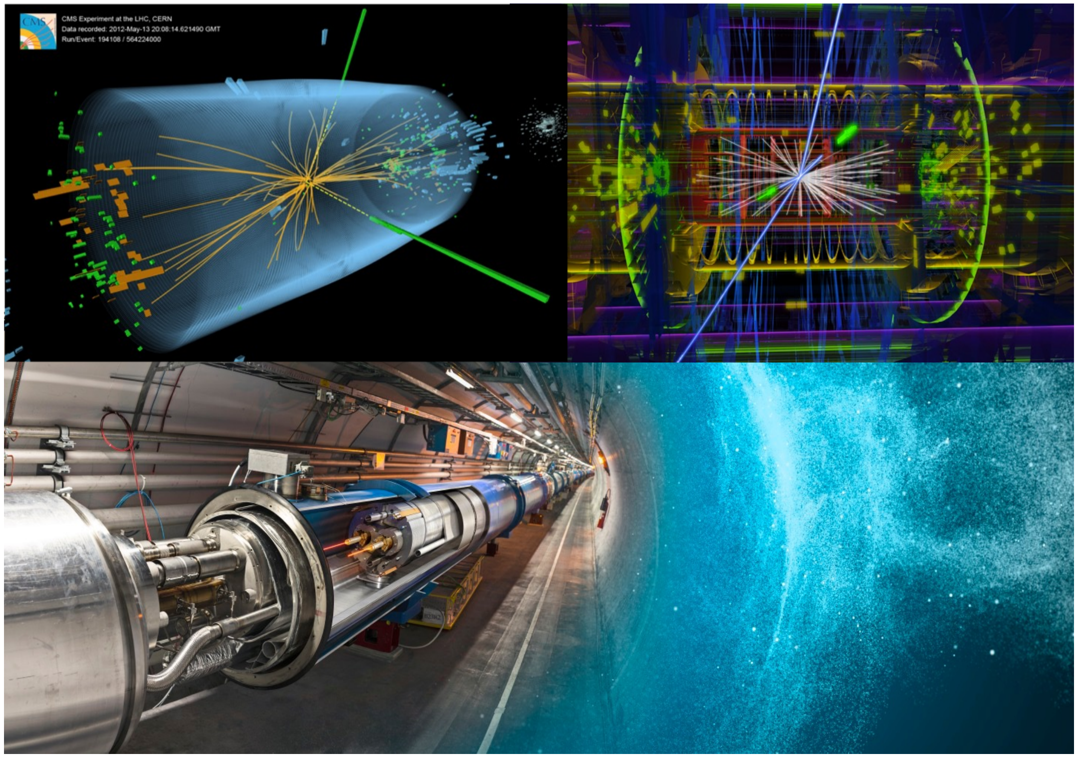
Quantum Information Science as a tool for exploring an extended Higgs Sector



Marcela Carena
Fermilab and UChicago

Scalars 2023, University of Warsaw, Faculty of Physics

Warsaw, September 16, 2023

Outline

- Introduction
- Concurrence and Entanglement Power of Unitary Operators
- Minimal Entanglers and Emergent Symmetries

  -- Neutron–proton scattering

  -- Two Higgs doublet Model
- Entanglement Suppression and emergent symmetry confronting experimental data
- Final thoughts

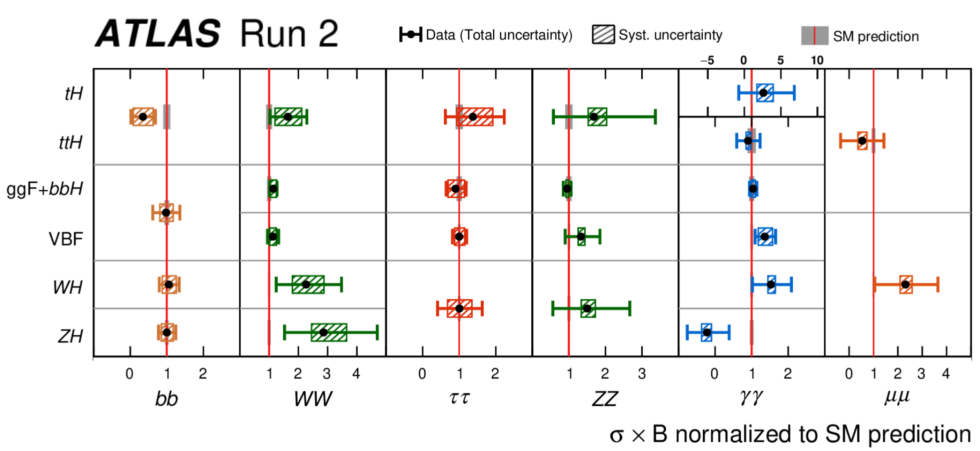
The great success of the Higgs Boson

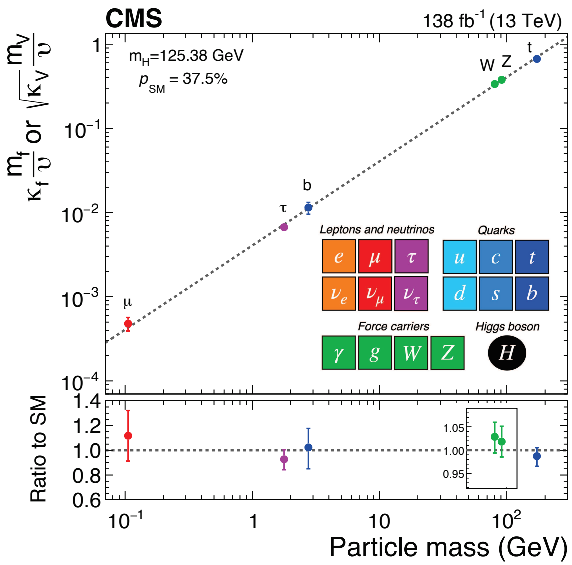
Its discovery and subsequent study of its properties at the LHC has provided *a first portrait* of the electroweak symmetry breaking mechanism

It ensures the calculability of the Standard Model of particle physics at high energies

With M_H = 125 GeV, its mass is at a lucky spot to maximally allow us to measure its interactions with other particles

The LHC data favors a SM-like Higgs boson



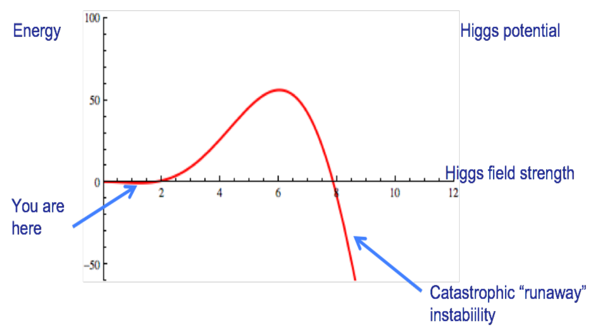


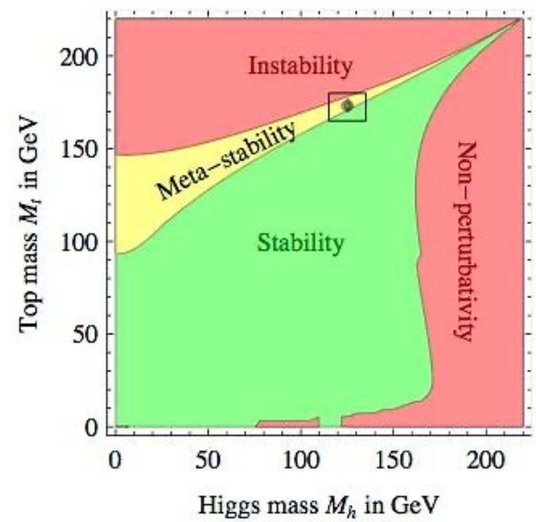
Nature 607, no. 7917, 52-59; 60-68 (2022) [arXiv:2207.00092]; [arXiv:2207.00043]

The SM does not explain the origins of electroweak symmetry breaking

We put **by hand** the condition for EWSB $V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$

- The SM does not explain how the Higgs mass parameter and self-coupling are determined
- Furthermore, once you include the effects of the Higgs coupling to fermions (especially to the top quark), the Higgs potential shows an instability



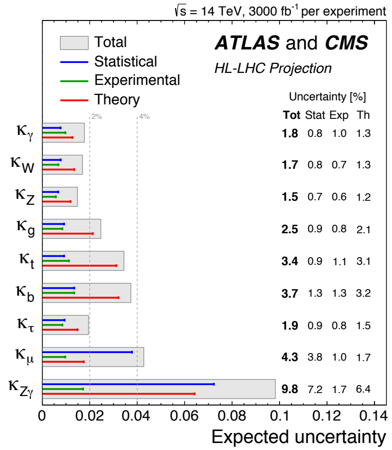


What is behind the EWSB mechanism?

Radiative Breaking (like in Supersymmetry) or Compositeness

LHC Precision Higgs Measurements and Di-Higgs Production

High Luminosity LHC Projections (2038)

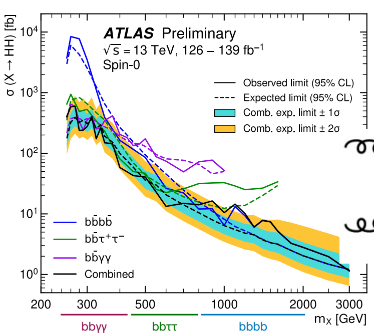


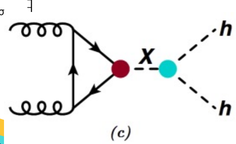
A powerful tool to explore new physics needed to explain many particle physics topics

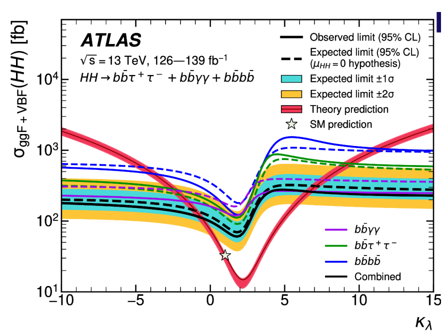
This could include **other Higgs bosons**, **new particles, new forces, and connections with invisible sectors**

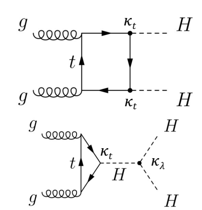
with 30 times more data at slightly higher energies

- Enhanced di-Higgs production being probed at LHC

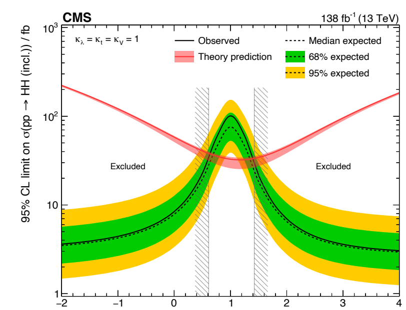


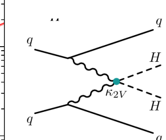






+ higher order EFT operators





- can shed light on understanding the Higgs potential and the EW phase transition
- may relate to the flavor structure and new sources of light-fermions-Higgs couplings

What can entanglement tell us about the possibility and properties of an extended Higgs sector, e.g. 2HDM?

- In this talk I will use entanglement to explore the possibility that there are two SU(2) Higgs doublets Φ_a, a = 1,2
- Each Higgs doublet has a charged and a neutral field

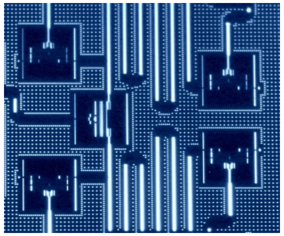
$$\Phi_a = (\Phi^+_{a=1,2}, \Phi^0_{a=1,2})$$

- I will treat the two charged components as defining one qubit, and the two neutral components as describing a second qubit
- I will look at the behavior of the S-Matrix when performing a tree level charged-neutral Higgs boson scattering

$$\mathcal{S}(\Phi^+_a, \Phi^0_b \to \Phi^+_c, \Phi^0_d)$$

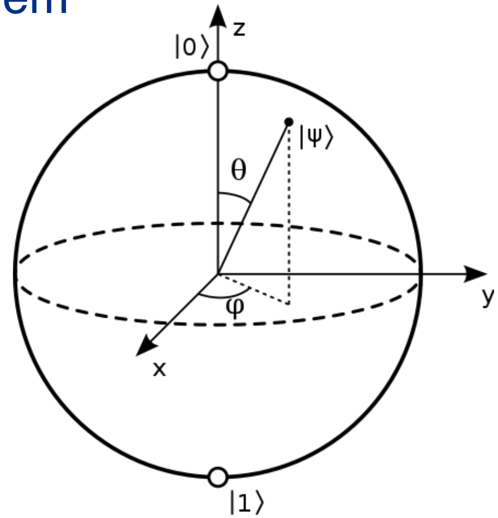
- I will relate the suppression of entanglement to an emergent approximate SO(8) symmetry in this 2HDM and explore its connection with LHC data

Qubits



- The simplest nontrivial quantum system (2-dim. Hilbert space)
- Call the two basis states $|0\rangle$ and $|1\rangle$
- A qubit state is in general a **quantum superposition** of them

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \mathrm{e}^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

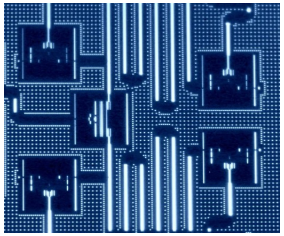


- You can think of the two angles as latitude and longitude of the surface of a sphere, called the **Bloch sphere**

- You can represent the basis states by 2-dim vectors:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Qubits



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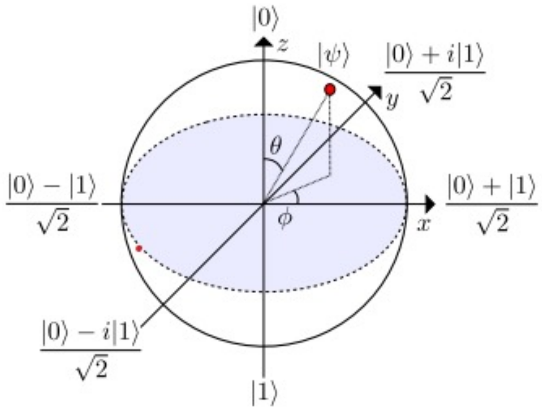


Quantum gates

- Starting from any single qubit state you can apply a unitary gate operation that rotates you to some other state on the surface of the Bloch sphere
- For example, the one qubit **Hadamard gate H** takes the l0> state to the l+> state, and the l1> state to the l-> state, where

$$|+\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

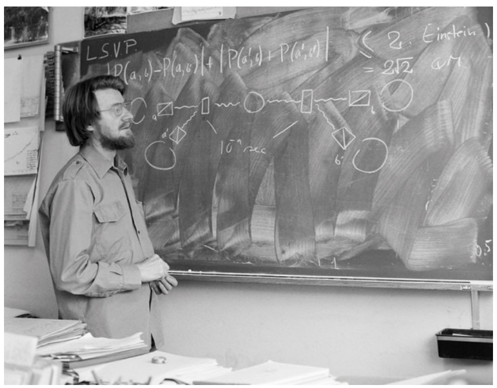
$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\hat{H}|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

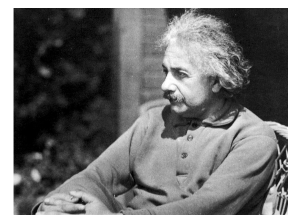
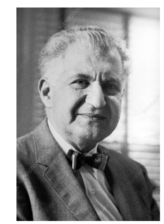
$$\hat{H}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Quantum entanglement

- A quantum state of two or more qubits can be **entangled**, meaning that the state cannot be written as a tensor product of single qubit states
- For two qubits a basis for entangled states is the four Bell states:



$$
\begin{aligned}
|\beta_{00}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\
|\beta_{01}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\
|\beta_{10}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\
|\beta_{11}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)
\end{aligned}
$$


Each of these states is maximally entangled, meaning that each qubit is sharing 100% of the information about its quantum state with the other qubit

Concurrence as Entanglement Measure of two qubits

We can write an arbitrary two qubit state as a 4-vector in the computational basis $|00\rangle,\ |01\rangle,\ |10\rangle,\ |11\rangle$

$$|\psi\rangle = \begin{pmatrix} c_{11} \\ c_{12} \\ c_{21} \\ c_{22} \end{pmatrix}$$

such that the density matrix is

$$\rho = \begin{pmatrix} |c_{11}|^2 & c_{11}^* c_{12} & c_{11}^* c_{21} & c_{11}^* c_{22} \\ c_{12}^* c_{11} & |c_{12}|^2 & c_{12}^* c_{21} & c_{12}^* c_{22} \\ c_{21}^* c_{11} & c_{21}^* c_{12} & |c_{21}|^2 & c_{21}^* c_{22} \\ c_{22}^* c_{11} & c_{22}^* c_{12} & c_{22}^* c_{21} & |c_{22}|^2 \end{pmatrix}$$

Doing the partial trace over the second qubit gives:

$$\rho_1 = \mathrm{Tr}_2 \rho = \begin{pmatrix} |c_{11}|^2 + |c_{12}|^2 & c_{11}^* c_{21} + c_{12}^* c_{22} \\ c_{21}^* c_{11} + c_{22}^* c_{12} & |c_{21}|^2 + |c_{22}|^2 \end{pmatrix}$$

From which we can compute $\mathrm{Tr}\rho_1^2$, which is an indicator of entanglement:

$$\mathrm{Tr}\rho_1^2 = 1 - 2|c_{11}c_{22} - c_{12}c_{21}|^2$$

If $\mathrm{Tr}\rho_1^2 < 1$ then ρ_1 is a mixed state and $|\psi\rangle$ is an entangled state

We can define a measure of entanglement of two qubits by the “concurrence” Δ

$$\Delta = 2|c_{11}c_{22} - c_{12}c_{21}| \qquad \mathrm{Tr}\rho_1^2 = 1 - \Delta^2/2$$

Like the von Neumann entropy $-\mathrm{Tr}\rho_1 \log_2 \rho_1$, the concurrence has the property that it vanishes for a tensor product state and equals 1 for a maximally entangled Bell state

Measure of Entanglement Power of a Unitary Operator

We are interested in the ability of a quantum operator U to generate entanglement

Its ”entanglement power” is defined averaging over all **direct product states** that U can act upon, averaging over each Bloch sphere

Zanardi, Phys. Rev. A 63, 040304 (2001).

$$E(U) = \overline{\Delta(U|\psi_A> \otimes |\psi_B>)}$$

Operators that can be written as product of single-qubit quantum gates V do not generate entanglement.

This defines an equivalent class among two qubit operators : $U \sim U'$ *if* $U = V_1 U' V_2$

Operators in the same equivalent class can have the same entanglement power

There are exactly **two distinct equivalence classes of unitaries that are minimally entangling**, i.e., for which the entanglement power vanishes:

Low, Mehen, Phys. Rev. D 104, 074014 (2021)

$$\mathbf{1}\,|ij\rangle = |ij\rangle$$

$$\mathrm{SWAP}\,|ij\rangle = |ji\rangle$$

$$\text{SWAP} = \frac{1}{2}(1 + \sigma\cdot\sigma) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sigma\cdot\sigma \equiv \sum_a \sigma^a \otimes \sigma^a$$

Minimal Entangler and Emergent Symmetry

Beane, Kaplan, Klco, Savage, arXiv:1812.03138

Simple example: low energy neutron-proton scattering

To see the relation between minimal entanglement and "accidental" symmetries of the system, consider low energy np scattering

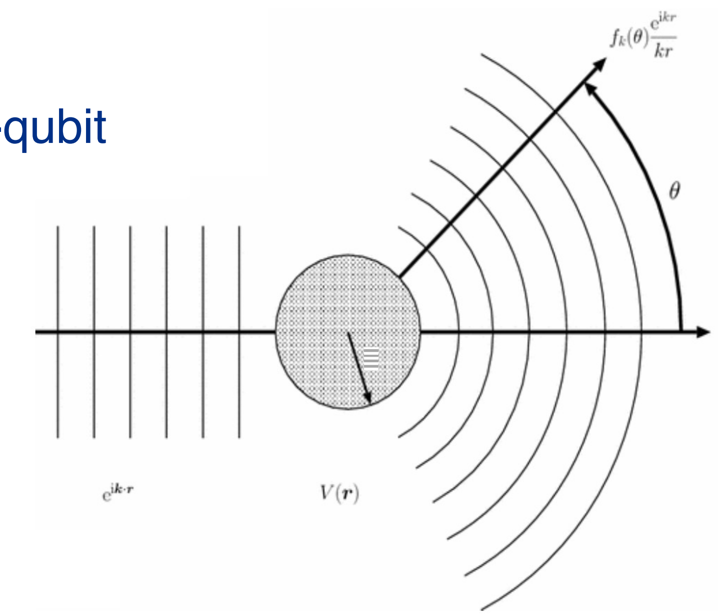
For low energies the s-wave scattering dominates, so the only angular momentum dependence is from the two spins, each of which can be represented as a qubit state

$$|\uparrow\rangle = |0\rangle \quad |\downarrow\rangle = |1\rangle$$

The total angular momentum operator for the two-qubit system can be written in terms of Pauli matrices:

$$J^a = \frac{1}{2}\sigma^a \otimes \mathbb{1} + \mathbb{1} \otimes \frac{1}{2}\sigma^a$$

The scattering has two spin channels, the singlet ${}^1S_0$ and the triplet ${}^3S_1$



Simple example: low energy neutron-proton scattering

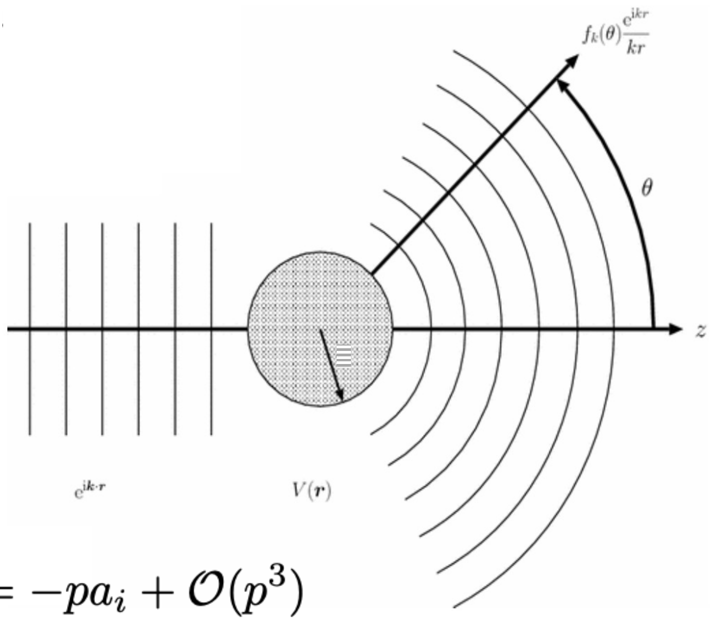
The S-matrix (restricting now to just s-wave scattering) is a unitary matrix in this 2-qubit space, and can be written in terms of phases and the projection operators $\boldsymbol{P_0}$, $\boldsymbol{P_1}$ onto the singlet $^1S_0$ and the triplet $^3S_1$ spin combinations, respectively:

$$\mathcal{S} = P_0\, \mathrm{e}^{i\delta_0} + P_1\, \mathrm{e}^{i\delta_1} \qquad \mathcal{S}^\dagger \mathcal{S} = 1 \qquad P_0^2 = P_0 \qquad P_1^2 = P_1 \qquad P_0 P_1 = 0 \qquad P_0 + P_1 = \mathbf{1}$$

$$P_0 = \frac{1}{4}(1 - \sigma\cdot\sigma) = \frac{1}{2}(1 - \mathrm{SWAP}) \qquad P_1 = \frac{1}{4}(3 + \sigma\cdot\sigma) = \frac{1}{2}(1 + \mathrm{SWAP})$$

The s-wave S-matrix can be written entirely in terms of minimal entanglers:

$$\mathcal{S} = \frac{1}{2}\left(\mathrm{e}^{2i\delta_1} + \mathrm{e}^{2i\delta_0}\right)\mathbb{1} + \frac{1}{2}\left(\mathrm{e}^{2i\delta_1} - \mathrm{e}^{2i\delta_0}\right)\mathrm{SWAP}$$



For sufficiently low momenta the phases δ_0, δ_1, can be written in terms of two scattering lengths a_0, a_1 and two "effective ranges" $r_0^{(0)}$, $r_0^{(1)}$

$$p\cot\delta_i(p) = -\frac{1}{a_i} + \frac{1}{2} r_0^{(i)} p^2 + \ldots \quad \rightarrow \quad \tan\delta_i(p) = -p a_i + \mathcal{O}(p^3)$$

The SWAP operator and Schrodinger symmetry

Now we can ask under what conditions will the S-matrix reduce to purely one or the other minimal entangler?

This analysis will connect the minimal entanglers with emergent symmetries

$$\mathcal{S} = \frac{1}{2}\left(e^{2i\delta_1} + e^{2i\delta_0}\right)\mathbb{1} + \frac{1}{2}\left(e^{2i\delta_1} - e^{2i\delta_0}\right)\mathrm{SWAP} \qquad \tan\delta_i(p) = -pa_i + \mathcal{O}(p^3)$$

To get just the SWAP operator, we would need (in the sense of RG flow to the infrared) $\delta_0 = 0;\ \delta_1 = \pm\frac{\pi}{2}$ or vice-versa

This corresponds to the scattering length vanishing in one channel and diverging in the other

This implies an approximate scale invariance, signaling an emergent “Schrodinger symmetry”, a nonrelativistic version of conformal symmetry

T. Mehen, I. Stewart, M. Wise, hep-th/9910025

The Identity operator and Wigner SU(4)

E. Wigner, Phys. Rev. 51, (1937) 106

$$\mathcal{S} = \frac{1}{2}\left(\mathrm{e}^{2i\delta_1} + \mathrm{e}^{2i\delta_0}\right)\mathbb{1} + \frac{1}{2}\left(\mathrm{e}^{2i\delta_1} - \mathrm{e}^{2i\delta_0}\right)\mathrm{SWAP} \qquad \tan\delta_i(p) = -pa_i + \mathcal{O}(p^3)$$

To get just the identity operator, we would need $\delta_0(p) = \delta_1(p)$

i.e., the two spin channels have the same phase shift

This implies the SU(4) Wigner symmetry, which we can see by writing the four n/p spin states as a 4-plet:

$$N = \begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$

The leading interaction terms in the effective Lagrangian are:

$$\mathcal{L}_{\mathrm{LO}} = -\frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger\sigma N)\cdot(N^\dagger\sigma N)$$

S. Weinberg, Phys. Lett. B251, (1990) 288

So $\delta_0 \simeq \delta_1$ implies $C_T << C_S$, thus an approximate SU(4)

T. Mehen, I. Stewart, M. Wise, hep-ph/9902370

What about actual QCD?

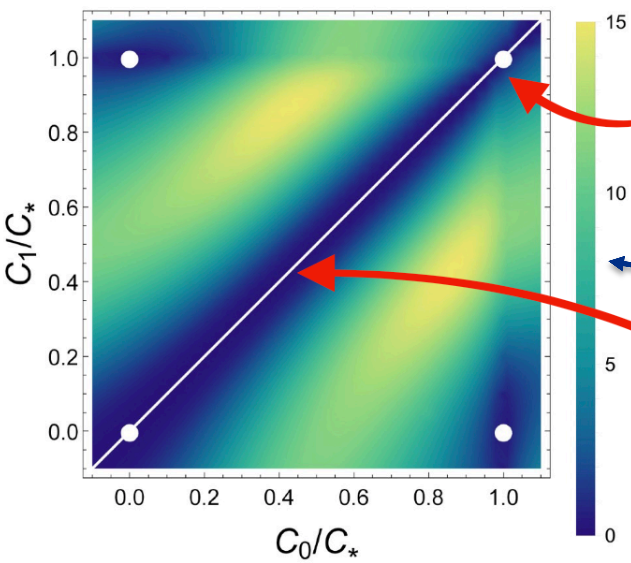
The actual data from low energy np scattering gives

$a_0 = -23.7\ \mathrm{fm} \quad a_1 = 5.4\ \mathrm{fm}$ both much larger than $1/m_\pi = 1.4\ \mathrm{fm}$

And fitting data to simple square wells for the nucleon-nucleon potential gives:

$C_T/C_S \simeq 0.05$ D.B. Kaplan, M. Savage, hep-ph/9509371

So low energy QCD has $\delta_0 \simeq \delta_1 \simeq \dfrac{\pi}{2}$ (corresponding to the Identity operator) and exhibits both the approximate Wigner SU(4) and Schrodinger symmetries



Conformal fixed points
(zero or infinite scattering lengths, $\delta_i = 0, \pi/2$)

$^1S_0: \quad \bar{C}_0 = (C_S - 3C_T)$

$^3S_1: \quad \bar{C}_1 = (C_S + C_T)$

Density plot of the entanglement power E(S) of the S-matrix

$\mathrm{SU(4)}_{\mathrm{Wigner}}$ symmetry line
($\delta_0 = \delta_1$)

Beane, Kaplan, Klco, Savage, arXiv:1812.03138

Exploring an extended Higgs sector via entanglement suppression and symmetry enhancement

- Based on observations in low-energy QCD of intriguing connections between the emergent global symmetries and suppression of entanglement in non-relativistic scattering of spin-1/2 baryons, we explore similar concepts in a two Higgs Doublet Model
- The 2HDM has two identical complex, hypercharge-one, $SU(2)_L$ doublet scalar fields

Branco et al., Phys. Rept. 516 (2012)

Pilaftsis, Phys. Lett.B 706 (2012)

2HDM talks at this conference

$$\Phi_i(x) \equiv (\Phi_i^+(x)\,,\ \Phi_i^0(x))^{\mathsf{T}}$$

$i \in \{1, 2\}$

$$\Phi_i = \begin{pmatrix} h_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i\ a_i) \end{pmatrix}$$

The potential reads:

$$\begin{aligned}\mathcal{V} = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}] \\ & + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right\} .\end{aligned}$$

After minimization of the potential: $\langle\Phi_i\rangle = v_i/\sqrt{2},$ with $v^2 = v_1^2 + v_2^2 = 246\ \mathrm{GeV}^2$

$$\tan\beta = v_2/v_1 \qquad 0 \le \beta \le \pi/2, \qquad c_\beta \equiv \cos\beta = v_1/v \qquad s_\beta \equiv \sin\beta = v_2/v.$$

The Higgs basis :

Davidson, Haber, Phys. Rev. D 72 (2005) [0504050]

It is convenient to introduce the Higgs basis fields H_1 and H_2: through a U(2) rotation

$$H_1 = c_\beta \Phi_1 + s_\beta \Phi_2 \quad H_2 = -s_\beta \Phi_1 + c_\beta \Phi_2$$ such that $\langle H_1^0 \rangle = v/\sqrt{2}$ and $\langle H_2^0 \rangle = 0.$

with $$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(H_{SM} + v + iG^0) \end{pmatrix} \qquad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H_{NSM} + iA^0) \end{pmatrix}$$

The potential is physically equivalent; parameters not invariant under the U(2) rotation

$$\mathcal{V} = Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2)$$
$$+ Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 \; + \left[Z_6 H_1^\dagger H_1 + Z_7 H_2^\dagger H_2 \right] H_1^\dagger H_2 + \text{h.c.} \right\}.$$

- Minimization of the scalar potential fixes

$$Y_1 = -Z_1 v_2 / 2$$
$$Y_3 = -Z_6 v^2 / 2$$

The Higgs basis (cont'd)

- It defines the masses in terms of the Zi's

$$m_{H^\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 v^2$$

For simplicity we assume CP conservation and λi as real parameters, although our results can be easily generalized to the CP-violating case. In this case we have:

$$m_A^2 = m_{H^\pm}^2 + \tfrac{1}{2}(Z_4 - Z_5)v^2$$

The masses of the CP-even scalars h and H (where $m_h < m_H$) are obtained by diagonalizing a 2 × 2 squared mass matrix

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix}$$

Recall we are in the Higgs basis state $\{H_{SM}, H_{NSM}\}$

one can express the real neutral Higgs field components in terms of the CP–even mass eigenstates, h and H, with α the angle of the U(2) rotation that diagonalizes the mass matrix in the original (weak eigenstates) basis Φ_i

$$H_{SM} = \sin(\beta - \alpha) h + \cos(\beta - \alpha) H$$
$$H_{NSM} = \cos(\beta - \alpha) h - \sin(\beta - \alpha) H$$

The Alignment limit

Gunion, Haber, Phys. Rev. D 67 (2003), [0207010].
M.C., Low, Shah, Wagner, JHEP 04 (2014) 015, [1310.2248]

Define the **Alignment Limit** as the condition for which one scalar mass eigenstate aligns with the direction of the scalar vev in field space, hence the SM Higgs field

Approximate alignment occurs for:

1) $m_A^2 \gg (Z_1 - Z_5)v^2$ → This is the decoupling limit, where h is SM-like

$$m_H^2 \sim m_A^2 \sim m_{H^\pm}^2 \gg m_h^2 \simeq Z_1 v^2$$

2) $|Z_6| \ll 1$ → Alignment without decoupling

h is SM-like if $m_A^2 + (Z_5 - Z_1)v^2 > 0$ (Otherwise, H is SM-like)

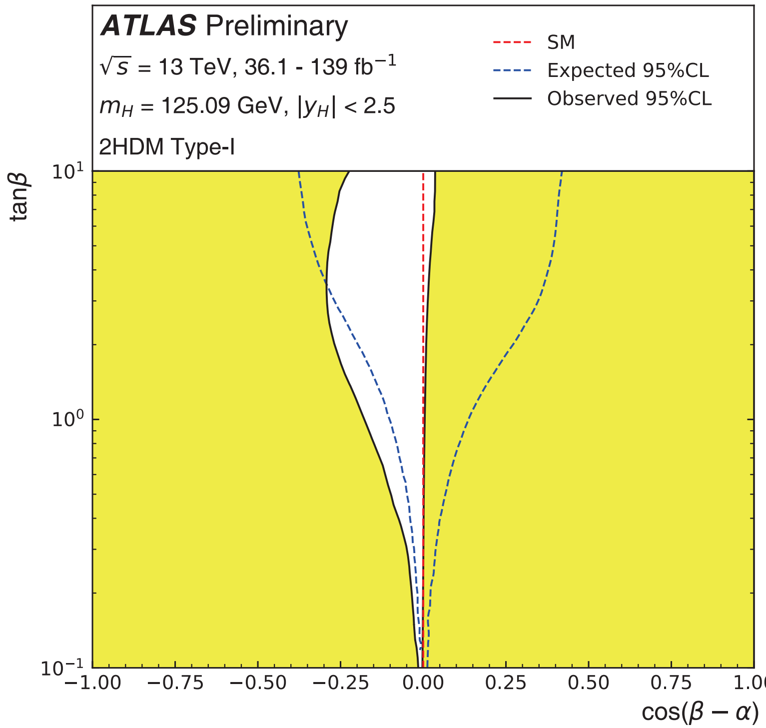
- h is SM-like if $|c_{\beta-\alpha}| \ll 1$

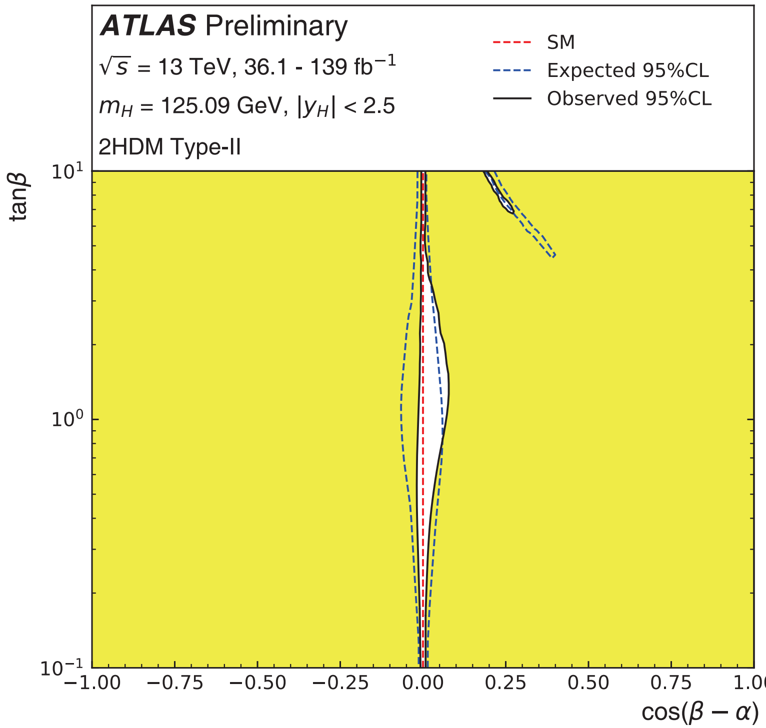
alignment with or without decoupling, depending on the magnitude of mA

$$|c_{\beta-\alpha}| = \frac{|Z_6|v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6|v^2}{m_H^2 - m_h^2} \ll 1$$

We will keep in mind that the LHC favors a SM-like Higgs boson

LHC constraints on Higgs alignment in the 2HDM





Regions excluded by fits to the measured rates of the productions and decay of the Higgs boson (assumed to be h of the 2HDM). Contours at 95% CL. ATLAS-CONF-2021-053

S-Matrix and perturbative entanglement suppression

Consider the information theoretic properties of the 2HDM Φ_a, a = 1,2 :
The two flavors, one for each $SU(2)_L$ doublet, serve as a quantum system – a qubit.
The scattering of the two isospin components of the two doublets is a two-qubit system

Two flavors of $SU(2)_L$ doublets $$\Phi_a = (\underbrace{\Phi^+_{a=1,2}}_{p_{\uparrow,\downarrow}}, \underbrace{\Phi^0_{a=1,2}}_{n_{\uparrow,\downarrow}})$$

Using the S-matrix – a Unitary Operator - as a logic quantum gate

we want to study suppression of flavor entanglement in the 2-to-2 scattering

$$\Phi_a^+ \Phi_b^0 \to \Phi_c^+ \Phi_d^0$$

Recall: $S = 1 + i\,T$ ➔ $\langle \Phi_c \Phi_d |\, iT \,| \Phi_a \Phi_b \rangle = i(2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_c) M_{ab,cd}$

$M_{ab,cd}$ are the scattering amplitudes one typically computes in perturbation theory.

Notice that the T matrix and the amplitude itself is not Unitary, so cannot be interpreted as a quantum gate

Unitarity of S requires: $i(T^\dagger - T) = TT^\dagger$

if T ~ O(λ) in perturbation, $T^\dagger T$ ~ O(λ^2) is higher order in the coupling constants; perturbative unitarity of the S-matrix is fulfilled at O(λ).

S-Matrix as an Identity quantum gate

We want to study suppression of flavor entanglement in this 2-to-2 scattering

Condition for entanglement suppression

Low and Mehen [2104.10835]

$$\begin{aligned} S &\sim [\mathbf{1}] &\Leftrightarrow\quad & T \sim [\mathbf{1}]\ , \\ S &\sim [\mathrm{SWAP}] &\Leftrightarrow\quad & T \sim i\,([\mathbf{1}] + [\mathrm{SWAP}]) \end{aligned}$$

Perturbatively, the S-Matrix can only be in **[1]**

[SWAP] requires a tree-level cancellation between the T-matrix, computed in perturbation theory, against the non-interacting part of the S-matrix. This only achievable in strongly coupled theory

Mehen, Stewart, Wise [9910025] Nishida and Son [0706.3746]

Let's work out the conditions on the amplitude such that S~ **[1]**

Starting from an initial product state in flavor space

$$|\Phi_a \Phi_b\rangle = (\kappa|1\rangle + \epsilon|2\rangle) \otimes (\gamma|1\rangle + \delta|2\rangle),$$

where $|\kappa|^2 + |\epsilon|^2 = |\gamma|^2 + |\delta|^2 = 1.$

After tracing over momentum variables, the outgoing state has the flavor structure

$$|\Phi_c \Phi_d\rangle = (\delta_{ac}\delta_{bd} + iM_{ab,cd})|\Phi_a\rangle \otimes |\Phi_b\rangle$$

Minimal Entanglement Conditions on the Amplitude

MC, Low, Wagner, Xiao [2307.08112]

Expanding the final state |ij>

$$|\Phi_c\Phi_d\rangle = (\delta_{ac}\delta_{bd} + iM_{ab,cd})|\Phi_a\rangle \otimes |\Phi_b\rangle = c_{ij}|ij\rangle$$

$$\begin{aligned}
c_{11} &= (1 + iM_{11,11})\,\kappa\gamma + iM_{12,11}\ \kappa\delta + iM_{21,11}\ \epsilon\gamma + iM_{22,11}\ \epsilon\delta \\
c_{12} &= iM_{11,12}\ \kappa\gamma + (1 + iM_{12,12})\,\kappa\delta + iM_{21,12}\ \epsilon\gamma + iM_{22,12}\ \epsilon\delta \\
c_{21} &= iM_{11,21}\ \kappa\gamma + iM_{12,21}\ \kappa\delta + (1 + iM_{21,21})\,\epsilon\gamma + iM_{22,21}\ \epsilon\delta \\
c_{22} &= iM_{11,22}\ \kappa\gamma + iM_{12,22}\ \kappa\delta + iM_{21,22}\ \epsilon\gamma + (1 + iM_{22,22})\,\epsilon\delta
\end{aligned}$$

We need to demand the concurrence $\Delta(|\Phi_c\Phi_d\rangle) = c_{11}c_{22} - c_{12}c_{21}$ to vanish

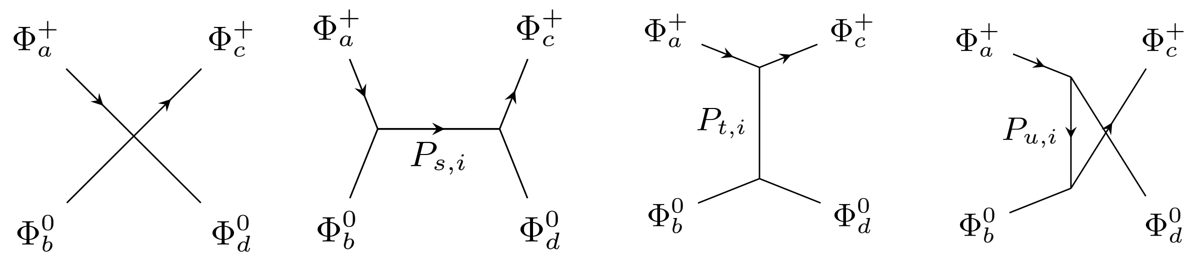
$$\begin{aligned}
\Delta(|\Phi_c\Phi_d\rangle) = {} & i\kappa\epsilon\gamma\delta(M_{11,11} - M_{12,12} - M_{21,21} + M_{22,22}) \\
& + i\kappa\epsilon(\gamma^2 - \delta^2)(M_{21,22} - M_{11,12}) + i(\kappa^2 - \epsilon^2)\gamma\delta(M_{12,22} - M_{11,21}) \\
& - iM_{12,21}\ \kappa^2\delta^2 - iM_{21,12}\ \epsilon^2\gamma^2 + iM_{11,22}\ \kappa^2\gamma^2 + iM_{22,11}\ \epsilon^2\delta^2 + O((M_{ab,cd})^2)
\end{aligned}$$

Since κ, ε, γ, δ are arbitrary, zero concurrence, for the S-Matrix to be in the equivalent class of the identity, implies

$$\boxed{\begin{aligned}
& M_{11,11} + M_{22,22} = M_{12,12} + M_{21,21}\ , \\
& M_{11,22} = M_{12,21} = M_{21,12} = M_{22,11} = 0 \\
& M_{11,12} = M_{21,22}\ , \quad M_{11,21} = M_{12,22}\ .
\end{aligned}}$$

we are working to first order in perturbation theory, keeping only linear terms in $M_{ab,cd}$

Amplitude for $\Phi_a^+\Phi_b^0 \to \Phi_c^+\Phi_d^0$ in the broken phase

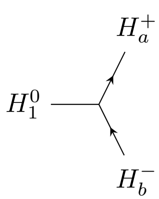


(a) (b) (c) (d)

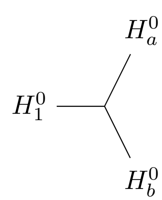
- Tree level contributions
- Gauge coupling turned off
- Yukawa couplings do not contribute at this order

We shall perform the calculation in the Higgs basis: such U(2) rotation - no mixing between Φ^0 and Φ^+ - corresponds to a single-qubit operation and does not change the entanglement power of the S-Matrix

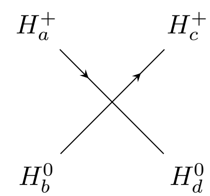
From the scalar potential the Feynman rules follow



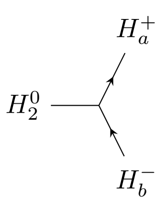
$$= \frac{iv}{\sqrt{2}}\begin{pmatrix} Z_1 & Z_6 \\ Z_6 & Z_3 \end{pmatrix}_{ab}$$



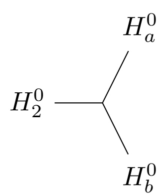
$$= \frac{iv}{\sqrt{2}}\begin{pmatrix} Z_1 & 2Z_6 \\ 2Z_6 & Z_5 \end{pmatrix}_{ab}$$



$$= i\begin{pmatrix} Z_1 & Z_6 & Z_6 & Z_5 \\ Z_6 & Z_3 & Z_4 & Z_7 \\ Z_6 & Z_4 & Z_3 & Z_7 \\ Z_5 & Z_7 & Z_7 & Z_2 \end{pmatrix}_{ab,cd}$$



$$= \frac{iv}{\sqrt{2}}\begin{pmatrix} Z_6 & Z_5 \\ Z_4 & Z_7 \end{pmatrix}_{ab}$$



$$= \frac{iv}{\sqrt{2}}\begin{pmatrix} Z_6 & Z_3+Z_4 \\ Z_3+Z_4 & Z_7 \end{pmatrix}_{ab}$$

Recall: $H_1^+ = G^+ \quad H_2^+ = H^+$

$$H_1^0 = \frac{H_{SM} + iG^0}{\sqrt{2}} \qquad H_2^0 = \frac{H_{NSM} + iA^0}{\sqrt{2}}$$

S-Matrix Minimal Entanglement and Emerging Symmetry

Full amplitude:

$$iM_{ab,cd} = iM^0_{ab,cd} - \frac{v^2}{2}\sum_i \sum_{r=s,t,u} M^r_i{}_{ab,cd}\, P_{r,i}$$

4-point contact interaction

$$M^s_i{}_{ab,cd} = M_{abi}M^*_{cdi}\ , \quad M^u_i{}_{ab,cd} = M_{adi}M^*_{cbi}$$

$$M^t_i{}_{ab,cd} = \sum_{j,k} \mathcal{R}_{ij} M_{ajc} (\mathcal{R}_{ik} M_{dkb,0})^* + \text{h.c.}\ ,$$

rotation matrix in the neutral sector

$$P_{t,i} = i/(t - m^2_{0,i}) \text{ and } P_{r,i} = i/(r - m^2_{+,i}),\ \ r = s, u.$$

$$m_{0,i} = \{m_h, m_H, 0, m_A\} \text{ and } m_{+,i} = \{m_{H^\pm}, 0\}$$

Every term should satisfy the conditions:

$$\begin{aligned} &M_{11,11} + M_{22,22} = M_{12,12} + M_{21,21}\ , \\ &M_{11,22} = M_{12,21} = M_{21,12} = M_{22,11} = 0 \\ &M_{11,12} = M_{21,22}\ , \quad M_{11,21} = M_{12,22}\ . \end{aligned}$$

$\rightarrow$

$$\begin{aligned} &Z_1 = Z_2 = Z_3 \equiv Z \\ &Z_i = 0\ , \quad i \neq 1,2,3 \\ &Y_1 = Y_2 \equiv Y = -Zv^2/2 \\ &Y_3 = 0 \end{aligned}$$

$\rightarrow$ $Z_6 = 0$

Alignment

MC, Low, Wagner, Xiao [2307.08112]

This leads to the scalar potential with maximal SO(8) symmetry:

Also noted in Dev, Pilaftsis, JHEP12 (2014)

$$\mathcal{V} = Y(H_1^\dagger H_1 + H_2^\dagger H_2) + \frac{Z}{2}(H_1^\dagger H_1 + H_2^\dagger H_2)^2$$

$$= \frac{Z}{2}\left(\sum_{i=1,2} |H_i^0|^2 + G^+G^- + H^+H^- - \frac{v^2}{2}\right)^2$$

Acting on the 8 real components of the two doublets

This is then broken spontaneously to SO(7) by the Higgs vev

Emergent Symmetry and LHC bounds on the Higgs spectra

Entanglement suppression for a 2HDM implies

- Existence of a **SM-like** Higgs boson with mass $m_h^2 = -2Y = Zv^2$ = 125 GeV
- Three massless Higgs bosons, incompatible with LHC data

$$m_{H^\pm}^2 = Y_2 + \tfrac{1}{2}Z_3 v^2 \qquad m_A^2 = m_{H^\pm}^2 + \tfrac{1}{2}(Z_4 - Z_5)v^2 \qquad m_H^2 = m_A^2 + Z_5 v^2$$

- However, gauge and Yukawa interaction will typically break the enhanced SO(8) explicitly

$$\mathcal{L} \supset Y_u \Phi_1 \bar{Q} u + Y_d \Phi_2^\dagger \bar{d} Q$$

- Still, we need to lift the zero mass of the charged Higgs boson by adding a soft mass term in the potential through Y_2

$$m_{H^\pm}^2 = Y_2 + \tfrac{1}{2}Z_3 v^2$$

This will equally lift the mass of the two non-SM CP-even Higgs bosons as well

Final Thoughts

Information-theoretic properties may provide insights on the origin of physical principles.

In the two Higgs doublet model: a prototypical example for BSM theories with extended Higgs sectors, the perturbative S-matrix, in the Identity class, suppresses flavor entanglement in

$$\Phi_a^+, \Phi_b^0 \to \Phi_c^+, \Phi_d^0$$

How does the entanglement behave:

When the gauge and Yukawa couplings are turned on?

When the emerging symmetry is softly broken in a realistic model?

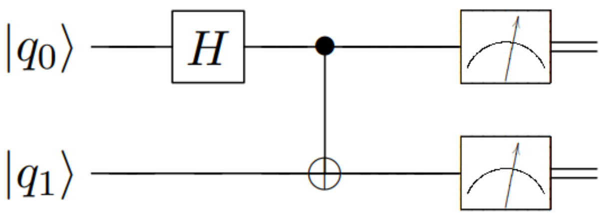
More general amplitudes and models (e.g. 3HDM with 2 qutrits) need to be investigated to explore possible entanglement suppression that may lead to emergent enhanced symmetries

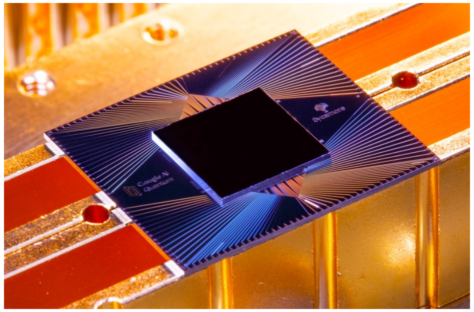
Thank You !

Extra Slides

Creating entangled states in a quantum circuit

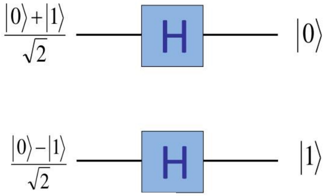
- Starting with a 2-qubit state in the computational basis, you can create a Bell state by applying the Hadamard gate and then a **CNOT**, which is a 2-qubit entangling gate





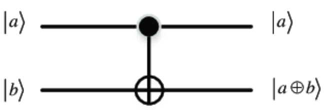
Recall Hadamard transforms the computational basis l0> and l1> states into the Hadamard basis states l+> and l->

Comment: interestingly, the Hadamard is its own inverse

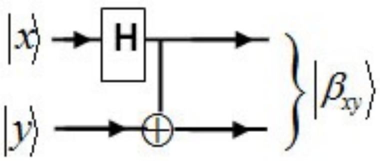


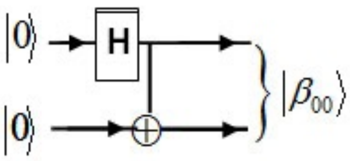
The CNOT gate act on two qubits

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



How it works:





$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

KAK decomposition of unitary operators on two qubits

Now we want to classify unitary operators that act on two qubits, according to how much entanglement they produce

Unitary operators on 2-qubit states are elements of SU(4), which has 15 hermitian generators

We will use the "KAK" decomposition of a general element of SU(4), first derived by Cartan to minimize the number of parameters relevant for entanglement

$$U = \mathrm{e}^{i\alpha_a^1 \sigma^a \otimes \mathbb{1}} \mathrm{e}^{i\alpha_a^2 \mathbb{1} \otimes \sigma^a} U_e \, \mathrm{e}^{i\alpha_a^3 \sigma^a \otimes \mathbb{1}} \mathrm{e}^{i\alpha_a^4 \mathbb{1} \otimes \sigma^a} \quad \text{where} \quad U_e = \mathrm{e}^{i\beta_a \sigma^a \otimes \sigma^a}$$

and where the "a" index runs over 3 values, and the σ^a are the Pauli matrices

The 15 real parameters describing an arbitrary unitary are $\alpha_a^1, \alpha_a^2, \alpha_a^3, \alpha_a^4, \beta_a$

but only 3 are relevant

2-qubit entangling operators

$$U = \mathrm{e}^{i\alpha_a^1 \sigma^a \otimes \mathbb{1}} \mathrm{e}^{i\alpha_a^2 \mathbb{1} \otimes \sigma^a} U_e \, \mathrm{e}^{i\alpha_a^3 \sigma^a \otimes \mathbb{1}} \mathrm{e}^{i\alpha_a^4 \mathbb{1} \otimes \sigma^a}$$

$$U_e = \mathrm{e}^{i\beta_a \sigma^a \otimes \sigma^a} \quad = \mathrm{e}^{i\beta_x X \otimes X + i\beta_y Y \otimes Y + i\beta_z Z \otimes Z}$$

Obviously only U_e can produce entanglement, so we can classify all 2-qubit "entanglers" by just looking at U_e, parametrized by $\beta_x, \beta_y, \beta_z$

First, one can readily see that the Bell states are eigenstates of U_e

$$U_e |\psi^i_{\text{Bell}}\rangle = \lambda_i |\psi^i_{\text{Bell}}\rangle = \mathrm{e}^{i\phi_i} |\psi^i_{\text{Bell}}\rangle$$

$$|\psi^1_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \qquad \phi_1 = \beta_x - \beta_y + \beta_z$$

$$|\psi^2_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \qquad \phi_2 = \beta_x + \beta_y - \beta_z$$

$$|\psi^3_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \qquad \phi_3 = -\beta_x - \beta_y - \beta_z$$

$$|\psi^4_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \qquad \phi_4 = -\beta_x + \beta_y + \beta_z$$

Entanglement power of operators on two-qubit states

Generically applying a unitary U_e to a two–qubit tensor product state $|\psi_A\rangle \otimes |\psi_B\rangle$ will produce an entangled state, but not always, and the amount of entanglement depends on the state you are acting on

We can define the “entanglement power” of a unitary by acting on all possible two-qubit tensor product states, and averaging the resulting value of the concurrence over the two Bloch spheres:

$$\mathcal{E}(U_e(\beta_a)) \equiv \overline{\Delta(U_e\,|\psi_A\rangle \otimes |\psi_B\rangle)}$$

Introducing more compact notation for the qubit states

$$|\psi_A\rangle = \cos\frac{\theta_A}{2}|0\rangle_A + \mathrm{e}^{i\phi_A}\sin\frac{\theta_A}{2}|1\rangle_A \quad = \kappa|0\rangle_A + \epsilon|1\rangle_A$$

$$|\psi_B\rangle = \cos\frac{\theta_B}{2}|0\rangle_B + \mathrm{e}^{i\phi_B}\sin\frac{\theta_B}{2}|1\rangle_B \quad = \gamma|0\rangle_B + \delta|1\rangle_B$$

$$|\psi_A\rangle \otimes |\psi_B\rangle = \begin{pmatrix} \kappa\gamma \\ \kappa\delta \\ \epsilon\gamma \\ \epsilon\delta \end{pmatrix}$$

Entanglement power of operators on two-qubit states

$$|\psi_A\rangle = \cos\frac{\theta_A}{2}|0\rangle_A + e^{i\phi_A}\sin\frac{\theta_A}{2}|1\rangle_A \quad = \kappa|0\rangle_A + \epsilon|1\rangle_A$$

$$|\psi_B\rangle = \cos\frac{\theta_B}{2}|0\rangle_B + e^{i\phi_B}\sin\frac{\theta_B}{2}|1\rangle_B \quad = \gamma|0\rangle_B + \delta|1\rangle_B$$

$$|\psi_A\rangle \otimes |\psi_B\rangle = \begin{pmatrix} \kappa\gamma \\ \kappa\delta \\ \epsilon\gamma \\ \epsilon\delta \end{pmatrix}$$

The easy way to compute the concurrence is to first rotate to the Bell state basis, then apply U_e , then rotate back to the computational basis, giving the following state:

$$U_e(\beta_a)\,|\psi_A\rangle \otimes |\psi_B\rangle = \frac{1}{2}\begin{pmatrix} (\kappa\gamma+\epsilon\delta)\lambda_1 + (\kappa\gamma-\epsilon\delta)\lambda_4 \\ (\kappa\delta+\epsilon\gamma)\lambda_2 + (\kappa\delta-\epsilon\gamma)\lambda_3 \\ (\kappa\delta+\epsilon\gamma)\lambda_2 - (\kappa\delta-\epsilon\gamma)\lambda_3 \\ (\kappa\gamma+\epsilon\delta))\lambda_1 - (\kappa\gamma-\epsilon\delta)\lambda_4 \end{pmatrix}$$

Computing the concurrence from this state:

$$\Delta = \frac{1}{2}\left|\sum_{i=1}^{4} v_i\lambda_i^2\right| \qquad v_1 = (\kappa\gamma+\epsilon\delta)^2 \quad v_3 = (\kappa\delta-\epsilon\gamma)^2$$

$$v_2 = -(\kappa\delta+\epsilon\gamma)^2 \quad v_4 = -(\kappa\gamma-\epsilon\delta)^2$$

Entanglement power of operators on two-qubit states

$$|\psi_A\rangle = \cos\frac{\theta_A}{2}|0\rangle_A + e^{i\phi_A}\sin\frac{\theta_A}{2}|1\rangle_A \quad = \kappa|0\rangle_A + \epsilon|1\rangle_A$$

$$|\psi_B\rangle = \cos\frac{\theta_B}{2}|0\rangle_B + e^{i\phi_B}\sin\frac{\theta_B}{2}|1\rangle_B \quad = \gamma|0\rangle_B + \delta|1\rangle_B$$

$$\Delta = \frac{1}{2}\left|\sum_{i=1}^{4} v_i \lambda_i^2\right|$$

$$v_1 = (\kappa\gamma + \epsilon\delta)^2 \qquad v_3 = (\kappa\delta - \epsilon\gamma)^2$$

$$v_2 = -(\kappa\delta + \epsilon\gamma)^2 \qquad v_4 = -(\kappa\gamma - \epsilon\delta)^2$$

Integrating over the 4 angles that parametrize the two Bloch spheres gives:

$$\frac{1}{2}\overline{\Delta(U_e(\beta_a))}^2 = \frac{1}{18}\left[3 - \cos(4\beta_x)\cos(4\beta_y) - \cos(4\beta_y)\cos(4\beta_z) - \cos(4\beta_z)\cos(4\beta_x)\right]$$

This gives us the entanglement power for any distinct two-qubit entangler, modulo equivalence classes from applying additional single qubit unitaries that don't contribute to entanglement

Minimal entanglers

$$\frac{1}{2}\overline{\Delta(U_e(\beta_a))}^2 = \frac{1}{18}\left[3 - \cos(4\beta_x)\cos(4\beta_y) - \cos(4\beta_y)\cos(4\beta_z) - \cos(4\beta_z)\cos(4\beta_x)\right]$$

From this expression we see immediately that there are exactly **two distinct equivalence classes of unitaries that are minimally entangling**, i.e., for which the entanglement power vanishes:

$\beta_x = \beta_y = \beta_z = 0$ corresponding to $U_e = \mathbb{1}$, the 2-qubit **Identity** operator

$\beta_x = \beta_y = \beta_z = \pi/4$ corresponding to the 2-qubit **SWAP** operator

$$\text{SWAP} = \frac{1}{2}(1 + \sigma\cdot\sigma) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sigma\cdot\sigma \equiv \sum_a \sigma^a \otimes \sigma^a \qquad U_e = \mathrm{e}^{\frac{i\pi}{4}\sigma\cdot\sigma} = \mathrm{e}^{-\frac{i\pi}{4}}\mathrm{e}^{\frac{i\pi}{2}\mathrm{SWAP}} = \mathrm{e}^{\frac{i\pi}{4}}\mathrm{SWAP}$$