

# One-Loop Flavour Observables and Two-Loop Higgs Masses for *free*

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Warsaw, 04./05. May 2015

## Disclaimer

The tools I'm speaking about are **not restricted to supersymmetric models**. Nevertheless, I'll focus mainly on supersymmetry!

# Outline

## Today:

1. Introduction: there is life beyond the MSSM
2. The SARAH framework to explore models
3. Flavour observables at one-loop

## Tomorrow:

4. Higgs masses at two-loop

# Supersymmetry

Supersymmetry is one of the best studied extensions of the SM.

## Appealing features

- ▶ Solves the hierarchy problem
- ▶ Predicts gauge coupling unification
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The **focus** was usually on the **MSSM**

## Public tools

Widely used **SUSY tools** (SoftSUSY, Suspect, Superiso, Susy\_Flavor, FeynHiggs,...) are **restricted** to the **MSSM** (and a few extensions).

## Reasons to look beyond the MSSM

- ▶ **Higgs mass/Naturalness** →  $F$ - or  $D$ -term enhanced tree mass?
- ▶ **Missing signals for SUSY at LHC**  
→ compressed spectra?  $R$ -parity violation? split-SUSY? ...
- ▶ **Neutrino masses** →  $R$ -parity violation? Seesaw mechanism?
- ▶ **The  $\mu$  problem** → effective  $\mu$  term?
- ▶ **Strong CP problem** → (gauged?) Peccei-Quinn symmetry?
- ▶  **$R$  symmetry** → Dirac Gauginos?
- ▶ **GUT/String model** → extended gauge sector?  $Z'$ ,  $W'$  in reach?
- ▶ ...

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### Generic SUSY tools needed

To **confront many models** with experimental data (e.g. **Higgs mass measurement**, **flavour observables**, **dark matter relic density**) a high level of **automatization** is needed.

# The SARAH framework to explore models

A large, light blue watermark logo is positioned on the left side of the slide. It features a stylized letter 'S' with a gear-like border and an arrow pointing to the right.



# SARAH

## SARAH

[FS,0806.0538,0909.2863,1002.0840,1207.0906,1309.7223,1503.04200]

**SARAH** is a Mathematica package to get from **a minimal input** all important properties of **SUSY** and **non-SUSY models**. Models are **defined** by

- ▶ gauge & global symmetries
- ▶ particle content
- ▶ (super)potential

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  - ▶ particle content
  - ▶ (super)potential
- 
- ▶ All **gauge (and gaugino) interactions** are **automatically** derived from quantum numbers
  - ▶ **Gauge fixing** terms in  $R_\xi$  gauge are **automatically** derived
  - ▶ **Soft SUSY breaking terms** are added **automatically**
- $(m^2 \phi \phi^*, M_\lambda \lambda \lambda, T \phi_i \phi_j \phi_k, B \phi_i \phi_j, L \phi_i)$

## Supported models

### Matter and gauge sector

The gauge sector can consist of an **arbitrary number of groups** and **all irreducible representations** can be used for matter fields.

---

<sup>1</sup>Susyino supports also  $SO(N)$ ,  $SP(2N)$ ,  $E_{6,7,8}$ ,  $G_2$ ,  $F_4$ , but the link is not yet well tested

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### Supported gauge groups

- ▶ Non-Abelian groups:  
SARAH links **SusyNo** to calculate **Clebsch-Gordan** coefficients, **generators** and **Casimir/Dynkin** for **non-fundamental irreps** of unbroken  $SU(N)$  <sup>1</sup> [Fonseca,1106.5016]
- ▶ Abelian groups:  
**kinetic mixing** for arbitrary numbers of  $U(1)$ 's included

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## Consistency check of a model

SARAH performs several checks

### Physical properties

- ▶ Check for gauge and Witten **anomalies**
- ▶ Check if all terms in the (super)potential are in agreement with **charge conservation**
- ▶ Check if **other (renormalizable) terms allowed** in the (super)potential by (gauge) symmetries
- ▶ Check if **other particles might mix**
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Also **formal checks** (syntax, self-consistency, ...) of the **implementation in SARAH** are done.

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- ▶ all Tadpole equations, Masses and Mass matrices
- ▶ all Vertices

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#### General QFT: Full CP and flavour structure

[Luo,Wang,Xiao,hep-ph/0211440]

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[Fonseca,Malinsky,FS,1308.1674]

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The **analytical expressions** derived by SARAH can be **exported**:

## Model files for Monte Carlo Tools

- ▶ **CalcHep/CompHep** (can be used with **MicrOmegas**)

[Pukhov et al.],[Boos et al.],[Belanger et al.]

- ▶ **WHIZARD**

[Kilian,Ohl,Reuter,0708.4233],[Moretti,Ohl,Reuter,0102195]

- ▶ **MadGraph & Herwig++ via UFO** [Alwall et al.,1106.0522], [Bellm et al.,1310.6877]

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## Interface to other tools

SARAH writes also model files

- ▶ [FeynArts/FormCalc](#)

[Hahn,hep-ph/0012260],[Hahn,Victoria,hep-ph/9807565]

- ▶ [Vevacious](#)

[Camargo-Molina,O'Leary,Porod,FS,1307.1477]

and generates Fortran code for

- ▶ [SPheno](#)

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## Interplay: Vevacious

Vevacious ...

[Camargo-Molina,O'Leary, Porod, FS,1307.1477]

...is a tool to find the **global minimum** of the **1-loop effective potential** and checks the stability of the 'correct' vacuum.

- ▶ Written in Python and C++; includes LHPC [O'Leary]
- ▶ Makes use of HOM4PS, pyminuit and CosmoTransitions [Lee, Yi, Tsai,Computing, 83, pp109-133],[Wainwright,1109.4189]]

[vevacious.hepforge.org](http://vevacious.hepforge.org)

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If you use it, you'll see that thumb rules like

$$A_t^2 < 3(m_{H_u}^2 + m_{t_l}^2 + m_{t_R}^2)$$

are not sufficient to identify CCB vacua in the MSSM! [1309.7212,1405.7376]

## SARAH and SPheno

'Spectrum Generator Generator'

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→ Implementation of new models in SPheno in a modular way  
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Features of 'SPheno by SARAH' versions

- ▶ Full 2-loop running of all parameters and all masses at 1-loop
- ▶ Complete 1-loop thresholds at  $M_Z$
- ▶ two-loop corrections to Higgs masses
- ▶ calculation of flavour and precision observables
- ▶ calculation of decay widths and branching ratios
- ▶ interface to HiggsBounds and HiggsSignals
- ▶ estimate of electroweak Fine-Tuning

## SARAH and SPheno

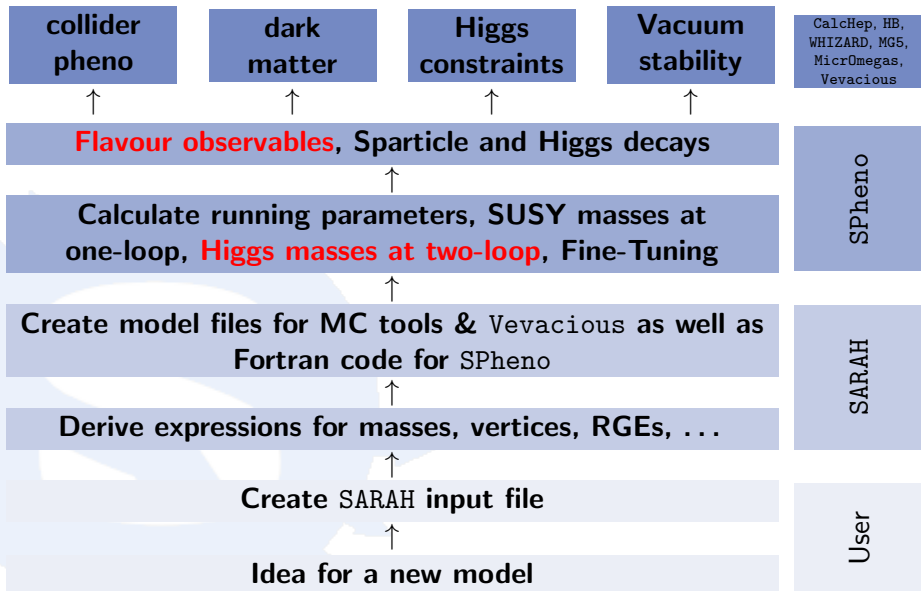
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# Flavour observables at one-loop

in collaboration with  
Werner Porod & Avelino Vicente

## Calculation of Flavour observables in a nutshell

To calculate flavour observables in a given model one needs

1. Expressions for vertices and masses
2. Expressions for Wilson coefficients<sup>2</sup>
3. Expressions for observables
4. Numerical values for everything

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Let's combine the different tools!

---

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# FlavorKit

[Porod,FS,Vicente,1405.1434]

1. SARAH calculates the necessary **vertices & masses** and includes them in the SPheno output
2. SPheno provides routines for the **numerical** evaluation of **Passarino-Veltman integrals**
3. The necessary **expressions** for the **form factors** and **observables** are still needed



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### FlavorKit

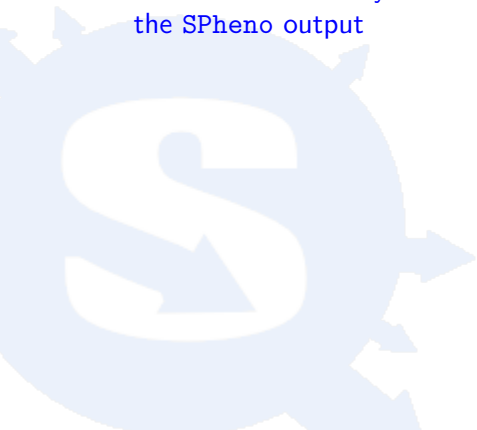
The calculation of flavour observables is **based on external files** parsed by SARAH which ...

- ▶ ... provide the **generic expressions of form factors** (function of masses, vertices, loop integrals)
- ▶ ... the **formulae for the observables** (function of form factors, masses, (hadronic) parameters, constants)

## New observables

To calculate **new observables** the user has to provide **two files**

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Both files have to be put into the **FlavorKit subdirectory** of SARAH

→ The **observables** are **included automatically** in the SPheno output.

## Example $l \rightarrow l_j \gamma$ : Steering file

The Steering file reads

```

1 NameProcess = "LLpGamma";
2
3 NameObservables = {{muEgamma, 701, "BR(mu->e gamma)"},
4                   {tauEgamma, 702, "BR(tau->e gamma)"},
5                   {tauMuGamma, 703, "BR(tau->mu gamma)"} };
6
7 NeededOperators = {K2L, K2R};
8
9 Body = "LLpGamma.f90";
    
```

**K2L, K2R** are the coefficients of the **dipole operator**

$$\mathcal{L}_{ll\gamma} = e \bar{\ell}_\beta \left[ i m_{\ell_\alpha} \sigma^{\mu\nu} q_\nu \left( K_2^{L,\beta\alpha} P_L + K_2^{R,\beta\alpha} P_R \right) \right] \ell_\alpha A_\mu + h.c.$$

which are known by SARAH.

## Example $l \rightarrow l_j \gamma$ : Fortran file

```

1 Real(dp) :: width
2 Integer :: i1, gt1, gt2
3
4 Do i1=1,3
5   If (i1.eq.1) Then           ! mu -> e gamma
6     gt1 = 2
7     gt2 = 1
8   Elseif (i1.eq.2) Then      ! tau -> e gamma
9     ...
10  End if
11
12  width=0.25_dp*mf_l(gt1)**5*(Abs(K2L(gt1,gt2))**2 &
13    & +Abs(K2R(gt1,gt2))**2)*Alpha
14
15  If (i1.eq.1) Then
16    muEgamma = width/(width+GammaMu)
17  Elseif (i1.eq.2) Then
18    ...
19  End if
20 End do
    
```

## Example $l \rightarrow l_j \gamma$ : Result

After running SARAH and compiling the SPheno module the spectrum files produced by SPheno include the new observable:

```

1 # SUSY Les Houches Accord 2 – NMSSM
2 # SPheno module generated by SARAH
3 ...
4 Block FlavorKitLFV # lepton flavor violating observables
5     701     1.61451131E-14 # BR(mu->e gamma)
6     702     5.67628390E-16 # BR(tau->e gamma)
7     703     2.15514014E-17 # BR(tau->mu gamma)
8     ...
    
```

## Coefficients of new operators

Input files for form factors look much more complicated:

```

1  Switch [prop ,
2  V, (* Vector penguins *)
3  Switch [top, (* Check topology *)
4  1,
5  Switch [type, (* Check the generic type of the diagram *)
6  SFF,
7  WriteString [file , " int1=B0(0._dp, mF12, mF22)\n" ];
8  WriteString [file , " int2=C0(mF22, mF12, mS12)\n" ];
9  WriteString [file , " int3=C0(mF22, mF12, mS12)\n" ];
10 WriteString [file , " PVOddIIVRR=PVOddIIVRR+ ↔
    ↔ chargefactor*coup1R*coup2L*coup4R*IMP2* ↔
    ↔ (-1.*coup3R*int3*mF1*mF2 + coup3L*(int1 - ↔
    ↔ 2.*int2 + int3*mS12))\n" ];
11 ...
    
```

(the files for  $(\bar{d}\Gamma d)(\bar{\ell}\Gamma\ell)$  have about 5000 lines like this)



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→ Nothing you want to implement by hand!

## PreSARAH

The [generic expressions](#) for the [coefficients of new operators](#) can be calculated with an [additional package \(PreSARAH\)](#):

- ▶ Easy way to define operators and colour flow
- ▶ Uses [FeynArts/FormCalc](#) to calculate [generic expressions](#)
- ▶ Writes [all necessary files](#) for SARAH

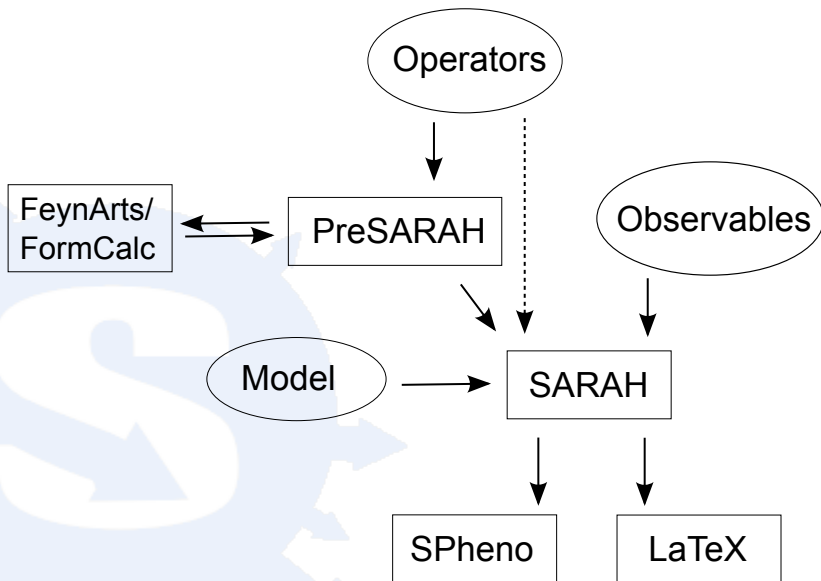
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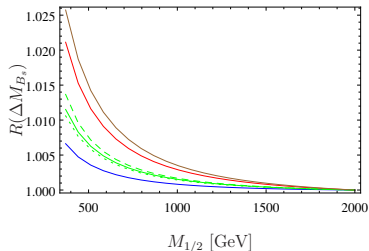
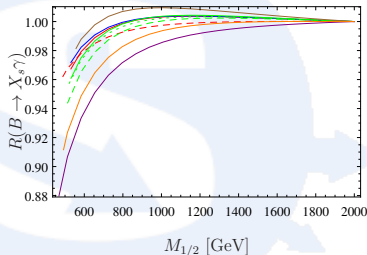
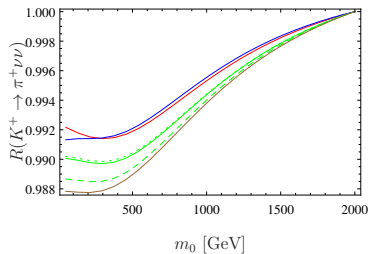
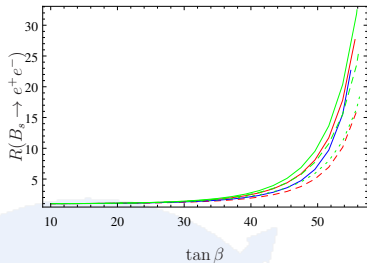
1 NameProcess="2d2L";
2
3 ConsideredProcess = "4Fermion";
4 FermionOrderExternal={2,1,4,3};
5 NeglectMasses={1,2,3,4};
6
7 ExternalFields= {DownQuark, bar[DownQuark],
8                  ChargedLepton, bar[ChargedLepton]};
9
10 AllOperators={{OddIIISLL, Op[7].Op[7]}, (* [d PL d][l PL l] *)
11              {OddIISRL, Op[6].Op[7]}, (* [d PR d][l PL l] *)
12              ...
13 };
    
```



## Implemented observables

We made use of this to (re-) implement in SARAH

- ▶  $\text{Br}(l_i \rightarrow l_j \gamma)$ ,  $\text{Br}(l \rightarrow 3l')$ ,  $\text{Br}(Z \rightarrow ll')$
- ▶  $\text{CR}(\mu - e, N)$  ( $N=\text{Al, Ti, Sr, Sb, Au, Pb}$ ),  $\text{Br}(\tau \rightarrow l + P)$  ( $P=\pi, \eta, \eta'$ )
- ▶  $\text{Br}(B \rightarrow X_s \gamma)$ ,  $\text{Br}(B_{s,d}^0 \rightarrow l\bar{l})$ ,  $\text{Br}(B \rightarrow sl\bar{l})$ ,  $\text{Br}(K \rightarrow \mu\nu)$
- ▶  $\text{Br}(B \rightarrow q\nu\nu)$ ,  $\text{Br}(K^+ \rightarrow \pi^+ \nu\nu)$ ,  $\text{Br}(K_L \rightarrow \pi^0 \nu\nu)$
- ▶  $\Delta M_{B_s, B_d}$ ,  $\Delta M_K$ ,  $\epsilon_K$ ,  $\text{Br}(B \rightarrow K \mu \bar{\mu})$
- ▶  $\text{Br}(B \rightarrow l\nu)$ ,  $\text{Br}(D_s \rightarrow l\nu)$



FlavorKit, SPhenoMSSM (dashed), SPheno 3.3, SUSY\_Flavor 1,  
 SUSY\_Flavor 2, MicrOmegas, SuperIso

# LFV in low-scale Seesaw models

[Abada,Krauss,Porod,FS,Vicente,Weiland,1408.0138]

## inverse Seesaw

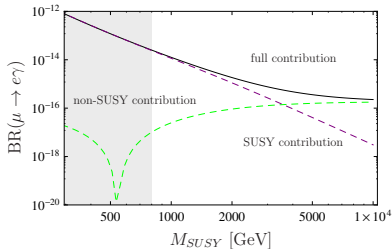
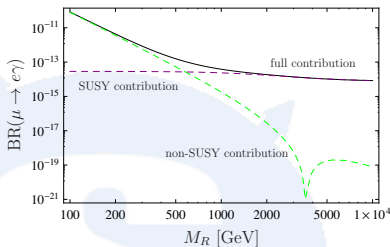
MSSM extended by 3 generations of **right-handed neutrinos** ( $\widehat{\nu}^C$ ) and **gauge singlets** ( $\widehat{X}$ )

$$W = W_{\text{MSSM}} + \varepsilon_{ab} Y_{\nu}^{ij} \widehat{\nu}_i^C \widehat{L}_j^a \widehat{H}_u^b + M_{R_{ij}} \widehat{\nu}_i^C \widehat{X}_j + \frac{1}{2} \mu_{X_{ij}} \widehat{X}_i \widehat{X}_j .$$

→ **Neutrino masses**  $M_{\nu} \simeq \frac{v_u^2}{2} Y_{\nu}^T M_R^{T-1} \mu_X M_R^{-1} Y_{\nu} ,$

$$\mu \rightarrow e\gamma$$

$$m_0 = M_{1/2} = 1 \text{ TeV}, A_0 = -1.5 \text{ TeV}, M_R = 2 \text{ TeV}, \tan \beta = 10, \mu > 0$$

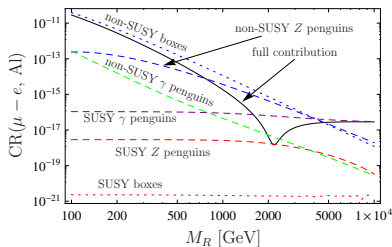
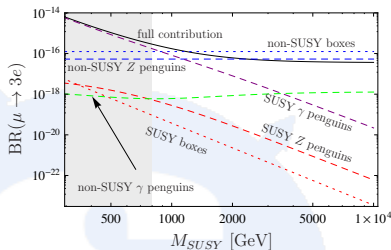


- ▶ Limits:  $5.7 \times 10^{-13}$  (present),  $6 \times 10^{-14}$  (future)
- ▶ light right-handed neutrinos can give dominant contributions
- ▶ Dependence of non-SUSY contributions on SUSY scale because of charged Higgs mass



### $\mu \rightarrow 3e$ and $\mu$ - $e$ conversion

$$m_0 = M_{1/2} = 1 \text{ TeV}, A_0 = -1.5 \text{ TeV}, M_R = 2 \text{ TeV}, \tan \beta = 10, \mu > 0$$



#### ► Limits

- $\mu \rightarrow 3e$ :  $1.0 \times 10^{-12}$  (present),  $10^{-16}$  (future)
- $\text{CR}(\mu - e, \text{Al})$ :  $10^{-15}$ – $10^{-18}$  (future)
- non-SUSY **box** contributions can **dominate**
- **Higgs** penguins contributions usually **negligible**

## Summary of Part I

- ▶ I have presented a **framework** to study **non-minimal SUSY models** based on the Mathematica package **SARAH**
- ▶ **Many aspects** needed to study BSM models **are automatized**



## Summary of Part I

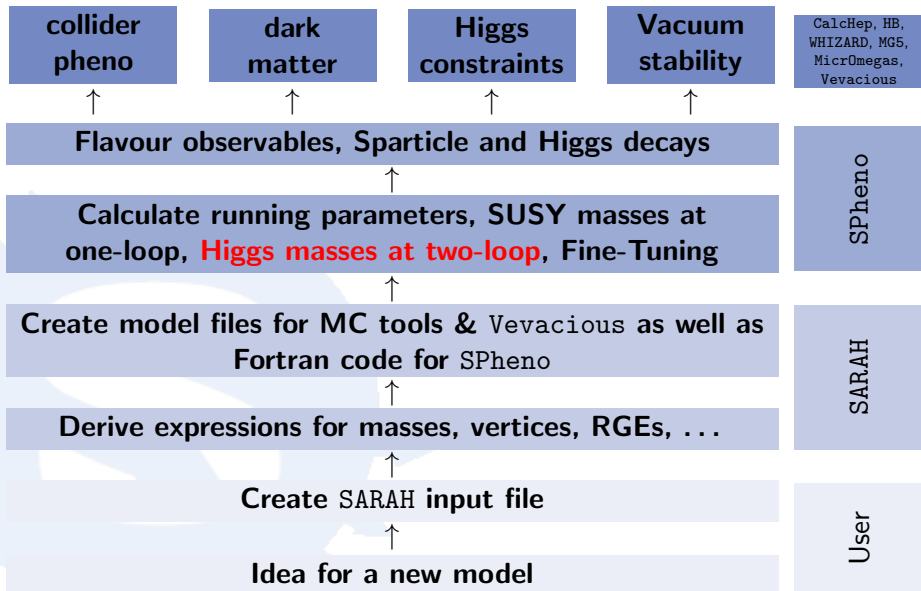
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- ▶ **Many aspects** needed to study BSM models **are automatized**
- ▶ I discussed today how SARAH in combination with SPheno and FeynArts/FormCalc **automatizes** the calculation of **flavour observables at one-loop**:
  - ▶ SPheno version created by SARAH provide **many flavour observables out of the box**
  - ▶ The **user can easily extend** the list of **observables** by combining the existing Wilson coefficients
  - ▶ Also **new coefficients** at full **one-loop** can be included

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  - ▶ The **user can easily extend** the list of **observables** by combining the existing Wilson coefficients
  - ▶ Also **new coefficients** at full **one-loop** can be included
- ▶ I have shown at the example of the inverse seesaw that **new/different features** compared to standard seesaw scenarios can show up

# Higgs masses at two-loop

in collaboration with  
Mark D. Goodsell & Kilian Nickel



## Higgs mass calculations 2015

- ▶ All BSM models have to be confronted today with the Higgs mass measurements.
  - ▶ However, including only new effects compared to the MSSM in an one-loop effective potential approach introduces a very large uncertainty.
- It is necessary to push non-minimal models to the MSSM precision!

# Higgs mass calculation with SARAH and SPheno

## Thresholds corrections

Full one-loop thresholds at  $M_Z$  to get running SM gauge and Yukawa couplings, in particular  $Y_{top}$





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All one-loop diagrams contributing to mass corrections of any particle in the model including full  $p^2$  dependence

## Two-loop contributions

[Goodsell,Nickel,FS,1411.0675]

Dominant two-loop contributions to CP even Higgs via effective potential approach

→ corresponds to precision available for the MSSM when using SoftSUSY, Suspect or SPheno

- ▶ Generic expressions for all two-loop diagrams are known

[Martin, hep-ph/0111209]

- ▶ Expressions have been translated into 4-component notation

[Goodsell, Nickel, FS, 1411.0675]



$SS$



$FFV$



$FFS$



$\overline{FFS}$



$SV$



$\overline{FFV}$



$SSS$



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- ▶ ew gauge contributions usually neglected
- ▶ Two-loop corrections calculated by

$$\delta t_i^{(2)} = \frac{\partial V^{(2)}}{\partial v_i}$$

$$\Pi_{ij}^{(2)} = \frac{\partial^2 V^{(2)}}{\partial v_i \partial v_j}$$



*SS*



*FFV*



*FFS*



$\overline{FFS}$



*SV*



$\overline{FFV}$



*SSS*



*SSV*



*VV*



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*VVS*



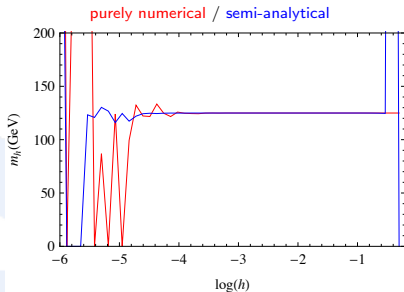
*GGV*

## First implementation in SARAH/SPheno

[Goodsell,Nickel,FS,1411.0675]

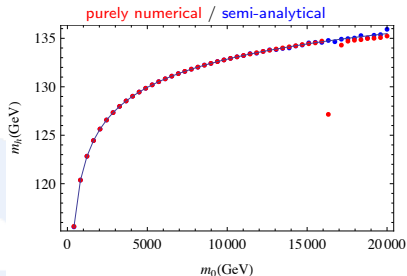
1. **Generic expressions** are implemented in **SARAH**
2. SARAH generates **all possible diagrams** for included topologies in **SPheno** output
3. **Self-energies / tadpoles** are calculated numerically:
  - 3.1 **Numerical derivation** of the **entire two-loop effective potential** with respect to VEVs
  - 3.2 Chain rule: **Analytical derivation** of **loop-functions** which respect to masses; **derivative of masses/couplings** with respect to VEVs **numerically**

## Numerical stability



- ▶ Numerical derivation dependence on initial step-size
  - ▶ There is a large plateau which can be used
  - ▶ we implemented a 'safe mode' which varies the step-size and checks the stability

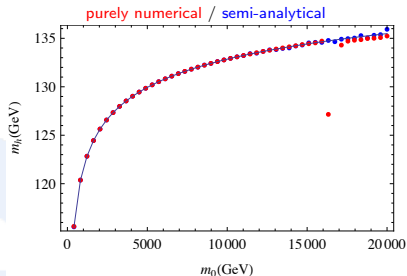
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- ▶ Numerics worse for  $M_{SUSY} \gg v$  No SUSY calculation should be used anyway!
- ▶ Problems can appear for models with small VEVs (e.g.  $RpV$ )

# Diagrammatic calculation

## Fully analytically expressions

[Goodsell,Nickel,FS,1503.03098]

One can take *all derivatives* of the eff. pot. *analytically* using e.g.

$$\frac{\partial}{\partial S_r} \left( \frac{1}{q^2 + \mathbf{m}^2} \right)_{ij} = - \left( \frac{1}{q^2 + \mathbf{m}^2} \right)_{ik} \frac{\partial m_{kk'}^2}{\partial S_r} \left( \frac{1}{q^2 + \mathbf{m}^2} \right)_{k'j}$$

$$m_{ij}^2(S) = \frac{\partial^2}{\partial S_i \partial S_j} V = m_i^2 \delta_{ij} + \lambda^{ijk} S_k + \frac{1}{2} \lambda^{ijkl} S_k S_l$$

→ each derivative introduces an additional propagator

→ equivalent to a diagrammatic calculation in the limit  $p^2 \rightarrow 0$ .

- ▶ We derived a new set of generic expressions
- ▶ The expressions are equivalent to S. Martin results in the limit  $p^2 \rightarrow 0$  but sometimes significantly shorter
- ▶ Results have been implemented in SARAH as third option to calculate two-loop masses

## Two-loop masses with SARAH/SPheno

There are **three options** to calculate the **two-loop masses** in SARAH/SPheno which can **easily be switched** in the numerical session:

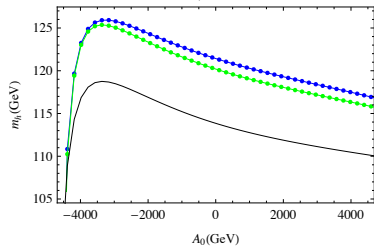
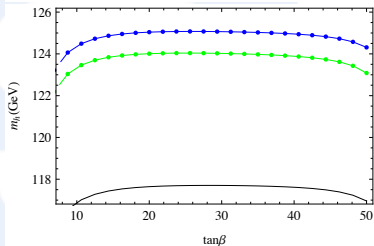
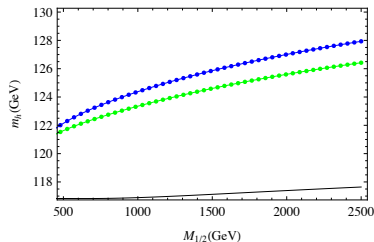
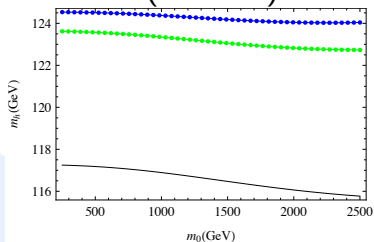
1. Effective potential with fully numerical derivation
2. Effective potential with semi-analytical derivation
3. Diagrammatic approach in the limit  $p^2 \rightarrow 0$

### Double check

The **third option** uses a **different ansatz** and was completely **independently implemented** as the other two  
→ possibility to internally **double checks results** by just using SARAH/SPheno!

That's **necessary** because there are **hardly other two-loop results** to compare with. However, a few are there ...

### Validation I (MSSM)

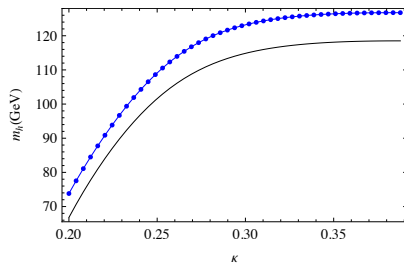
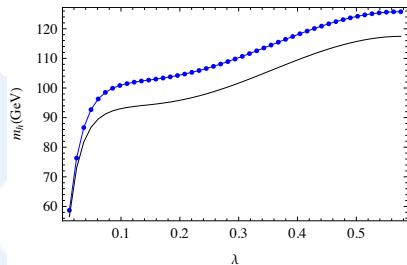


full lines: SARAH, dots: Brignole, Dedes, Degrassi, Slavich, Zwirner ([[hep-ph/0112177,0206101,0212132,0305127](http://hep-ph/0112177,0206101,0212132,0305127)])

1-loop /  $\alpha_S(\alpha_b + \alpha_t)$  / full 2-loop

## Validation II

NMSSM:

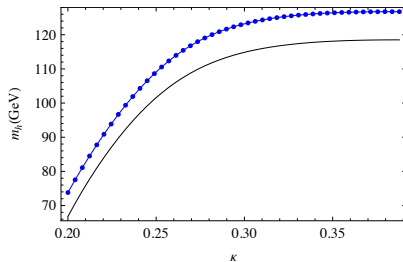
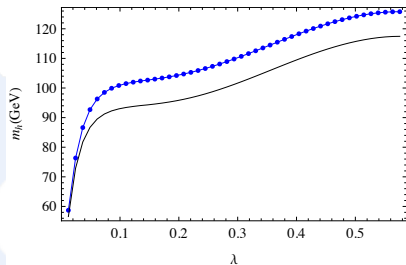


full lines: SARAH, dots: Degrassi, Slavich ([0907.4682])

1-loop /  $\alpha_S(\alpha_b + \alpha_t)$

## Validation II

NMSSM:



full lines: SARAH, dots: Degrassi, Slavich ([0907.4682])

1-loop /  $\alpha_S(\alpha_b + \alpha_t)$

Dirac Gauginos:

full agreement with non-public code for  $\alpha_S(\alpha_b + \alpha_t)$  corrections

[Goodsell, Slavich]

## Problems with massless states

There is an intrinsic problem in the eff. pot. in Landau gauge

### Goldstone boson catastrophe

The **second derivative** of the **one-loop** effective potential

$$V^{(1)} \sim (m^2)^2 [\log(m^2/Q^2) + c]$$

**diverges for massless particles**

$$\Pi^{(1)} \equiv \frac{\partial^2 V^{(1)}}{\partial m^2 \partial m^2} \rightarrow \infty \quad \text{for } m^2 \rightarrow 0$$

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- ▶ Problematic are the **Goldstones of broken groups**  
 → **ew corrections** are **not considered** in the **MSSM at 2-loop**
- ▶ In **BMSSM** also other **very light scalars** can cause **similar problems**

# Outlook

The **diagrammatic calculation** gives not only a **very important cross check**, but allows for **future improvements**:

- ▶ Corrections to **CP odd** states can be included
- ▶ Full coverage of **CP violation** at two-loop
- ▶ To include **momentum dependence**, only the loop functions but not the generic structures have to be changed

We hope that we can release improvements in these directions in the not too far future!

# Vectorlike top partners

[Goodsell,Nickel,FS,in prep.]

## MSSM with vectorlike top partners

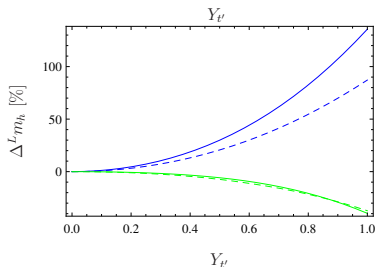
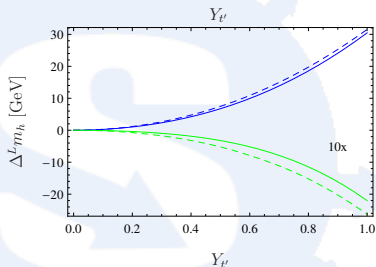
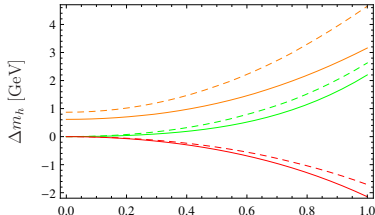
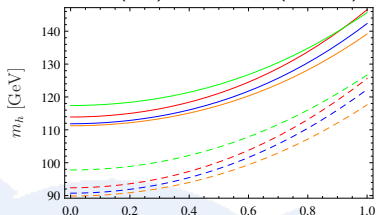
$$W = W_{MSSM} + Y_{t'}^i \hat{Q}_i \hat{T}' \hat{H}_u + M_{T'} \hat{T}' \hat{T}' + m_{t'}^i \hat{U}_i \hat{T}'.$$

→ it is well known that  $Y_{t'}^3 \equiv Y_{t'}$  can give a large push to the Higgs mass.

Using SARAH/SPheno one can easily improve existing calculations in three aspects:

1. **one-loop thresholds** to calculate  $Y_{top}$  at  $M_Z$
2. **momentum dependence** at one-loop
3. **dominant two-loop** corrections

$\tan \beta = 10$  (full),  $\tan \beta = 2$  (dashed),  $M_{T'} = 1.0$  TeV,  $B_{T'} = 0$



top left: 1-loop eff.pot, 1-loop with  $p^2$ , 1-loop  $p^2$  and thresholds, two-loop

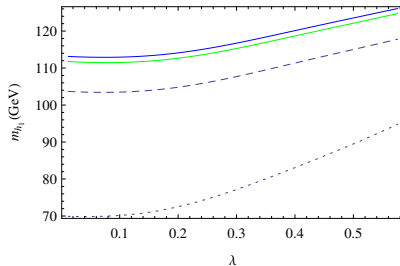
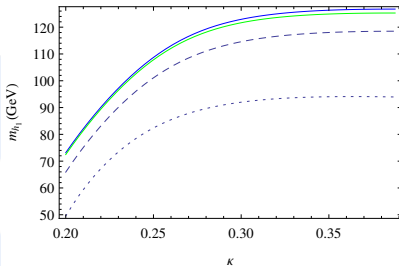
top right: shift by momentum dependence, thresholds, two-loop

bottom: absolute shifts (left) by 1- and 2-loop corrections, and normalized to MSSM contributions (right)

# NMSSM results I: heavy singlet & moderate $\lambda$

[Goodsell,Nickel,FS,1411.4665]

$$\begin{array}{llll}
 m_0 = 1.4 \text{ TeV} & M_{1/2} = 1.4 \text{ TeV} & \tan \beta = 2.9 & A_0 = -1.35 \text{ TeV} \\
 \kappa = 0.33 & A_\lambda = -390 \text{ GeV} & A_\kappa = -280 \text{ GeV} & \mu_{eff} = 200 \text{ GeV}
 \end{array}$$



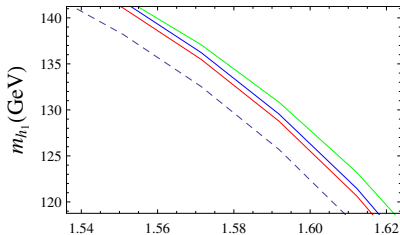
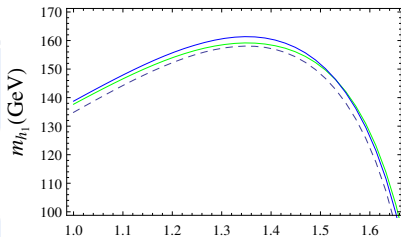
1-loop /  $\alpha_S(\alpha_b + \alpha_t)$  / full

- ▶ **Corrections** beyond  $\alpha_S(\alpha_t + \alpha_b)$  give **negative** contribution of a few GeV
- ▶ Corrections often **MSSM-like** and dominated by (s)quarks

# NMSSM results II: heavy singlet & large $\lambda$

[Goodsell,Nickel,FS,1411.4665]

$$\begin{array}{cccccc}
 \kappa = 1.6 & \tan \beta = 3 & T_\lambda = 600 \text{ GeV} & T_\kappa = -2650 \text{ GeV} & \mu_{eff} = 614 \text{ GeV} & \\
 m_{\tilde{f}}^2 = 2 \cdot 10^6 \text{ GeV}^2 & T_i = 0 & M_1 = 200 \text{ GeV} & M_2 = 400 \text{ GeV} & M_3 = 2000 \text{ GeV} & 
 \end{array}$$


 $\lambda$ 
 $\lambda$ 

1-loop /  $\alpha_S(\alpha_b + \alpha_t)$  / full / MSSM approx.

- ▶ Additional corrections can be **positive** for **very large  $\lambda$**
- ▶ Using **MSSM** results not a good approximation any more

# NMSSM results III: light singlet

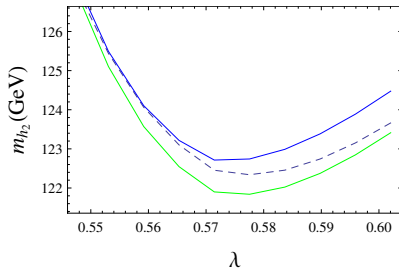
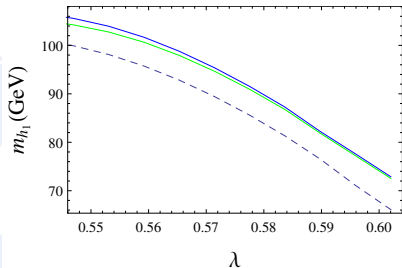
[Goodsell,Nickel,FS,1411.4665]

$$\kappa = 0.596 \quad T_\lambda = -27 \text{ GeV}$$

$$T_\kappa = -240 \text{ GeV} \quad \mu_{eff} = 130 \text{ GeV}$$

$$T_t = -3050 \text{ GeV} \quad T_b = T_\tau = -1000 \text{ GeV}$$

$$m_{\tilde{t}_L}^2 = 9.0 \cdot 10^5 \text{ GeV}^2 \quad m_{\tilde{t}_R}^2 = 1.05 \cdot 10^6 \text{ GeV}^2$$


 1-loop /  $\alpha_S(\alpha_b + \alpha_t)$  / full

- ▶ Corrections can be **larger than the ones**  $\sim \alpha_S$
- ▶ Again, using MSSM results not a good approximation any more

# MSSM with trilinear $RpV$

[Dreiner, Nickel, FS, 1411.3731]

## MSSM with trilinear $RpV$

$$W = W_{MSSM} + \frac{1}{2} \lambda_{ijk} \mathbf{L}_i \mathbf{L}_j \bar{\mathbf{E}}_k + \lambda'_{ijk} \mathbf{L}_i \mathbf{Q}_j \bar{\mathbf{D}}_k + \frac{1}{2} \lambda''_{ijk} \bar{\mathbf{U}}_i \bar{\mathbf{D}}_j \bar{\mathbf{D}}_k .$$

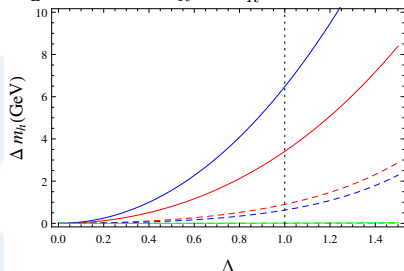
- ▶  $RpV$  contributions to Yukawas at one-loop
- ▶  $RpV$  contributions to effective potential at two-loop



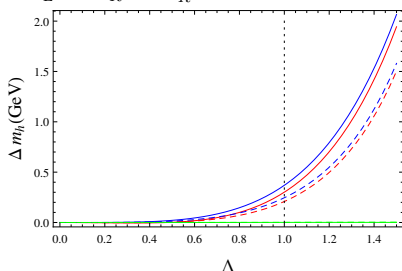
# MSSM with trilinear $R_pV$

[Dreiner, Nickel, FS, 1411.3731]

$$m_{\tilde{t}_L} = 1.5 \text{ TeV}, \quad m_{\tilde{t}_R} = m_{\tilde{b}_R} = 0.5 \text{ TeV}$$



$$m_{\tilde{t}_L} = m_{\tilde{t}_R} = m_{\tilde{b}_R} = 2.5 \text{ TeV}$$



$$T_t = -2.5 \text{ TeV}, \quad T_\Lambda = -2.5\Lambda \text{ TeV}$$

 $\lambda''_{313}, \lambda''_{312}, \lambda''_{213}, \lambda'_{333}$  (dashed),  $\lambda'_{331}$  (dashed),  $\lambda'_{313}$  (dashed)

- ▶ Corrections only **important** if **stops** are involved
- ▶ For **light stops** the corrections can be **several GeV**
- ▶ Often **couplings beyond the perturbativity limit** needed

## Summary Part II

- ▶ The precise measurement of the Higgs mass is a challenge for many BSM models
- ▶ To confront models with this measurement a precise prediction of the Higgs mass in a given model is necessary
- ▶ The combination SARAH/SPheno provides a prediction for a wide range of SUSY models which is comparable with the MSSM precision of standard spectrum generators by including:
  - ▶ All new effects in the one-loop thresholds to determine the running gauge and Yukawa couplings
  - ▶ The full momentum dependence at one-loop
  - ▶ The (most likely!) dominant two-loop corrections
- ▶ By providing  $O((\alpha_\lambda + \alpha_\kappa)\alpha_x)$  ( $x = s, t, b, \tau, \lambda, \kappa$ ) corrections, the NMSSM version of SPheno generated by SARAH is even more precise than dedicated tools