Effective action from M-theory on twisted connected sum G_2-manifolds

Thaisa Guio
with Albrecht Klemm, Hans Jockers & Hung-Yu Yeh

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Bonn-Cologne Graduate School of Physics and Astronomy

universitätbonn

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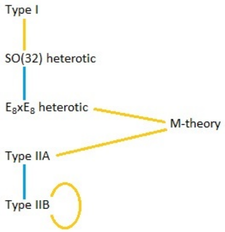
Why M-theory on twisted connected sum G_2?

- Type IIA & heterotic $E_8 \times E_8$ @ g_s large

$$\Downarrow$$

M-theory

Central position in string dualities!



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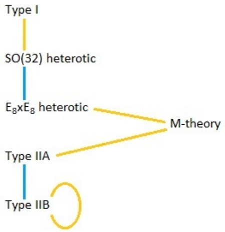

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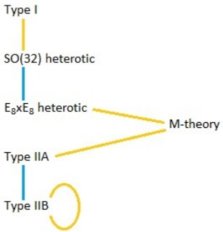

- M-theory on 7d G_2-manifolds $\Rightarrow$ **4d $\mathcal{N} = 1$ SUGRA**

- Twisted connected sum construction allows (Kovalev limit)

$\rightarrow$ singularities that lead to non-Abelian gauge symmetry enhancement
$\rightarrow$ interesting matter content
$\rightarrow$ transitions within $\mathcal{N} = 1$ effective theories

in context of compact G_2!

Outline

- Dimensional reduction of M-theory on a G_2
- Building the G_2-manifold: twisted connected sum
- The Kovalev limit
- M-theory on TCS G_2-manifolds
- Summary & Future Prospects

Dimensional reduction of M-theory on a G_2

G_2-manifold:

- a compact 7d manifold
- $\pi_1(G_2)$ finite
- Ricci-flat $g = g(\varphi)$
  φ is torsion-free 3-form G_2-structure

  [Joyce '00]

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- Dimensional reduction on $\mathcal{M}^{1,10} = \mathbb{R}^{1,3} \times G_2$
  - **Starting point**: 11d $\mathcal{N} = 1$ SUGRA – 3-form C, g_{11}, Ψ – [Cremmer *et al* '78]

Multiplicity	Massless 4d component fields bosonic fields	fermionic fields	Massless 4d $\mathcal{N}=1$ multiplets
1	metric $g_{\mu\nu}$	gravitino ψ_μ, ψ^*_μ	gravity multiplet
$i = 1, \ldots, b_3(Y)$	scalars (S^i, P^i)	spinors χ^i, χ^{*i}	chiral multiplets Φ^i
$I = 1, \ldots, b_2(Y)$	vectors A^I_μ	gauginos λ^I_α	vector multiplets V^I

$$K(\phi, \bar{\phi}) = -3\log\left(\frac{1}{7}\int_{G_2} \varphi \wedge *_{g(\varphi)}\varphi\right)$$

$$f_{IJ}(\phi) = 2V_{Y_0}\sum_k \kappa_{IJk}\phi^k$$

$$W(\phi^i) = \frac{1}{4V_{Y_0}}\sum_i \phi^i \int_{\Gamma^i} G \wedge (C + i\varphi)$$

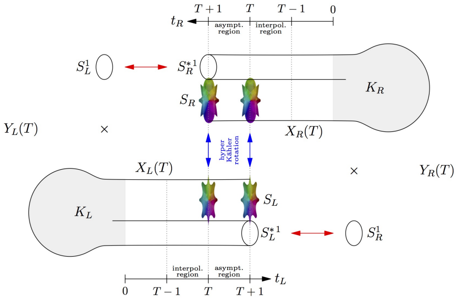
- **M-theory moduli space $\mathcal{M}_\mathbb{C}$**
  $\rightarrow$ locally parametrized by $b_3(G_2)$ complex chiral scalars $\phi^i = P^i + iS^i$.

Building the G_2-manifold: twisted connected sum

[Kovalev '01], [Corti *et al* '12]

1) Construct two non-compact **asymptotically cylindrical CY 3-folds** & product w/ circle (truncate at $T+1$)

$$Y_{L/R} = X_{L/R} \times S^1_{L/R}$$
$$= \Delta^{\mathrm{cyl}}_{L/R} \times S_{L/R} \times S^1_{L/R}$$



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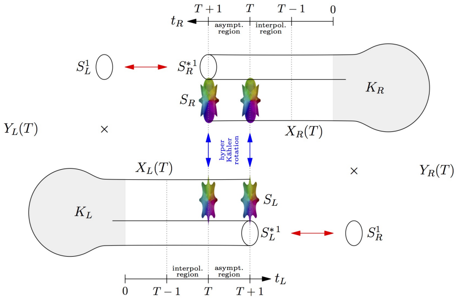
$$r^*\omega^I_R = \omega^J_L \ , \quad r^*\omega^J_R = \omega^I_L \ , \quad r^*\omega^K_R = -\omega^K_L$$



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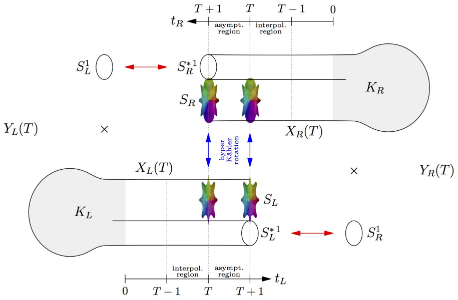

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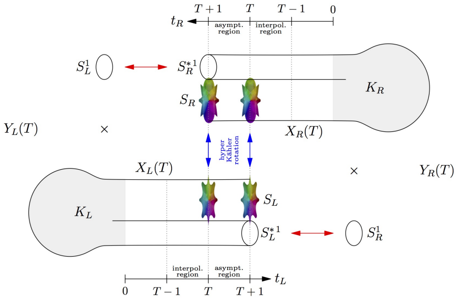

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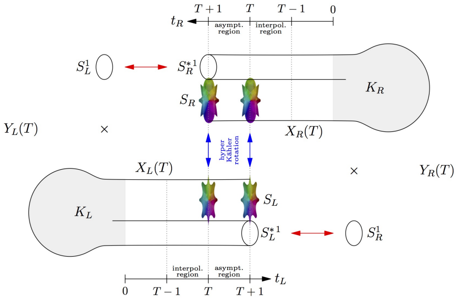

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G_2-manifold $Y = Y_L \cup Y_R = X_L \ \cup_{S^1_{L/R}} \ X_R$

- compact 7d ✓   • $\pi_1(Y)$ finite ✓
- torsion-free G_2-structure φ approaches TCS G_2-structure $\widetilde{\varphi}$ ✓

$$\varphi(T, R, \tilde{S}) = \widetilde{\varphi}(T, R, \tilde{S}) + \mathrm{d}\epsilon(T, R, \tilde{S}) \quad \text{with} \quad \left|\nabla^{k\in\mathbb{Z}>0}\epsilon(T, R, \tilde{S})\right| = O(e^{-RT})$$

Moduli: $T :=$ length of Δ^{cyl} $R :=$ overall volume modulus of $X_{L/R}$ $\tilde{S} :=$ remaining G_2 moduli

The Kovalev limit

- $X_{L/R}$ metrics are good approximations to G_2-metrics.

$$\varphi(T,R,\tilde{S}) = \widetilde{\varphi}(T,R,\tilde{S}) + \mathrm{d}\epsilon(T,R,\tilde{S}) \quad \text{with} \quad \left|\nabla^{k\in\mathbb{Z}>0}\epsilon(T,R,\tilde{S})\right| = O(e^{-RT})$$

4d Planck const. $\kappa_4 = \kappa_4(V_Y)$ remains const.

$+$

exponential correction terms suppressed

$$V_Y(T,R,\tilde{S}) \propto R^7 V_S \left(2T + \alpha(\tilde{S})\right) + O(e^{-RT})$$

$\Longleftrightarrow$

$\tilde{S}$ fixed

$+$

$T \to \infty$

- Analogous to **large volume limit** of CY compactifications of type II.

M-theory on TCS G_2-manifolds

Using Kovalev limit!

Universal $\mathcal{N} = 1$ chiral moduli multiplets ν and $\varkappa$

- From $H^3(Y)$:

$$H^3_1(Y) \Rightarrow \textbf{universal volume modulus } v$$
$$\text{K3 fibration} \Rightarrow \textbf{squashing modulus } b$$

- Torsion-free G_2-structure now written in terms of

$$\mathrm{Re}(\nu) = v \qquad \mathrm{Re}(\varkappa) = vb$$

$\varkappa :=$ **Kovalevton** as Kovalev limit is given by $\varkappa \to \infty$

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The Kähler potential (for const. $\tilde{S}$)

$$K(\nu, \bar{\nu}, \varkappa, \bar{\varkappa}) = -\log[(\nu + \bar{\nu})^4 (\varkappa + \bar{\varkappa})^3]$$

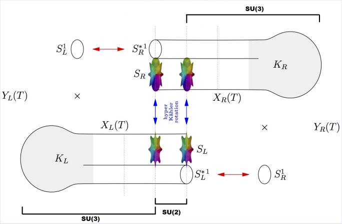
- No-scale inequality $g^{i\bar{\jmath}} \partial_i K \partial_{\bar{\jmath}} K - 3 \geq 0$ ✓

$\Rightarrow$ $\mathbf{\Lambda > 0}$ **(No AdS-SUSY vacua!)**

Gauge sectors on TCS G_2-manifolds

$$H^2(Y) \simeq \underbrace{(k_L \oplus k_R)}_{2\ \mathcal{N}=2} \oplus \underbrace{(N_L \cap N_R)}_{\mathcal{N}=4}$$

$$H^3(Y) \supset \overbrace{(k_L \oplus k_R)}$$



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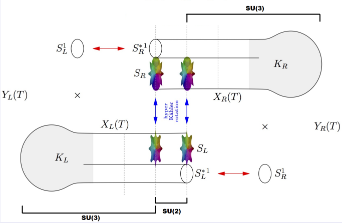

- $k_{L/R}$: local geometry $Y_{L/R}$

  $\mathcal{N} = 1$ vector + neutral $\mathcal{N} = 1$ chiral multiplets $\Rightarrow$ **4d $\mathcal{N} = 2$ gauge theory**

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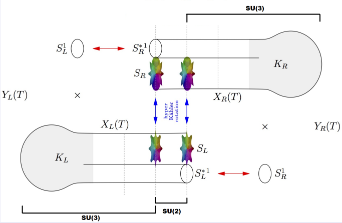

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- $N_L \cap N_R$: local geometry asymp. region $Y_L(T) \cap Y_R(T)$

  $\mathcal{N} = 1$ vector + 3 $\mathcal{N} = 1$ chiral multiplets $\Rightarrow$ **4d** $\mathcal{N} = 4$ **gauge theory**

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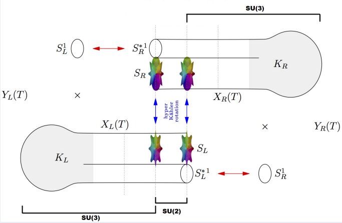

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- $N_L \cap N_R$: local geometry asymp. region $Y_L(T) \cap Y_R(T)$

  $\mathcal{N}=1$ vector + 3 $\mathcal{N}=1$ chiral multiplets $\Rightarrow$ **4d $\mathcal{N}=4$ gauge theory**

  $\theta_{L/R}$ of $S^1_L \times S^1_R \simeq T^2$ & smooth bump fct. $h(t) \in (0,1)$

$$\underbrace{\omega^{(2)}}_{\mathcal{N}=1\text{ vector}} \in (N_L \cap N_R) \text{ of K3} \Longrightarrow \underbrace{\omega^{(2)} \wedge h(t)d\theta_L\ ,\ \ \omega^{(2)} \wedge h(t)d\theta_R\ ,\ \ \omega^{(2)} \wedge h(t)dt}_{\substack{\text{3 scalar fields} \\ + \\ \text{3 scalar deformations of } K3 \text{ metric} \\ \Downarrow \\ 3\ \mathcal{N}=1 \text{ chiral multiplets}}}$$

Singularities on TCS G_2-manifolds

Real codimension 4 $\rightarrow$ non-Abelian gauge theory enhancement ✓

Real codimension 6 $\rightarrow$ non-trivial matter reps. ✓

Real codimension 7 $\rightarrow$ chiral matter ?

$S^1_{L/R}$ prevent real cod. 7 singularities

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Examples

Focus on left-side only.

$P :=$ 3-fold , $-K_P :=$ anticanonical div. of P, $\mathcal{C}_i :=$ curves given by sections of $-K_P$

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	Gauge group	$\mathcal{N} = 2$ hypermultiplet spectrum
1) $P = \mathbb{P}^3$:	$SU(4)$	$3\times$ **adj**
2) $P = \mathbb{P}^1 \times \mathbb{P}^2$:	$SU(3) \times SU(2) \times U(1)$	$2 \times (\mathbf{adj}, \mathbf{1})$, $(\mathbf{1}, \mathbf{adj}), 3 \times (\mathbf{3}, \mathbf{2})_{+1}$

Summary & Future Prospects

* **Twisted connected sum G_2-manifolds (Kovalev)**
  - Compatible gluing of a pair of asymptotically cylindrical Calabi-Yau 3-folds
* **Moduli spaces & 4d low-energy effective theory**
  - Kovalev limit: $X_{L/R}$ metric is good approximation to G_2-metric (Kähler potential, gauge sectors, singularities, G_2-transitions)

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  - Phenomenology behind K, W, f_{IJ}?
  - $X_{L/R}$ sectors: hidden/visible sectors to mediate SUSY breaking?
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