

# Electroweak and Dark matter scalegenesis from a bilinear scalar condensate



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Based on

J, Kubo and **M. Y.**, arXiv:1505.05971

J, Kubo and **M. Y.**, PTEP **2015** 093B01 (arXiv:1506.06460)

# Summary of my talk

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- Classically scale invariant model
  - Prohibits the mass term
  - Introduce a new scalar field coupled to a non-abelian gauge field in the hidden sector.
  - The Higgs mass term is generated due to the strong dynamics.
- Dark matter candidate exists.
  - Flavor symmetry makes it stable.
- Strong 1<sup>st</sup> order EW phase transition

# Model

J, Kubo and **M. Y.**, arXiv:1505.05971

cf. J. Kubo, K. S. Lim and M. Lindner Phys. Rev. Lett. 113. 091604

## □ Strongly interacting Hidden sector

- $SU(N_c) \times U(N_f)$  invariant + classically scale invariant

$$\begin{aligned} \mathcal{L}_{\text{HQCD}} = & -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + ([D_\mu S_i]^\dagger D^\mu S_i) \\ & -\hat{\lambda}_S(S_i^\dagger S_i)(S_j^\dagger S_j) - \hat{\lambda}'_S(S_i^\dagger S_j)(S_j^\dagger S_i) \\ & +\hat{\lambda}_{HS}(S_i^\dagger S_i)H^\dagger H \end{aligned}$$

$$m_H^2 = -\lambda_{HS}\langle S^\dagger S \rangle$$

$$\langle S^\dagger S \rangle$$

Dynamical  
Symmetry  
Scale  
Breaking

$\mathcal{L}_{\text{SM}}|_{m_H \rightarrow 0}$

$-\lambda_{HS}(S^\dagger S)H^\dagger H$

$\mathcal{L}_{\text{HQCD}}$

# Effective theory

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## □ Low energy effective Lagrangian

$$\mathcal{L}_{\text{eff}} = ([\partial_\mu S_i]^\dagger \partial^\mu S_i) + \lambda_{HS} (S_i^\dagger S_i) H^\dagger H \\ - \lambda_S (S_i^\dagger S_i) (S_j^\dagger S_j) - \lambda'_S (S_i^\dagger S_j) (S_j^\dagger S_i)$$

- Describe the **D**ynamical **S**cale **S**ymmetry **B**reaking.
  - The order parameter is  $\langle S^\dagger S \rangle$
- Assume that the DSSB is more dominant than the scale anomaly.
- Scale invariant Lagrangian.
- $\lambda_S, \lambda'_S$  and  $\lambda_{HS}$ : **effective** coupling constants.
  - which contain the quantum effects of hidden gluon.
- Renormalizable.
- Analyzed by the mean-field approx. (non-perturbative method).

# Dark matter candidate is $\phi^\alpha$

- The excitation fields from the vacuum  $\langle S^\dagger S \rangle$ 
  - Assume the **unbroken**  $U(N_f)$  flavor symmetry:

$$\langle \Omega | (S_i^\dagger S_j) | \Omega \rangle = f_0 \delta_{ij} + \delta_{ij} Z_\sigma^{\frac{1}{2}} \sigma + t_{ji}^\alpha Z_\phi^{\frac{1}{2}} \phi^\alpha$$

- c.f. chiral condensate

$$\langle \Omega | \bar{\psi}_i (1 - \gamma^5) \psi_j | \Omega \rangle = f_0 \delta_{ij} + \delta_{ij} Z_\sigma^{\frac{1}{2}} \sigma + t_{ji}^\alpha Z_\pi^{\frac{1}{2}} \pi^\alpha$$

- Lagrangian

$$\mathcal{L}'_{\text{MFA}} \supset \phi^\alpha \text{ --- } t_{ij}^\alpha \begin{cases} \text{--- } S_i \\ \text{--- } S_j \end{cases}$$

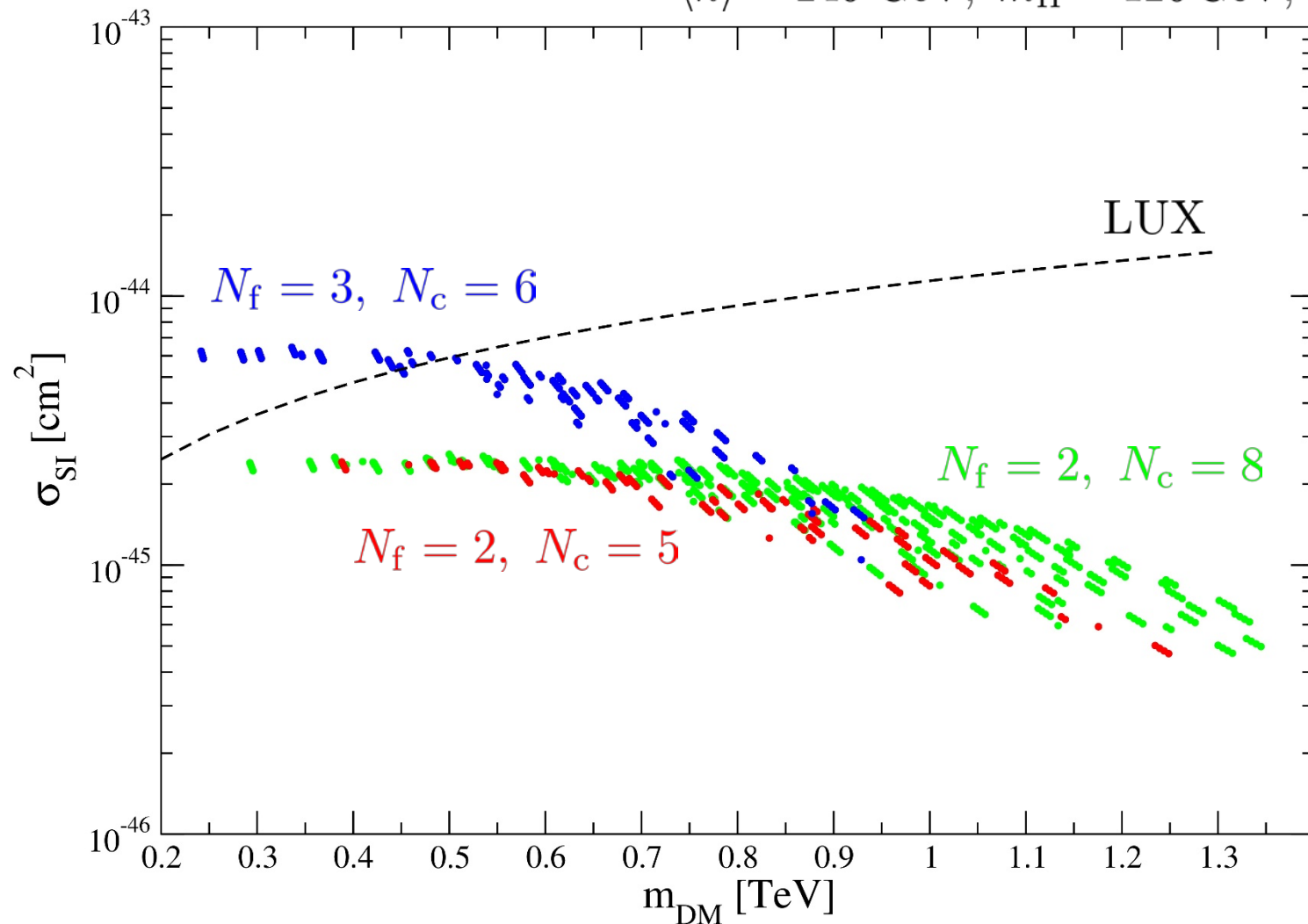
$$\phi^\alpha \text{ --- } \text{--- } \begin{array}{c} \text{--- } S_i \\ \text{--- } h \\ \text{--- } h \end{array}$$

Forbidden by flavor symmetry

$\phi^\alpha$  is stable.

# $\sigma_{\text{SI}}$ VS. $m_{\text{DM}}$

$\langle h \rangle = 246 \text{ GeV}$ ,  $m_{\text{H}} = 126 \text{ GeV}$ ,  $\Omega \hat{h}^2 \sim 0.12$



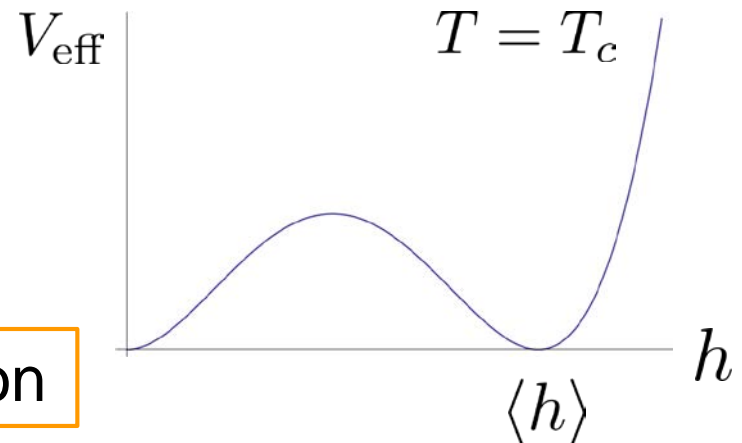
# EW Baryogenesis scenario

## □ Sakharov conditions

1. Baryon number violation
2. C-symmetry and CP-symmetry violation
3. **Interactions out of thermal equilibrium.**

## □ Electroweak strong first-order phase transition

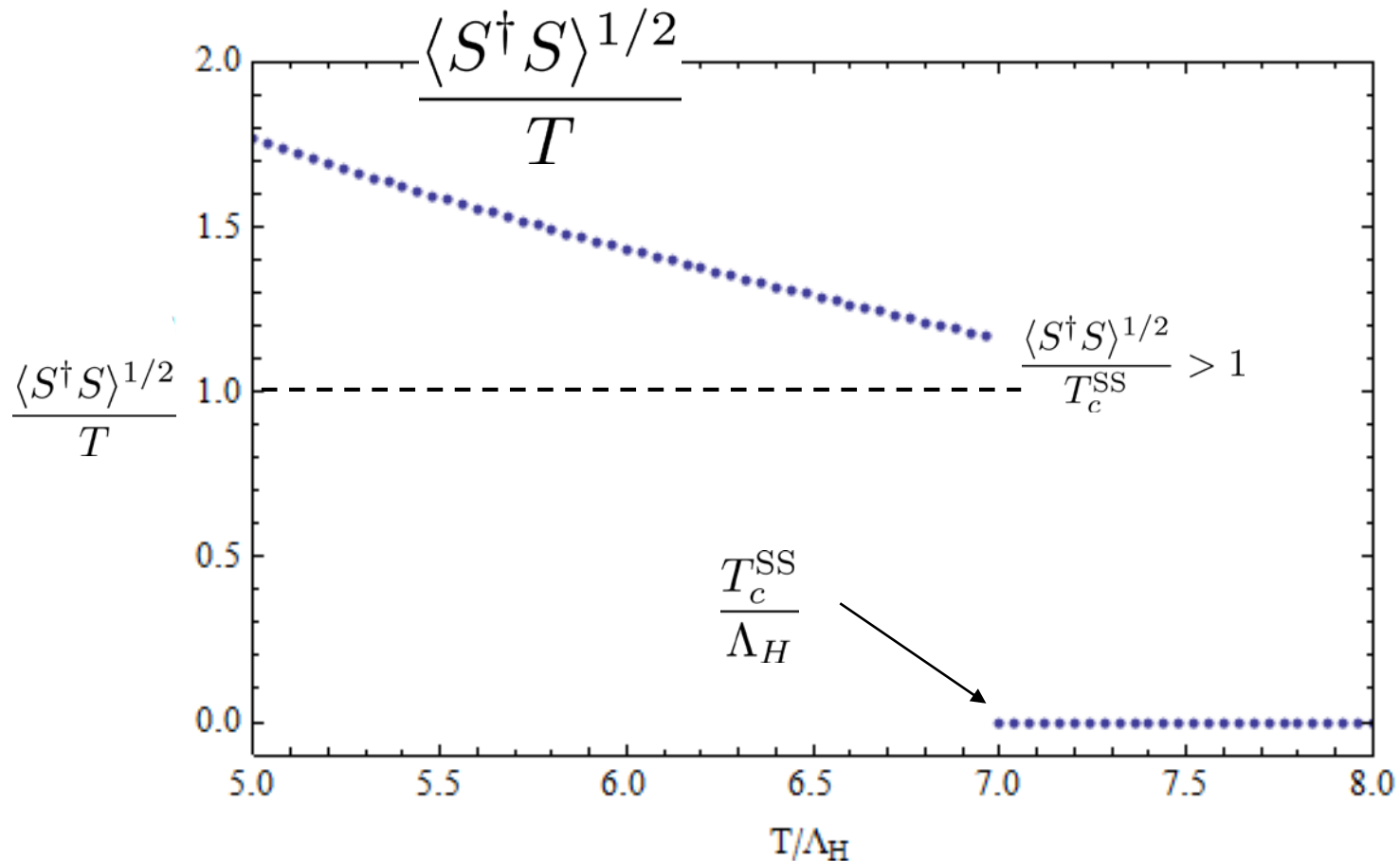
$$\frac{\langle h \rangle}{T_c} \gtrsim 1$$



The SM cannot satisfy this condition

# Scale transition is strong 1<sup>st</sup> order.

J, Kubo and M. Y., PTEP **2015** 093B01 (arXiv:1506.06460)



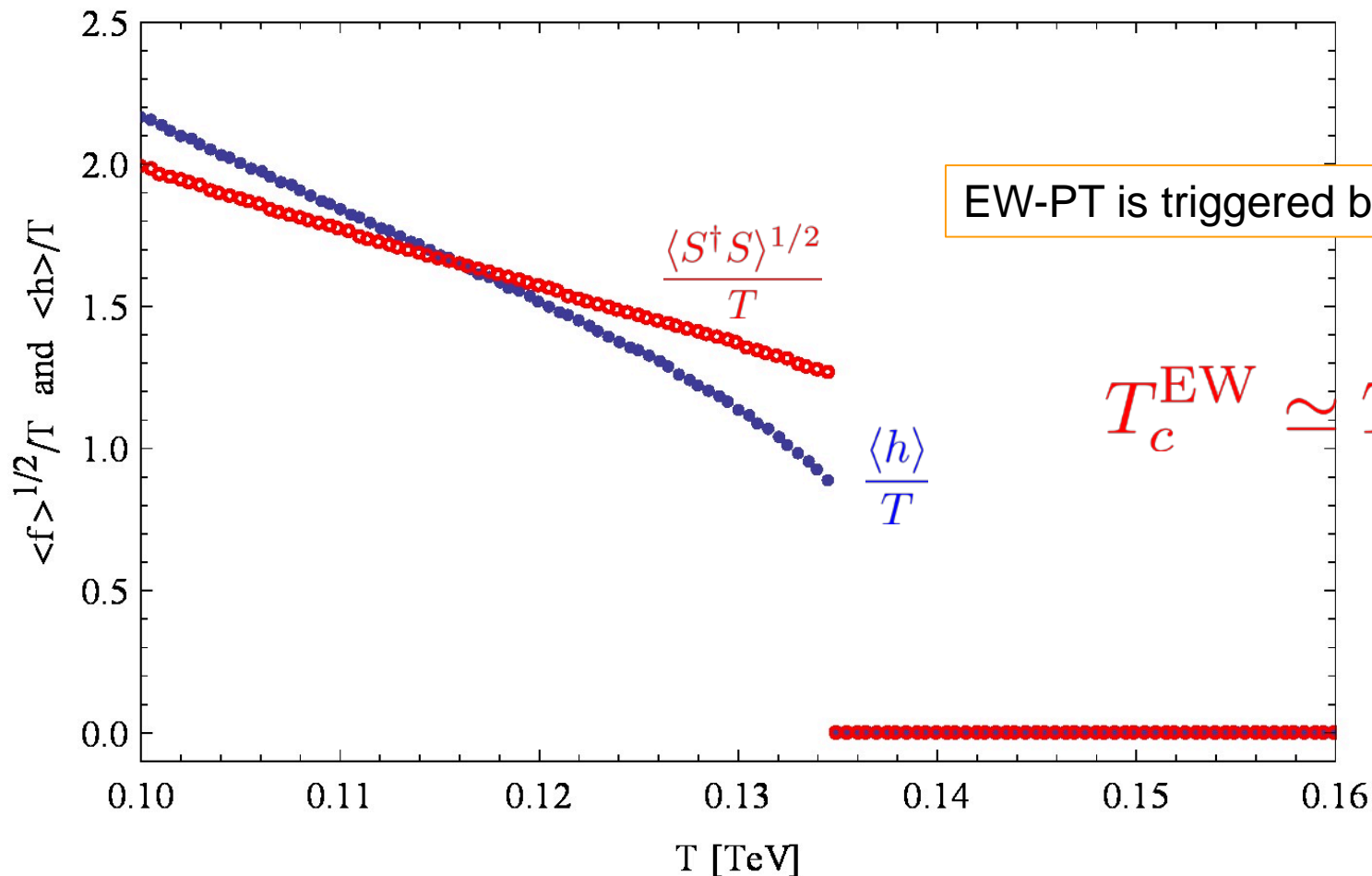


Without dark matter case:  $N_f = 1$

EW phase transition becomes strong 1<sup>st</sup> order

$$\langle h \rangle = 246 \text{ GeV}, m_H = 126 \text{ GeV}$$

$$N_c = 6$$

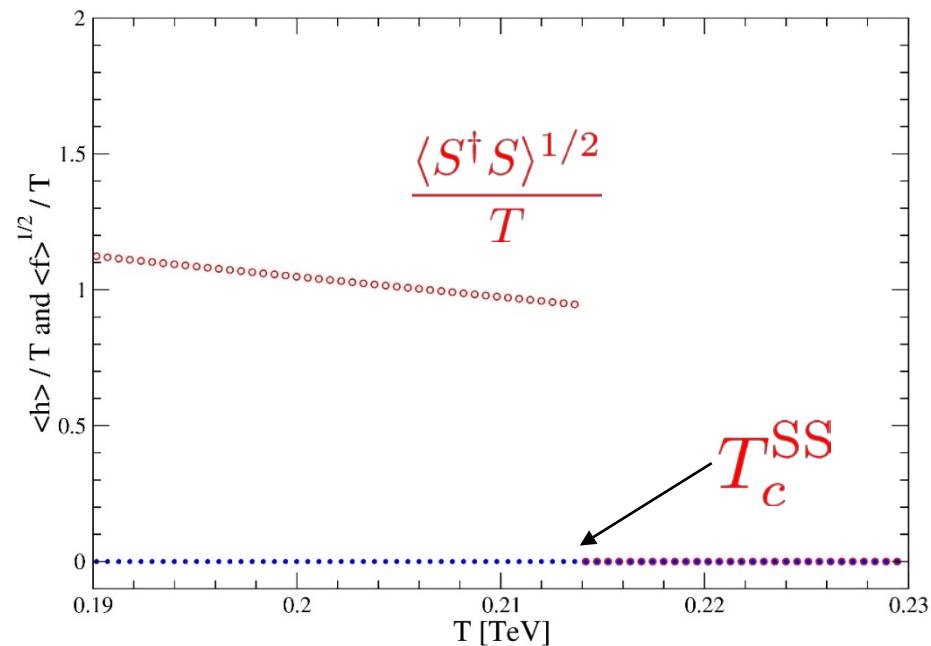
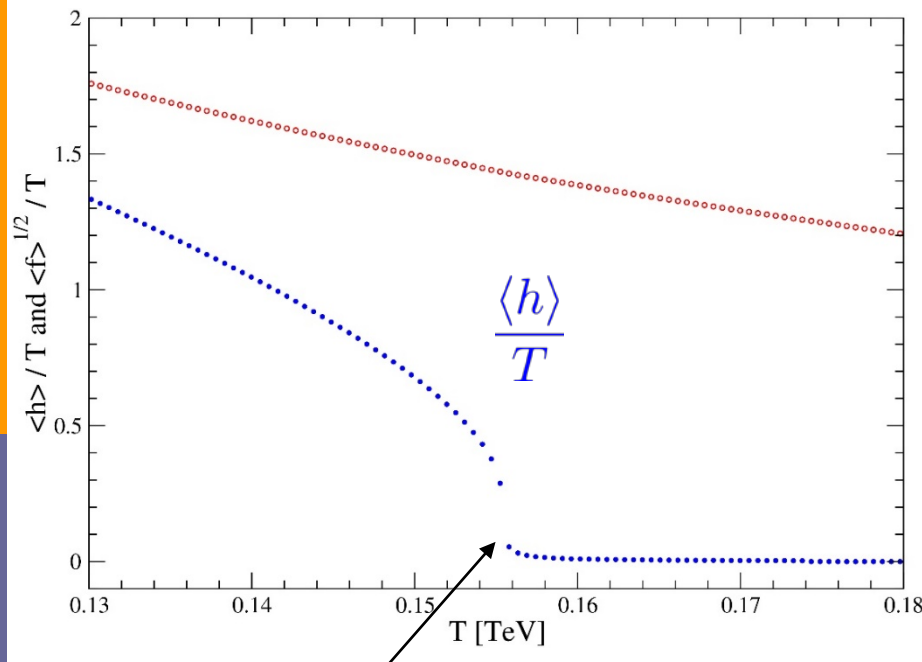


# With dark matter case: $N_f = 2$

## EW phase transition becomes **weak** 1<sup>st</sup> order

$$\langle h \rangle = 246 \text{ GeV}, \quad m_H = 126 \text{ GeV}, \quad \Omega \hat{h}^2 \sim 0.12$$

$$N_c = 6$$



# Summary

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- We suggested a new model based on classically scale invariance.
  - Strongly interacting hidden sector with the scalar field
  - Explain the mechanism of generation of “scale”
  - Dynamical Scale Symmetry Breaking  $\langle S^\dagger S \rangle \neq 0$
  - The EW symmetry breaking  $\langle h \rangle \neq 0$

“Scalegenesis” is realized!

# Summary

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- We suggested a new model based on classically scale invariance.
  - Strongly interacting hidden sector with the scalar field
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  - Dynamical Scale Symmetry Breaking  $\langle S^\dagger S \rangle \neq 0$
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“Scalegenesis” is realized!

- Dark matter candidate exists.
- The EW 1<sup>st</sup> order phase transition

# Prospects

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- The impacts of higher order operators
- More precise analysis is needed.
  - Lattice simulation
- Why is the scale transition 1<sup>st</sup> order?
- Is the hidden sector UV complete?
  - Working with H. Goto and H. Kawauchi
- C and CP violation

# Appendix

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# Hierarchy problem

- Nothing between  $\Lambda_{EW}$  and  $\Lambda_{pl}$ ?
  - $\Lambda_{EW} \sim \mathcal{O}(10^2) \text{ GeV} \Leftrightarrow \Lambda_{pl} \sim \mathcal{O}(10^{19}) \text{ GeV}$



- Fine-tuning problem

$$m_R^2 = m_0^2 - \left( \frac{\lambda}{16\pi^2} + \dots \right) \Lambda_{pl}^2$$

$$(10^2 \text{ GeV})^2 = (10^{19} \text{ GeV})^2 - (10^{19} \text{ GeV})^2$$

- Fermion and gauge field have not the problem.

- Gauge symmetry:

~~$$m_0^2 A_\mu A^\mu$$~~

$$m_Z^2 \propto \langle h \rangle^2 \sim \Lambda_{EW}^2$$

- Chiral symmetry:

~~$$m_0 \bar{\psi} \psi$$~~

$$m_q^2 \propto \langle \bar{\psi} \psi \rangle \sim \Lambda_{QCD}^2$$

# Argument by Bardeen



W.A. Bardeen, On naturalness in the standard model, **FERMILAB-CONF-95-391** (1995).

- The quadratic divergences are **spurious**.
  - $\Lambda$  always is subtracted by renormalization.
  - The dimensional regularization automatically subtracts the quadratic divergence.
  
- Only **logarithmic** terms related to the **scale anomaly** **survive** in the perturbation.
  - ➔ The non-zero beta function  $\beta \neq 0$



# Argument by Bardeen



W.A. Bardeen, On naturalness in the standard model, **FERMILAB-CONF-95-391** (1995).

## □ The RG equation of Higgs mass

$$\frac{dm^2}{d \log \mu} = \frac{\textcolor{red}{m}^2}{16\pi^2} \left( 12\lambda + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_1^2 \right)$$

- If  $m(\Lambda_{\text{pl}}) = 0$ , the mass does not run.

## □ If the Higgs field is coupled to a new particle with mass $M$ ,

$$m_R^2 = m_0^2 + \frac{\lambda'}{16\pi^2} M^2 \log \left( \frac{\mu^2}{M^2} \right) + \dots$$

- If  $M \sim \mathcal{O}(\text{TeV})$ , fine-tuning is not needed.
  - Even if so, the origin of  $m_0$  with TeV order is **unknown**.
- If  $M \gg \text{TeV}$ , fine-tuning problem appears.

# Classical scale invariance

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- The classical scale invariance prohibits  $m_0$ .

- Boundary condition:  $m_0 = m(\Lambda_{\text{pl}}) = 0$

- The origin of observed mass is **radiative corrections with TeV scale.**

$$m_R^2 = \frac{\lambda'}{16\pi^2} M^2 \log \left( \frac{\mu^2}{M^2} \right)$$

- The classical scale invariance is one of candidates for the solution of fine-tuning problem.

How to generate **radiative corrections**?

# Two ways

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## □ Perturbative way

- Coleman-Weinberg mechanism
- **Scale anomaly**
- e.g. CW potential

## □ Non-perturbative way

- Strong dynamics
- The mass **dynamically** is generated.
- e.g. chiral symmetry breaking

# Advantages of our model

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- The number of parameters is less.
- The mediator is the strongly interacting particle.
  - Observing the hidden sector is easier than other models such as the hidden (quark) model.
    - $\langle \bar{\psi}\psi \rangle \rightarrow \langle S \rangle \rightarrow m_H \rightarrow \langle h \rangle$
    - $\langle S^\dagger S \rangle \rightarrow m_H \rightarrow \langle h \rangle$
  - The DM candidate is CP even.
    - c.f. The DM in hidden (quark) QCD is CP odd.
- Strong 1<sup>st</sup> order of EW phase transition can be realized.(will see later)

# Comparison

$\mathcal{L}_{\text{HQCD}}$

Scale  
invariant

$\mathcal{L}_{\text{QCD}}$

Chiral  
invariant

Effective model

$$V_{\text{eff}} = \lambda_S (S_i^\dagger S_i) (S_j^\dagger S_j) \\ + \lambda'_S (S_i^\dagger S_j) (S_j^\dagger S_i) \\ - \lambda_{HS} (H^\dagger H) (S^\dagger S)$$

$$V_{\text{eff}} = G [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau^\alpha\psi)^2]$$

Order parameter

$$\langle S^\dagger S \rangle \neq 0$$

$$\langle \bar{\psi}\psi \rangle \neq 0$$

meson

$\sigma$

$\phi^\alpha$

$\sigma$

$\pi^\alpha$

# Strong interaction is difficult...

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- It is hard to analytically solve the strongly interacting system.
- In QCD, **effective model approaches** are successful.
  - e.g. **N**ambu—**J**ona-**L**asinio (NJL) model for  $D\chi$ SB
- We formulate an effective theory of our model.

# How to formulate?

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- An effective model describing dynamical **scale symmetry** breaking (DSSB)
- Scale invariance is broken by scale anomaly.
- The breaking is only **logarithmic**.
  - The non-perturbative **scale breaking** due to the condensation  $\langle S^\dagger S \rangle \neq 0$  is dominant.
  - Ignore the breaking by scale anomaly.

# Effective potential

## □ The mean-field approximated effective potential

- Integrate out  $\chi$  (Gauss integral)  $S_i \rightarrow S_i + \chi_i$

$$V_{\text{MFA}} = M^2 (S_i^\dagger S_i) + \lambda_H (H^\dagger H)^2 - N_f (N_f \lambda_S + \lambda'_S) f^2 + \frac{N_f N_c}{32\pi^2} M^4 \ln \frac{M^2}{\Lambda_H^2}$$

$M^2 = 2(N_f \lambda_S + \lambda'_S) f - \lambda_{HS} H^\dagger H$

$\text{Tr log} \left( \chi \right)$   $\xrightarrow{\text{MS scheme}}$

## □ Solving the gap equations

$$\langle S \rangle = 0, \quad \langle f \rangle \neq 0, \quad \langle H \rangle \neq 0$$



# Input & free parameters

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## □ Input

- Higgs mass
- EW vacuum
- DM relic abundance

$$m_H = 126 \text{ GeV}$$

$$\langle h \rangle = 246 \text{ GeV}$$

$$\Omega \hat{h}^2 \sim 0.12$$



## □ 7 free parameters.

$$\lambda_S$$

$$\lambda'_S$$

$$\lambda_{HS}$$

$$\lambda_H$$

$$N_f$$

$$N_c$$

$$\Lambda_H$$

# Where is the vacuum?

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- Minimum of  $V_{\text{MFA}}$ ; Solving gap equations:

$$\frac{\partial}{\partial S_i^a} V_{\text{MFA}} = 0, \quad \frac{\partial}{\partial f} V_{\text{MFA}} = 0, \quad \frac{\partial}{\partial H} V_{\text{MFA}} = 0$$

- Three solutions:

- i.  $\langle S_i^a \rangle \neq 0, \langle M^2 \rangle = 0, G = 0$
- ii.  $\langle S_i^a \rangle = 0, \langle M^2 \rangle = 0 \longrightarrow \langle V_{\text{eff}} \rangle = 0$
- iii.  $\langle S_i^a \rangle = 0, \langle M^2 \rangle \neq 0, G > 0 \longrightarrow \langle V_{\text{eff}} \rangle < 0$

$$M^2 = 2(N_f \lambda_S + \lambda'_S) f - \lambda_{HS} H^\dagger H$$
$$G = 4N_f \lambda_H \lambda_S - N_f \lambda_{HS}^2 + 4\lambda_H \lambda'_S$$

The solution (iii) is suitable.

# Solutions

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## □ The vacuum of Higgs

$$\langle h \rangle = \frac{N_f \lambda_{HS}}{G} \Lambda_H^2 \exp \left( \frac{32\pi^2 \lambda_H}{N_c G} - \frac{1}{2} \right)$$

## □ The scalar condensate

$$\langle S^\dagger S \rangle = \langle f \rangle = \frac{2\lambda_H}{G} \Lambda_H^2 \exp \left( \frac{32\pi^2 \lambda_H}{N_c G} - \frac{1}{2} \right)$$

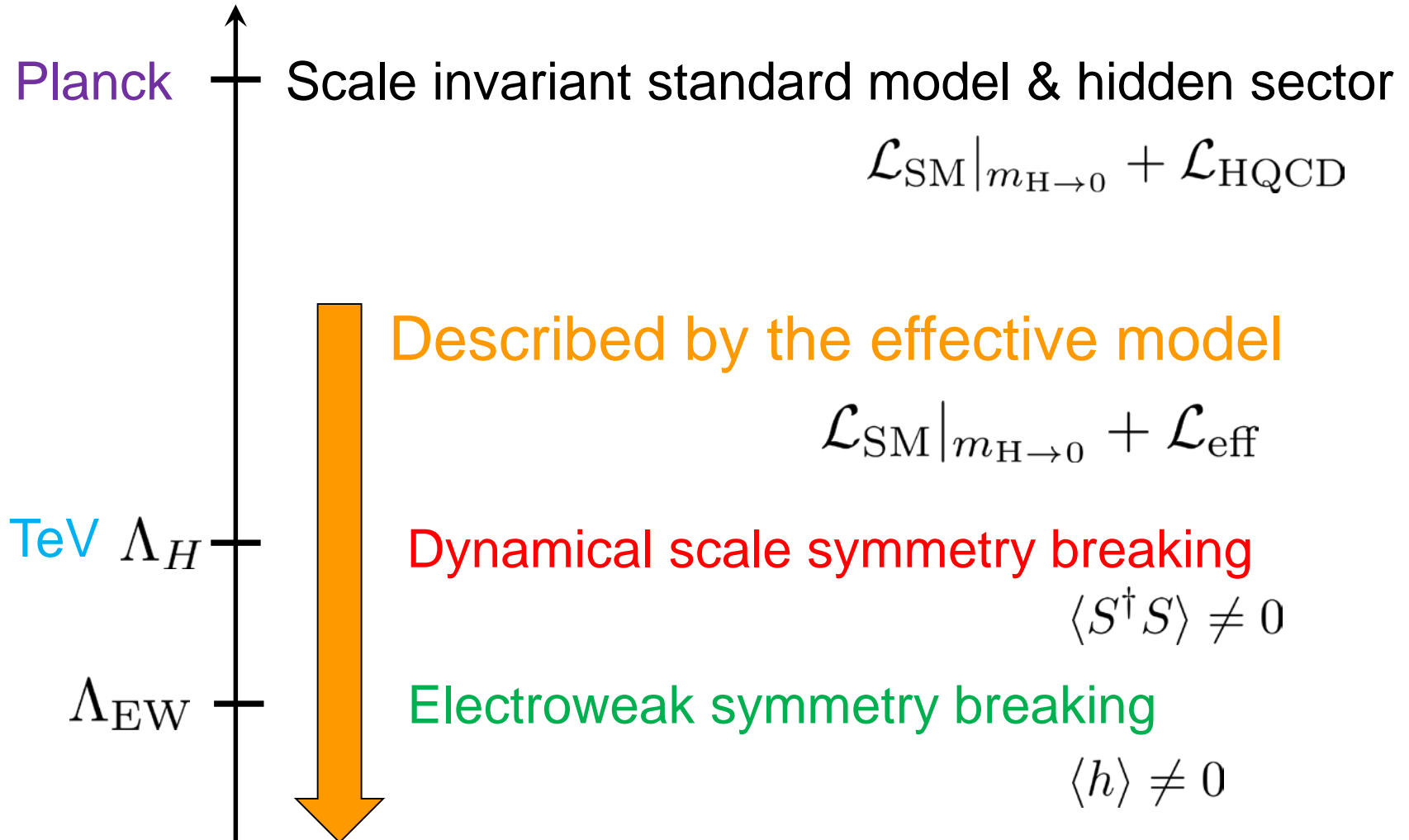
## □ Constituent scalar mass

$$M^2 = \Lambda_H^2 \exp \left( \frac{32\pi^2 \lambda_H}{N_c G} - \frac{1}{2} \right)$$

$$G = 4N_f \lambda_H \lambda_S - N_f \lambda_{HS}^2 + 4\lambda_H \lambda'_S$$

# Summary so far

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# How to evaluate physical values?

Review: T. Hatsuda and T. Kunihiro, Phys. Rep. 247 221 (1994)

## □ Mean-field approximation (MFA)

- Many body system is reduced to 1 body system.

- Methods:

1. Introduce a “BCS” vacuum  $|\Omega\rangle$  and a mean field:

$$f_{ij} \equiv \langle \Omega | S_i^\dagger S_j | \Omega \rangle$$

2. Apply the following replacements to  $\mathcal{L}_{\text{eff}}$

$$(S_i^\dagger S_j)(S_j^\dagger S_i) \rightarrow : (S_i^\dagger S_j)(S_j^\dagger S_i) : + 2f_{ij}(S_j^\dagger S_i) - |f_{ij}|^2$$

Normal ordering

3. We obtain

$$\langle \Omega | : \mathcal{L}_{\text{Int}} : | \Omega \rangle = 0$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{MFA}} + : \mathcal{L}_{\text{Int}} :$$

# Mean-field approximation

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□ Bogoliubov-Valatin vacuum  $|\Omega\rangle$

$$\langle\Omega|(S_i^\dagger S_j)|\Omega\rangle = f_0\delta_{ij} + Z_\sigma^{1/2}\delta_{ij}\sigma + Z_\phi^{1/2}t_{ji}^\alpha\phi^\alpha$$

$$\langle S_i S_j \rangle = \left\langle \sum_{a=1}^{N_c} S_i^a S_j^a \right\rangle$$

□ Wick contractions

$$(S_i^\dagger S_j) =: (S_i^\dagger S_j) : + f_{ij}$$

$$\langle\Omega| : \mathcal{O} : |\Omega\rangle = 0$$

$$(S_i^\dagger S_j)(S_j S_i) =: (S_i^\dagger S_j)(S_j^\dagger S_i) : + 2f_{ij}(S_j^\dagger S_i) - |f_{ij}|^2$$

# Mean-field approximation

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□ Lagrangian  $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{MFA}} + \mathcal{L}_I$   $\langle \Omega | \mathcal{L}_I | \Omega \rangle = 0$

$$\begin{aligned} \mathcal{L}_{\text{MFA}} = & (\partial^\mu S^\dagger \partial_\mu S) - M^2 (S_i^\dagger S_j) \\ & + N_f (N_f \lambda_S + \lambda'_S) Z_\sigma \sigma^2 + \frac{\lambda'_S}{2} Z_\phi \phi^\alpha \phi^\alpha \\ & - 2(N_f \lambda_S + \lambda'_S) Z_\sigma^{1/2} \sigma (S_i^\dagger S_i) - 2\lambda'_S Z_\phi^{1/2} (S_i^\dagger t_{ij}^\alpha \phi^\alpha S_j) \\ & + \lambda_{HS} (S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2 \end{aligned}$$

□ Constituent scalar mass

$$M^2 = 2(N_f \lambda_S + \lambda'_S) f - \lambda_{HS} H^\dagger H$$

# Effective potential

---

$$V_{\text{MFA}} = M^2(S_i^\dagger S_i) + \lambda_H(H^\dagger H)^2 - N_f(N_f\lambda_S + \lambda'_S)f^2 + \frac{N_c N_f}{32\pi^2} M^4 \log \frac{M^2}{\Lambda_H^2}$$

$$H = \begin{pmatrix} \chi^+ \\ \langle h \rangle + h + i\chi^0 \end{pmatrix}$$



# Mass of dark matter

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- Mass = a pole of two point function
  - Inverse two point function of  $\phi^\alpha$  (dark matter)

$$\begin{aligned}\Gamma_{\phi\phi}^{\alpha\beta}(p^2) &= \overline{\phi^\alpha} \phi^\beta + \overline{\phi^\alpha} \begin{array}{c} \xrightarrow{p} \\ \text{---} \text{---} \text{---} \end{array} \text{---} \phi^\beta \\ &= \delta^{\alpha\beta} \left[ Z_\phi \lambda'_S + Z_\phi \lambda'^2_S N_c \Gamma(p^2) \right]\end{aligned}$$

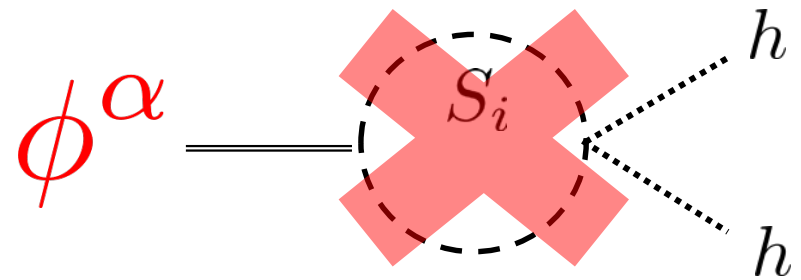
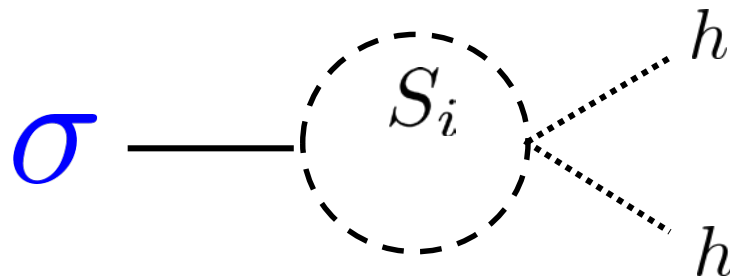
- Find zero

$$\Gamma_{\phi\phi}^{\alpha\beta}(p^2 = m_{\text{DM}}^2) = 0$$

# Dark matter candidate is $\phi^\alpha$

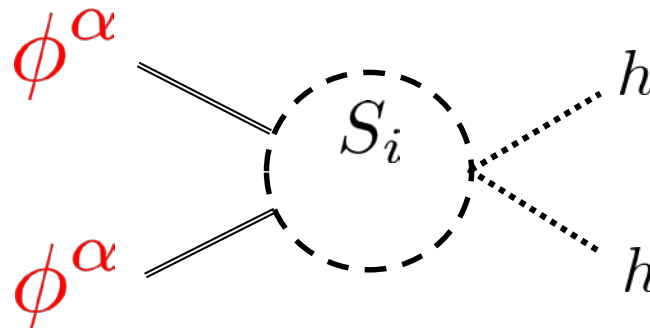
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- Decay into Higgs through  $S$  loop



Forbidden by flavor symmetry

- Coannihilation



# Coannihilation

---

$$\begin{aligned}
 \kappa_{S(t)} \delta_{\alpha\beta} = & \quad \begin{array}{c} \phi^\alpha \\ \text{---} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} \phi^\beta \\ \text{---} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} h \\ \text{---} \end{array} \begin{array}{c} h \\ \text{---} \end{array} \quad + \text{cross} \\
 & + \begin{array}{c} \phi^\alpha \\ \text{---} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} \phi^\beta \\ \text{---} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} h \\ \text{---} \end{array} \begin{array}{c} h \\ \text{---} \end{array} \\
 & + \begin{array}{c} \phi^\alpha \\ \text{---} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} \phi^\beta \\ \text{---} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} h \\ \text{---} \end{array} \begin{array}{c} h \\ \text{---} \end{array} \quad + \text{crosses}
 \end{aligned}$$

The diagram illustrates the coannihilation process. It shows three terms in a sum, each representing a different Feynman diagram. Each diagram features a central dashed circle labeled  $S_i$ . In the first term, two incoming lines (one solid, one dashed) connect to the circle, and two outgoing lines (one solid, one dashed) emerge from it. The second term is a cross of the first, where the incoming and outgoing lines are swapped. The third term is another cross, where the incoming and outgoing lines are swapped again. The labels  $\phi^\alpha$ ,  $\phi^\beta$ , and  $h$  are placed near the lines to identify the particles involved.

# Velocity averaged annihilation cross section

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$$\langle v\sigma \rangle = \frac{1}{32\pi m_{\text{DM}}^3} \sum_{I=W,Z,t,h} (m_{\text{DM}}^2 - m_I^2)^{1/2} a_I + \mathcal{O}(v^2)$$

$$a_{W(Z)} = 4(2)[\text{Re}(\kappa_s)]^2 \Delta_h^2 m_{W(Z)}^4 \left( 3 + 4 \frac{m_{\text{DM}}^4}{m_{W(Z)}^4} - 4 \frac{m_{\text{DM}}^2}{m_{W(Z)}^2} \right)$$

$$a_t = 24[\text{Re}(\kappa_s)]^2 \Delta_h^2 m_t^2 (m_{\text{DM}}^2 - m_t^2)$$

$$a_h = [\text{Re}(\kappa_s)]^2 \left( 1 + 24\lambda_H \Delta_h \frac{m_W^2}{g^2} \right)^2$$

$$\Delta_h = (4m_{\text{DM}}^2 - m_h^2)^{-1}$$

# Dark matter candidate is $\phi^\alpha$

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  - Assume the **unbroken**  $U(N_f)$  flavor symmetry:

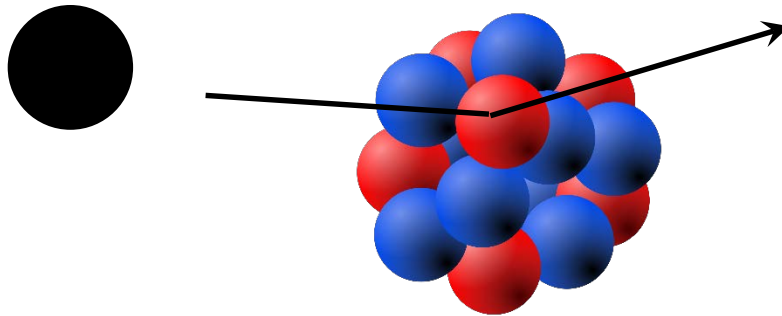
$$\langle \Omega | (S_i^\dagger S_j) | \Omega \rangle = f_0 \delta_{ij} + \delta_{ij} Z_\sigma^{\frac{1}{2}} \sigma + t_{ji}^\alpha Z_\phi^{\frac{1}{2}} \phi^\alpha$$

- Mean-field Lagrangian (before integrating  $S$ )

$$\begin{aligned} \mathcal{L}'_{\text{MFA}} = & (\partial_\mu S_i)^2 - M^2 (S_i^\dagger S_i) + N_f (N_f \lambda_S + \lambda'_S) Z_\sigma \sigma^2 + \frac{\lambda'_S}{2} Z_\phi (\phi^\alpha)^2 \\ & - 2(N_f \lambda_S + \lambda'_S) Z_\sigma^{1/2} \sigma (S_i^\dagger S_i) - 2\lambda'_S Z_\phi^{1/2} (S_i^\dagger t_{ij}^\alpha \phi^\alpha S_j) \\ & + \lambda_{HS} (S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2 \end{aligned}$$

# Direct detection

## □ Scattering off the Nuclei



Inverse two-point function

$$\Gamma_{\phi\phi}^{\alpha\beta}(p^2 = m_{\text{DM}}^2) = 0$$

## ■ Spin independent cross section

$$\sigma_{\text{SI}} = \frac{1}{4\pi} \left( \frac{\kappa_t \hat{r} m_N^2}{m_{\text{DM}} m_h^2} \right)^2 \left( \frac{m_{\text{DM}}}{m_N + m_{\text{DM}}} \right)^2$$

$$\kappa_t =$$

$m_N$ : nucleon mass

$\hat{r}$ : nucleonic matrix element  $\hat{r} \sim 0.3$

# $\sigma_{\text{SI}}$

---

$$\sigma_{\text{SI}} = \frac{1}{4\pi} \left( \frac{\kappa_t \hat{r} m_N^2}{m_{\text{DM}} m_h^2} \right)^2 \left( \frac{m_{\text{DM}}}{m_N + m_{\text{DM}}} \right)^2$$

$$\hat{r} \sim 0.3$$

# Dark matter relic abundance

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## □ DM relic abundance

$$\Omega \hat{h}^2 = (N_f^2 - 1) \frac{Y_\infty s_0 m_{\text{DM}}}{\rho_c / \hat{h}^2}$$

### ■ Entropy density

$$s_0 = 2890 \text{ cm}^{-3}$$

### ■ Critical density/Hubble parameter

$$\rho_c / \hat{h}^2 = 1.05 \times 10^{-5} \text{ GeV cm}^{-3}$$

### ■ DM number density

$$g_* = 106.75 + N_f^2 - 1$$

$$\frac{dY}{dx} = -0.264 g_*^{1/2} \frac{m_{\text{DM}} M_{\text{Pl}}}{x^2} \langle \sigma v \rangle (Y^2 - \bar{Y}^2)$$



# At finite temperature

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## □ Momentum integral

$$\int \frac{d^4 p}{(2\pi)^4} f(p_0, \vec{p}) \quad \longrightarrow \quad T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} f(\omega_n, \vec{p})$$

## ■ Matsubara frequency

$$\omega_n = \begin{cases} 2n\pi T & \text{(boson loop)} \\ (2n+1)\pi T & \text{(fermion loop)} \end{cases}$$

# Effective potential

- There are four components.

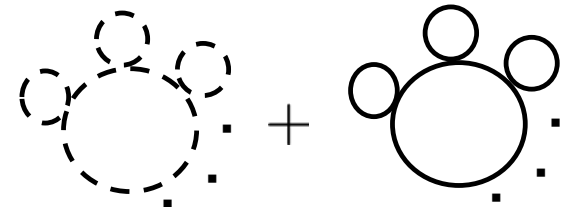
$$V_{\text{eff}}(f, h; T) =$$

$$V_{\text{MFA}}(f, h) + V_{\text{CW}}(h) \quad \leftarrow \text{Zero temp. part}$$

$$+ \underline{V_{\text{FT}}(f, h; T)} + V_{\text{RING}}(h; T) \quad \leftarrow \text{Finite temp. part}$$

$$T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} \left[ \text{dashed circle } S + \text{solid circle All SM particles} \right]$$

Summation of **thermal mass**  
(remove the IR divergence)



# Phase transition

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- $V_{\text{eff}}$  at zero temperature

$$V_{\text{eff}}(f, h; T = 0) = V_{\text{MFA}}(f, h) + V_{\text{CW}}(h)$$

$$\longrightarrow \langle h \rangle = 246 \text{ GeV} \quad \langle f \rangle \neq 0$$
$$m_{\text{H}} = 126 \text{ GeV}$$

- $V_{\text{eff}}$  at critical temperature  $T_c^{\text{EW}}$  (EWPT)

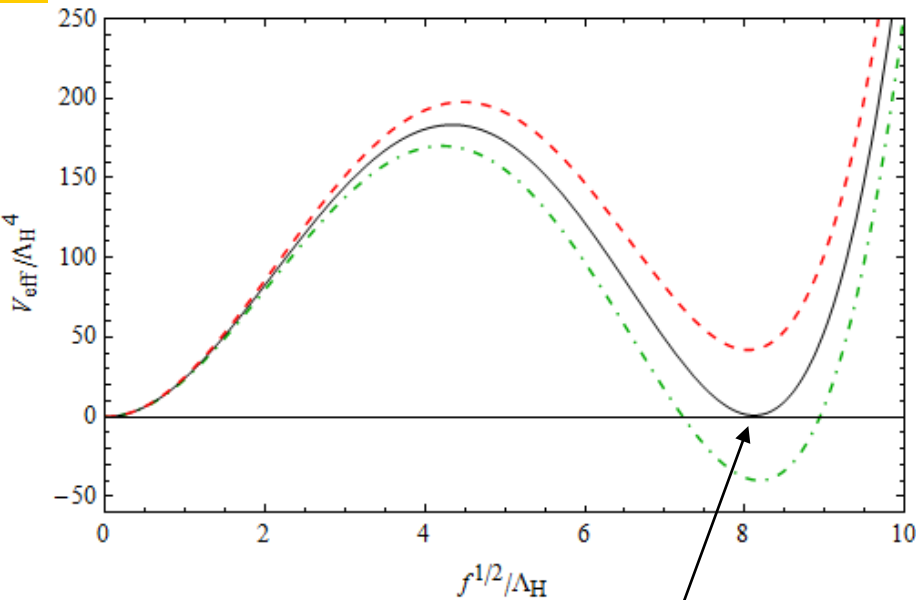
$$V_{\text{eff}}(f, h; T = T_c^{\text{EW}}) \longrightarrow \langle h \rangle = 0$$

- $V_{\text{eff}}$  at critical temperature  $T_c^{\text{SS}}$  (SSPT)

$$V_{\text{eff}}(f, h; T = T_c^{\text{SS}}) \longrightarrow \langle f \rangle = \langle S^\dagger S \rangle = 0$$

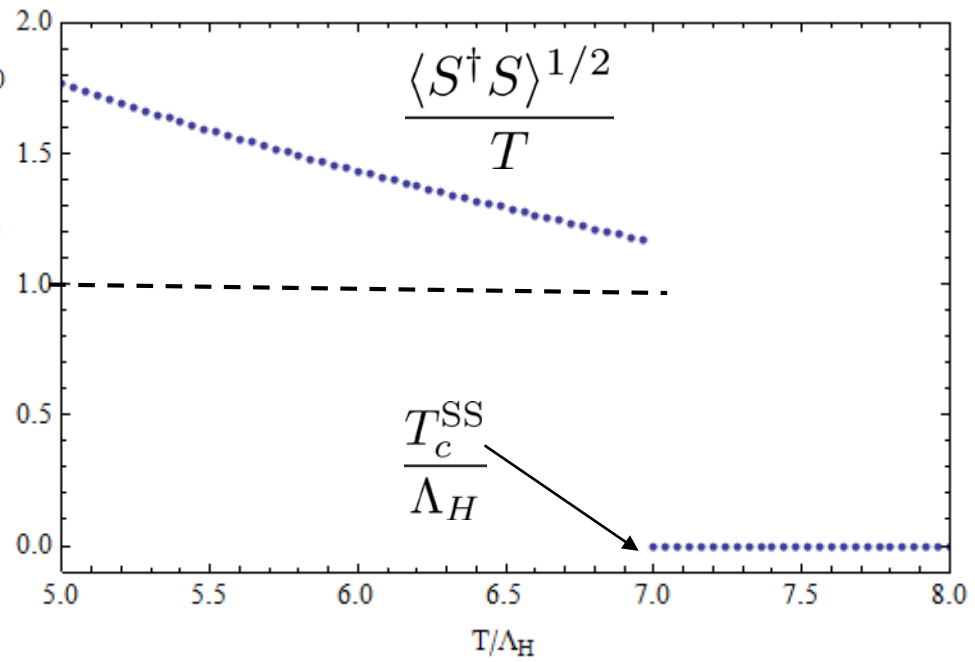
# Scale transition is strong 1<sup>st</sup> order.

J, Kubo and M. Y., PTEP **2015** 093B01 (arXiv:1506.06460)



$$T_c^{\text{SS}} / \Lambda_H = 7.0$$

$$\frac{\langle S^\dagger S \rangle^{1/2}}{\Lambda_H}$$

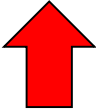


# Difference between two cases


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- The Higgs portal is important

$$-\lambda_{HS}(S^\dagger S)H^\dagger H$$

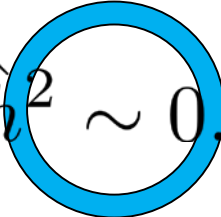
  $\lambda_{HS}$

EW-PT is triggered.

$\Omega \hat{h}^2 \sim 0.12$  

  $\lambda_{HS}$

Not enough to trigger

$\Omega \hat{h}^2 \sim 0.12$  

- Need more precisely analysis