum

$$\frac{m_a}{|W_0|} = \frac{1}{3}\frac{\sqrt{3+3q\tau_0}}{(1+\frac{2}{3}q\tau_0)} \Longrightarrow m_a \sim \Lambda_{\mathrm{EW}}$$

Setting $q = 2\pi$ and stabilizing T one finds that the axion mass is of size the electroweak scale. Thus, we see that single modulus KKLT is incompatible with an ultralight axion.

Large Volume Scenario [14] (LVS)

In LVS we will consider a Kahler potential with an α' correction

$$K = -\log(S+\bar{S}) - 2\log(\mathcal{V}+\alpha) + K_{cs}(U,\bar{U})\,,$$

where $\alpha = \frac{1}{2}\xi S_1^{3/2}$, $S_1 = Re(S)$, $\xi = \zeta(3)\chi/2(2\pi)^3$, and χ is the topological Euler characteristic of the internal space X. We consider the case of two complex moduli

$$T_a = \tau_a + i\theta_a, \;\; T_b = \tau_b + i\theta_b.$$

Further, we consider the case when $\mathcal{V}$ has the strong cheese form

$$\mathcal{V} = k\,(\tau_b^{3/2} - \tau_s^{3/2})\,.$$

Here τ_b controls the overall volume of the cheese, and τ_s controls the size of a hole in the cheese and k is a numeric $O(1)$.

[14] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, JHEP **0503**, 007 (2005).

In LVS we use the approximations

$$\frac{\tau_s}{\tau_b} \ll 1, \quad \frac{\alpha}{\mathcal{V}} \ll 1\,.$$

$$K \approx -2\log(\mathcal{V}) - 2\,\frac{\alpha}{\mathcal{V}}\,.$$

In standard LVS one effectively ignores any non-perturbative effects that depend on τ_b so that

$$W \simeq W_0 + A_s e^{-a_s T_s}\,.$$

- However, in this approximation the axion θ_b is massless.
- A mass for θ_b can be generated if we include correction to W from T_b. Let us consider a correction to the superpotential of the form
$$\Delta W = A\, e^{-a_b T_b}\,.$$

With inclusion of ΔW correction from T_b, the axion potential has the form

$$V = c + c_1 \cos(a_s\theta_s) + c_2 \cos(a_b\theta_b) + c_3 \cos(a_b\theta_b - a_s\theta_s)\,.$$

The axions are stabilized at $\theta_b = \pi/a_b, \theta_s = \pi/a_s$. A concrete example:

$$\begin{aligned}
&h^{1,1} = 2, h^{2,1} = 171,\\
&W_0 = 10^{-12}, A_s = A_b = 1,\\
&a_s = a_b = 2\pi/6,\ k = 0.07785,\ S_1 = 10.71\,.
\end{aligned}$$

- A non-supersymmetric AdS minimum is found. The moduli are stabilized at $\boldsymbol{\tau_b = 187.39}$, and $\boldsymbol{\tau_s = 32.5}$ and the volume is stabilized at $\boldsymbol{\mathcal{V} = 187}$. Both axion decay constants are $\boldsymbol{\mathcal{O}(10^{16})}$ GeV.
- Canonically normalized axions $(\boldsymbol{a_1, a_2})$, axinos $(\boldsymbol{\tilde{a}_1, \tilde{a}_2})$, and saxions $(\boldsymbol{\rho_1, \rho_2})$ have the following masses [15]

Light	**Heavy**
$\boldsymbol{a_{\text{light}} = 3.9 \times 10^{-22}\,\text{eV}}$	$\boldsymbol{a_{\text{heavy}} = 26\ \text{TeV}}$
	$\boldsymbol{\rho_1 = 590\ \text{GeV}}$
	$\boldsymbol{\rho_2 = 280\ \text{TeV}}$
	$\boldsymbol{\tilde{a}_1 = 13\ \text{TeV}}$
	$\boldsymbol{\tilde{a}_2 = 26\ \text{TeV}}$
	$\boldsymbol{m_{3/2} = 13\ \text{TeV}}$

[15] J. Halverson, C. Long and PN, arXiv:1703.07779 [hep-ph].

Relic density of the axion.

Axion field in FRW universe

$$\ddot{a} + 3H\dot{a} + m^2 \sin a = 0,$$

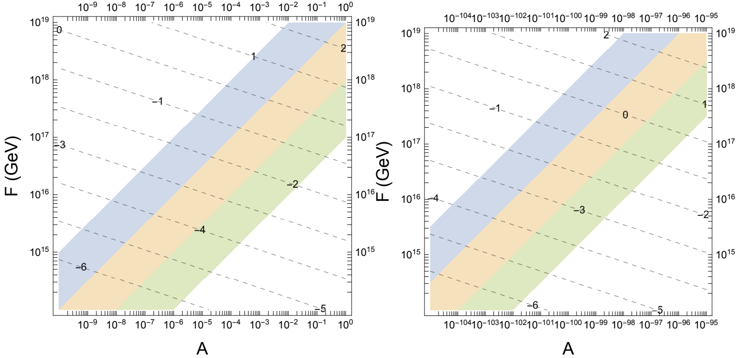
$H = \dot{R}/R$ is the Hubble parameter.

- When H falls below m, a begins to oscillate with a frequency $mc^2/\hbar$. The temperature at which this happens is $T_0 = 500\,\mathrm{eV}$. This is the radiation dominated era.
- The oscillating a can be thought of as a Bose condensate of ultralight particles of zero spatial momentum. During the oscillation the energy density ρ scales like cold dark matter, i..e, as $1/R^3$.
- If the initial value of the axion field is close to the decay constant, it would lead to a relic density

$$\Omega_a \sim 0.1 \left(\frac{m}{10^{-22}\,\mathrm{eV}} \right)^{1/2} \left(\frac{F}{10^{17}\,\mathrm{GeV}} \right)^2 .$$

Relic density

The space of the Wilson coefficient $\boldsymbol{A}$ and $\boldsymbol{F}$ where the relic density is satisfied [16].



Left: n=0 (instantons); Right: n=3 (higher dim).

$$\begin{aligned} \boldsymbol{log_{10}(m_a(\text{eV}))}\ &\textbf{:Blue}(\boldsymbol{-23,-22}),\\ &\textbf{Orange}(\boldsymbol{-22,-21}),\\ &\textbf{Green}(\boldsymbol{-21,-20}). \end{aligned} \tag{3}$$

[16] J. Halverson, C. Long and PN, arXiv:1703.07779 [hep-ph].

Conclusion

A boson of de Broglie wavelength $\mathbf{1kpc}$ may help resolve problems in cosmology at scales order $\mathbf{10kpc}$. A possible candidate is an ultralight axion of mass in the range $\mathbf{10^{-21} - 10^{-22}\ eV}$.

- The ultra light axion could arise from instanton effects which break the shift symmetry.
- They can also arise from higher dimensional operators involving ratio of the electroweak scale and the Planck scale.
- Quite remarkably an ultralight axion can produce the right amount of dark matter by the misalignment mechanism.

Future Prospects

- Tests of axionic dark matter have been discussed over the years [17].
- Tests specific to FDM may come from more studies of cosmology at small scales[18].
- It has also been suggested that future gravitational wave detectors such as LISA (Laser Interferometer Space Antenna), eLISA and DECIGO(DECi hertz Interferometer Gravitational wave Observatory) could be used for detection of ultralight axionic dark matter[19].

[17] P. Sikivie, Phys. Rev. Lett. 51, 1415 (1983)
A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, and J. March-Russell, Phys. Rev., D81, 123530 (2010)

[18] Tula Bernal, Lizbeth M. Fernandez-Hernandez, Tonatiuh Matos, Mario A. Rodriguez-Mez, arXiv:1701.00912v1 [astro-ph.GA].

[19] A. Aoki and J. Soda, arXiv:1608.05933 [astro-ph.CO]; J. Crowder and N. J. Cornish, Phys. Rev. D **72**, 083005 (2005 doi:10.1103/PhysRevD.72.083005 [gr-qc/0506015]. P. Amaro-Seoane *et al.*, GW Notes **6** (2013) 4 [arXiv:1201.3621

P. S. B. Dev, M. Lindner and S. Ohmer, arXiv:1609.03939 [hep-ph].

The remaining spectra

In addition to $\boldsymbol{a_-}$ there remain the following spectra

$$\begin{aligned} \boldsymbol{\rho_+, a_+, \xi_+} \\ \boldsymbol{\rho_-, \xi_-} \end{aligned}$$

The saxion $\boldsymbol{\rho_+}$ will decay via the effective interaction

$$\boldsymbol{W_3 = \lambda_+ S_+ H_1 H_2 + \cdots} \,. \tag{4}$$

The interaction allows the decay $\boldsymbol{\rho_+ \to \tilde{H}_1 \tilde{H}_2}$ with a lifetime consistent with the BBN constraints with mass $\mathbf{(50 - 100)}$ TeV. The lifetime for $\boldsymbol{a_+, \xi_+}$ are of similar size and consistent with the BBN constrains. A similar analysis holds for $\boldsymbol{\rho_-}$.

There is no efficient production mechanism to generate the relic density for $\boldsymbol{\xi_-}$ comparable to the $\boldsymbol{a_-}$ and thus dark matter is dominated by the ultralight axion .