In search of natural Higgs alignment without decoupling in the 2HDM

Howard E. Haber

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The Higgs alignment limit: approaching the SM Higgs boson

Consider an extended Higgs sector with n hypercharge-one Higgs doublets Φ_i and m additional singlet Higgs fields ϕ_i.

After minimizing the scalar potential, we assume that only the neutral Higgs fields acquire vacuum expectation values (in order to preserve $\mathrm{U}(1)_{\mathrm{EM}}$),

$$\langle\Phi_i^0\rangle = v_i/\sqrt{2}\,, \qquad\qquad \langle\phi_j^0\rangle = x_j\,.$$

Note that $v^2 \equiv \sum_i |v_i|^2 = 4m_W^2/g^2 = (246~\mathrm{GeV})^2$.

The Higgs basis

Define new linear combinations of the hypercharge-one doublet Higgs fields (the so-called *Higgs basis*). In particular,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \frac{1}{v}\sum_i v_i^* \Phi_i\,, \qquad\qquad \langle H_1^0\rangle = v/\sqrt{2}\,,$$

and $H_2, H_3, \ldots, H_n$ are the other linear combinations of doublet scalar fields such that $\langle H_i^0\rangle = 0$ (for $i = 2, 3, \ldots, n$).

That is H_1^0 is aligned in field space with the direction of the Higgs vacuum expectation value (vev). Thus, if $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is a mass-eigenstate, then the tree-level couplings of this scalar to itself, to gauge bosons and to fermions are precisely those of the SM Higgs boson, h^0. This is the exact alignment limit.

A SM-like Higgs boson

In general, $\sqrt{2}\,\mathrm{Re}(H_1^0)-v$ is not a mass-eigenstate due to mixing with other neutral scalars. Nevertheless, a SM-like Higgs boson exists if either:

- the diagonal squared masses of the other Higgs basis scalar fields are all large compared to the mass of the observed Higgs boson (the so-called *decoupling limit*).

and/or

- the elements of the neutral scalar squared-mass matrix that govern the mixing of $\sqrt{2}\,\mathrm{Re}(H_1^0)-v$ with other neutral scalars are suppressed.

The decoupling limit of an extended Higgs sector

In the SM, $m_h^2 = \lambda v^2$ where $v = 246$ GeV and λ is the Higgs self-coupling, which should not be much larger than $\mathcal{O}(1)$. Thus, we expect $m_h \sim \mathcal{O}(v)$.

In extended Higgs sectors, there can be a new mass parameter, $M \gg v$, such that all physical Higgs masses with one exception are of $\mathcal{O}(M)$. The Higgs boson, with $m_h \sim \mathcal{O}(v)$, is SM-like, due to approximate alignment. This is the decoupling limit.

Integrating out all the heavy degrees of freedom at the mass scale M, one is left with a low-energy effective theory which consists of the SM particles, including a single neutral scalar boson. This low-energy effective theory is precisely the SM!

Alignment without decoupling[1]

The alignment limit is most naturally achieved in the decoupling regime. However, in this case the additional Higgs boson states are very heavy and may be difficult to observe at the LHC.

In the case of approximate alignment without decoupling (due to suppressed scalar mixing), non-SM-like Higgs boson states need not be very heavy and thus may be more easily accessible at the LHC.

The exact alignment limit is defined as the limit in which $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is a mass-eigenstate, i.e. it does not mix with any of the other Higgs basis states H_k^0 (for $k = 2, 3, \ldots$).

[1] J.F. Gunion and H.E. Haber, hep-ph/0207010; N. Craig, J. Galloway and S. Thomas, arXiv:1305.2424.

Exact alignment as a consequence of a symmetry?

We wish to explore the possibility of approximate Higgs alignment without decoupling. Typically, alignment arises due to a (fine-tuned) choice of Higgs sector parameters. In such cases, the exact alignment limit is not a consequence of a symmetry.

In the case of *natural Higgs alignment*, exact alignment limit is the result of a symmetry of the Lagrangian. We shall explore whether natural Higgs alignment is possible, and whether small deviations from exact alignment can explain the observation of a SM-like Higgs boson.

Consider a Higgs sector comprised entirely of n Higgs doublets. The scalar potential expressed in terms of Higgs basis fields has the form

$$\mathcal{V} = Y_1 H_1^\dagger H_1 + \big(Y_{3i} H_1^\dagger H_i + \text{h.c.}\big) + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 + H_1^\dagger H_1 \big(Z_{6i} H_1^\dagger H_i + \text{h.c.}\big) + \cdots\,,$$

where there is an implicit sum over the index $i = 2, 3, \ldots, n$.

In light of $\langle H_1 \rangle = v$ and $\langle H_i \rangle = 0$ (for $i = 2, 3, \ldots, n$), the minimum conditions $\partial\mathcal{V}/\partial H_k = 0$ yield,

$$Y_1 = -\tfrac{1}{2} Z_1 v^2 \,, \qquad\qquad Y_{3i} = -\tfrac{1}{2} Z_{6i} v^2 \,.$$

Exact alignment implies that $Z_{6i} = 0$, which via the minimum condition yields $Y_{3i} = 0$.

Exact alignment can therefore be achieved by imposing a discrete $\mathbb{Z}_2$ symmetry in the Higgs basis,

$$H_1 \to +H_1 \,, \qquad\qquad H_i \to -H_i \ (\text{for } i = 2, 3, \ldots, n) \,.$$

Assuming that all SM fermions and gauge bosons are even under the $\mathbb{Z}_2$ symmetry, the end result is a generalization of the inert doublet model in which $h = \sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is identified as the SM Higgs boson.

However, this symmetry *cannot* be softly broken, so Higgs alignment is either exact or not present. We now explore Higgs alignment further in the 2HDM.

Theoretical structure of the 2HDM

Consider the most general renormalizable 2HDM potential,

$$\begin{aligned}\mathcal{V} = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 \\ & + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} .\end{aligned}$$

After minimizing the scalar potential, $\langle \Phi_i^0 \rangle = v_i/\sqrt{2}$ (for $i = 1, 2$) with $v \equiv (|v_1|^2 + |v_2|^2)^{1/2} = 2m_W/g = 246$ GeV.

Define the scalar doublet fields of the Higgs basis,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^* \Phi_1 + v_2^* \Phi_2}{v} , \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2 \Phi_1 + v_1 \Phi_2}{v} ,$$

such that $\langle H_1^0 \rangle = v/\sqrt{2}$ and $\langle H_2^0 \rangle = 0$. The Higgs basis is uniquely defined up to an overall rephasing, $H_2 \to e^{i\chi} H_2$.

In the Higgs basis, the scalar potential is given by:

$$\begin{aligned}\mathcal{V} = &\, Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 \\ & + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \big[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \big] H_1^\dagger H_2 + \text{h.c.} \right\} ,\end{aligned}$$

where Y_1, Y_2 and $Z_1, \ldots, Z_4$ are real and uniquely defined, whereas Y_3, Z_5, Z_6 and Z_7 are complex and transform under the rephasing of H_2,

$$[Y_3, Z_6, Z_7] \to e^{-i\chi}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi} Z_5 \,.$$

After minimizing the scalar potential, $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$.

Remarks:

1. Generically, the Z_i are $\mathcal{O}(1)$ parameters.

2. Exact alignment corresponds to $Z_6 = 0$.

The Higgs alignment limit in the general 2HDM

In the general 2HDM, the scalar potential is generically CP-violating. In this case, the neutral Higgs mass-eigenstates are linear combinations of $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\ \mathrm{Re}\,H_2^0\,,\ \mathrm{Im}\,H_2^0\}$, which are determined by diagonalizing the 3×3 real symmetric squared-mass matrix

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \mathrm{Re}(Z_6) & -\mathrm{Im}(Z_6) \\ \mathrm{Re}(Z_6) & \tfrac{1}{2}Z_{345} + Y_2/v^2 & -\tfrac{1}{2}\mathrm{Im}(Z_5) \\ -\mathrm{Im}(Z_6) & -\tfrac{1}{2}\mathrm{Im}(Z_5) & \tfrac{1}{2}Z_{345} - \mathrm{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re}(Z_5)$. The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}, such that θ_{12} and θ_{13} are invariant whereas $\theta_{23} \to \theta_{23} - \chi$ under the rephasing of H_2.[2]

The alignment limit again corresponds to two cases:

1. $Y_2 \gg v^2$, corresponding to the decoupling limit.
2. $|Z_6| \ll 1$, corresponding to alignment with or without decoupling.

[2]See H.E. Haber and D. O'Neil, arXiv: hep-ph/0602242.

The alignment limit of the general 2HDM in equations

To obtain the conditions in which h_1 is the SM-like Higgs boson, noting that:

$$\frac{g_{h_1VV}}{g_{h_{\rm SM}VV}} = c_{12}c_{13}\,, \qquad \text{where } V = W \text{ or } Z\,,$$

where $h_{\rm SM}$ is the SM Higgs boson, we demand that

$$s_{12}\,,\, s_{13} \ll 1\,.$$

Here, $s_{12} \equiv \sin\theta_{12}$, $c_{12} \equiv \cos\theta_{12}$, etc. We denote the masses of the neutral Higgs mass eigenstates by m_1, m_2 and m_3. It follows that:

$$Z_1 v^2 = m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2\,,$$

$${\rm Re}(Z_6\, e^{-i\theta_{23}})\, v^2 = c_{13}s_{12}c_{12}(m_2^2 - m_1^2)\,,$$

$${\rm Im}(Z_6\, e^{-i\theta_{23}})\, v^2 = s_{13}c_{13}(c_{12}^2 m_1^2 + s_{12}^2 m_2^2 - m_3^2)\,,$$

$${\rm Re}(Z_5\, e^{-2i\theta_{23}})\, v^2 = m_1^2(s_{12}^2 - c_{12}^2 s_{13}^2) + m_2^2(c_{12}^2 - s_{12}^2 s_{13}^2) - m_3^2 c_{13}^2\,,$$

$${\rm Im}(Z_5\, e^{-2i\theta_{23}})\, v^2 = 2s_{12}c_{12}s_{13}(m_2^2 - m_1^2)\,.$$

Assuming no mass degeneracies in the neutral scalar sector, it then follows that in the alignment limit,

$$\boxed{\begin{aligned} s_{12} \equiv \sin\theta_{12} &\simeq \frac{\mathrm{Re}(Z_6 e^{-i\theta_{23}})v^2}{m_2^2 - m_1^2} \ll 1\,, \\ s_{13} \equiv \sin\theta_{13} &\simeq -\frac{\mathrm{Im}(Z_6 e^{-i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,, \end{aligned}}$$

One additional small quantity characterizes the alignment limit,

$$\mathrm{Im}(Z_5 e^{-2i\theta_{23}}) \simeq \frac{(m_2^2 - m_1^2)s_{12}s_{13}}{v^2} \simeq -\frac{2\,\mathrm{Im}(Z_6^2 e^{-2i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,.$$

Finally, the following mass relations in the alignment limit are noteworthy,

$$\begin{aligned} m_1^2 &\simeq Z_1 v^2\,, \\ m_2^2 - m_3^2 &\simeq \mathrm{Re}(Z_5 e^{-2i\theta_{23}})v^2\,. \end{aligned}$$

The alignment limit of the CP-conserving 2HDM

Recall that the most relevant terms of the Higgs basis scalar potential are:

$$\mathcal{V} \ni \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + Z_6(H_1^\dagger H_1)(H_1^\dagger H_2) + \text{h.c.}\right\} .$$

In the CP-conserving 2HDM, one can rephase the field H_2 such that all the parameters of the scalar potential are real.

We identify the CP-odd Higgs boson as $A = \sqrt{2}\,\text{Im}\,H_2^0$ with squared-mass $m_A^2 = Y_2 + \frac{1}{2}(Z_3 + Z_4 - Z_5)v^2$. After eliminating Y_2 in favor of m_A^2, the CP-even Higgs squared-masses are obtained by diagonalizing the corresponding 2×2 squared-mass matrix, $\mathcal{M}_H^2$. The results are most transparent in the Higgs basis.

With respect to Higgs basis states, $\{\sqrt{2}\,\mathrm{Re}\ H_1^0 - v\,,\sqrt{2}\,\mathrm{Re}\ H_2^0\}$,

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix} .$$

The CP-even Higgs bosons are h and H with $m_h \leq m_H$. The couplings of $\sqrt{2}\,\mathrm{Re}\ H_1^0 - v$ coincide with those of the SM Higgs boson. Alignment arises two limiting cases:

1. $m_A^2 \gg (Z_1 - Z_5)v^2$. This is the *decoupling limit*, where h is SM-like and $m_A^2 \sim m_H^2 \sim m_{H^\pm}^2 \gg m_h^2 \simeq Z_1 v^2$.

2. $|Z_6| \ll 1$. Then, h is SM-like if $m_A^2 + (Z_5 - Z_1)v^2 > 0$. Otherwise, H is SM-like.

In particular, the CP-even mass eigenstates are:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\ \mathrm{Re}\ H_1^0 - v \\ \sqrt{2}\,\mathrm{Re}\ H_2^0 \end{pmatrix},$$

where $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$ are defined in terms of the mixing angle α that diagonalizes the CP-even Higgs squared-mass matrix when expressed in the Φ_1–Φ_2 basis of scalar fields, $\{\sqrt{2}\,\mathrm{Re}\,\Phi_1^0 - v_1\,,\ \sqrt{2}\,\mathrm{Re}\,\Phi_2^0 - v_2\}$, and $\tan\beta \equiv v_2/v_1$.

Since the SM-like Higgs boson must be approximately $\sqrt{2}\,\mathrm{Re}\ H_1^0 - v$, it follows that

- h is SM-like if $|c_{\beta-\alpha}| \ll 1$ (alignment with or without decoupling, depending on the magnitude of m_A),
- H is SM-like if $|s_{\beta-\alpha}| \ll 1$ (alignment without decoupling).

The alignment limit in equations

The CP-even Higgs squared-mass matrix yields,

$$Z_1 v^2 = m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2\,,$$

$$Z_6 v^2 = (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha}\,,$$

$$Z_5 v^2 = m_H^2 s_{\beta-\alpha}^2 + m_h^2 c_{\beta-\alpha}^2 - m_A^2\,.$$

If h is SM-like, then $m_h^2 \simeq Z_1 v^2$ (i.e., $Z_1 \simeq 0.26$) and

$$|c_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1\,,$$

If H is SM-like, then $m_H^2 \simeq Z_1 v^2$ (i.e., $Z_1 \simeq 0.26$) and

$$|s_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(Z_1 v^2 - m_h^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1\,.$$

A symmetry origin for alignment without decoupling

Consider the CP-conserving 2HDM in the Φ_1–Φ_2 basis. Then, $\lambda_1, \lambda_2, \ldots, \lambda_7$ are related to the corresponding Higgs basis parameters. For example,

$$Y_3 = \tfrac{1}{2}(m_{11}^2 - m_{22}^2)s_{2\beta} + m_{12}^2 c_{2\beta}\,,$$

$$Z_6 = -\tfrac{1}{2}\big[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}\big]s_{2\beta} + \lambda_6 c_\beta c_{3\beta} + \lambda_7 s_\beta s_{3\beta}\,,$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$. If the alignment condition $Z_6 = 0$ holds independently of $\tan\beta$, then it follows that

$$\lambda_1 = \lambda_2 = \lambda_{345}\,, \qquad \lambda_6 = \lambda_7 = 0\,,$$

which is called the *natural alignment condition*.[3]

In order to associate natural alignment with a symmetry, we shall make use of the alignment condition $Y_3 = 0$. If this condition holds independently of $\tan\beta$, then,

$$m_{11}^2 = m_{22}^2\,, \qquad m_{12}^2 = 0\,.$$

[3] See P.S. Bhupal Dev and A. Pilaftsis, arXiv:1408.3405.

Family and Generalized CP symmetries of the 2HDM

The scalar potential of the most general 2HDM is governed by 11 free parameters: 1 vev, 8 real parameters and two relative phases. It is possible to impose a discrete or continuous global symmetry on the Higgs potential [beyond the hypercharge U(1)$_Y$] to reduce the number of 2HDM parameters.

symmetry	m_{22}^2	m_{12}^2	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
$\mathbb{Z}_2$		0					0	0
Π_2	m_{11}^2	real	λ_1			real		λ_6^*
U(1)		0				0	0	0
SO(3)	m_{11}^2	0	λ_1		$\lambda_1 - \lambda_3$	0	0	0
CP1		real				real	real	real
CP2	m_{11}^2	0	λ_1					$-\lambda_6$
CP3	m_{11}^2	0	λ_1			$\lambda_1 - \lambda_3 - \lambda_4$ (real)	0	0

Classification of 2HDM scalar potential symmetries and their impact on the coefficients of the scalar potential in a basis where the symmetry is manifest [Ivanov; Ferreira, Haber and Silva].

Higgs family symmetries

$\mathbb{Z}_2$: $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$

Π_2 : $\Phi_1 \longleftrightarrow \Phi_2$

U(1)$_{\rm PQ}$ [Peccei-Quinn]: $\Phi_1 \to e^{-i\theta}\Phi_1$, $\Phi_2 \to e^{i\theta}\Phi_2$

SO(3): $\Phi_a \to U_{ab}\Phi_b$, $U \in {\rm U}(2)/{\rm U}(1)_Y$

Generalized CP (GCP) transformations

CP1 : $\Phi_1 \to \Phi_1^*$, $\Phi_2 \to \Phi_2^*$

CP2 : $\Phi_1 \to \Phi_2^*$, $\Phi_2 \to -\Phi_1^*$

CP3 : $\Phi_1 \to \Phi_1^* c_\theta + \Phi_2^* s_\theta$, $\Phi_2 \to -\Phi_1^* s_\theta + \Phi_2^* c_\theta$, for $0 < \theta < \frac{1}{2}\pi$

where $c_\theta \equiv \cos\theta$ and $s_\theta \equiv \sin\theta$. Some observations of note:

1. Π_2 symmetry is equivalent to $\mathbb{Z}_2$ symmetry in a different basis.
2. Applying $\mathbb{Z}_2$ and Π_2 simultaneously $\Longleftrightarrow$ CP2 in a different basis.
3. Applying U(1)$_{\rm PQ}$ and Π_2 simultaneously $\Longleftrightarrow$ CP3 in a different basis.

Exceptional region of the parameter space (ERPS)

An exceptional region of the 2HDM parameter space (first identified by Davidson and Haber) consists of:

$$\text{ERPS}: \qquad m_{22}^2 = m_{11}^2\,, \qquad m_{12}^2 = 0\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_7 = -\lambda_6$$

The corresponding conditions in the Higgs basis are,

$$Y_2 = Y_1\,, \qquad Y_3 = Z_6 = Z_7 = 0\,, \qquad Z_1 = Z_2\,.$$

Indeed, in the ERPS one of the two fine-tuning conditions is removed.

The ERPS includes SO(3), CP3 (equivalent to U(1)$_{\rm PQ}\otimes\Pi_2$ in another basis), and CP2 (equivalent to $\mathbb{Z}_2\otimes\Pi_2$ in another basis). To avoid an extra massless Goldstone boson, one must softly-break the SO(3) and CP3 symmetries.

However, none of the ERPS models can be extended to the Yukawa interactions without generating a massless quark or some other phenomenologically untenable feature [P.M. Ferreira and J.P. Silva, Eur. Phys. J. C **69**, 45 (2010)].

Exact alignment due to a symmetry

We have noted previously that exact alignment can be achieved by imposing $m_{11}^2 = m_{22}^2$ and $m_{12}^2 = 0$. This leads to three possible symmetry choices:

symmetry	m_{22}^2	m_{12}^2	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP2	m_{11}^2	0	λ_1					$-\lambda_6$
CP3	m_{11}^2	0	λ_1			$\lambda_1 - \lambda_3 - \lambda_4$ (real)	0	0
SO(3)	m_{11}^2	0	λ_1		$\lambda_1 - \lambda_3$	0	0	0

Note that CP2 is not "natural alignment" as defined by Dev and Pilaftsis, since a particular $\tan\beta$ is chosen by imposing $Z_6 = 0$. That is, setting $\lambda_1 = \lambda_2$ and $\lambda_7 = -\lambda_6$ yields

$$Z_6 = -\tfrac{1}{4}\big(\lambda_1 - \lambda_{345}\big)s_{4\beta} + \lambda_6 c_{4\beta}\,,$$

which satisfies $Z_6 = 0$ only for the specific choice, $\tan 4\beta = 4\lambda_6/(\lambda_1 - \lambda_{345})$.

However, none of these symmetries can be extended to the Yukawa sector.

The CP2-symmetric 2HDM with mirror fermions

Consider the 2HDM with a CP2-symmetric scalar potential, which can be realized as a $\mathbb{Z}_2 \otimes \Pi_2$ discrete symmetry. To extend this symmetry to the Yukawa sector, we introduce mirror fermions.[4] SM fermions are denoted by lower case letters (e.g. left-handed doublet fields q and right-handed singlet fields u and d); mirror fermions by upper case letters.

We take the top sector to transform under the discrete symmetries as follows,

$$\Pi_2: \; q \Leftrightarrow q, \quad u \Leftrightarrow U, \qquad \overline{U} \Leftrightarrow \overline{U}\,,$$
$$\mathbb{Z}_2: \; q \Leftrightarrow q, \quad u \Leftrightarrow -u, \quad U \Leftrightarrow U, \quad \overline{U} \Leftrightarrow \overline{U}\,.$$

where $\overline{U}$ is in the representation conjugate to U (to avoid anomalies). The Yukawa couplings consistent with the $\mathbb{Z}_2 \otimes \Pi_2$ discrete symmetry are

$$\mathscr{L}_{\rm Yuk} \supset y_t \left(q\Phi_2 u + q\Phi_1 U\right) + {\rm h.c.}$$

[4] P. Draper, H.E. Haber and J.T. Ruderman, JHEP **1606**, 124 (2016) [arXiv:1605.03237 [hep-ph]].

The model is not phenomenologically viable due to experimental limits on mirror fermion masses. Thus, we introduce a vectorlike mass,

$$\mathscr{L}_{\text{mass}} \supset M_U U\overline{U} + \text{h.c.}$$

which preserves the $\mathbb{Z}_2$ but explicitly breaks the Π_2 discrete symmetry. This symmetry breaking is soft, so that $m_{22}^2 - m_{11}^2$ is protected from quadratic sensitivity to the cutoff scale Λ.

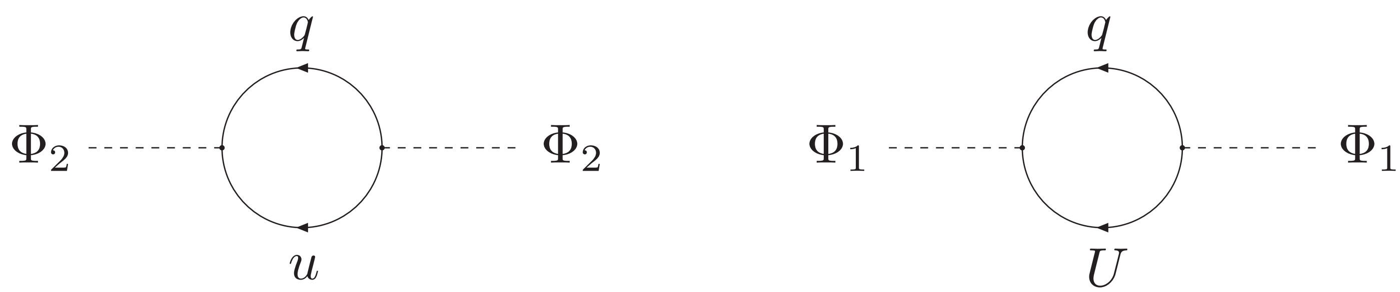
The other SM fermions can also be included by introducing the appropriate mirrors such that

$$\begin{aligned} \Pi_2: &\ d \Leftrightarrow D, \quad && e \Leftrightarrow E, \quad && \overline{D} \Leftrightarrow \overline{D}, \quad && \overline{E} \Leftrightarrow \overline{E} \\ \mathbb{Z}_2: &\ d \Leftrightarrow -d, \quad && e \Leftrightarrow -e, \quad && D \Leftrightarrow D, \quad && E \Leftrightarrow E. \end{aligned}$$

The corresponding Yukawa couplings and vectorlike fermion masses are

$$\mathscr{L} \supset y_b \left(q\Phi_2^* d + q\Phi_1^* D\right) + y_\tau \left(\ell\Phi_2^* e + \ell\Phi_1^* E\right) + M_D D\overline{D} + M_E E\overline{E}\,.$$

Effects of the softly-broken Π_2 discrete symmetry



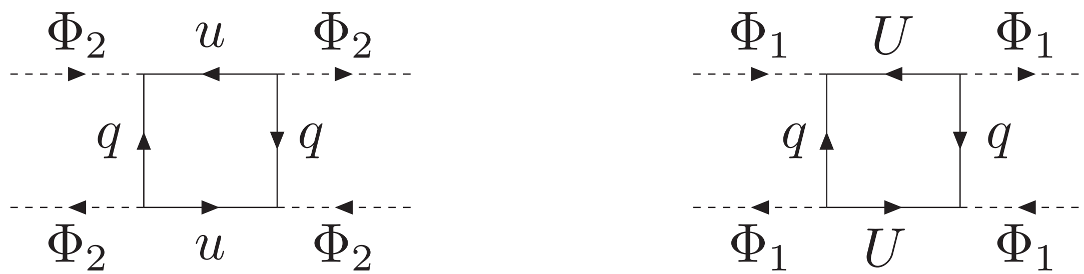
$$\Delta m^2 \equiv m_{22}^2 - m_{11}^2 \sim -\frac{3y_t^2 M_U^2}{4\pi^2} \ln(\Lambda/M_U) \,,$$

neglecting finite thresholds proportional to M_U^2. Since $\mathbb{Z}_2$ is unbroken (or at worst spontaneously broken if $v_2 \neq 0$), m_{12}^2 is not generated in this approximation. Assuming that $\ln(\Lambda/M_U)$ is not much larger than $\mathcal{O}(1)$, we see that there is no second fine tuning if $\Delta m^2 \lesssim \mathcal{O}(v^2)$, or roughly

$$M_U \lesssim \frac{\pi v^2}{m_t} \,,$$

which is satisfied for M_U less than a few TeV. Note that the other mirror masses are far less constrained since the corresponding SM fermion masses are significantly less than m_t.

Integrating out the mirror fermions below the scale M_U, one generates a splitting between λ_1 and λ_2. Above the scale M, the diagrams



contribute equally to $\lambda_2(\Phi_2^\dagger\Phi_2)^2$ and $\lambda_1(\Phi_1^\dagger\Phi_1)^2$, respectively. Below the scale M_U, the diagrams with internal U lines decouple, which then yields

$$\Delta\lambda \equiv |\lambda_1 - \lambda_2| \sim \frac{3y_t^4}{4\pi^2}\log(M_U/m_t) \sim 0.1\,,$$

for $M_U \sim 1$ TeV. Note: λ_6 and λ_7 are not generated due to the unbroken $\mathbb{Z}_2$.

Henceforth, we write m_{11}^2 and m_{22}^2 (at the scale m_t) in terms of

$$m^2 \equiv \tfrac{1}{2}(m_{11}^2 + m_{22}^2)\,, \qquad \Delta m^2 \equiv m_{22}^2 - m_{11}^2\,.$$

We denote $\tan\beta \equiv v_2/v_1$ and we neglect the effects of $\Delta\lambda$ which are small.

Local minima of the 2HDM scalar potential

We define $\lambda \equiv \lambda_1 = \lambda_2$ and

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5 \,, \qquad R = \frac{\lambda_{345}}{\lambda} \,,$$

and demand that $\lambda > 0$ and $R > -1$ to ensure that the vacuum is bounded from below. Solving for the potential minimum, there are two possible phases:

1. The inert phase

Assuming that $\Delta m^2 < -2m^2$, the Higgs vacuum is

$$\left\langle \Phi_1^0 \right\rangle^2 = \tfrac{1}{2} v^2 = -\left(\frac{m^2 + \frac{1}{2}\Delta m^2}{\lambda} \right) \qquad \langle \Phi_2 \rangle = 0.$$

In this case, $\mathbb{Z}_2$ is unbroken by the vacuum.

2. The mixed phase[5]

If both $v_1 \neq 0$ and $v_2 \neq 0$, then the $\mathbb{Z}_2$ is spontaneously broken. Minimizing the scalar potential yields

$$m^2 = -\tfrac{1}{4}\lambda(1+R)v^2\,, \qquad \tan\beta = \sqrt{\frac{1-\epsilon}{1+\epsilon}}\,,$$

where

$$\epsilon \equiv \frac{2\Delta m^2}{\lambda(1-R)v^2}\,.$$

The positivity of v_1^2 and v_2^2 and the curvature at the extremum requires

$$|R| < 1\,, \qquad |\epsilon| < 1\,.$$

Given the constraint on R, the constraint on ϵ can also be written

$$m^2 < 0\,, \quad \Delta m^2 < -2m^2\left(\frac{1-R}{1+R}\right)\,.$$

[5] There is a parameter regime in which both the inert phase and the mixed phase coexist. However, one can check that in this case, the mixed phase minimum is deeper than the inert phase minimum.

Scalar spectrum of the inert phase

The physical neutral Higgs bosons are eigenstates of CP.

$$
\begin{aligned}
m_h^2 &= \lambda v^2\,, \\
m_H^2 &= -\tfrac{1}{2}\lambda v^2(1-R) - \Delta m^2\,, \\
m_A^2 &= m_H^2 - \lambda_5 v^2\,, \\
m_{H^\pm}^2 &= m_H^2 - \tfrac{1}{2}(\lambda_4+\lambda_5)v^2\,,
\end{aligned}
$$

where the couplings of h are precisely those of the SM Higgs boson.

Since we are interested in the case where all Higgs boson masses are of $\mathcal{O}(v)$, we restrict $\Delta m^2 \sim \mathcal{O}(v^2)$ as previously stated. Of course, if $M_U \gg v$, then we can make $-\Delta m^2$ arbitrarily large (which is an allowed regime of the inert phase), in which case H, A and $H^\pm$ become arbitrarily heavy.

Scalar spectrum of the mixed phase

In the convention where the ratio of the vevs is real, it follows from the scalar potential minimum conditions that $\lambda_5 \leq 0$. The Higgs mass spectrum is:

$$
\begin{aligned}
m^2_{h,H} &= \tfrac{1}{2}\lambda v^2 \big(1 \mp \sqrt{R^2 + (1-R^2)\epsilon^2}\big)\,, \\
m^2_A &= -\lambda_5 v^2\,, \\
m^2_{H^\pm} &= -\tfrac{1}{2}(\lambda_4 + \lambda_5) v^2\,.
\end{aligned}
$$

Requiring h to be SM-like, it follows that $|\cos(\beta-\alpha)| \ll 1$ [the so-called alignment limit], assuming that[6]

$$
-1 < R < -\frac{\epsilon^2}{1-\epsilon^2}\,,
$$

and $\alpha - \beta$ is the CP-even mixing angle in the Higgs basis, with

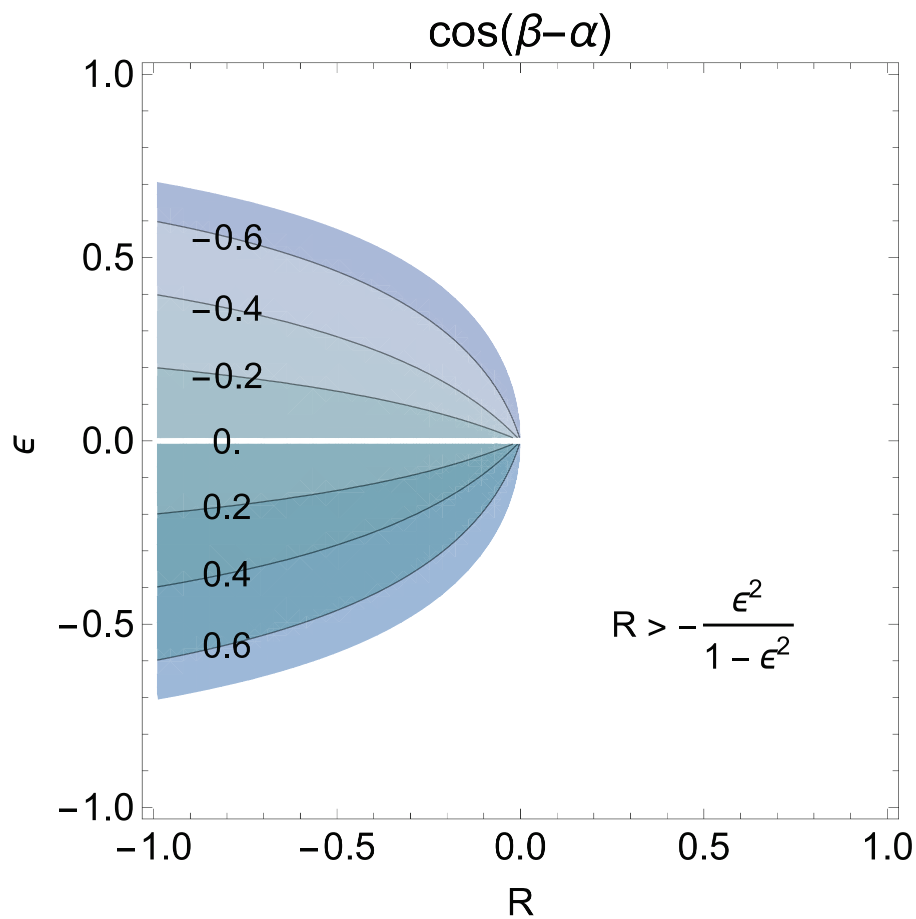
$$
\sin(\beta-\alpha)\cos(\beta-\alpha) = \frac{\epsilon(\epsilon^2-1)(1-R)}{2\sqrt{R^2+\epsilon^2(1-R^2)}}\,.
$$

[6] Otherwise, H is SM-like and $|\sin(\beta-\alpha)| \ll 1$.

When ϵ and $|\cos(\beta-\alpha)|$ are small [in a convention where $\sin(\beta-\alpha) \geq 0$], then

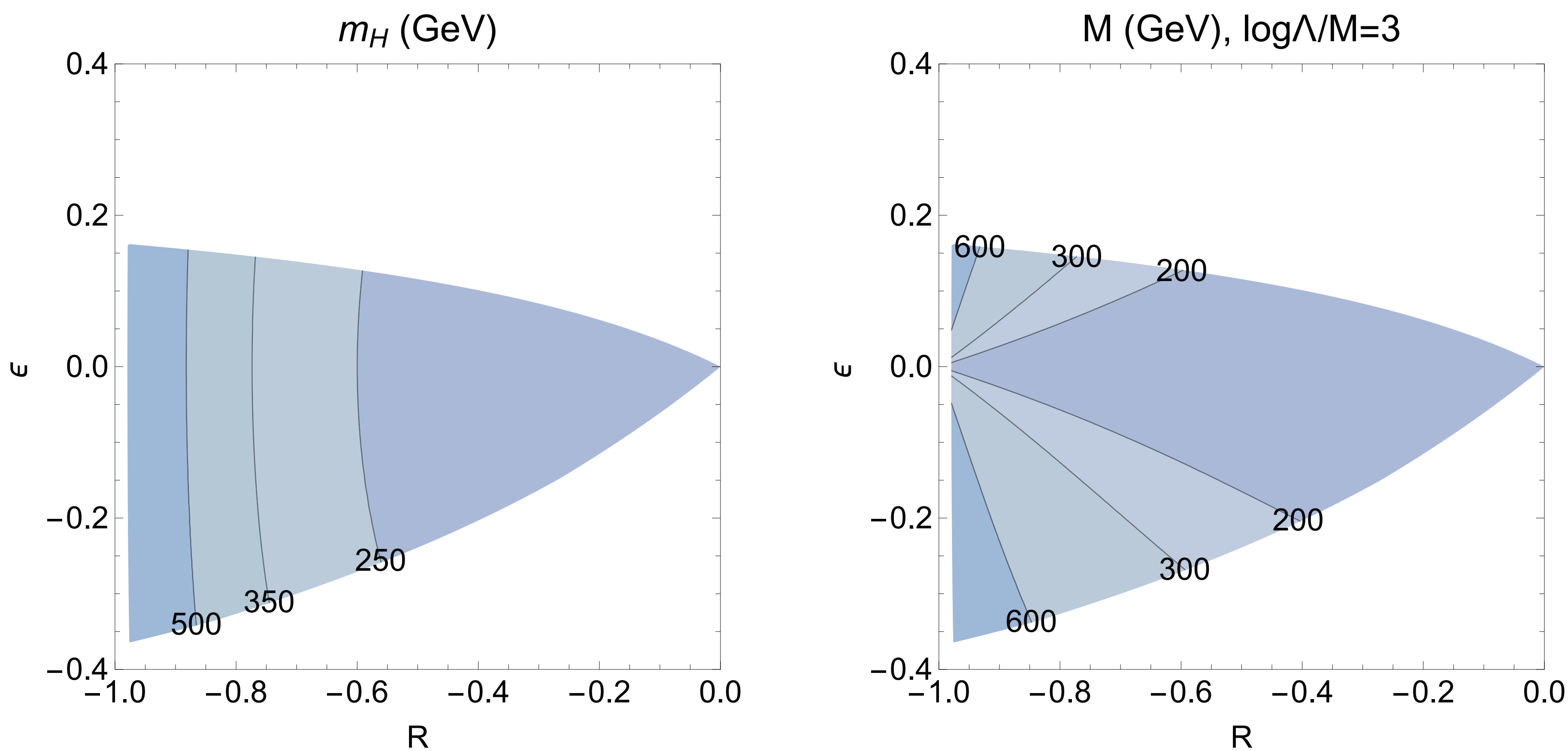
$$\cos(\beta-\alpha) \simeq -\frac{\epsilon(1-R)}{2|R|}\,.$$

In particular, the alignment limit favors small $|\epsilon|$, which yields $\tan\beta \sim \mathcal{O}(1)$.



It is convenient to rewrite m_H in terms of m_h,

$$m_H^2 = m_h^2 \left(\frac{1 + \sqrt{R^2 + (1 - R^2)\epsilon^2}}{1 - \sqrt{R^2 + (1 - R^2)\epsilon^2}} \right) \ .$$



The shaded regions are consistent with the Higgs couping fits taken from N. Craig et al., JHEP **1506**, 137 (2015).

Phenomenological constraints and implications

- Below the scale of M_U, the effective theory is that of a Type-I 2HDM.

- In the inert phase, the lightest scalar in the Φ_2 doublet is a stable dark matter candidate. There is no mixing of U with SM quarks due to the exact discrete $\mathbb{Z}_2$ symmetry. But $U \to q\Phi_2$ is a possible decay, which can be discovered in the $t\bar{t}+$missing energy channel. Current LHC limits yield $m_U \gtrsim 500$ GeV for $m_H \lesssim 150$ GeV.

- In the mixed phase, the discrete $\mathbb{Z}_2$ symmetry is broken and U can mix with the top quark. In this case $U \to Wb$, Zt and ht are possible decays. LHC experimental limits require $m_U \gtrsim 700$ GeV if no other decay modes are present. If tH and bH^+ are kinematically allowed, they will dominate and the experimental limits must be reconsidered.

- In the regime of the mixed phase where the non-minimal Higgs states have masses below 1 TeV, $\tan\beta$ is moderate, of order a few. This is a very difficult regime for the LHC. Perhaps $H \to hh$ and the production of $t\bar{t}H$, $t\bar{t}A$ and $t\bar{b}H^-$ provide the best opportunities for discovery.

The CP3 Model of Natural Alignment

The CP3-symmetric 2HDM scalar potential yields,[7]

$$m_{11}^2 = m_{22}^2\,, \quad m_{12}^2 = 0\,, \quad \lambda_1 = \lambda_2 = \lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5\,, \quad \mathrm{Im}\lambda_5 = \lambda_6 = \lambda_7 = 0\,.$$

This results in one neutral scalar $m_A^2 = -\lambda_5 v^2$ and a neutral scalar squared-mass matrix,

$$\mathcal{M}^2 \equiv \lambda_1 v^2 \begin{pmatrix} c_\beta^2 & s_\beta c_\beta \\ s_\beta c_\beta & s_\beta^2 \end{pmatrix}\,,$$

which yields one massless scalar $(m_h = 0)$ and a second scalar of squared-mass $m_H^2 = \lambda_1 v^2$. The massless scalar is to be expected, since it corresponds to a Goldstone boson of a spontaneously broken Peccei-Quinn (PQ) symmetry.

In the Higgs basis, we have $Y_1 = Y_2 = m_{11}^2 = -\frac{1}{2}\lambda_1 v^2$, $Y_3 = 0$ and

$$Z_1 = Z_2 = Z_{345} = \lambda_1\,, \quad Z_i = \lambda_i \quad (\text{for } i = 3, 4, 5)\,, \quad \mathrm{Im} Z_5 = Z_6 = Z_7 = 0\,.$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re} Z_5$.

[7] F. D'Eramo, P. Draper, and H.E. Haber, in preparation.

It is more convenient to analyze this model in a different basis in which the U(1)$_{\rm PQ} \otimes \Pi_2$ symmetry is manifest. This yields.

$$m_{11}^2 = m_{22}^2\,, \quad m_{12}^2 = 0\,, \quad \lambda_1 = \lambda_2 \neq \lambda_3 + \lambda_4\,, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0\,,$$

which yields

$$Z_1 = Z_2 = Z_{345} = \tfrac{1}{2}(\lambda_1 + \lambda_3 + \lambda_4)\,, \qquad {\rm Im}Z_5 = Z_6 = Z_7 = 0\,,$$
$$Z_5 = \tfrac{1}{2}(\lambda_3 + \lambda_4 - \lambda_1)\,,$$

and

$$Y_1 = Y_2 = m_{11}^2 = -\tfrac{1}{4}v^2(\lambda_1 + \lambda_3 + \lambda_4)\,, \qquad Y_3 = 0\,.$$

It then follows that $m_A = 0$ and the two other scalar squared-masses are

$$m_{H,h}^2 = \tfrac{1}{2}v^2\left[\lambda_1 \pm (\lambda_3 + \lambda_4)\right]\,.$$

That is the SM Higgs boson has mass $m_h^2 = Z_1 v^2$, and the two other neutral boson squared masses are 0 and $|Z_5|v^2$.

To avoid the massless scalar, the $\mathrm{U}(1)_{\mathrm{PQ}}$ must be softly broken. Indeed, if we allow arbitrary soft-breaking of the CP3-symmetric scalar potential, then $m_{11}^2 \neq m_{22}^2$ and $m_{12}^2 \neq 0$. It is convenient to write,

$$m_{11}^2 = m^2 - \tfrac{1}{2}\Delta m^2 \,, \qquad\qquad m_{22}^2 = m^2 + \tfrac{1}{2}\Delta m^2 \,.$$

Then,

$$\begin{aligned} Y_1 &= m^2 - \tfrac{1}{2}\Delta m^2 c_{2\beta} - m_{12}^2 s_{2\beta} \\ Y_2 &= m^2 + \tfrac{1}{2}\Delta m^2 c_{2\beta} + m_{12}^2 s_{2\beta} \,, \\ Y_3 &= \tfrac{1}{2}\Delta m^2 s_{2\beta} - m_{12}^2 c_{2\beta} \,. \end{aligned}$$

After imposing the the scalar potential minimum conditions, it follows that $Y_1 = -\frac{1}{2}Z_1 v^2$ and $Y_3 = 0$. The latter condition fixes the angle β and yields

$$\tan\beta = \frac{\sqrt{(\Delta m^2)^2 + (2m_{12}^2)^2} - \Delta m^2}{2m_{12}^2} \,,$$

in a convention in which $\tan\beta$ is real and non-negative.

The resulting neutral scalar mass spectrum is:

$$m_A^2 = \sqrt{(\Delta m^2)^2 + (2m_{12}^2)^2}\,,$$
$$m_{h,H}^2 = \left\{ Z_1 v^2\,,\ \sqrt{(\Delta m^2)^2 + (2m_{12}^2)^2} + |Z_5|v^2 \right\},$$

which can be rewritten as:

$$m_{h,H,A}^2 = \left\{ Z_1 v^2\,,\ Y_2 + \tfrac{1}{2}(Z_3 + Z_4 + |Z_5|)\,,\ Y_2 + \tfrac{1}{2}(Z_3 + Z_4 - |Z_5|) \right\}.$$

Note that even with the most general soft-CP3-breaking squared mass terms, the resulting scalar sector still respects $Y_3 = Z_6 = Z_7 = 0$, i.e. it corresponds to the inert phase.

Remark: In the inert phase, the CP quantum numbers of H and A are not individually well-defined, although they are *relatively* odd with respect to CP.

The CP3-symmetric 2HDM with mirror fermions

We introduce mirror fermions as before. In the top sector,

$$\mathbb{Z}_2^m: \; q \Longleftrightarrow q, \quad u \Longleftrightarrow U \quad \bar{U} \Longleftrightarrow \bar{U}$$

$$\mathrm{U}(1)_{\mathrm{PQ}}: \; q \Longrightarrow q, \quad u \Longrightarrow e^{-i\theta} u, \quad U \longleftrightarrow e^{i\theta} U, \quad \bar{U} \Longrightarrow e^{-i\theta} \bar{U}$$

The corresponding Yukawa couplings take the form,

$$V \supset y_t \left(q\Phi_2 u + q\Phi_1 U \right) + \mathrm{h.c.}$$

We again introduce a vectorlike mass,

$$\mathscr{L}_{\mathrm{mass}} \supset M_U U \overline{U} + \mathrm{h.c.}$$

which preserves the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry but explicitly breaks the Π_2 discrete symmetry. This symmetry breaking is soft, so that $m_{22}^2 - m_{11}^2$ is protected from quadratic sensitivity to the cutoff scale Λ.

The other SM fermions can also be included by introducing the appropriate mirrors such that

$$\mathbb{Z}_2^m: \; d \Longleftrightarrow D, \quad \ell \Longleftrightarrow \ell, \quad e \Longleftrightarrow E, \quad \bar{D} \Longleftrightarrow \bar{D}, \quad \bar{E} \Longleftrightarrow \bar{E}$$

$$\mathrm{U}(1)_{\mathrm{PQ}}: \; d \Longrightarrow e^{i\theta} d, \quad \ell \Longrightarrow \ell, \quad e \Longrightarrow e^{i\theta} e, \quad D \Longrightarrow e^{-i\theta} D, \quad \bar{D} \Longrightarrow e^{i\theta} D,$$

$$E \Longrightarrow e^{-i\theta} E, \quad \bar{E} \Longrightarrow e^{i\theta} E.$$

The corresponding Yukawa couplings and vectorlike fermion masses are

$$V \supset y_b \left(q\Phi_2^* d + q\Phi_1^* D\right) + y_\tau \left(\ell\Phi_2^* e + \ell\Phi_1^* E\right) + M_D D\bar{D} + M_E E\bar{E}.$$

Due to the soft-breaking of Π_2, we again generate $\Delta m^2 = m_{22}^2 - m_{11}^2 \neq 0$ and $\Delta\lambda \equiv \lambda_1 - \lambda_2 \neq 0$. We propose to break the U(1)$_{\mathrm{PQ}}$ symmetry by introducing a complex singlet S, and adding a term

$$-\mathscr{L}_S = \lambda_s(|S|^2 - \mu^2)^2 + (\kappa S^2 \Phi_1^\dagger \Phi_2 + \mathrm{h.c.})$$

to the scalar sector Lagrangian. After integrating out S, one obtains an effective m_{12}^2 parameter.

As noted previously, including non-zero Δm^2 and m_{12}^2 alone still respects $Y_3 = Z_6 = Z_7 = 0$, corresponding to the inert phase. However, one also generates a non-zero $\Delta\lambda$ due to the soft-breaking of Π_2 and a non-zero λ_5 due to the soft-breaking of U(1)$_{\rm PQ}$. As a result, the local minima of the corresponding 2HDM potential will yield a solution corresponding to the mixed phase, thereby producing a departure from the Higgs alignment limit.

The numerical analysis of this case is currently underway, but it is not yet complete (so no results are shown here). We anticipate smaller departures from the alignment limit as compared with the CP2 model considered previously since the corrections to $\Delta\lambda$ and λ_5 are insensitive to the cutoff Λ and depend logarithmically on the mass of the mirror top, M_U.