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Implication of quadratic divergences cancellation in the two Higgs doublet model

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Cancellation of the quadratic divergences

- One of the basic problems of the SM is **Hierarchy problem**.
- There are unnatural tuning between the tree-level Higgs mass and the mass from the radiative corrections.

$$m^2(p^2) = m_o^2 + \underbrace{\text{[wavy line loop]}_p}_{J=1} + \underbrace{\text{[circle loop]}_p}_{J=1/2} + \underbrace{\text{[bubble loop]}_p}_{J=0}$$

$$(M_{\text{Higgs}})_{\text{experiment-tree level}} \ll (M_{\text{Higgs}})_{\text{naturalness}}$$

- These radiative corrections are sensitive to the largest scale in the theory.
- Veltman $\rightarrow$ These radiative corrections to the scalar mass either vanish or are kept at a manageable level.
- Implication of Cancellation quadratically Divergences in the 2HDM.

Two Higgs Doublet Model (2HDM)

- 2HDM consists of two SU(2) scalar doublets ϕ_1 and ϕ_2 .

$$Z_2 \text{ transformation : } \phi_1 \rightarrow \phi_1, \quad \phi_2 \rightarrow -\phi_2$$

- Potential for 2HDM (**soft Z_2 symmetry breaking**)

$$\begin{aligned} V = & \frac{\lambda_1}{2}(\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) \\ & + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \left(\frac{1}{2}\lambda_5(\phi_1^\dagger \phi_2)^2 + h.c.\right) \\ & - \frac{m_{11}^2}{2} (\phi_1^\dagger \phi_1) - \frac{m_{22}^2}{2} (\phi_2^\dagger \phi_2) - \left(\frac{m_{12}^2}{2}(\phi_1^\dagger \phi_2) + h.c.\right). \end{aligned}$$

- Positivity and the perturbativity constraints

$$\begin{aligned} \lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}, \quad |\lambda_5| < \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2} \\ \lambda < 4\pi. \end{aligned}$$

- **2HDM II** for Yukawa interaction

$$\langle \phi_1 \rangle = v_1/\sqrt{2} \neq 0, \quad \langle \phi_2 \rangle = v_2/\sqrt{2} \neq 0$$

$$v^2 = v_1^2 + v_2^2, \quad \tan \beta = v_2/v_1$$

$$\text{D quarks} \rightarrow \phi_1, \quad \text{U quarks} \rightarrow \phi_2$$

There are 5 physical Higgs particle with these masses: $M_h, M_H, M_A, M_{H^\pm}$

Implication of cancellation of one loop quadratic divergences in the 2HDM

- Applying Cancellation of one loop quadratic divergences in the 2HDM leads to two set of conditions [Newton&Wu 1994]:

$$6M_W^2 + 3M_Z^2 + v^2 (3\lambda_1 + 2\lambda_3 + \lambda_4) = \frac{12}{\cos^2\beta} m_D^2,$$

$$6M_W^2 + 3M_Z^2 + v^2 (3\lambda_2 + 2\lambda_3 + \lambda_4) = \frac{12}{\sin^2\beta} m_U^2.$$

- The SM-like h and H scenarios are defined as:

- $M_H = 125$ GeV (SM-like $H_{\pm}$ scenario)

$$\cos(\beta - \alpha) \sim \pm 1$$

- $M_h = 125$ GeV (SM-like h scenario)

$$\sin(\beta - \alpha) \sim +1$$

Approximate solutions of the cancelation conditions

- For the SM-like scenarios, with $\sin(\beta - \alpha) = 1$, $\cos(\beta - \alpha) = 1$:

$$SM - like\ h \quad : \quad \lambda_1 - \lambda_2 = \left(\tan^2 \beta - \frac{1}{\tan^2 \beta} \right) \left(\frac{M_H^2}{v^2} - \nu \right),$$

$$SM - like\ H_+ \quad : \quad \lambda_1 - \lambda_2 = \left(\tan^2 \beta - \frac{1}{\tan^2 \beta} \right) \left(\frac{M_h^2}{v^2} - \nu \right).$$

- The solutions of equations, in terms of masses and angle β are:

$$\lambda_1 - \lambda_2 = \frac{4}{v^2} \left(\frac{m_b^2}{\cos^2 \beta} - \frac{m_t^2}{\sin^2 \beta} \right) = \frac{4m_b^2}{v^2} \left(1 - \frac{m_t^2/m_b^2}{\tan^2 \beta} \right) (1 + \tan^2 \beta).$$

- The expressions for masses squared of the partner of the SM-like Higgs particle is:

$$M^2 = 4m_b^2 \frac{\tan^2 \beta - \frac{m_t^2}{m_b^2}}{\tan^2 \beta - 1} + \nu v^2.$$

SM-like h and H scenarios

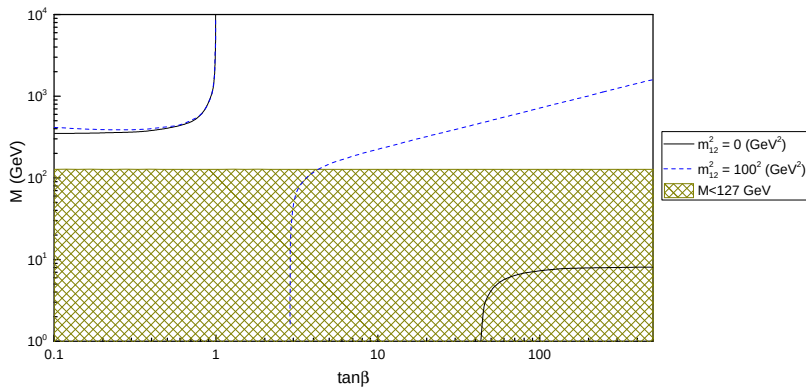


Figure: The mass of the partner of the SM-like $h(H)$ Higgs particle versus $\tan\beta$ for $m_{12}^2 = 0$ and $m_{12}^2 = 100^2$ GeV².

SM-like h and H scenarios

$$6M_W^2 + 3M_Z^2 + v^2 (3\lambda_1 + 2\lambda_3 + \lambda_4) = \frac{12}{\cos^2 \beta} m_D^2,$$

$$6M_W^2 + 3M_Z^2 + v^2 (3\lambda_2 + 2\lambda_3 + \lambda_4) = \frac{12}{\sin^2 \beta} m_U^2.$$

- Inserting λ in the term of mass and including only top and bottom quarks:

$$\delta_1 = \frac{12m_b^2}{\cos^2 \beta} - 6M_W^2 - 3M_Z^2 - 2M_{H^\pm}^2 - M_A^2 + \frac{m_{12}^2}{2 \sin \beta \cos \beta} [1 + 3 \tan^2 \beta],$$

$$\delta_2 = \frac{12m_t^2}{\sin^2 \beta} - 6M_W^2 - 3M_Z^2 - 2M_{H^\pm}^2 - M_A^2 + \frac{m_{12}^2}{2 \sin \beta \cos \beta} [1 + 3 \cot^2 \beta],$$

$$\begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} M_h^2 \\ M_H^2 \end{pmatrix}$$

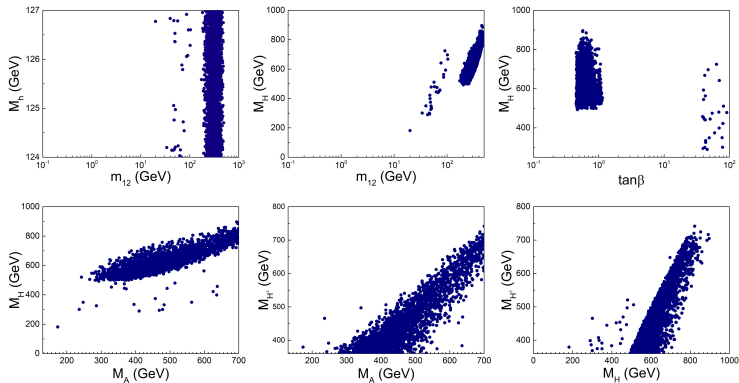
Scanning:

$$\begin{aligned} -\pi/2 < \alpha < \pi/2, & \quad M_{H^\pm} \in [360, 800] \text{ GeV}, & \quad m_{12}^2 \in [-400^2, 400^2] \text{ GeV}^2, \\ 0 < \beta < \pi/2, & \quad M_h < M_H \leq 1000 \text{ GeV}, & \quad M_A \in [130, 700] \text{ GeV}. \end{aligned}$$

Results:

Solution exist only for masses in SM-like h scenario

Considering only theoretical constraint:



Experiment constraints:

► ATLAS+CMS collaboration:

- * Higgs mass: 125.09 ± 0.24 GeV
- * The relative Higgs couplings:

$$\chi_i = \frac{g_i}{g_i^{SM}}$$

$$\chi_t^h = 1.43_{-0.22}^{+0.23} \quad \chi_b^h = 0.57 \pm 0.16$$

- * Higgs signal strength:

$$\mathcal{R}_{XX} = \frac{\sigma(gg \rightarrow h_1)}{\sigma(gg \rightarrow \phi_{SM})} \frac{\text{BR}(h_1 \rightarrow XX)}{\text{BR}(\phi_{SM} \rightarrow XX)} \simeq \frac{\Gamma(h_1 \rightarrow gg)}{\Gamma(\phi_{SM} \rightarrow gg)} \frac{\text{BR}(h_1 \rightarrow XX)}{\text{BR}(\phi_{SM} \rightarrow XX)}.$$

$$R_{ZZ}^h = 1.29_{-0.23}^{+0.26}, \quad R_{\gamma\gamma}^h = 1.14_{-0.18}^{+0.19}, \quad R_{Z\gamma}^h < 9$$

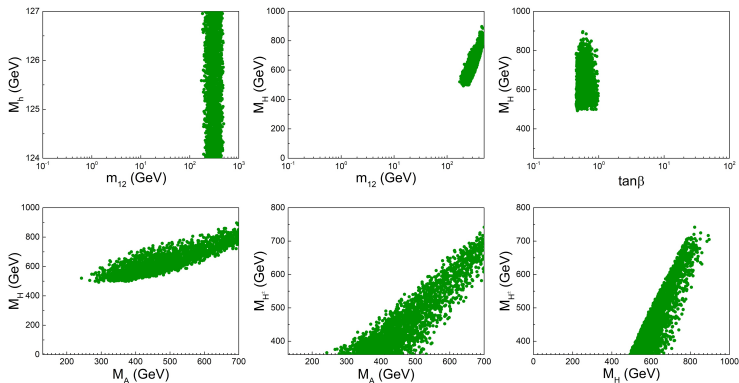
- * The total Higgs decay width:

$$\frac{\Gamma_h^{tot}}{\Gamma_h^{tot}|_{SM}} < 5.4$$

► Gfitter Group Collaboration:

$$S_{exp} = 0.05 \pm 0.11, \quad T_{exp} = 0.09 \pm 0.1, \quad U_{exp} = 0.01 \pm 0.11$$

Considering theoretical and experiment constraints:



- Solutions exist only in low $\tan\beta$ below 1.
- Lower and upper bounds on M_H between: $500 \text{ GeV} < M_H < 900 \text{ GeV}$
- Lower bound on M_A : $250 \text{ GeV} < M_A$
- Lower bound on m_{12}^2 : $170 \text{ GeV}^2 < m_{12}^2$

Benchmarks

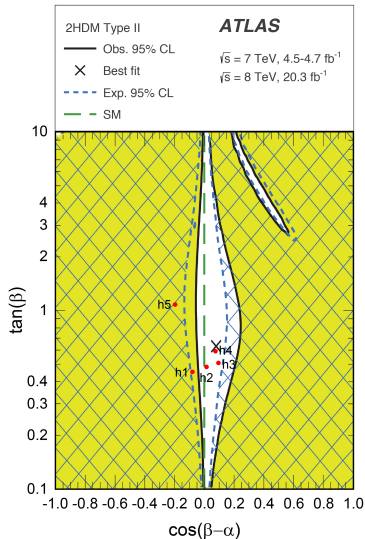


Table: SM-like h for $\sin(\beta - \alpha) \sim 1$, Higgs bosons masses (in GeV)

B mark	M_h	M_H	M_A	$M_{H^\pm}^\pm$	m_{12}^2
h1	124.426	573.832	444.16	454.424	$(281.0690)^2$
h2	124.082	505.298	266.59	375.488	$(191.9640)^2$
h3	124.242	736.961	567.37	598.392	$(352.9160)^2$
h4	125.252	826.947	650.08	645.560	$(421.8410)^2$
h5	125.771	605.931	448.48	438.628	$(309.6770)^2$

Benchmarks

Table: SM-like h for $\sin(\beta - \alpha) \sim 1$, Higgs bosons masses (in GeV)

B point	M_H	χ_t^H	χ_b^H	$R_{\gamma\gamma}^H$	$R_{Z\gamma}^H$	$\frac{\Gamma_H^{tot}}{\Gamma_H^{totSM}}$
h1	573.832	-2.30	0.34	69.14	0.62	0.88
h2	505.298	-2.06	0.49	14.88	0.31	1.09
h3	736.961	-1.86	0.59	152.71	1.21	0.78
h4	826.947	-1.61	0.66	49.95	1.44	0.53
h5	605.931	-1.10	0.85	72.79	0.79	0.28

- These are calculated with respect to the would be SM Higgs with the considered mass.

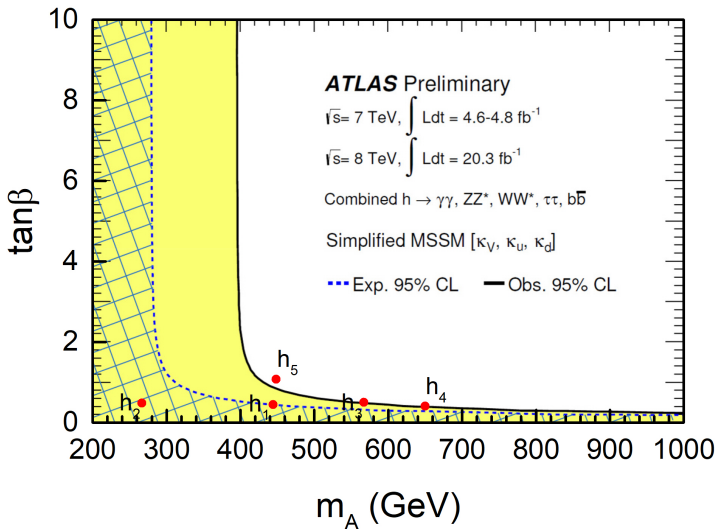
Summary and Conclusion

- ▶ Applying cancellation of one loop quadratic divergences in the 2HDM leads to two set of conditions.
- ▶ We have considered two possible scenarios, with the SM like-h and the SM like-H bosons.
- ▶ We have found that solution exist in SM-like h scenario (within the shown boundaries).
- ▶ We found theoretical constraints for mass of heavy scalars in SM-like h scenario.
- ▶ Our solution are in agreement with the recent experimental data.

Back up

Problems of the Veltman condition

- The Veltman condition, which claims to be a way to ameliorate the hierarchy problem, is known not to satisfy this goal.
- Cutoff regularization assumes that the cutoff is the same on all sectors of the theory that couple to the Higgs.
- In ultraviolet completions like a supersymmetric SM, the cutoff in each sector is related to the mass of a physical particle, which can be different for each sector.
- It is further not clear how to extend the Veltman condition beyond one loop.



The ratio of the coupling constant (g_i) of the neutral Higgs boson to the corresponding SM coupling g_i^{SM} , called the relative couplings

$$\chi_i = \frac{g_i}{g_i^{SM}},$$

	$\chi_V(W \text{ and } Z)$	$\chi_u(\text{up-type quarks})$	$\chi_d(\text{down-type quarks})$
h	$\sin(\beta - \alpha)$	$\sin(\beta - \alpha) + \frac{1}{\tan \beta} \cos(\beta - \alpha)$	$\sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha)$
H	$\cos(\beta - \alpha)$	$\cos(\beta - \alpha) - \frac{1}{\tan \beta} \sin(\beta - \alpha)$	$\cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha)$
A	0	$-i\gamma_5 \cot \beta$	$-i\gamma_5 \tan \beta$

Tree-level couplings of the neutral Higgs bosons to gauge bosons and fermions in 2HDM (II).

Two Higgs Doublet Model(2HDM)

- The minimization conditions:

$$m_{11}^2 = v_1^2 \lambda_1 + v_2^2 (\lambda_{345} - 2\nu)$$

$$m_{22}^2 = v_2^2 \lambda_2 + v_1^2 (\lambda_{345} - 2\nu)$$

$$\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5, \quad \nu \equiv m_{12}^2 / (2v_1 v_2)$$

- There are five Higgs particles:

$M_{h,H}^2$ are the eigenvalues of the mass squared matrix $\mathcal{M}^2$

$$\mathcal{M}^2 = \begin{bmatrix} \cos^2 \beta \lambda_1 + \sin^2 \beta \nu & (\lambda_{345} - \nu) \cos \beta \sin \beta \\ (\lambda_{345} - \nu) \cos \beta \sin \beta & \sin^2 \beta \lambda_2 + \cos^2 \beta \nu \end{bmatrix} v^2$$

$$\mathcal{M}^2 = \begin{bmatrix} M_h^2 \sin^2 \alpha + M_H^2 \cos^2 \alpha & (M_H^2 - M_h^2) \sin \alpha \cos \alpha \\ (M_H^2 - M_h^2) \sin \alpha \cos \alpha & M_H^2 \sin^2 \alpha + M_h^2 \cos^2 \alpha \end{bmatrix}.$$

$$M_A^2 = (\nu - \lambda_5) v^2$$

$$M_{H^\pm}^2 = \left(\nu - \frac{1}{2} (\lambda_4 + \lambda_5) \right) v^2$$