Naturalness and the Weak Gravity Conjecture

Grant N. Remmen

Walter Burke Institute for Theoretical Physics
California Institute of Technology

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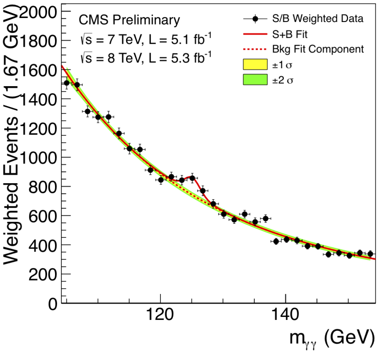
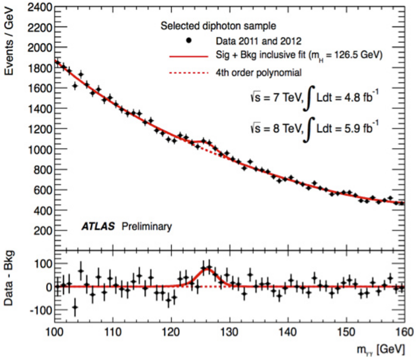
Naturalness

The Challenge Confronting Naturalness

A new scalar has been discovered at the LHC.





Where are the “naturalons”?

Old & New Ideas

- The principle of **naturalness** is the requirement that operators not protected by a symmetry are unstable to quantum corrections induced at the cutoff of the effective field theory.
- This means that the numerical coefficients are $\mathcal{O}(1)$.
  - No delicate cancellation allowed without additional symmetry (e.g., SUSY).
- Naturalness has been a motivator of new physics for decades.
- Currently being revisited:
  - regulator tricks
  - modified naturalness, ultraviolet conformal symmetry
  - meso-tuning

The Weak Gravity Conjecture

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, & Vafa (hep-th/0601001, JHEP)

- The **Weak Gravity Conjecture (WGC)** is an ultraviolet consistency condition for quantum gravity.
- **WGC statement:** For any Abelian gauge theory coupled to quantum gravity, there exists a state in the spectrum with charge q and mass m such that

$$\frac{q}{m} > \frac{1}{m_{\mathrm{Pl}}}.$$

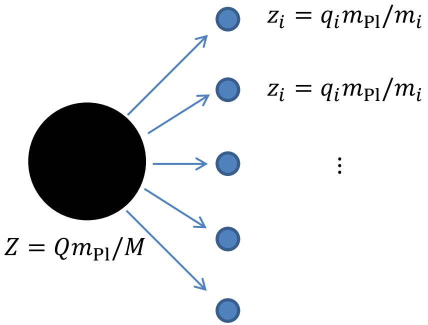
- In other words, "gravity is the weakest force."
- **Low-Cutoff Conjecture:** There is a low cutoff scale

$$\Lambda \sim q m_{\mathrm{Pl}},$$

at which 4D QFT breaks down completely.

Evidence from Black Holes

A black hole decay thought experiment:



- Theory with spectrum $\{q_i, m_i\}$.
- Black hole decaying to one species i via Hawking or Schwinger process.
- Charge conservation: Q/q_i particles produced.
- Energy conservation: $M > m_i Q/q_i$.
- Decay requires $z_i > Z$.

Evidence from Black Holes, continued

- For extremal black holes ($Z = 1$) to decay, there must be a particle in the spectrum with $z_i > 1$. $\implies$ WGC.
- Stable black holes $\implies$ *very large number* of stable states in the theory.
  - Thermodynamic pathologies
  - Virtual black hole loops in Feynman diagrams
  - Tension with holography

Other Evidence for the WGC

- Arkani-Hamed et al. supported the WGC with a host of examples in field theory and string theory.
  - $SU(2) \to U(1)$ gauge theory: W-bosons and monopoles
  - States in $SO(32)$ heterotic string theory
  - Problems with DGP gravity and embedding extranatural inflation in string theory
- If no WGC, then the $q \to 0$ limit of a gauge theory yields an exact global symmetry.
  - Conflict with no-hair theorems: black holes labeled only by overall mass, spin, and charge.

The Limits of Naturalness

Scalar QED: Naturalness vs. Weak Gravity

- Simple example: scalar $U(1)$-charged particle with a hierarchy problem:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu \phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4,$$

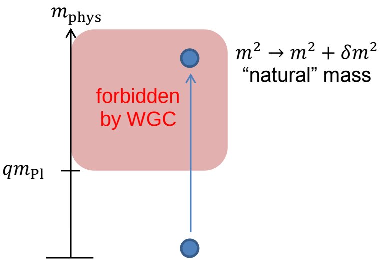
where $D_\mu = \partial_\mu + iqA_\mu$.

- WGC: $q(\mu) > m(\mu)/m_{\rm Pl}$ — running with renormalization scale:
  - $q(\mu)$ logarithmically
  - $m(\mu)$ quadratically
  - Presages tension with naturalness. Evaluate μ at physical mass.
- Loop corrections to mass: $m^2 \to m^2_{\rm phys} = m^2 + \delta m^2$, where

$$\delta m^2 = \frac{\Lambda^2}{16\pi^2}\left(aq^2 + b\lambda\right).$$

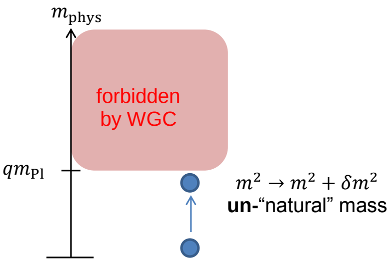
- Naturalness: a and b are incalculable, $\mathcal{O}(1)$ coefficients.

Scalar QED: Naturalness vs. Weak Gravity



- Technically natural region of parameter space: $q^2 \ll \lambda$.
- Then naturalness $\implies \delta m^2 \gg$WGC bound.
- Conflict between ultraviolet consistency and low-energy effective field theory!

Scalar QED: Naturalness vs. Weak Gravity



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Quantifying the Tension

Charge-to-mass ratio for ϕ:

$$z = \frac{4\pi m_{\rm Pl}}{\Lambda} \frac{1}{\sqrt{a + b\lambda/q^2}} > 1$$

so

$$\Lambda < \begin{cases} \frac{4\pi m_{\rm Pl}}{\sqrt{a}}, & q^2 \gg \lambda \\ 4\pi m_{\rm Pl}\sqrt{\frac{q^2}{b\lambda}}, & q^2 \ll \lambda \end{cases}.$$

- $q^2 \gg \lambda \implies$ sub-Planckian cutoff (reasonable).
- $q^2 \ll \lambda \implies$ mandatory, quantifiable fine-tuning to satisfy the WGC.

Small coupling q is technically natural: q runs only logarithmically.

Options for Reconciliation

How can naturalness and the WGC be reconciled in a theory containing charged scalars?

- Forbid coupling hierarchies?
  - Perhaps $q^2 \ll \lambda$ is strictly forbidden (e.g. SUSY D-terms), though SUSY is not enough to ensure this.
- Better options:
  1. Higgs phase
  2. New physics below the Planck scale

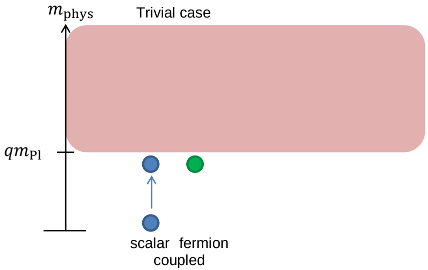
Higgsing the Theory

- If the mass term $m^2|\phi|^2$ becomes tachyonic after loop corrections, the gauge field A_μ acquires a mass.
- The original WGC argument of Arkani-Hamed et al. was: "no stable extremal black holes" $\implies$ WGC.
- No-hair theorems: No stationary black hole solutions supporting classical hair from a massive photon.
    - A black hole charged under a massive $U(1)$ becomes "bald" on timescale of order $1/m_\gamma$.
- So the WGC is not justified for a Higgsed $U(1)$.

New Physics

Simplest and most interesting option for reconciling the WGC and naturalness in scalar QED: **Introduce new physics!**

- If the EFT has cutoff at $\Lambda \sim q m_{\rm Pl}$, then the WGC can still be satisfied.
- This is exactly the cutoff conjectured by Arkani-Hamed et al.
- Here, we see the first known effective-field-theoretic evidence for the weak interpretation of the low-cutoff conjecture.



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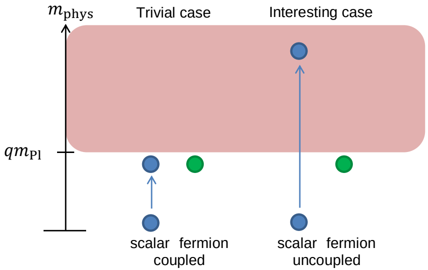


More Forces, More Particles

More Forces, More Particles

To be more realistic, we'd like to generalize the WGC to a much broader class of theories than a single $U(1)$.

Consider:

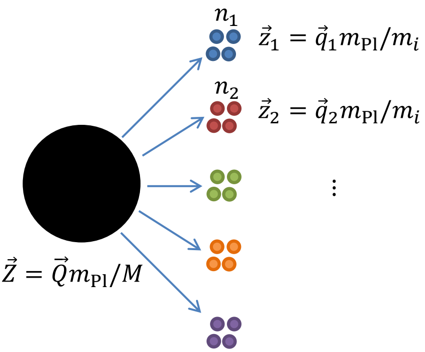
- A product gauge group $\Pi_{a=1}^{N} U(1)_a$ with N Abelian factors.
- States i
  - masses m_i
  - charges $q_{ia} \equiv \vec{q}_i$, vectors of $SO(N)$
  - charge-to-mass vectors $\vec{z}_i = q_{ia} m_{\text{Pl}}/m$

What is the correct justification of the WGC?

- At least one species i with $|\vec{z}_i| > 1$? Not sufficient!
- A $SO(N)$ basis of such species? Not sufficient!
- True generalized WGC is even stronger!

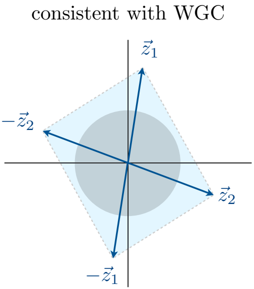
Back to Black Hole Kinematics

A black hole decay thought experiment:



- Theory with spectrum $\{\vec{q}_i, m_i\} \to \vec{z}_i$.
- Black hole decaying to n_i particles of species i.
- Charge conservation: $\vec{Q} = \sum_i n_i \vec{q}_i$.
- Energy conservation: $M > \sum_i n_i m_i$.
- Decay requires $\vec{Z} = \sum_i \sigma_i \vec{z}_i$, where $\sigma_i = n_i m_i / M$ and $\sum_i \sigma_i < 1$.

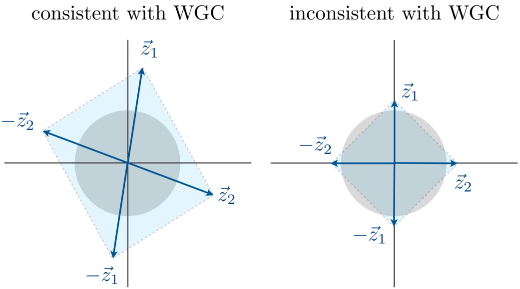
Geometrical Picture



We can understand the generalized WGC bound geometrically:

- Draw charge-to-mass vectors $\pm\vec{z}_i$ for all particles in the theory
- Unit ball $|\vec{Z}| \leq 1$: all black hole states
- The convex hull spanned by $\vec{z}_i$ must contain the unit ball

Geometrical Picture



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A Stronger WGC Bound

The generalized WGC bound is parametrically stronger than the bound for a single $U(1)$.

- Example: $U(1)^2$, with states $\vec{z}_1$ and $\vec{z}_2$ requires

$$\left(\vec{z}_1^2 - 1\right)\left(\vec{z}_2^2 - 1\right) > \left(1 + |\vec{z}_1 \cdot \vec{z}_2|\right)^2 .$$

- For $U(1)^N$ with $z_{ia} = \delta_{ia} z$, we require $z > \sqrt{N}$.
- Considering an N-charge theory of N scalars, where

$$\delta m_i^2 = \frac{\Lambda^2}{16\pi^2}\left(a_i \vec{q}_i^2 + b_i \lambda_i\right),$$

we require

$$\Lambda < \frac{4\pi m_{\text{Pl}}}{\mathcal{O}(\sqrt{N})} \times \begin{cases} \frac{1}{\sqrt{a_i}}, & \vec{q}_i^2 \gg \lambda_i \quad [\text{sub} - \text{Planckian cutoff}] \\ \sqrt{\frac{\vec{q}_i^2}{b_i \lambda_i}} & \vec{q}_i^2 \ll \lambda_i \quad [\text{tension with naturalness}] \end{cases} .$$

The Hierarchy Problem

The Standard Model Hierarchy Problem

- We've seen that naturalness can contradict the WGC.
- Let's now apply this tension to address the electroweak scale.
  - In the SM, the Higgs mass gets loop corrections that make $v \sim 246 \text{ GeV}$ unnatural.
  - Extra symmetry, SUSY, usually invoked to avoid this.
  - We'll try to just use the WGC and as little new low-energy physics as possible.
- It's tempting to charge the Higgs...
  - We can't do this, since that would give the photon a mass!

Example Model for the SM Hierarchy Problem

- Charge SM fermions under unbroken $U(1)_{B-L}$ Abelian gauge symmetry.
- To cancel anomalies, add right-handed neutrino ν_{R}, with Dirac mass $m_\nu \sim y_\nu v \lesssim 0.1 \text{ eV}$:

$$m_\nu \bar{\nu}_{\mathrm{L}} \nu_{\mathrm{R}} + \text{h.c.}$$

- Set q very small:

$$q \sim m_\nu / m_{\mathrm{Pl}} \sim 10^{-29} \qquad \text{(technically natural)}$$

- Then the lightest neutrino has the largest z and just marginally satisfies the WGC.
- At fixed Yukawa coupling y_ν, a heavier, more natural, electroweak scale v is forbidden!

Discussion

- This model is a proof of concept, but it does have a **prediction**: a new, weakly coupled massless gauge boson.
- Not ruled out by torsion balance and equivalence principle tests:

$$q \lesssim 10^{-24}$$

- May be probed in future!
- Makes the naturalness principle **directly experimentally testable**.
- Assuming naturalness in this theory requires Higgs phase or low cutoff $\Lambda \lesssim \text{keV}$.
- Given predictions in the string landscape, such a fifth-force discovery would also falsify string theory.
- Can be adapted to other models to solve the SM hierarchy problem: charge weak-scale dark matter with $q \sim 10^{-16}$ under a $U(1)$ dark force.

Conclusions

Summary

- We showed that ultraviolet consistency can be at odds with naturalness.
- We showed that certain natural parameter regions of scalar QED are actually in the swampland.
- We extended the WGC to generic theories.
- We exhibited models in which the SM hierarchy is mandated by the WGC.
    - Lesson: Big hierarchies can be misleading without knowledge of the UV completion!

Upcoming Work — arXiv:14xx.xxxx

- The WGC is still a **conjecture**.
- All current evidence is either:
  - Specific examples in string theory
  - UV-dependent reasoning (black hole remnants, etc.)
- We'd like to understand the WGC from a low-energy effective field theory perspective.
- We're developing WGC-like bounds on q/m by observing violation of
  - analyticity
  - causality
  - unitarity

in the effective photon-graviton theory if the WGC fails.