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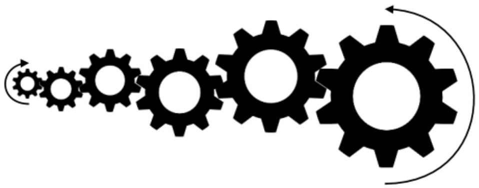
WARSAW OCHOTA CAMPUS

Phenomenology of the clockwork solution to the hierarchy problem

Yevgeny Kats

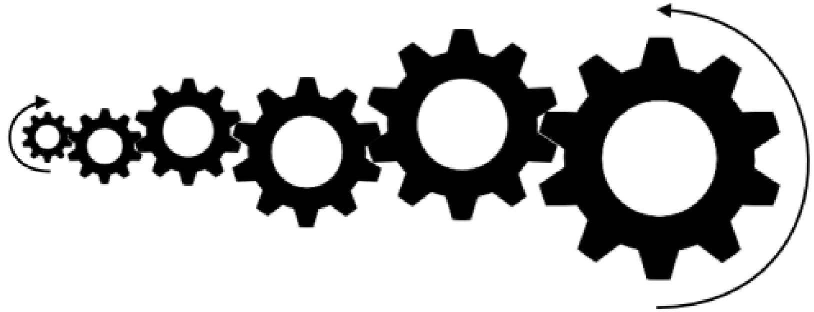
Work in progress
in collaboration with:

Gian Giudice
Matthew McCullough
Riccardo Torre
Alfredo Urbano



Horlogerie CERN, Genève

The clockwork mechanism



A generator of tiny couplings.

First proposed to generate a tiny coupling to a **scalar** in **relaxion** models.

Choi, Im [1511.00132]
Kaplan, Rattazzi [1511.01827]

Later,

- ❑ Generalized to **fermions, gauge bosons, gravitons.**
- ❑ Obtained from deconstruction of an **extra dimension**.
- ❑ Applied to the **electroweak-Planck hierarchy** directly.

Giudice, McCullough [1610.07962]

Further discussion: Craig, Garcia Garcia, Sutherland [1704.07831]
Giudice, McCullough [1705.xxxxx]

The clockwork mechanism

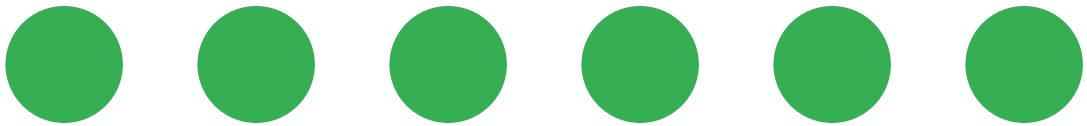
- Consider a particle P kept massless by a symmetry S.

  For example:

  - Shift symmetry for a spin-0 particle
  - Chiral symmetry for a spin-1/2 particle
  - Gauge symmetry for a spin-1 particle
  - Diffeomorphism invariance for a spin-2 particle

The clockwork mechanism

- Consider $N + 1$ such particles P_i $(i = 0, \ldots, N)$ kept massless by symmetries S_i .



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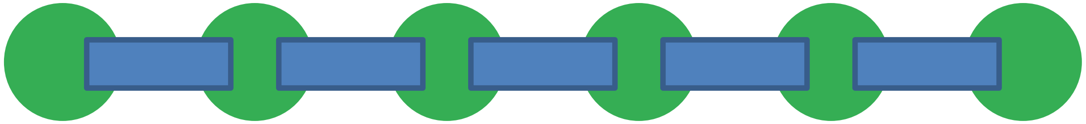

- Break the symmetries by nearest-neighbor mass mixings. One combination

$$\mathcal{P} = \sum c_i P_i$$

remains massless.

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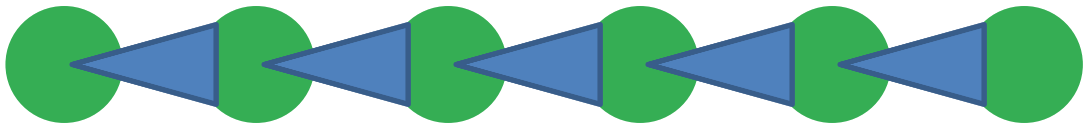

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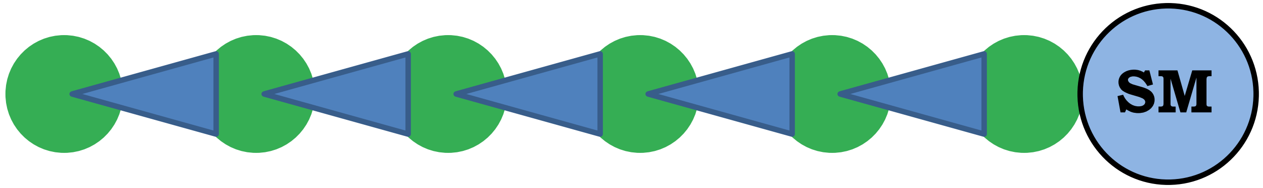

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- If the breaking is asymmetric, c_i vary with i exponentially.

- Coupling external fields to P_N will result in their exponentially suppressed coupling to $\mathcal{P}$.

See McCullough talk for details.

Linear dilaton scenario

- $N \to \infty$ clockwork: site i ⟺ spatial coordinate y

$$ds^2 = e^{\frac{4}{3}k|y|}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\right)$$

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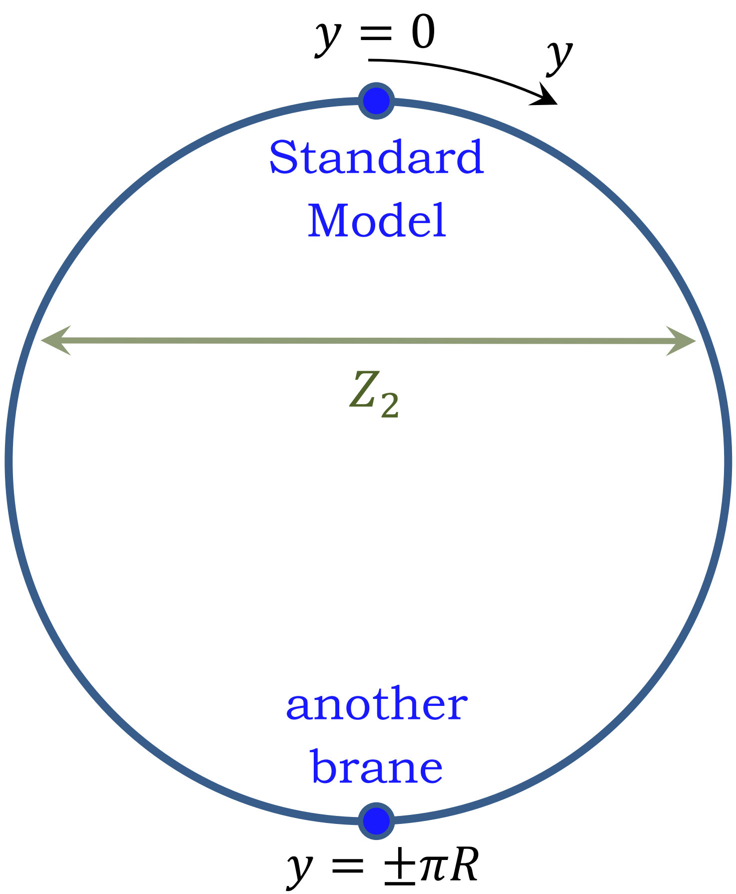

gravity g_{MN} ($M_5 \sim$ TeV)

scalar S (dilaton)

$$S(y) = 2k|y|$$

Linear dilaton profile sources the metric.

See McCullough talk for details.

Solution to the hierarchy problem

$$M_P^2 = \frac{M_5^3}{k}\left(e^{2\pi kR} - 1\right) \quad \Longrightarrow \quad R(M_5, k)$$

For $M_5 \sim \mathcal{O}(10\text{ TeV})$: $\quad kR \approx 10$

Comparison with other extra dimensional scenarios

LED	$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu + dy^2$	$M_P^2 = L_5 M_5^3$
RS	$ds^2 = e^{2ky}\, \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$	$M_P^2 \approx e^{2k\pi R}\, \frac{M_5^3}{k}$
Clockwork/LD	$ds^2 = e^{\frac{4}{3}ky}\left(\eta_{\mu\nu} dx^\mu dx^\nu + dy^2\right)$	$M_P^2 \approx L_5\, e^{\frac{4}{3}k\pi R}\, \frac{M_5^3}{3}$

Stringy connection of the linear dilaton

Stack of D3 branes

→ *4d* strongly coupled SCFT

→ dual to gravitational theory in $AdS_5 \times S_5$ Maldacena [hep-th/9711200]

→ **Randall-Sundrum** setup with two branes to explain the TeV-Planck hierarchy Randall, Sundrum [hep-ph/9905221]

Stack of NS5 branes

→ *6d* strongly coupled non-local theory: Little String Theory (LST)

Berkooz, Rozali, Seiberg [hep-th/9704089]; Seiberg [hep-th/9705221]

→ dual to *7d* gravitational theory w/linearly varying dilaton

Aharony, Berkooz, Kutasov, Seiberg [hep-th/9808149]

Giveon, Kutasov [hep-th/9909110]

→ **LST at a TeV (linear dilaton)** setup with two branes to explain the TeV-Planck hierarchy Antoniadis, Dimopoulos, Giveon [hep-th/0103033]

Previous studies of phenomenology

Antoniadis, Arvanitaki, Dimopoulos, Giveon [1102.4043]

KK gravitons for large k @ Tevatron, LHC

Baryakhtar [1202.6674]

KK gravitons @ LHC, beam dump, supernova, BBN

Cox, Gherghetta [1203.5870]

KK dilatons / radion for large k @ LHC

Giudice, Plehn, Strumia [hep-ph/0408320]

Franceschini, Giardino, Giudice, Lodone, Strumia [1101.4919]

KK gravitons in a low-k RS scenario @ LEP, LHC

The clockwork KK graviton spectrum and interaction strengths for $n \gg kR$ are the same as in RS with $k_{\rm RS} = \frac{1}{\pi R} \approx \frac{k}{30}$

KK modes

KK graviton masses

$$m_0^2 = 0 \qquad m_n^2 = k^2 + \frac{n^2}{R^2} \qquad n = 1, 2, 3, \ldots$$

KK graviton couplings

$$\mathcal{L} \supset -\frac{1}{\Lambda_n} h_{\mu\nu}^{(n)} T^{\mu\nu} \qquad \Lambda_0^2 = M_P^2 \qquad \Lambda_n^2 = M_5^3 \pi R \left(1 + \left(\frac{kR}{n}\right)^2\right)$$

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KK dilaton / radion masses and couplings

$$m_0^2 = \tfrac{8}{9} k^2 \qquad m_n^2 = k^2 + \frac{n^2}{R^2} \qquad n = 1, 2, 3, \ldots$$

$$\mathcal{L} \supset -\frac{1}{\Lambda_n} \phi^{(n)} T_\mu^\mu \qquad \Lambda_0^2 \simeq \frac{18 M_5^3}{k} \qquad \Lambda_n^2 = \tfrac{3}{4} M_5^3 \pi R \left(10 + \left(\frac{kR}{n}\right)^2 + 9\left(\frac{n}{kR}\right)^2\right)$$

Model dependence in the case of non-rigid stabilization or Higgs-curvature coupling.

Kofman, Martin, Peloso [hep-ph/0401189]
Cox, Gherghetta [1203.5870]

KK mode splittings

$$m_n^2 = k^2 + \frac{n^2}{R^2} \qquad n = 1, 2, 3, \ldots$$

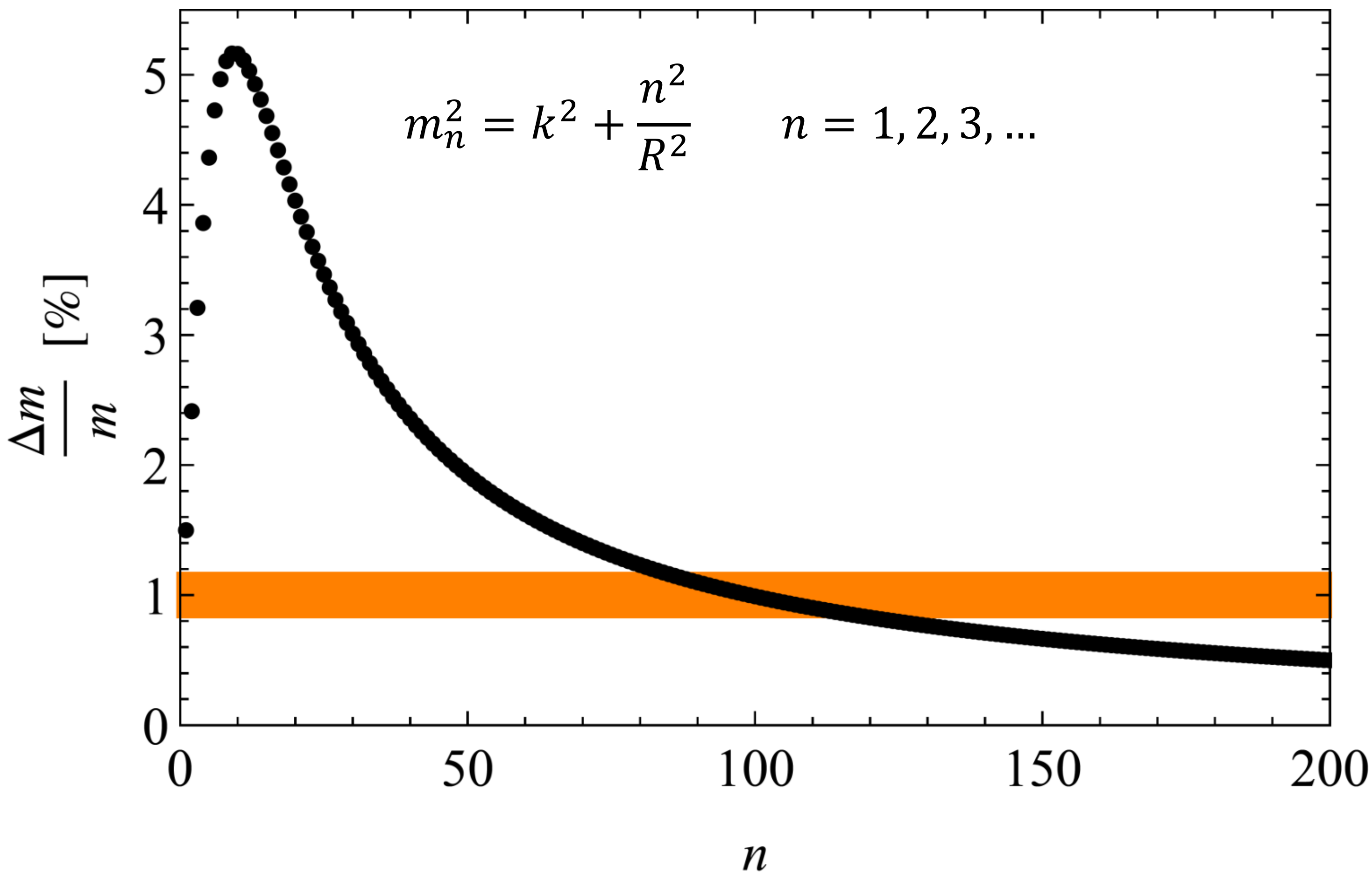
The relative mass splitting starts from

$$\frac{m_2 - m_1}{m_1} \approx \frac{3}{2(kR)^2} \approx 1.5\% \,,$$

then grows, but eventually falls as

$$\frac{m_{n+1} - m_n}{m_n} \simeq \frac{1}{n}$$

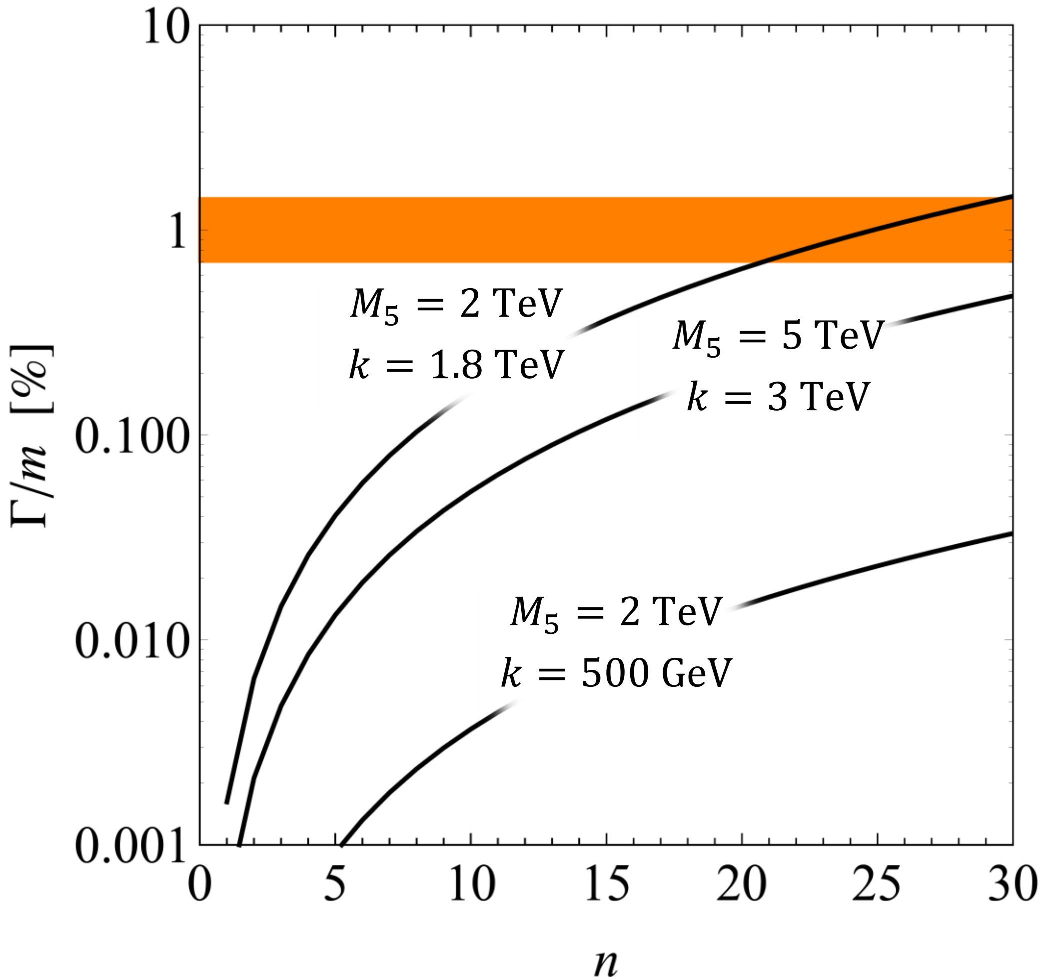
KK mode splittings



For $n \lesssim 100$, i.e. $k \lesssim m_n \lesssim 10k$, the individual modes can be resolved in the $\boldsymbol{\gamma\gamma}$ and $\boldsymbol{e^+e^-}$ channels in ATLAS and CMS!

KK mode splittings

The intrinsic widths of at least the first ~30 modes are smaller than the resolution in the relevant range of parameters.



KK graviton decays

Decays to SM particles:

gg	$\sum_i q_i\bar{q}_i$	W^+W^-	ZZ	hh	$\gamma\gamma$	$\sum_i \ell_i^+\ell_i^-$	$\sum_i \nu_i\bar{\nu}_i$
34%	38%	9.2%	4.6%	0.35%	4.2%	6.4%	3.2%

*In the regime where phase space suppressions are negligible.

Easiest decays to see: $\gamma\gamma$, e^+e^-, $(\mu^+\mu^-)$

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Total rate to SM particles (for $n \gg kR$, $m_n \gg m_t$):

$$\Gamma_{n\to \mathrm{SM}} \simeq \frac{283}{960\pi^2}\frac{m_n^3}{RM_5^3}$$

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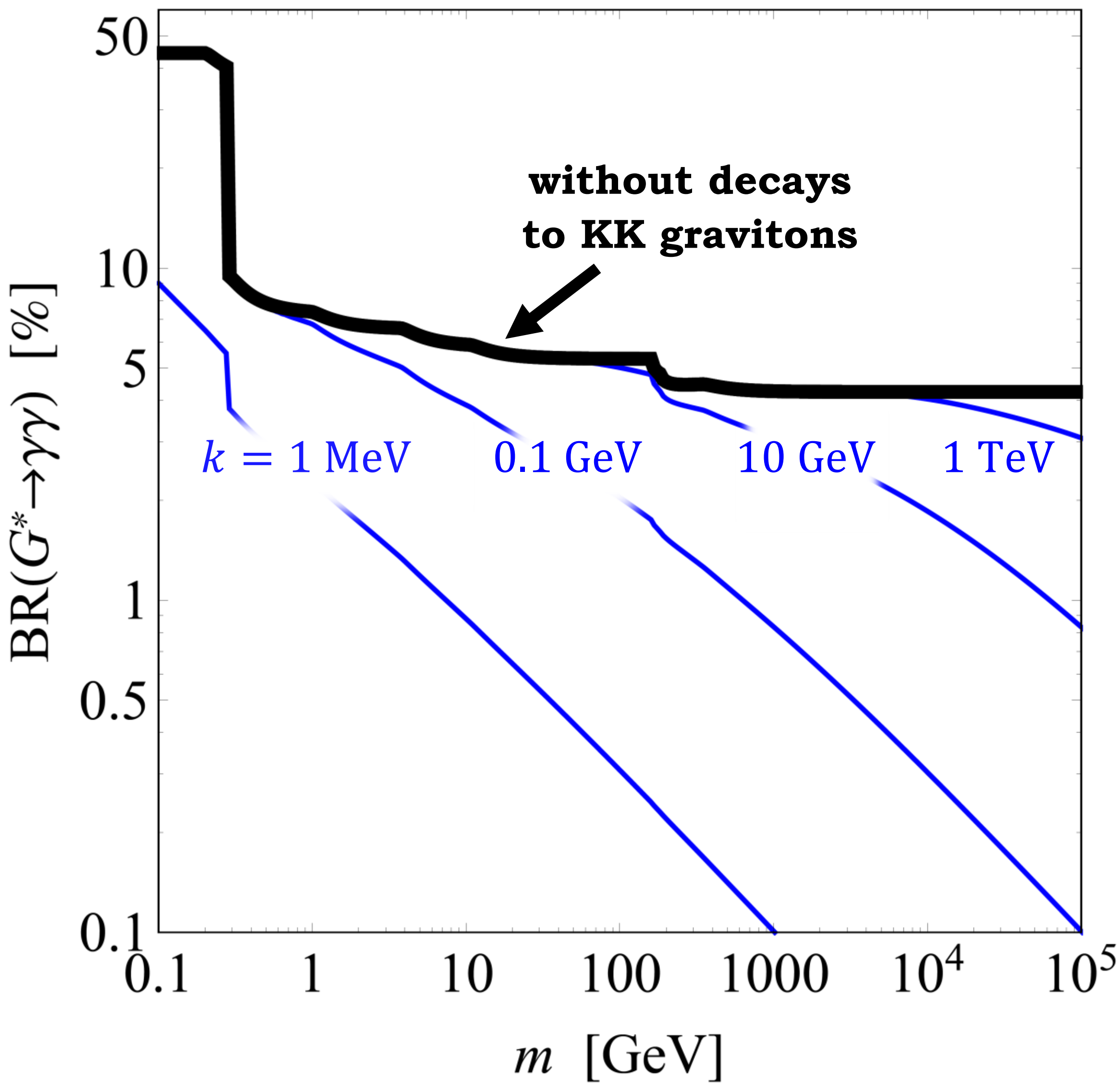
There are also decays to lighter KK gravitons. For $n \gg kR \gg 1$:

$$\Gamma_{n\to \mathrm{KK}} \simeq \frac{5\cdot 7\cdot 17}{3\cdot 2^{14}\pi^2}\frac{\sqrt{km_n}\,m_n^3}{kRM_5^3} \quad \Rightarrow \quad \frac{\Gamma_{n\to \mathrm{KK}}}{\Gamma_{n\to \mathrm{SM}}} \approx 0.04\sqrt{\frac{m_n}{k}}$$

A huge effect for low k!

KK graviton decays



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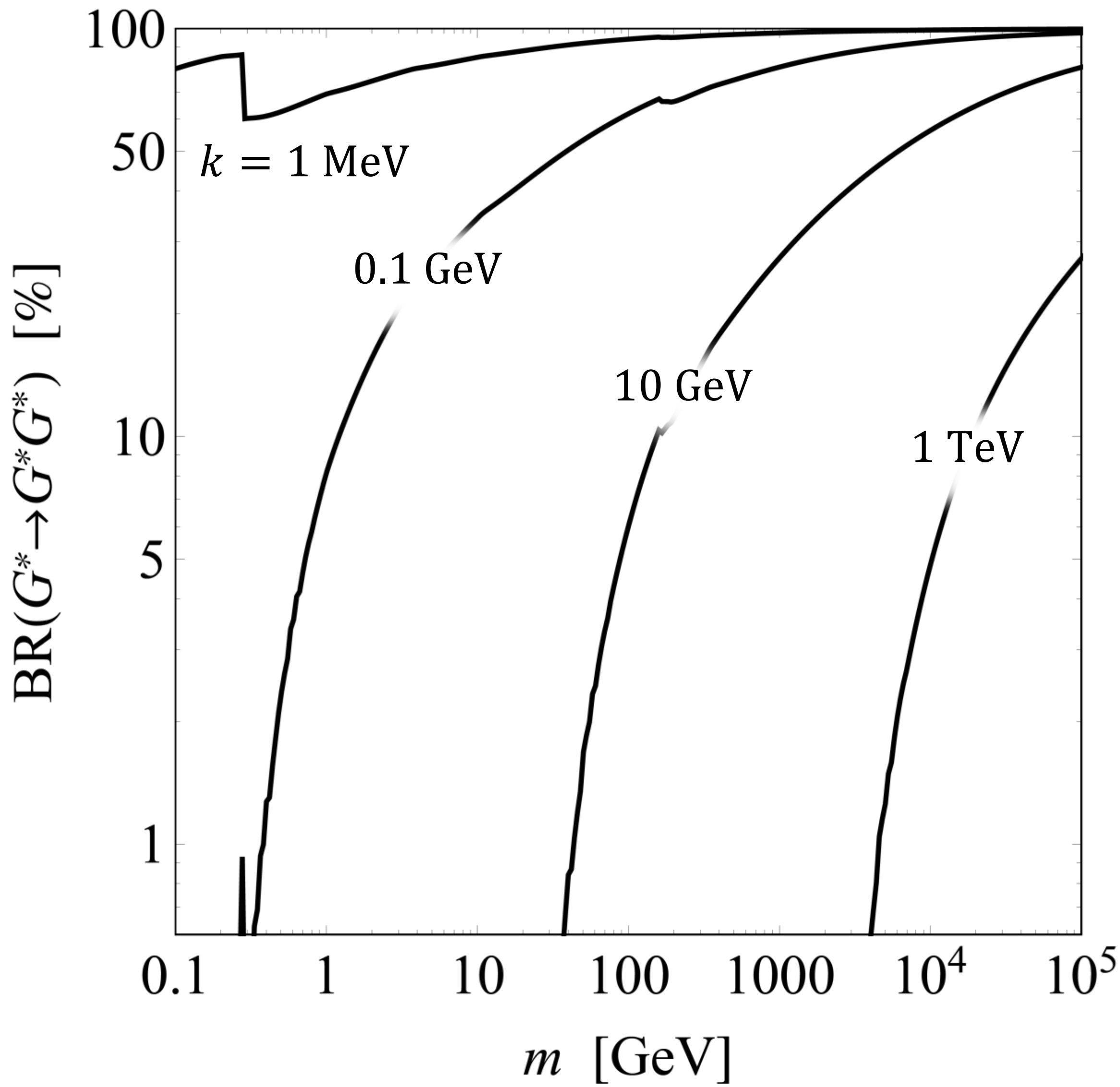

Production cross sections

Single KK graviton:

$$\sigma_n = \frac{\pi}{48\Lambda_n^2}\left(3\mathcal{L}_{gg}(m_n^2) + 4\sum_q \mathcal{L}_{q\bar{q}}(m_n^2)\right)$$

KK graviton tower approximated by a continuum:

$$\frac{d\sigma}{dm} \simeq \frac{\pi}{48M_5^3}\sqrt{1-\frac{k^2}{m^2}}\left(3\mathcal{L}_{gg}(m^2) + 4\sum_q \mathcal{L}_{q\bar{q}}(m^2)\right)$$

Independent of k for $m \gg k$.

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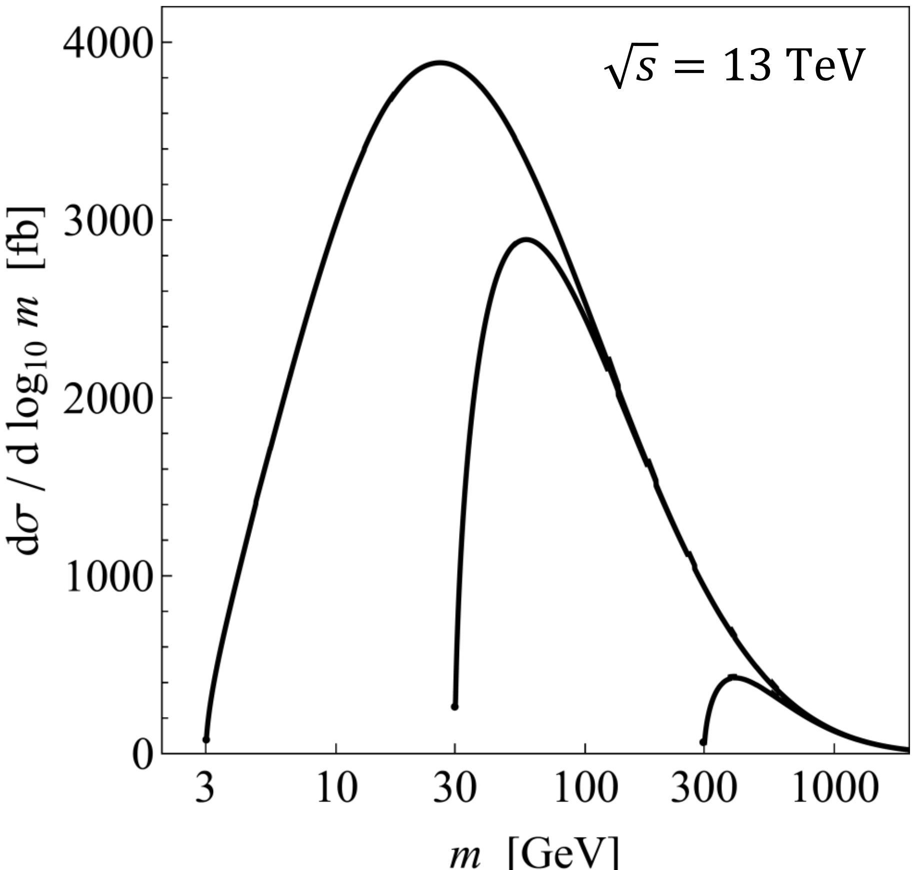
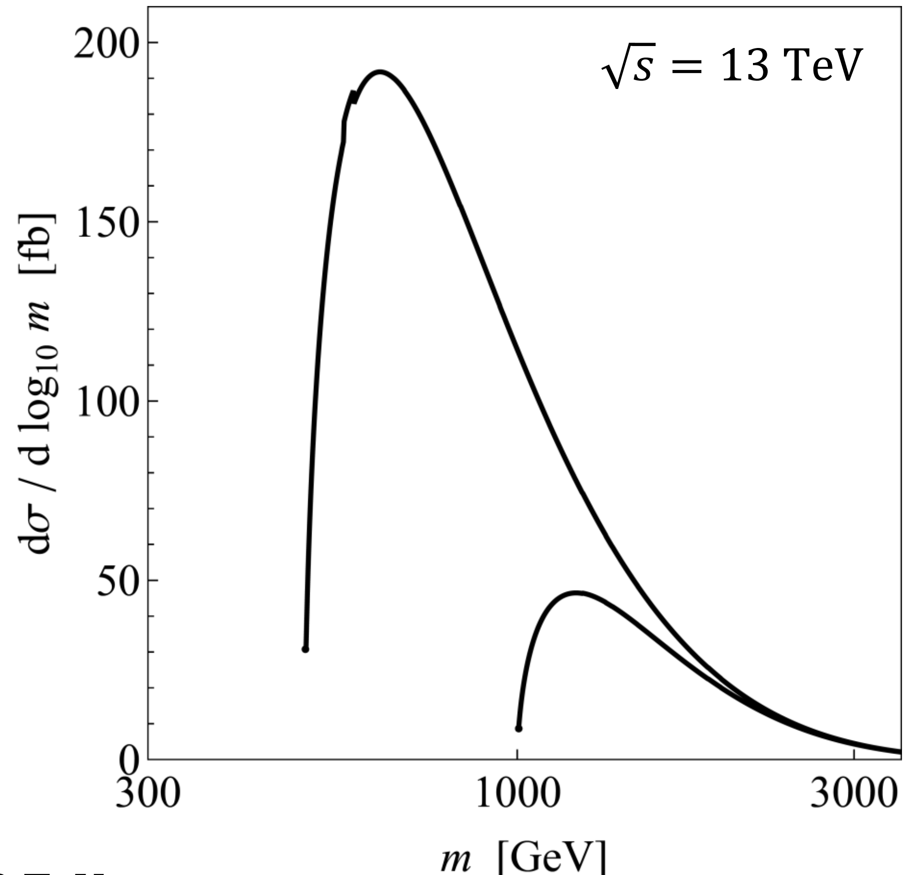
KK dilaton tower:

$$\frac{d\sigma}{dm} \simeq \frac{49\alpha_s^2}{864\pi^2 M_5^3}\sqrt{1-\frac{k^2}{m^2}}\left(1-\frac{8}{9}\frac{k^2}{m^2}\right)^{-1}\frac{k^2}{m^2}\,\mathcal{L}_{gg}(m^2)$$

Production cross sections

KK graviton



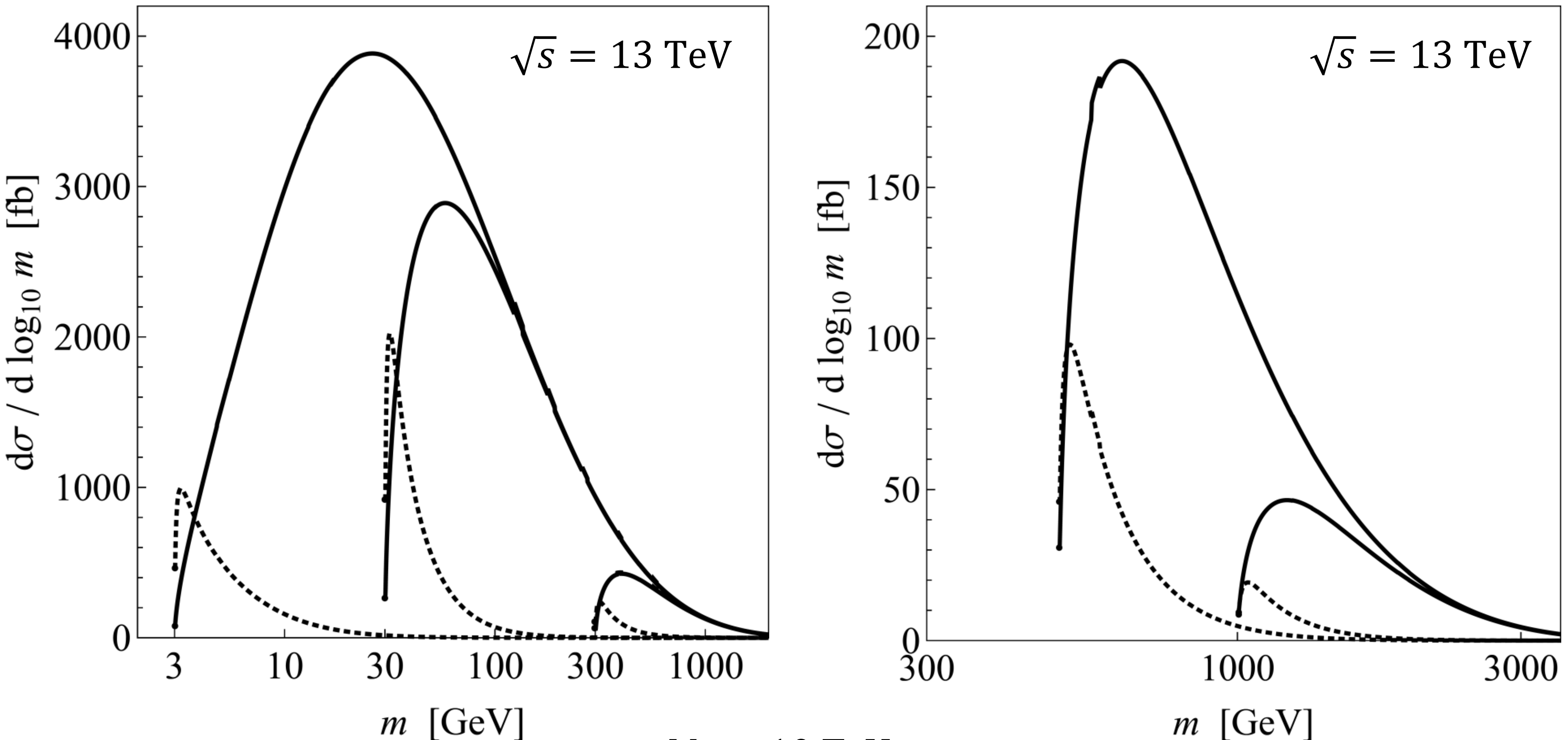


$M_5 = 10$ TeV

$k = 3, 30, 300, 500, 1000$ GeV

Production cross sections

KK graviton and KK dilaton (× 500, dashed)



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Production cross sections and lifetimes

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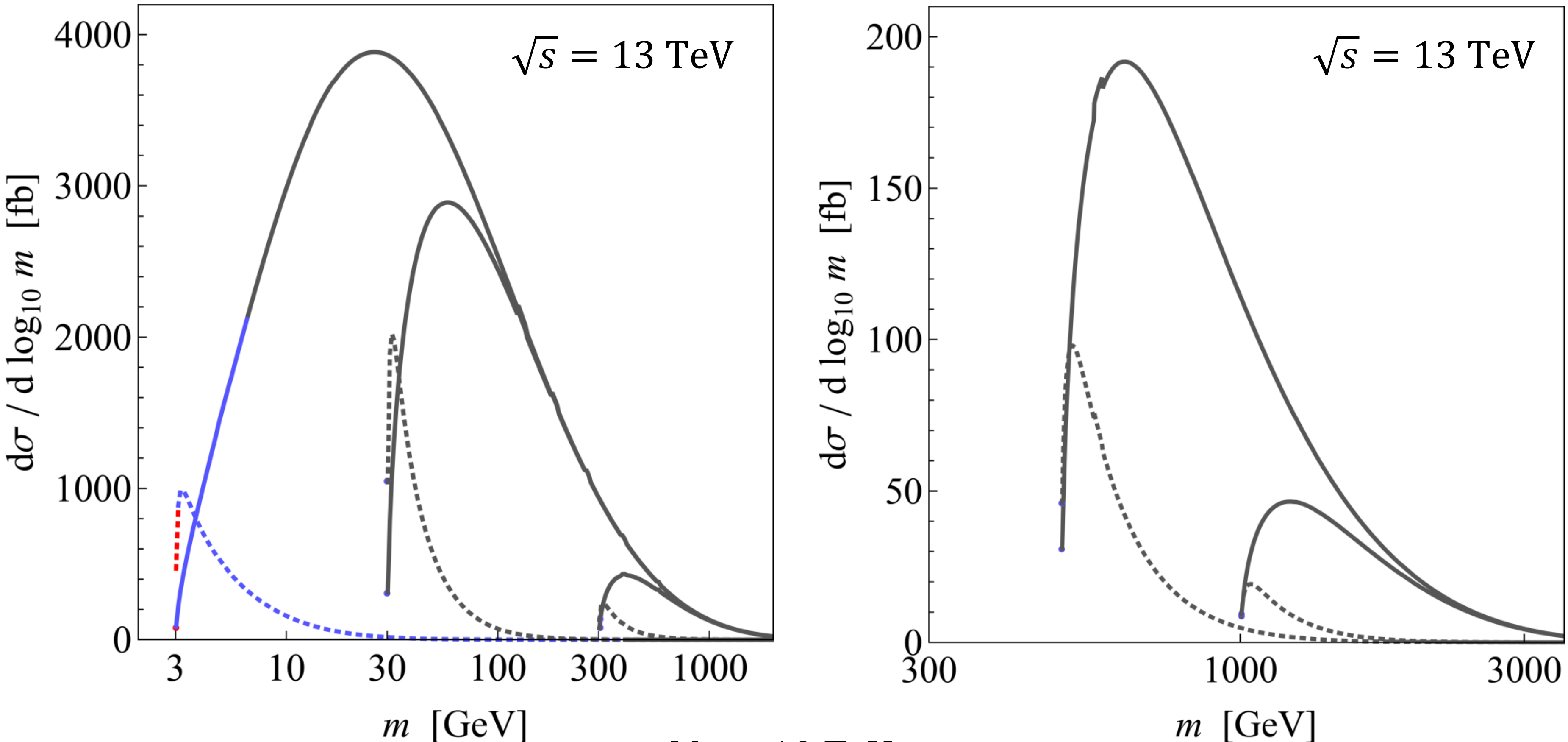
prompt **displaced** **detector-stable**



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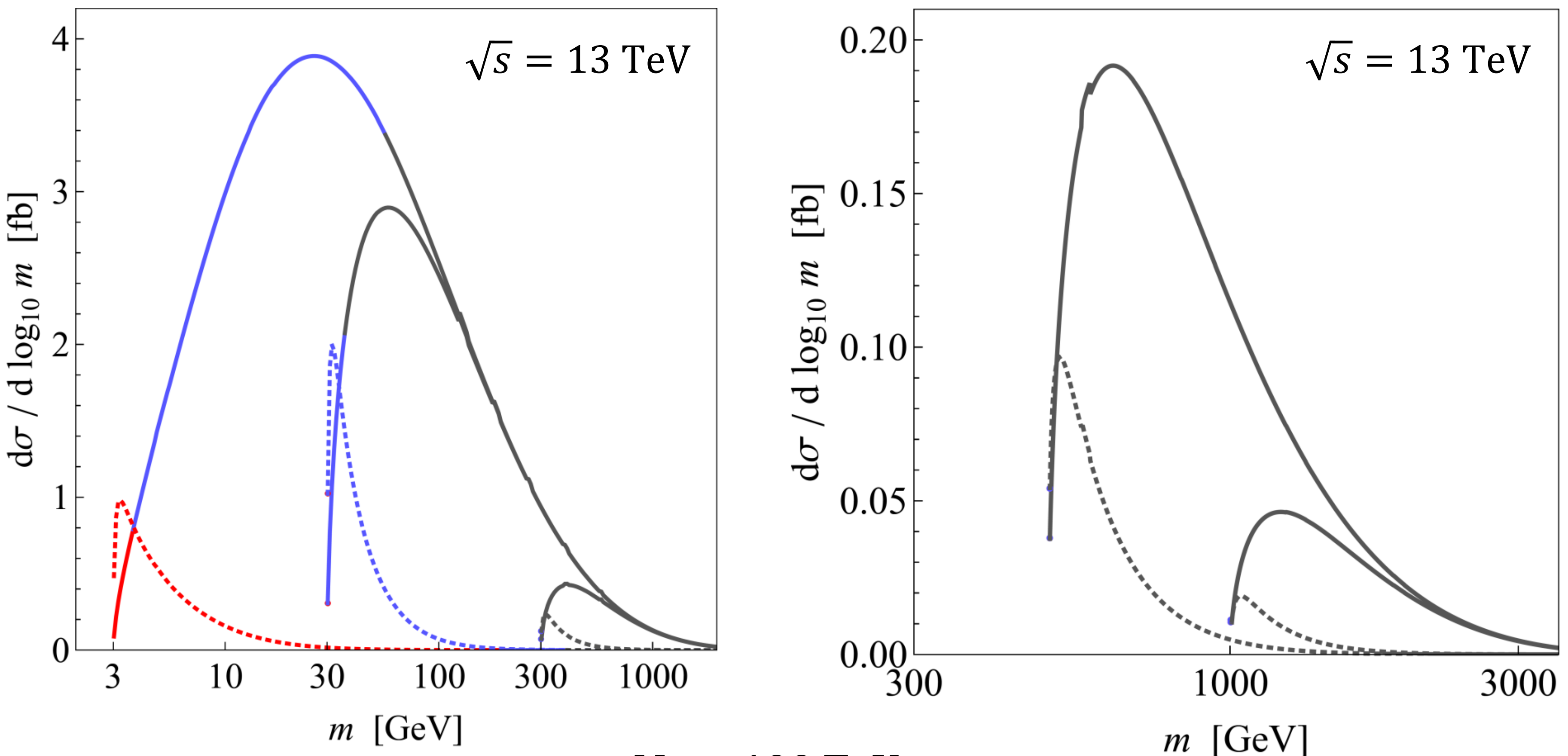

$M_5 = 100$ **TeV**

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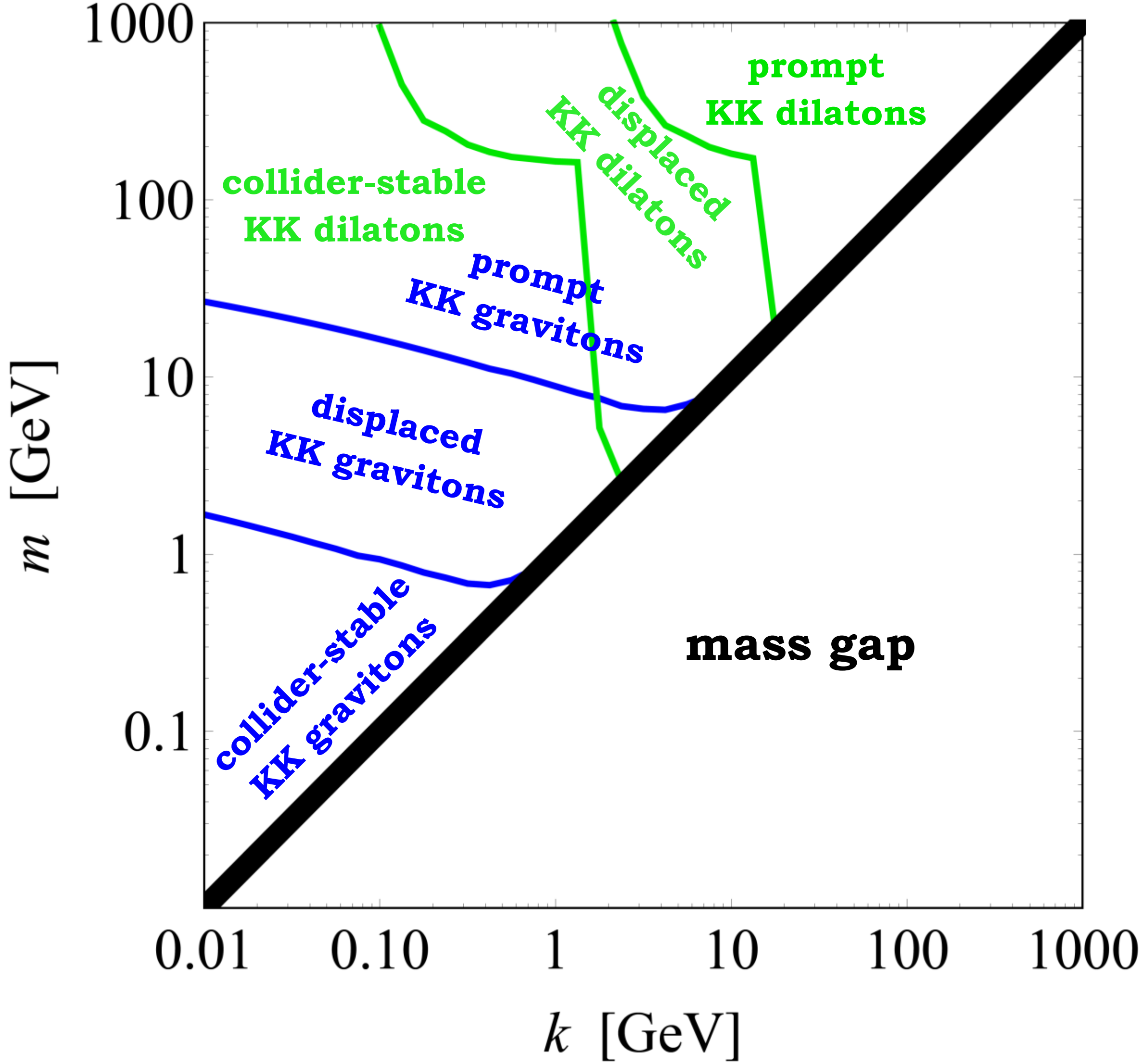

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Standard signatures

- Enhancement of the $\gamma\gamma$, e^+e^-, $\mu^+\mu^-$ spectra at high mass.

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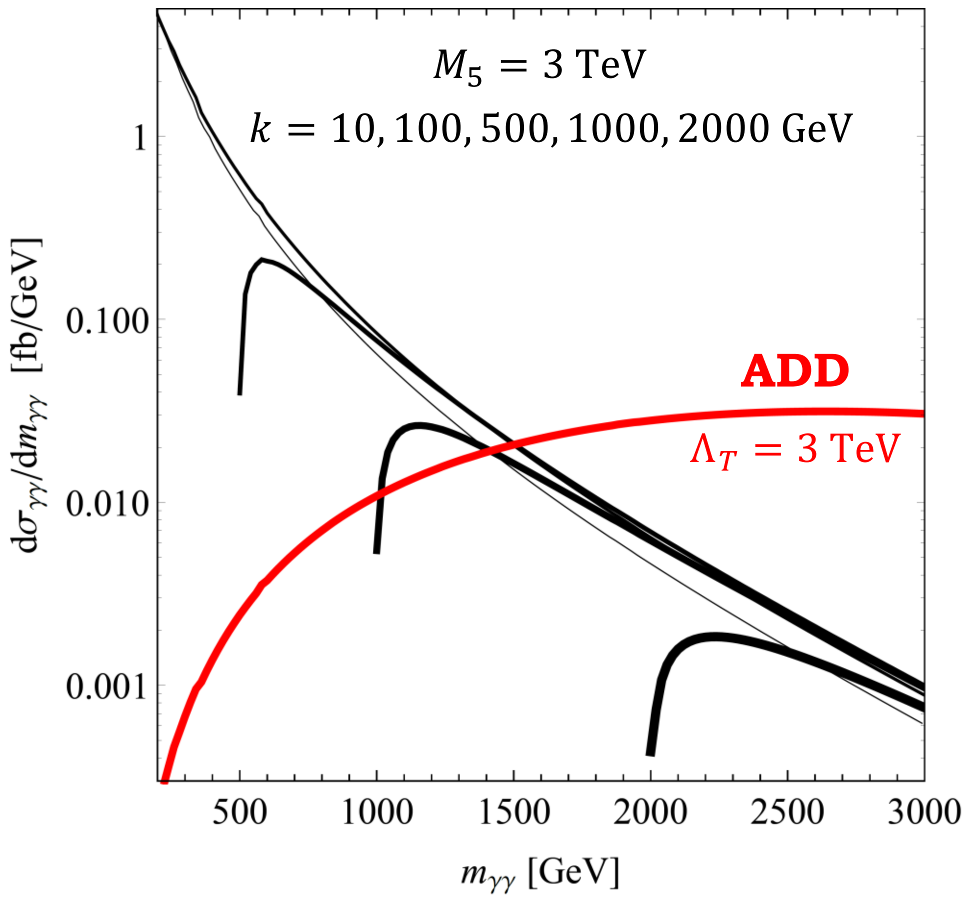

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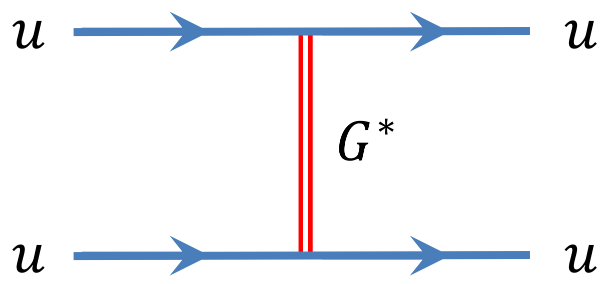

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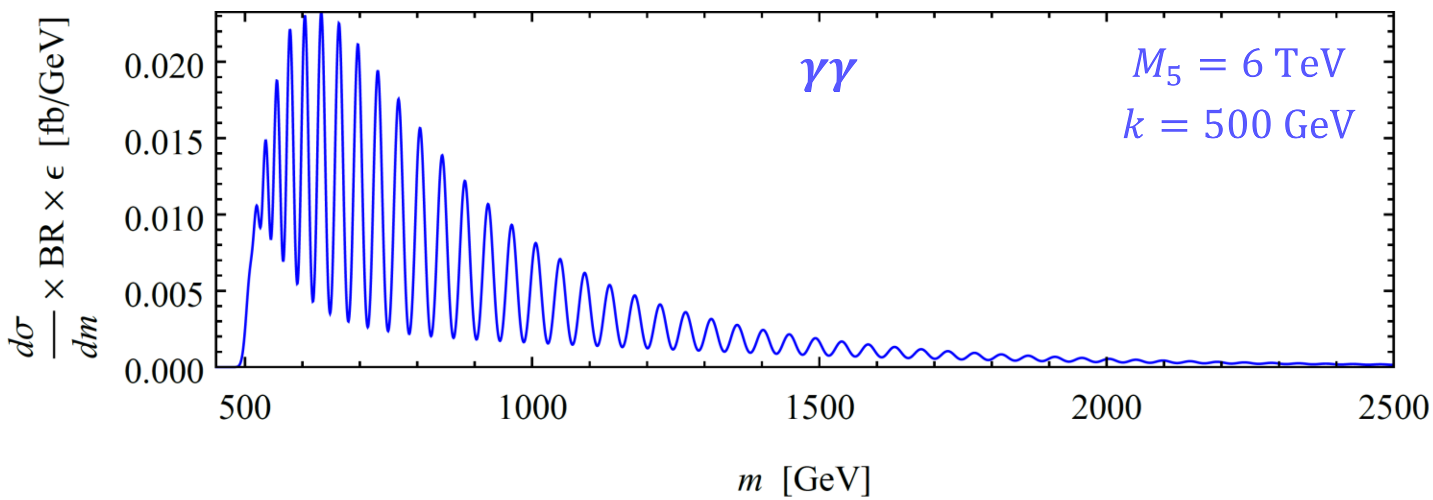

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  However, to what extent are resonance searches affected by nearby peaks?
- Strong gravity signatures (black holes etc.) around $m \sim M_5$.
  As in other scenarios, unknown and model dependent.

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Novel signatures

- Periodic peaks in $\gamma\gamma$ and e^+e^- spectra,
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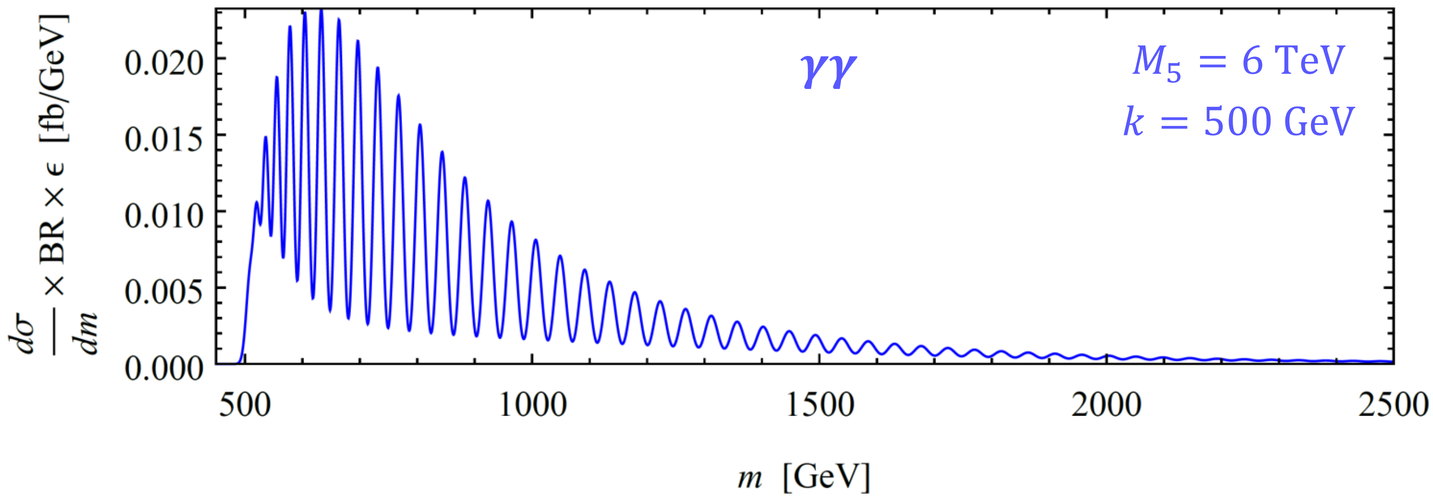

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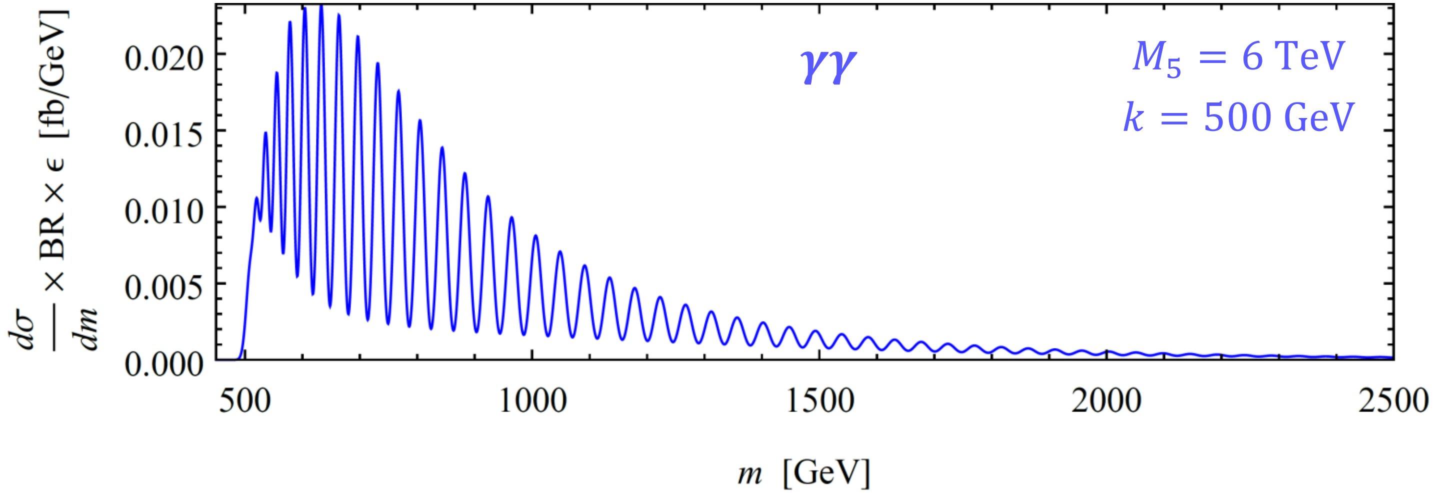

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- Cascades within the KK graviton and KK dilaton towers:
  - Final states with **high object multiplicity**.
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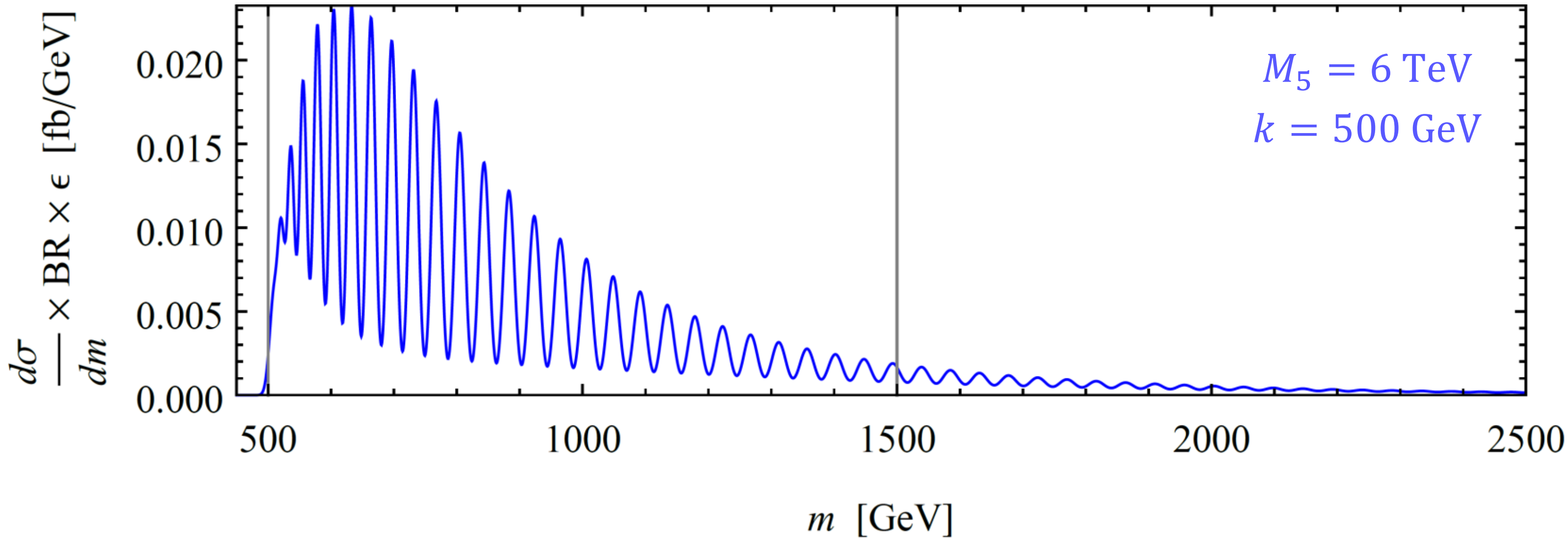
- Resonant production of somewhat long-lived (although not very boosted) light KK gravitons.

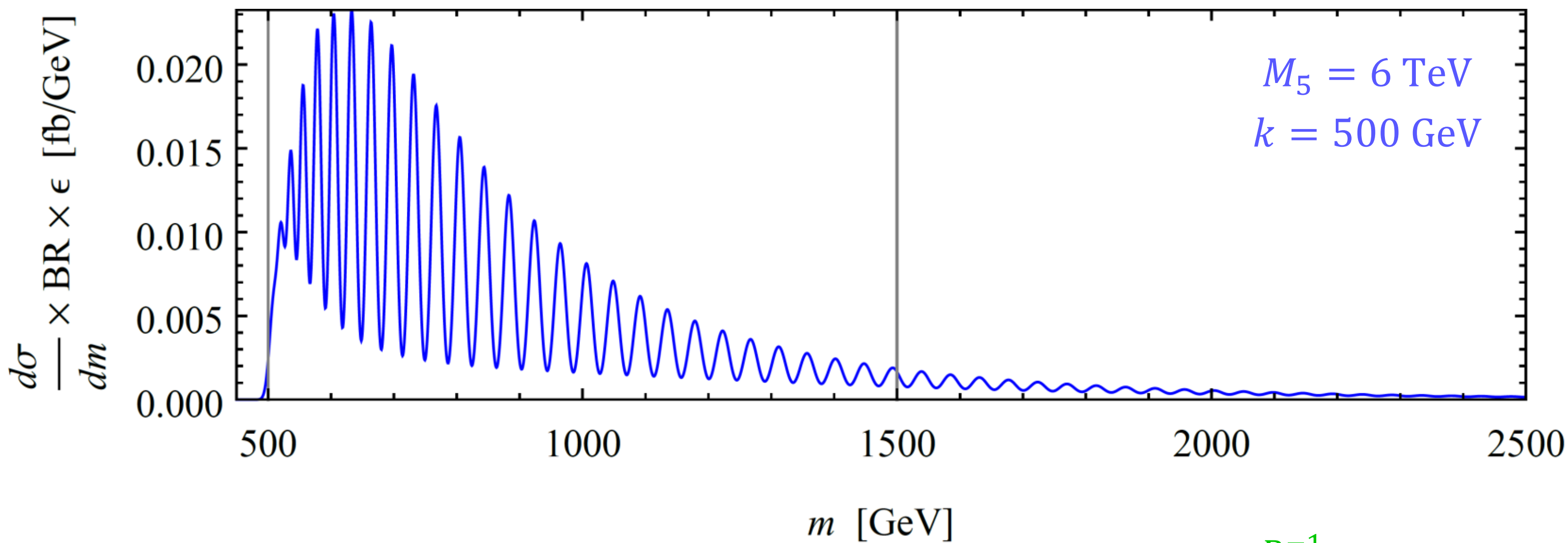
Fourier analysis of the $\gamma\gamma$ spectrum

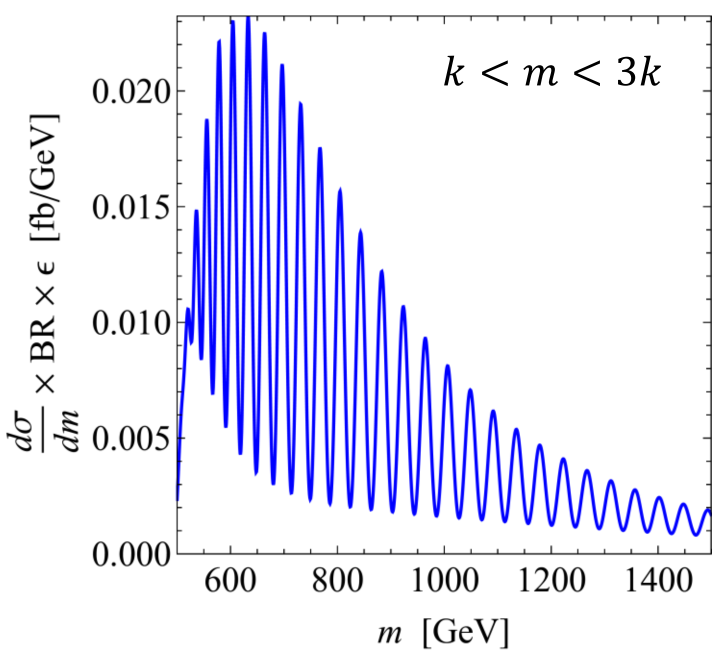


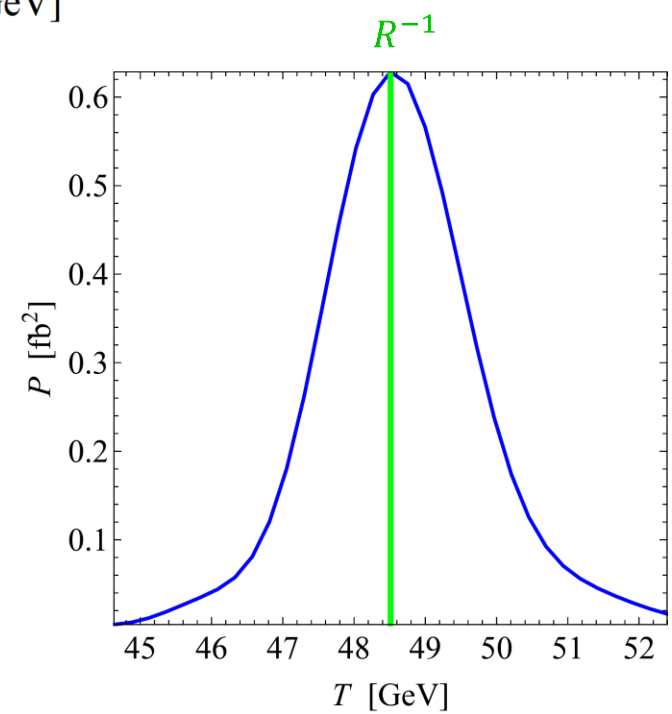
Is it possible to detect the periodic structure by analyzing the $\gamma\gamma$ spectrum in Fourier space?

$$P(T) \equiv \left| \frac{1}{\sqrt{2\pi}} \int_{m_{\min}}^{m_{\max}} dm \, \frac{d\sigma}{dm} \exp\left(i \, \frac{2\pi\sqrt{m^2 - k^2}}{T} \right) \right|^2$$

Fourier analysis of the $\gamma\gamma$ spectrum







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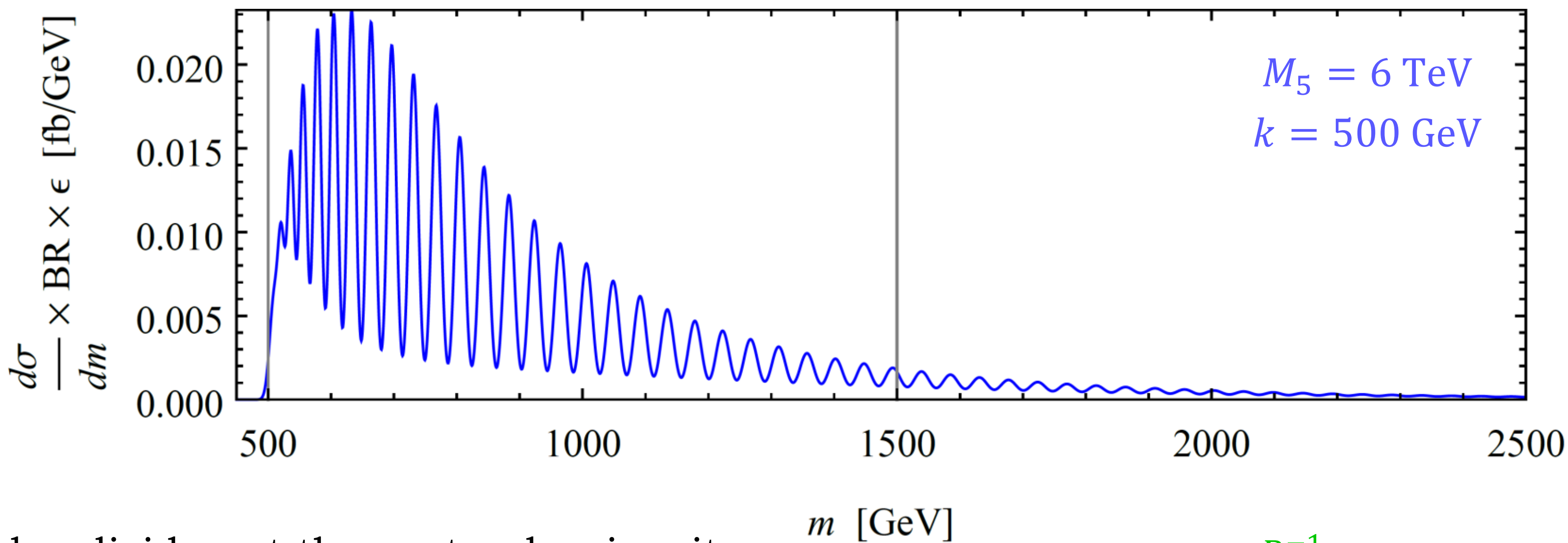
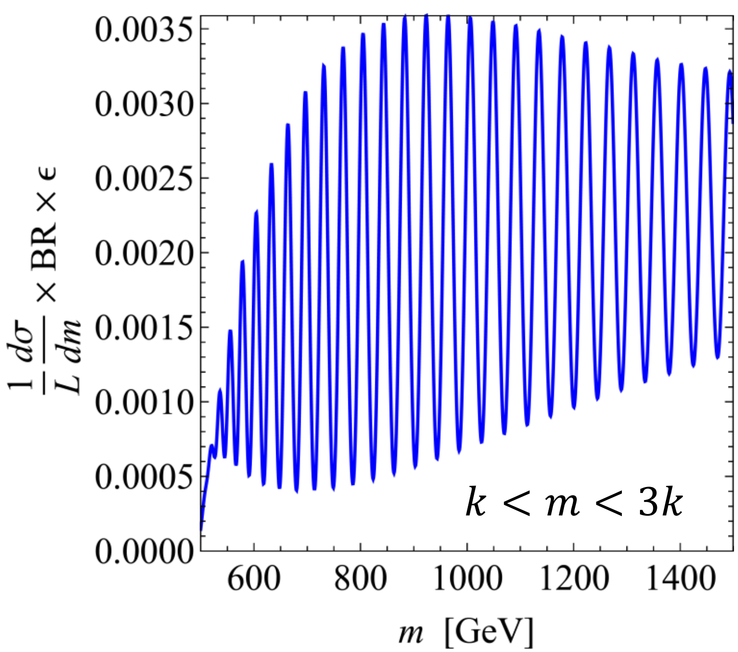
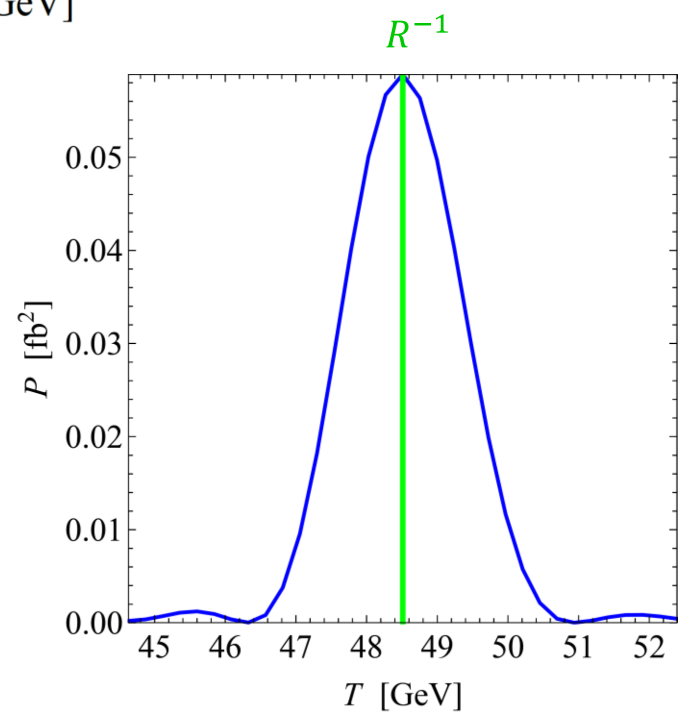


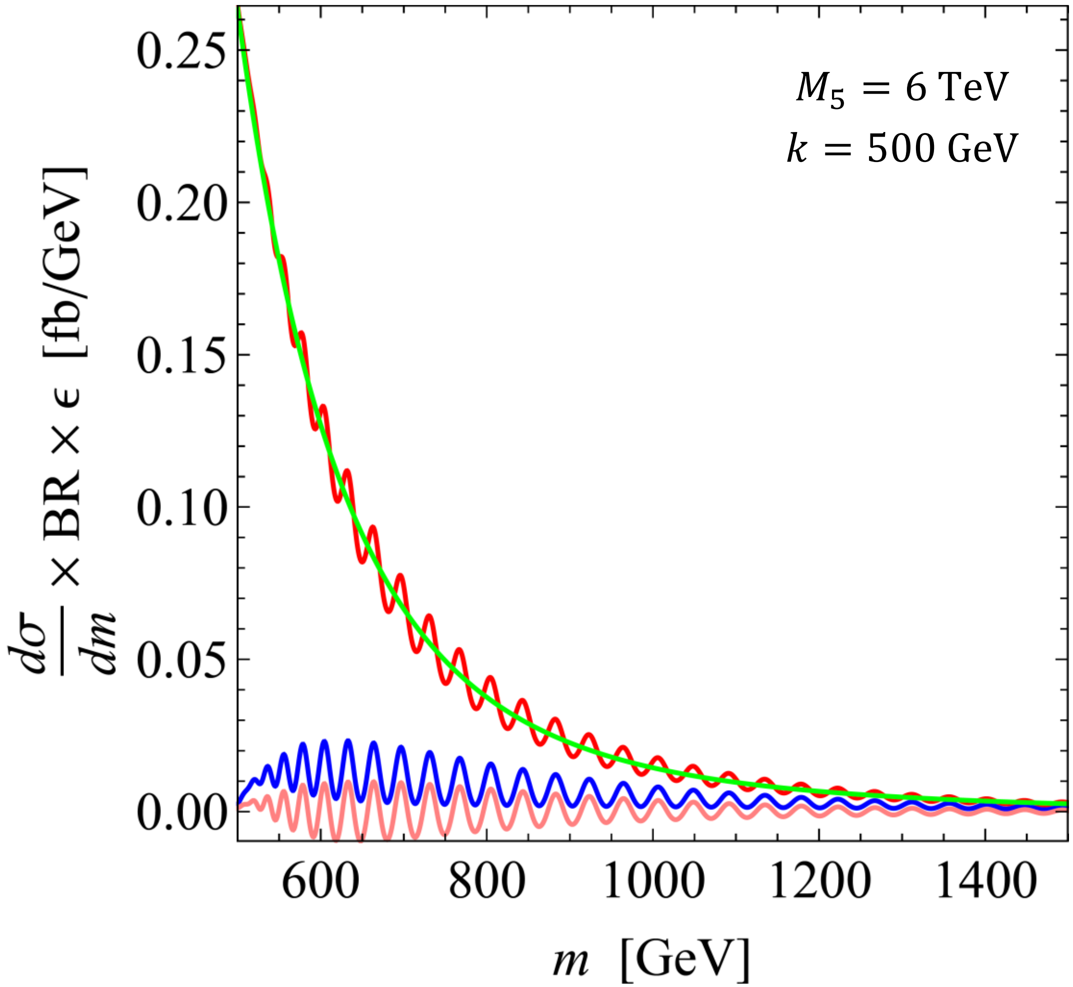
Also divide out the parton luminosity:





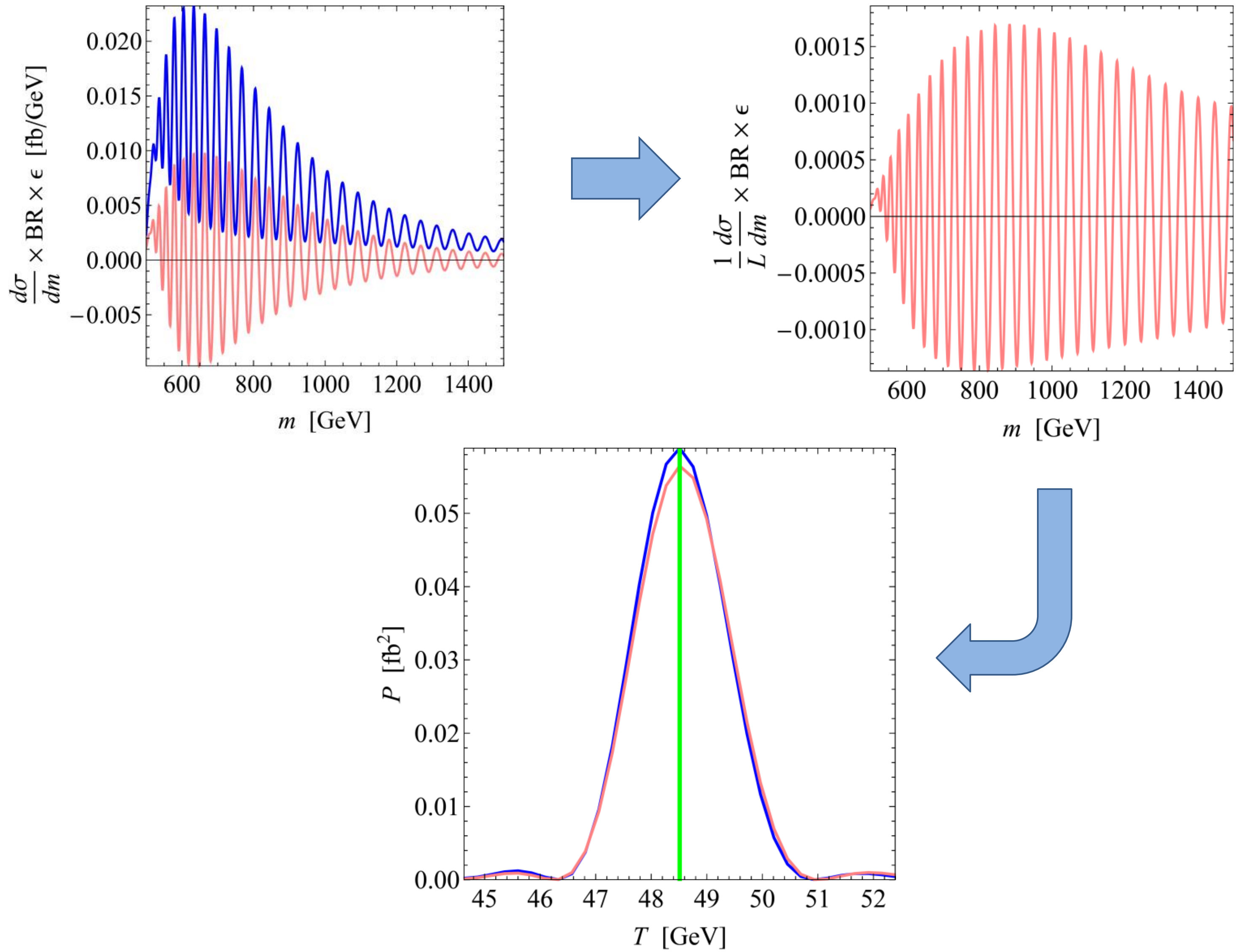
Fourier analysis of the $\gamma\gamma$ spectrum

Adding background and subtracting a fit to a smooth function.



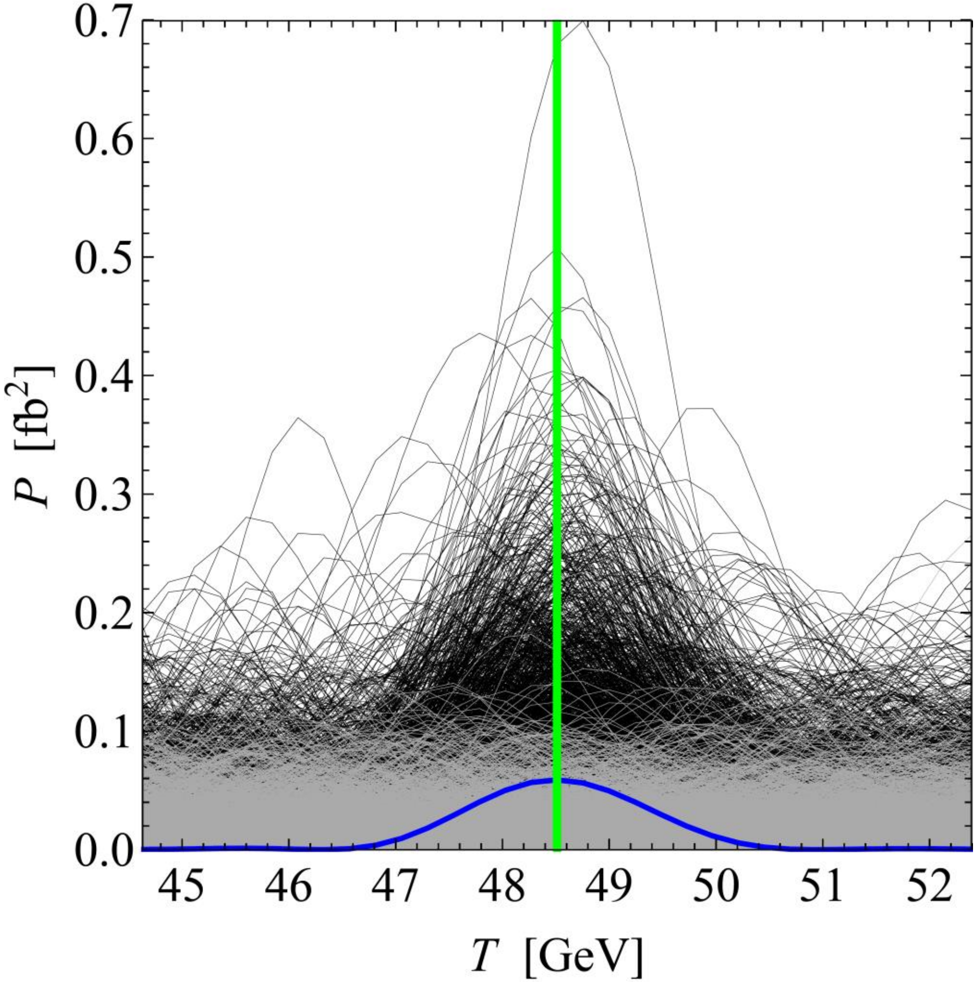
Fourier analysis of the $\gamma\gamma$ spectrum

Dividing out the parton luminosity and Fourier transforming.



Fourier analysis of the $\gamma\gamma$ spectrum

Generating multiple realizations of signal+background (black) and background alone (gray) to quantify significance.

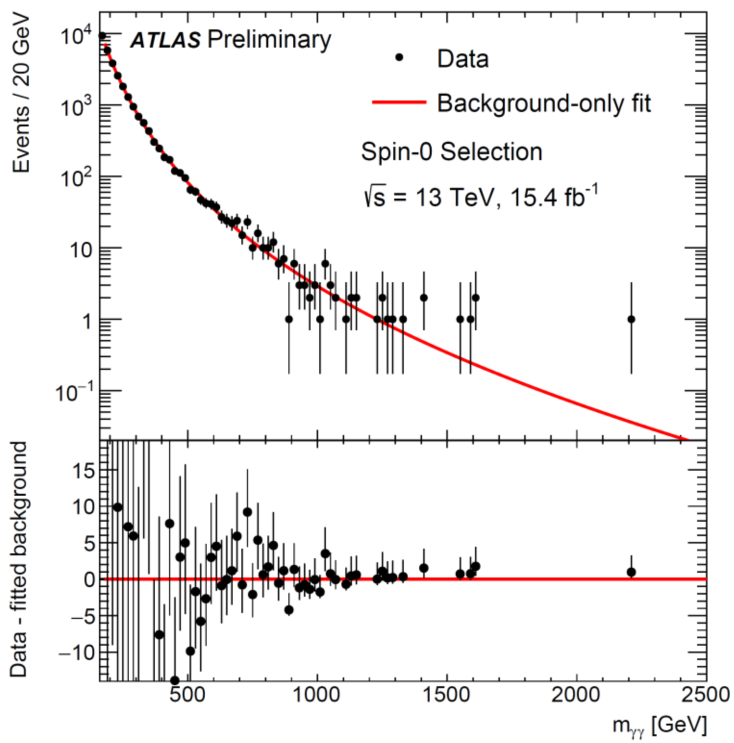


Limits from high-mass $\gamma\gamma$ continuum

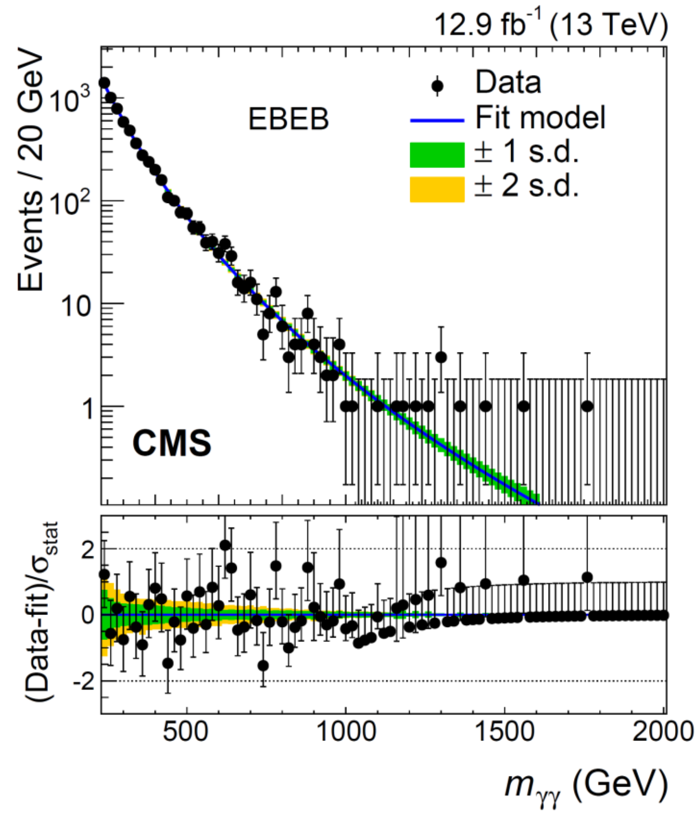
Unfortunately, no searches released since 7 TeV.

ATLAS-CONF-2012-087 (4.9 fb^{-1}); CMS [1112.0688] (2.2 fb^{-1})

Let us then examine more recent data ourselves:

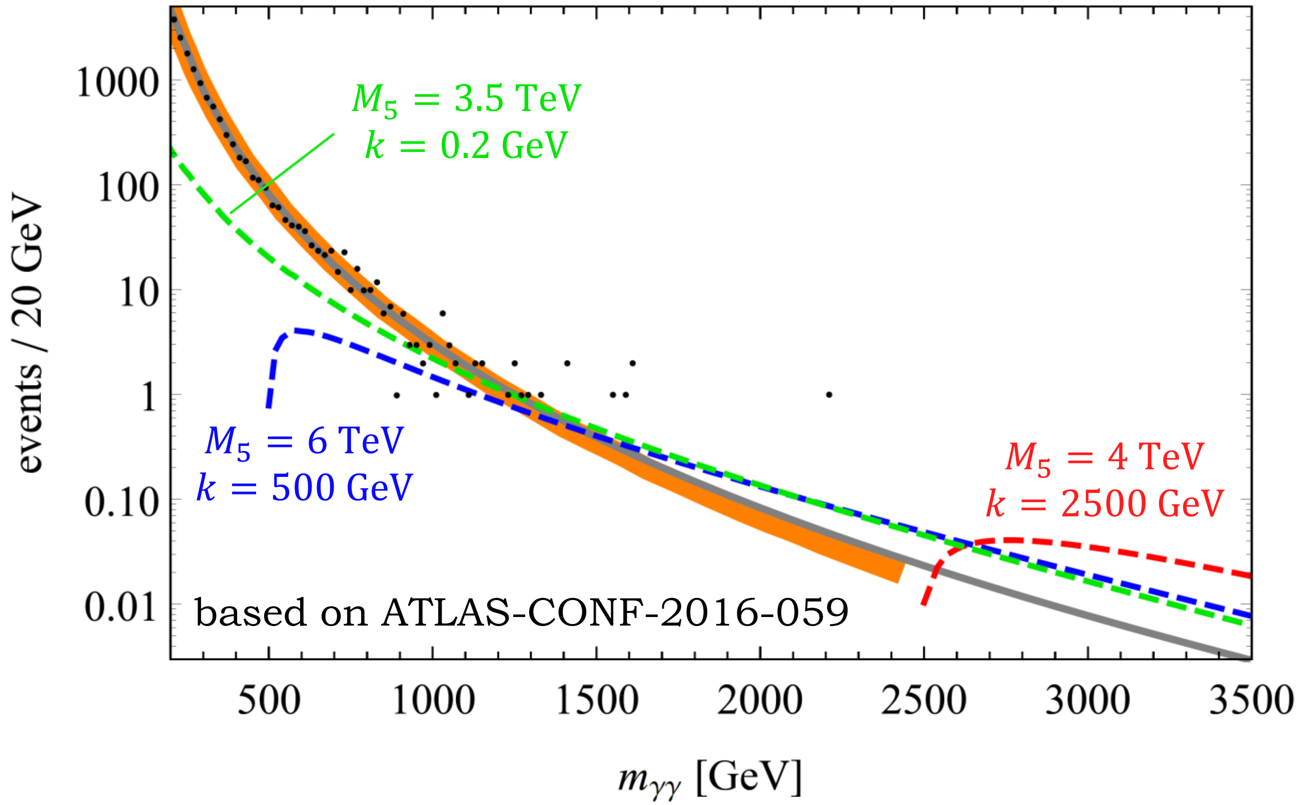


ATLAS-CONF-2016-059



arXiv:1609.02507

Limits from high-mass $\gamma\gamma$ continuum

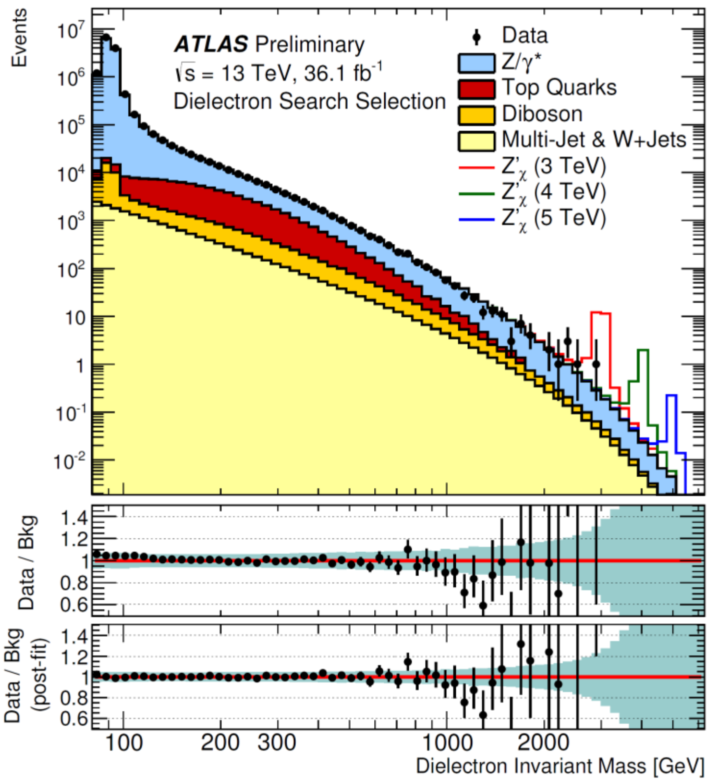


Will derive: (1) **conservative limits** assuming background is completely unknown

(2) **approximate expected limits** assuming signal is absent and statistics-dominated uncertainties

Search regions: $m_{\gamma\gamma} > 500,\ 1000,\ 2000$ GeV

Limits from high-mass $\ell^+\ell^-$ continuum



m_{ee} [GeV]	80–120	120–250	250–400	400–500	500–700
Drell-Yan	11800000 ± 700000	216000 ± 11000	17230 ± 1000	2640 ± 180	1620 ± 120
Top Quarks	28600 ± 1800	44600 ± 2900	8300 ± 600	1130 ± 80	560 ± 40
Dibosons	31400 ± 3300	7000 ± 700	1300 ± 140	228 ± 25	146 ± 16
Multi-jet & W+jets	11000 ± 9000	5600 ± 2000	780 ± 80	151 ± 21	113 ± 17
Total SM	11900000 ± 700000	273000 ± 12000	27600 ± 1100	4150 ± 200	2440 ± 130
Data	12415434	275711	27538	4140	2390
Z'_χ (4 TeV)	0.00635 ± 0.00021	0.0390 ± 0.0015	0.0564 ± 0.0025	0.0334 ± 0.0027	0.064 ± 0.004
Z'_χ (5 TeV)	0.00305 ± 0.00012	0.0165 ± 0.0006	0.0225 ± 0.0010	0.0139 ± 0.0007	0.0275 ± 0.0015

m_{ee} [GeV]	700–900	900–1200	1200–1800	1800–3000	3000–6000
Drell-Yan	421 ± 34	176 ± 17	62 ± 7	8.7 ± 1.3	0.34 ± 0.07
Top Quarks	94 ± 8	27.9 ± 2.8	5.1 ± 0.7	< 0.001	< 0.001
Dibosons	39 ± 4	16.9 ± 2.1	5.8 ± 0.8	0.74 ± 0.11	0.028 ± 0.004
Multi-jet & W+jets	39 ± 6	16.1 ± 2.0	7.9 ± 2.3	1.6 ± 1.2	0.08 ± 0.27
Total SM	590 ± 40	237 ± 17	81 ± 7	11.0 ± 1.8	0.45 ± 0.28
Data	589	209	61	10	0
Z'_χ (4 TeV)	0.0585 ± 0.0035	0.074 ± 0.005	0.121 ± 0.011	0.172 ± 0.017	2.57 ± 0.27
Z'_χ (5 TeV)	0.0218 ± 0.0013	0.0295 ± 0.0021	0.040 ± 0.004	0.040 ± 0.004	0.280 ± 0.030

… and analogously for muons.

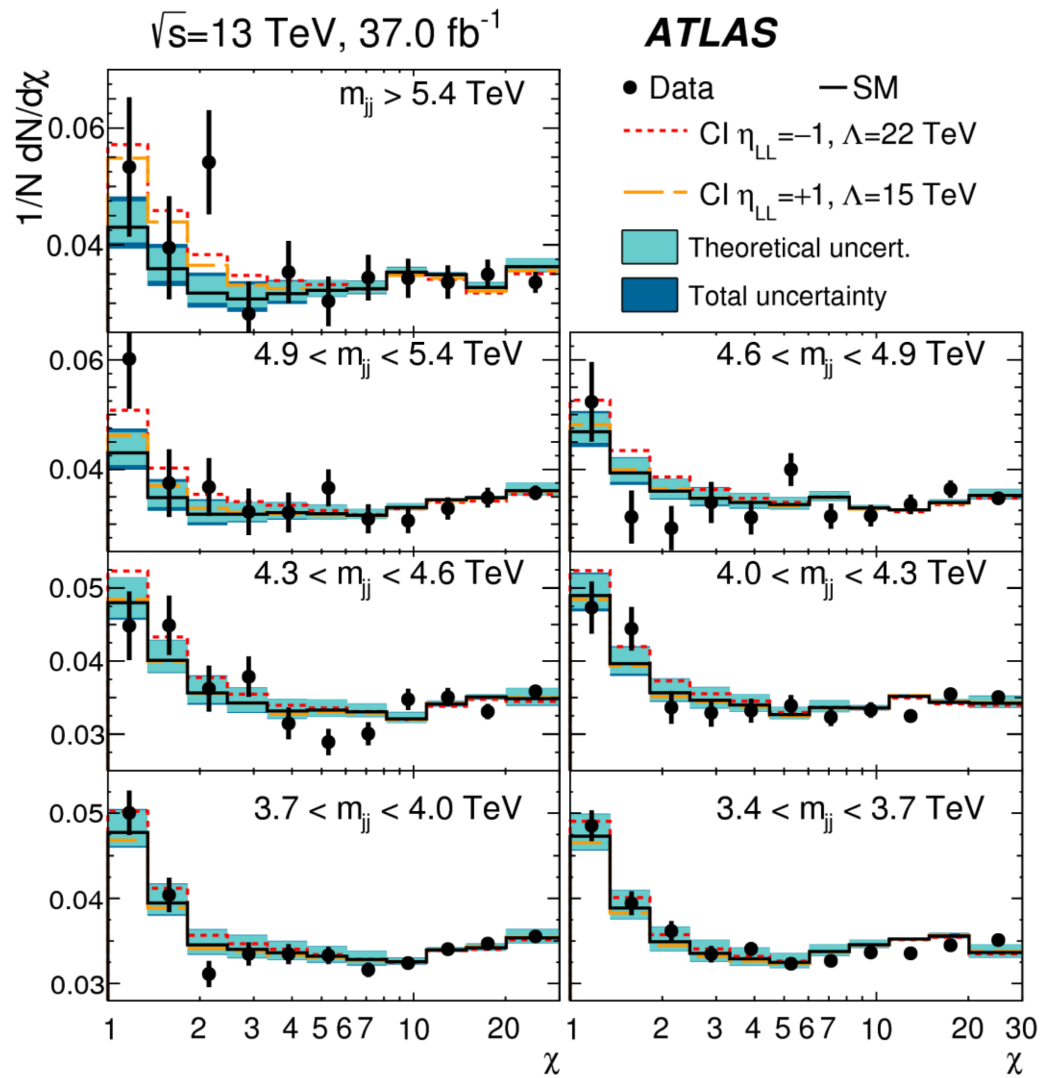
ATLAS-CONF-2017-027

Limits from dijet angular distributions

Searches look at angular distributions in m_{jj} bins, using the variable

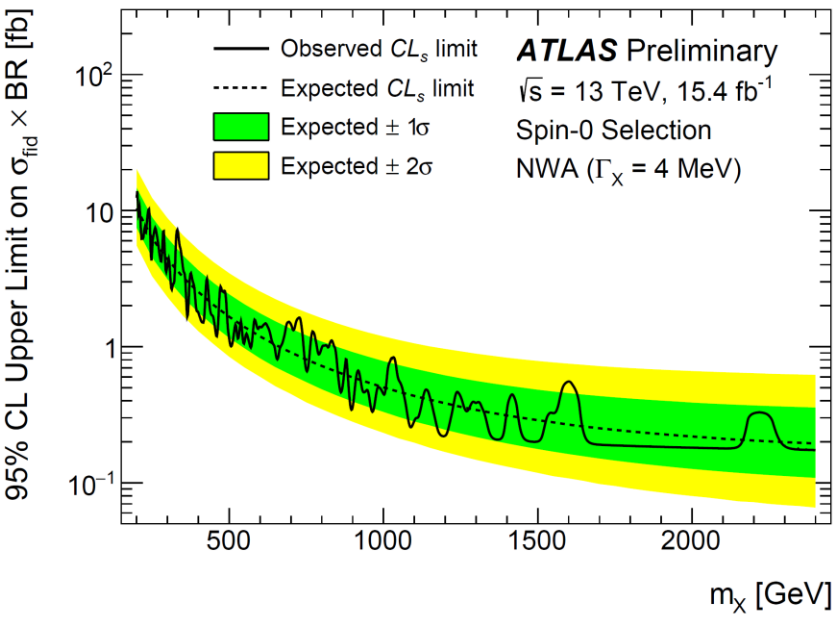
$$\chi = \exp(|y_1 - y_2|)$$

ATLAS, we can't read many of the numbers here. 😕

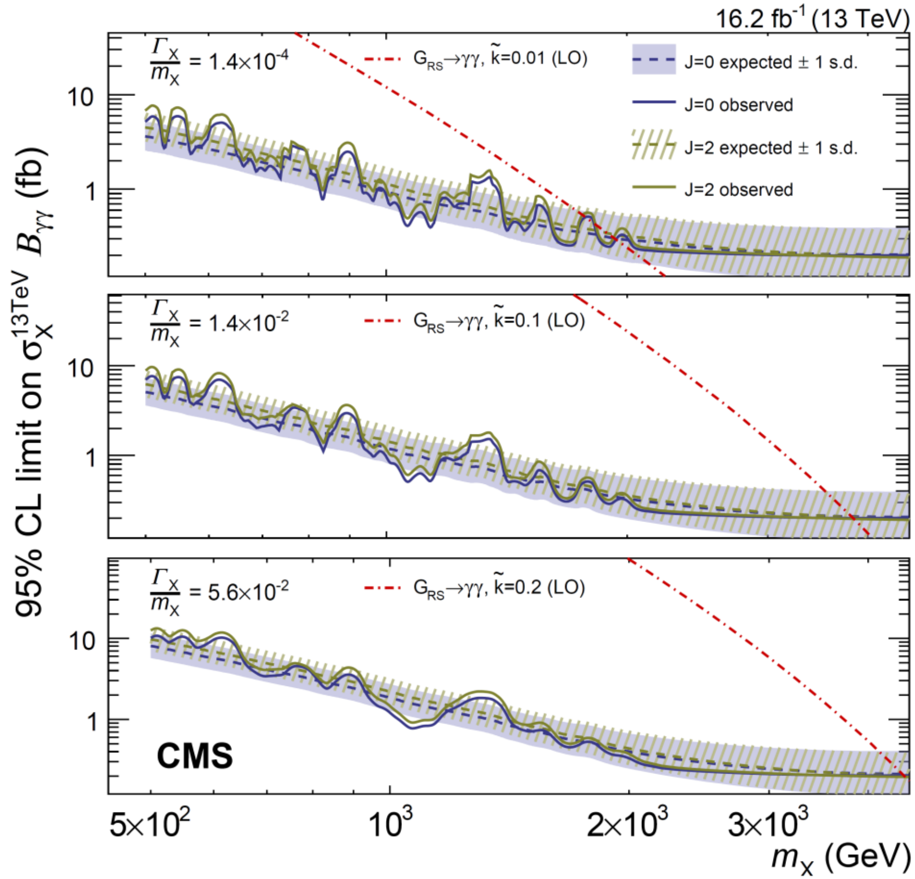


arXiv:1703.09127

Limits from $\gamma\gamma$ resonance searches



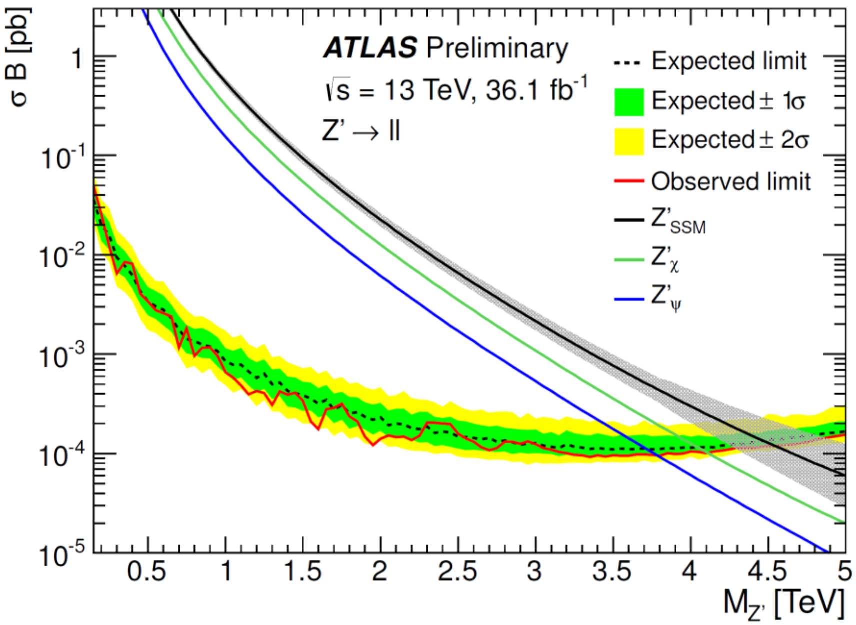
ATLAS-CONF-2016-059



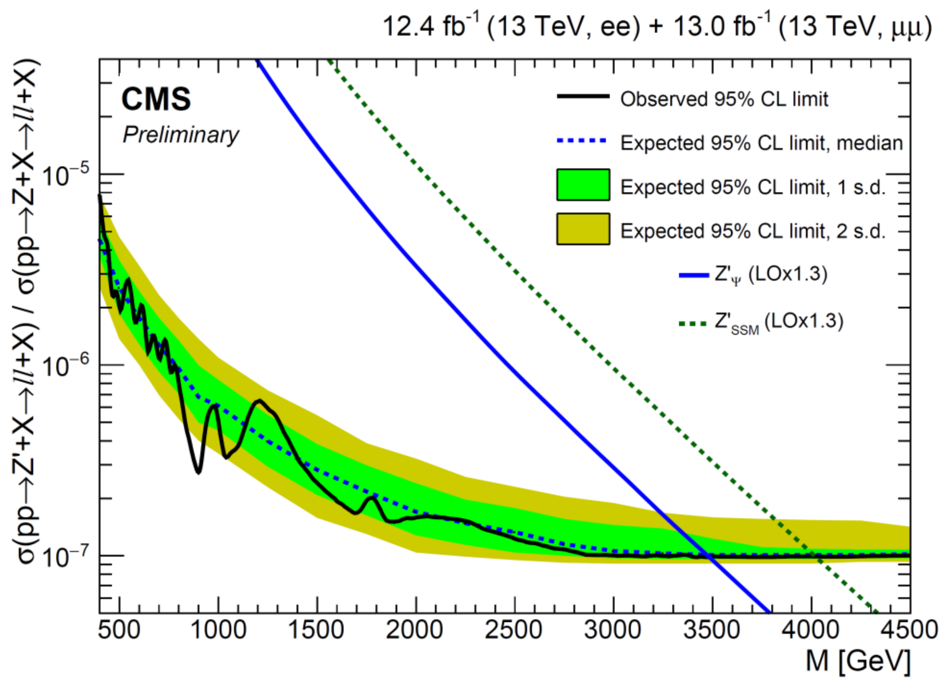
arXiv:1609.02507

Caveats:
1. Bump hunting might not work as expected due to the additional nearby peaks.
2. Intrinsic background due to the rest of the KK tower is not taken into account.

Limits from $\ell^+\ell^-$ resonance searches



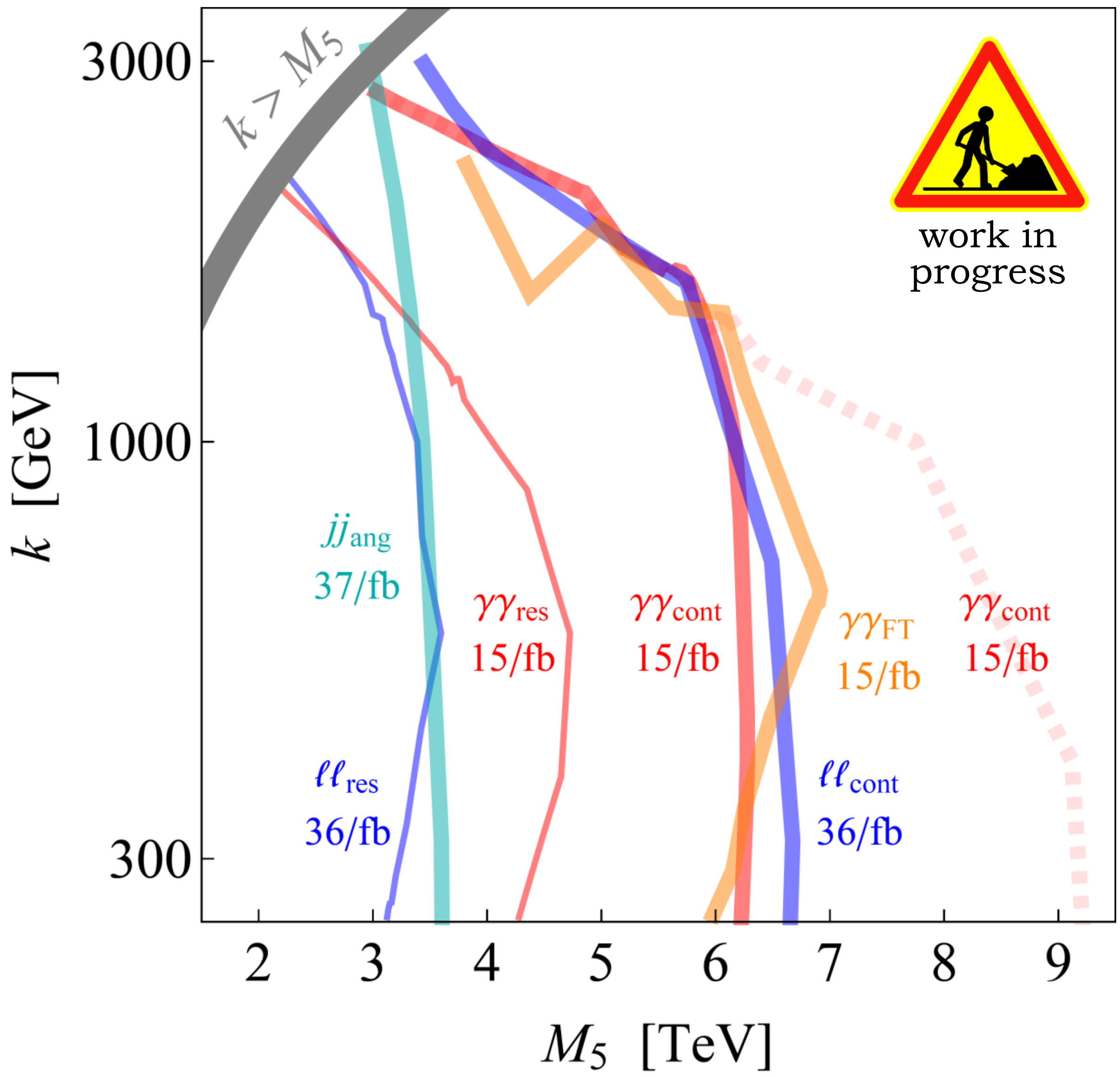
ATLAS-CONF-2017-027



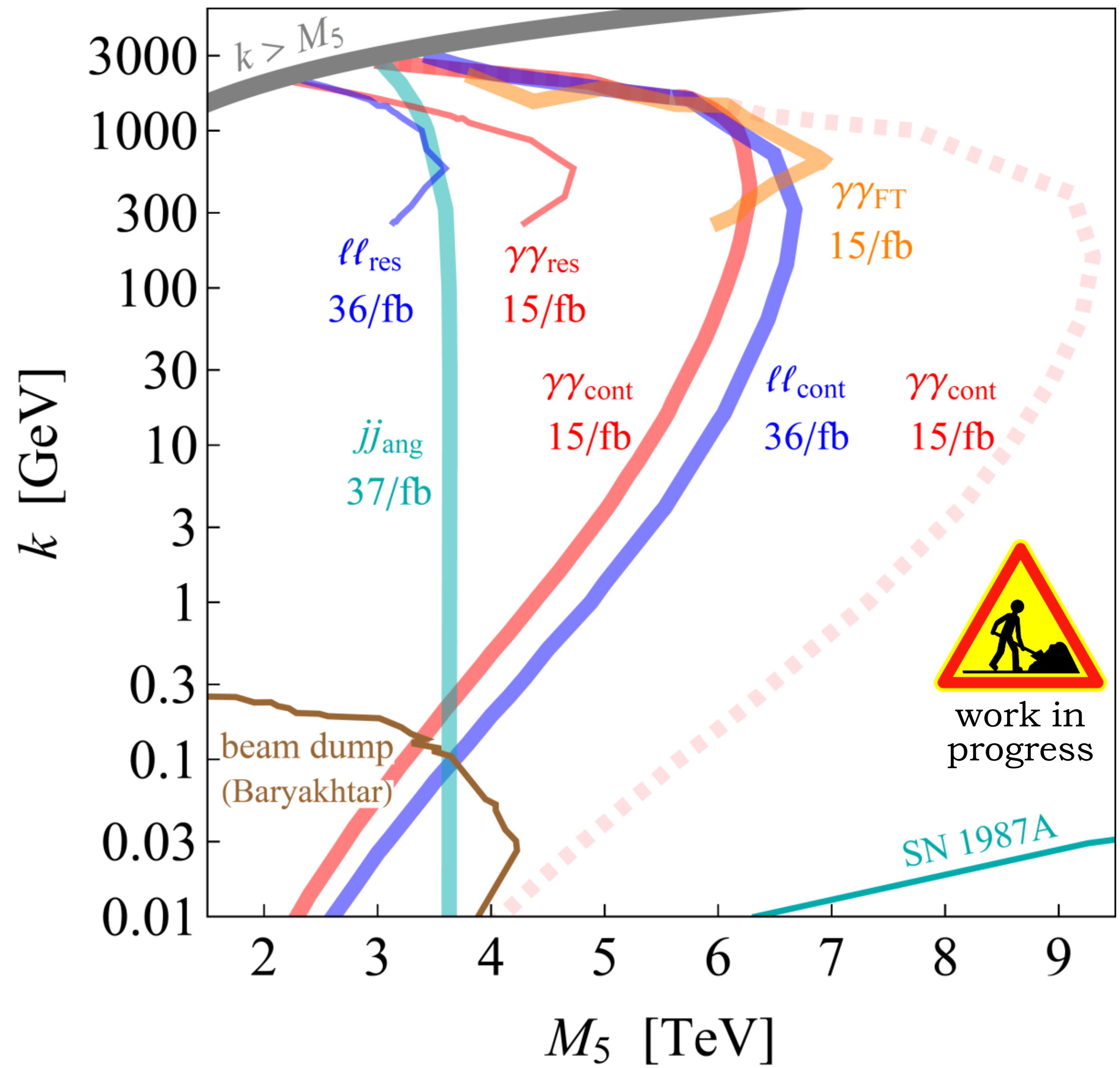
CMS-PAS-EXO-16-031

Caveats:
1. Bump hunting might not work as expected due to the additional nearby peaks.
2. Intrinsic background due to the rest of the KK tower is not taken into account.

Sensitivity of some of the channels



Sensitivity of some of the channels

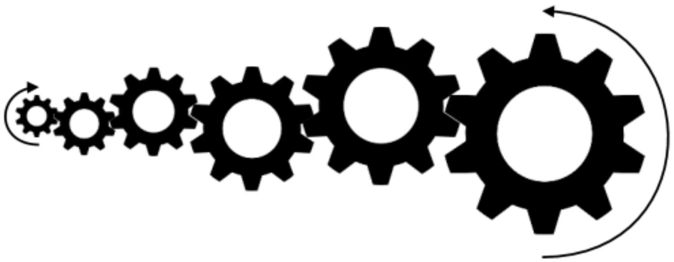


Conclusions

The clockwork / linear dilaton solution to the hierarchy problem features novel LHC signatures:

- Effects on diphoton / dilepton / dijet spectra qualitatively different from ADD benchmark models.
- Motivation for searches in Fourier space.
- Interesting benchmark models for high-multiplicity final states.
- Interesting benchmark models for displaced decays.

Thank you for your time!



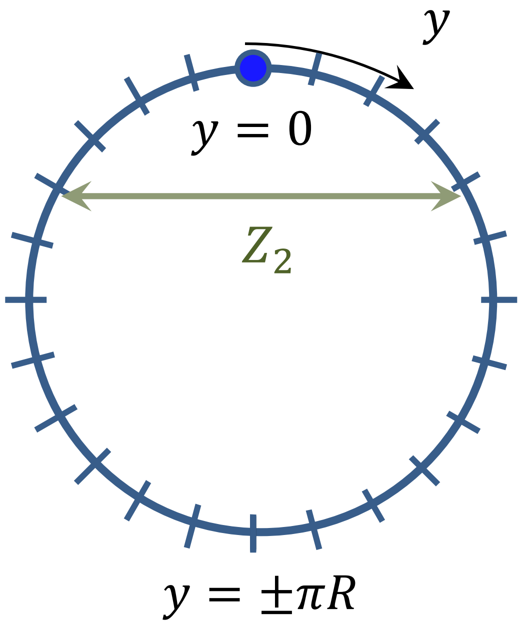
Supplementary Slides

$N \to \infty$ clockwork as an extra dimension

Consider a compact extra dimension and a metric of the form

$$ds^2 = X(|y|)\,\eta_{\mu\nu}dx^\mu dx^\nu + Y(|y|)\,dy^2$$

and discretize it.



$N \to \infty$ clockwork as an extra dimension

Consider a compact extra dimension and a metric of the form

$$ds^2 = X(|y|)\,\eta_{\mu\nu}dx^\mu dx^\nu + Y(|y|)\,dy^2$$

The action for a massless scalar (for simplicity) can be written as

$$S = \int d^4x\,dy\sqrt{-g}\left(-\frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi\right)$$

$$\to -\frac{1}{2}\int d^4x\left[\sum_i(\partial_\mu\phi_i)^2 + \sum_i m_i^2(\phi_i - q_i\phi_{i+1})^2\right]$$

where

$$m_i^2 \equiv \frac{N^2X_i}{\pi^2R^2Y_i}, \qquad q_i \equiv \frac{X_i^{1/2}Y_i^{1/4}}{X_{i+1}^{1/2}Y_{i+1}^{1/4}}$$

To have m_i^2 and q_i uniform across the sites, can take

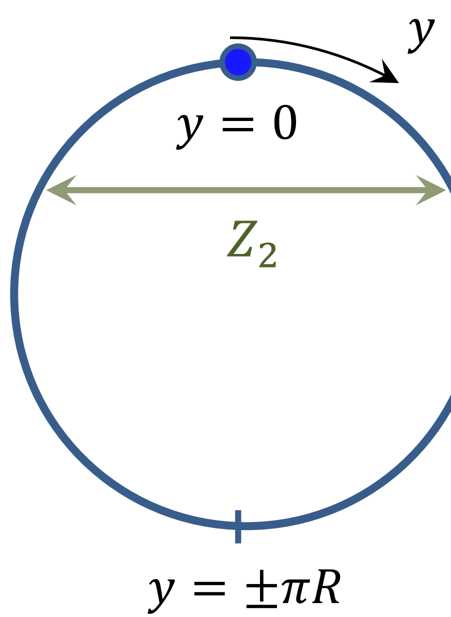
$$X(|y|) \propto Y(|y|) \propto e^{-\frac{4}{3}k|y|}$$

where k is a free parameter, with which $q^N = e^{k\pi R}$.

$N \to \infty$ clockwork as an extra dimension

What kind of physics would create such a metric?

$$ds^2 = e^{\frac{4}{3}k|y|}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\right)$$



A background scalar field linear in y:

$$S(y) = 2k|y|$$

Known in the string theory literature as the **linear dilaton background**

Clockwork / linear dilaton setup

$$S = \int dy d^4x \sqrt{-g}\, \frac{M_5^3}{2} e^S (R + (\nabla S)^2 + 4k^2) + \sum_{i=\mathrm{SM,h}} e^{S(y_i)} \int d^4x \sqrt{-g}\, (\mathcal{L}_i - \Lambda_i)$$

where in LST

$$M_5 \simeq \frac{M_s^3 V_6^{1/3}}{N^{1/6}}, \qquad k = \frac{M_s}{2\sqrt{N}} \,.$$

Going from Jordan to Einstein frame ($g_{MN} \to e^{-2S/3} g_{MN}$):

$$S = \int dy d^4x \sqrt{-g}\, \frac{M_5^3}{2} \left(R - \tfrac{1}{3}(\nabla S)^2 - V(S)\right) - \sum_{i=\mathrm{SM,h}} e^{-S(y_i)/3} \int d^4x \sqrt{-g}\, (\mathcal{L}_i - \Lambda_i)$$

where $V(S) = -4k^2 e^{-2S/3}$.

Background solution:

$$ds^2 = e^{\frac{4}{3}k|y|} (\eta_{\mu\nu} dx^\mu dx^\nu + dy^2) \qquad S(y) = 2k|y|$$

assuming $\Lambda_\mathrm{h} = -\Lambda_\mathrm{SM} = 4M_5^3 k.$

Clockwork vs. RS geometry

Randall-Sundrum

$$ds^2 = e^{2kz}\,\eta_{\mu\nu}dx^{\mu}dx^{\nu} + dz^2 \qquad\qquad 0 \leq z \leq \pi R$$

$$R = -20k^2$$

Clockwork / linear dilaton

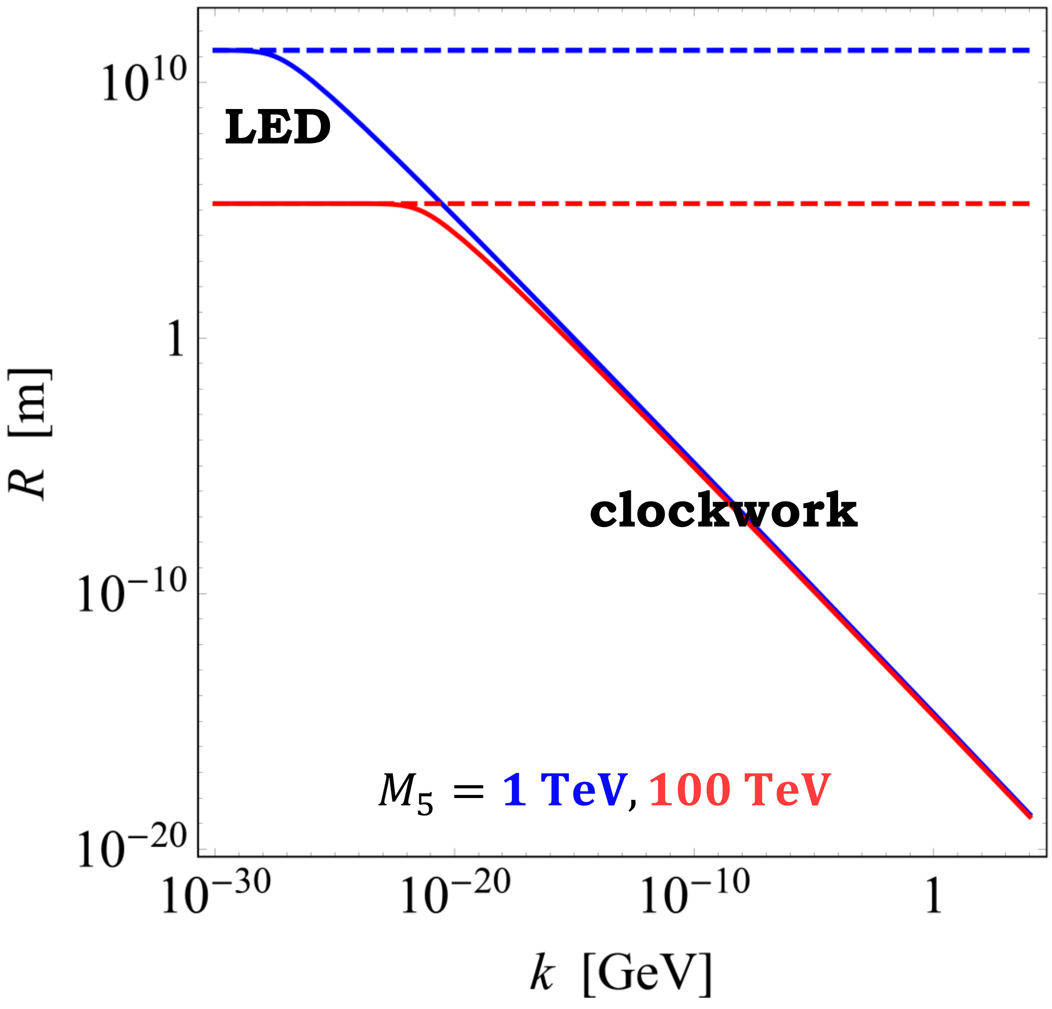
$$ds^2 = e^{\frac{4}{3}ky}\left(\eta_{\mu\nu}dx^{\mu}dx^{\nu} + dy^2\right) \qquad\qquad 0 \leq y \leq \pi R$$

$$= \left(1 + \tfrac{2}{3}kz\right)^2 \eta_{\mu\nu}dx^{\mu}dx^{\nu} + dz^2 \qquad\qquad 1 \leq \tfrac{2}{3}kz \leq e^{\frac{2}{3}k\pi R}$$

$$R = -\frac{12}{z^2}$$

4d Planck scale

$$M_P^2 = \frac{M_5^3}{k}\left(e^{2\pi kR} - 1\right) \quad \Longrightarrow \quad R(M_5, k)$$

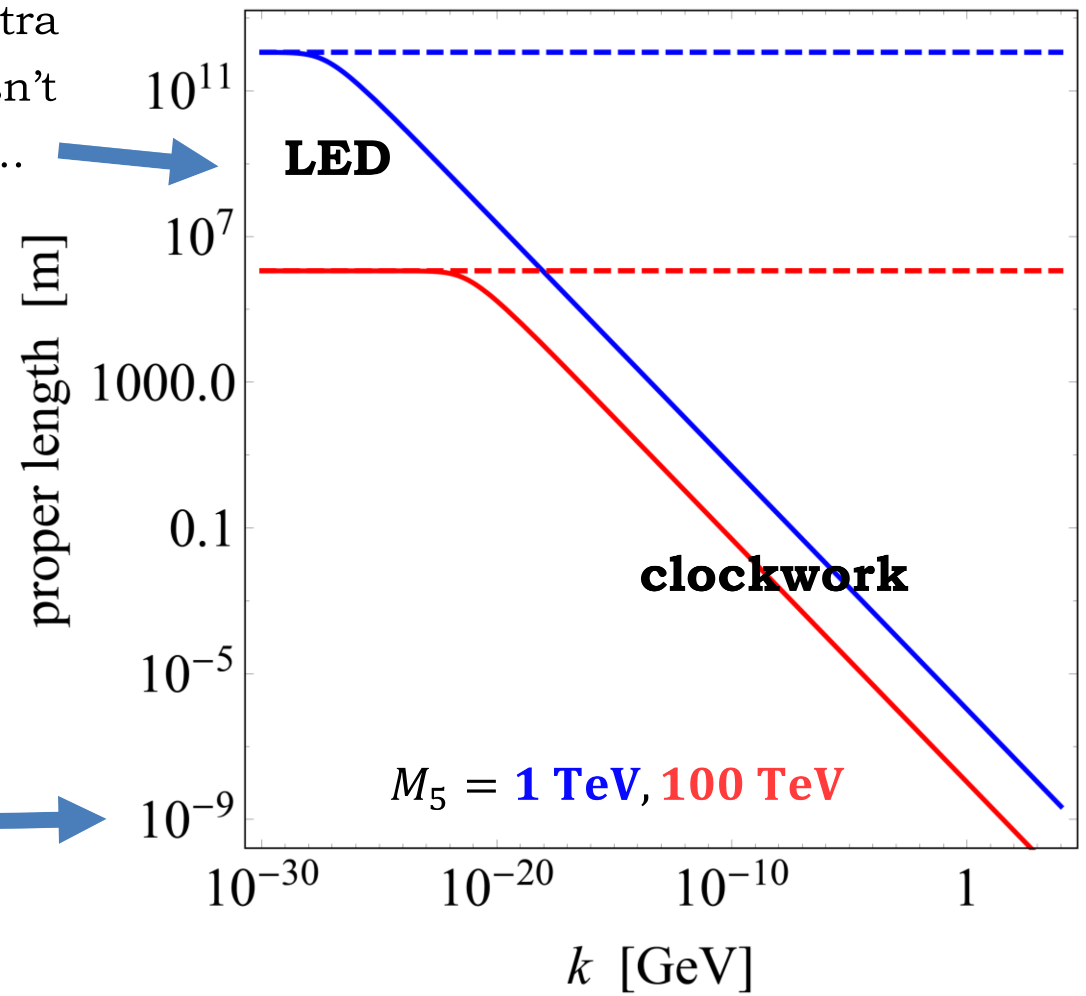


4d Planck scale

$$M_P^2 = \frac{M_5^3}{k}\left(e^{2\pi kR} - 1\right) \quad \Longrightarrow \quad R(M_5, k)$$

Such a large extra dimension doesn't exist in nature…

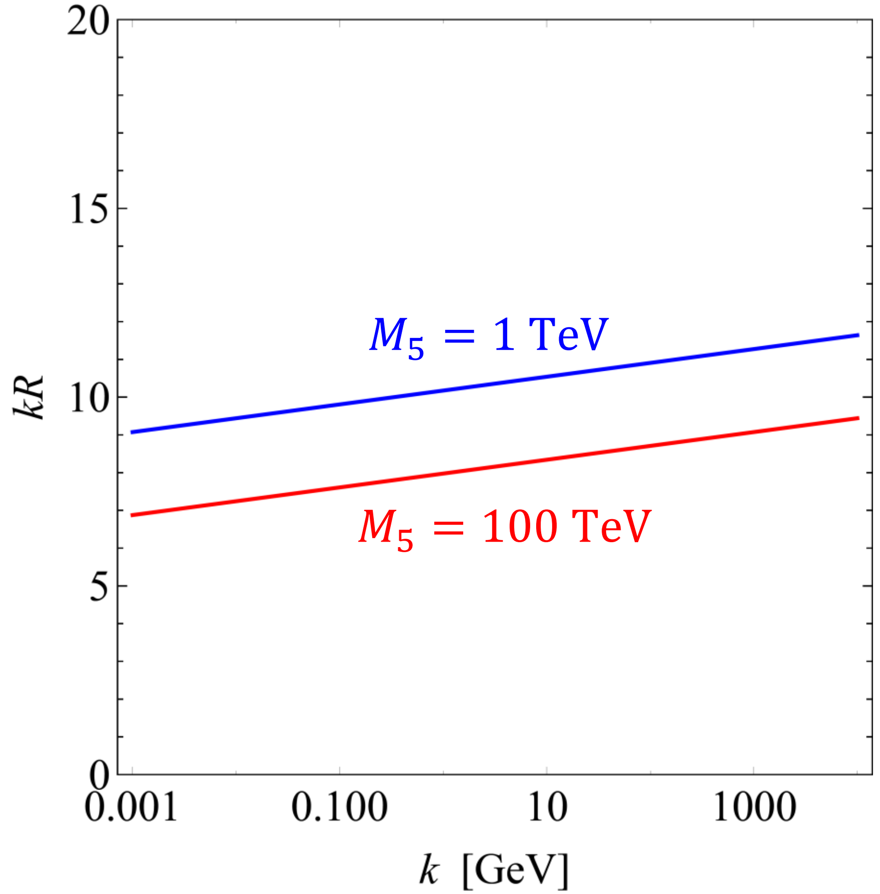
… but this could work



4d Planck scale

$$M_P^2 = \frac{M_5^3}{k}\left(e^{2\pi kR} - 1\right) \quad \Longrightarrow \quad R(M_5, k)$$

For $M_5 \sim \mathcal{O}(10\ \text{TeV})$: $\boldsymbol{kR \approx 10}$



KK graviton width vs. mass splitting

Even with significant decays to lighter KK gravitons, the width

$$\Gamma_{n\to \mathrm{KK}} \simeq \frac{5\cdot 7\cdot 17}{3\cdot 2^{14}\pi^2}\frac{\sqrt{k m_n} m_n^3}{kRM_5^3}$$

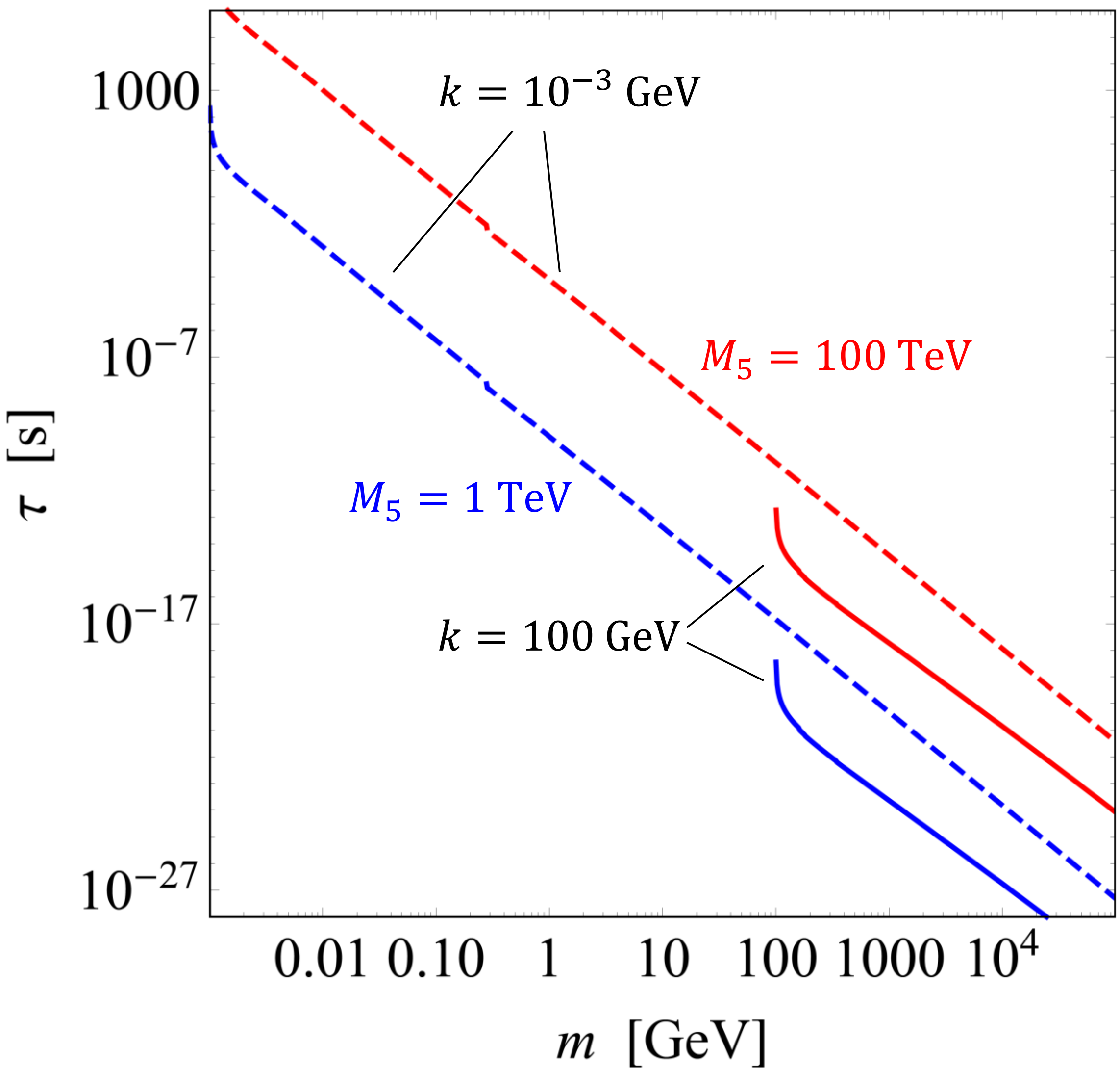
is smaller than the mass splitting, $1/R$, as long as

$$m_n \lesssim 6.8\left(\frac{k}{M_5}\right)^{1/7} M_5 \,.$$

This is satisfied for all $m_n < M_5$ as long as

$$k \gtrsim 1.5\times 10^{-6}\ M_5 \approx 15\ \mathrm{MeV}\left(\frac{M_5}{10\ \mathrm{TeV}}\right)$$

KK graviton lifetimes



KK dilaton decays

For coupling to T_μ^μ only and neglecting phase space suppressions

$$\Gamma_{WW} = \frac{\Gamma_0}{2} \qquad \Gamma_{ZZ} = \Gamma_{hh} = \frac{\Gamma_0}{4}$$

$$\Gamma_{gg} = 49\left(\frac{\alpha_s}{2\pi}\right)^2 \Gamma_0 \approx 0.012\,\Gamma_0$$

$$\Gamma_{\gamma\gamma} = \frac{289}{72}\left(\frac{\alpha}{2\pi}\right)^2 \Gamma_0 \approx 6\times 10^{-6}\,\Gamma_0$$

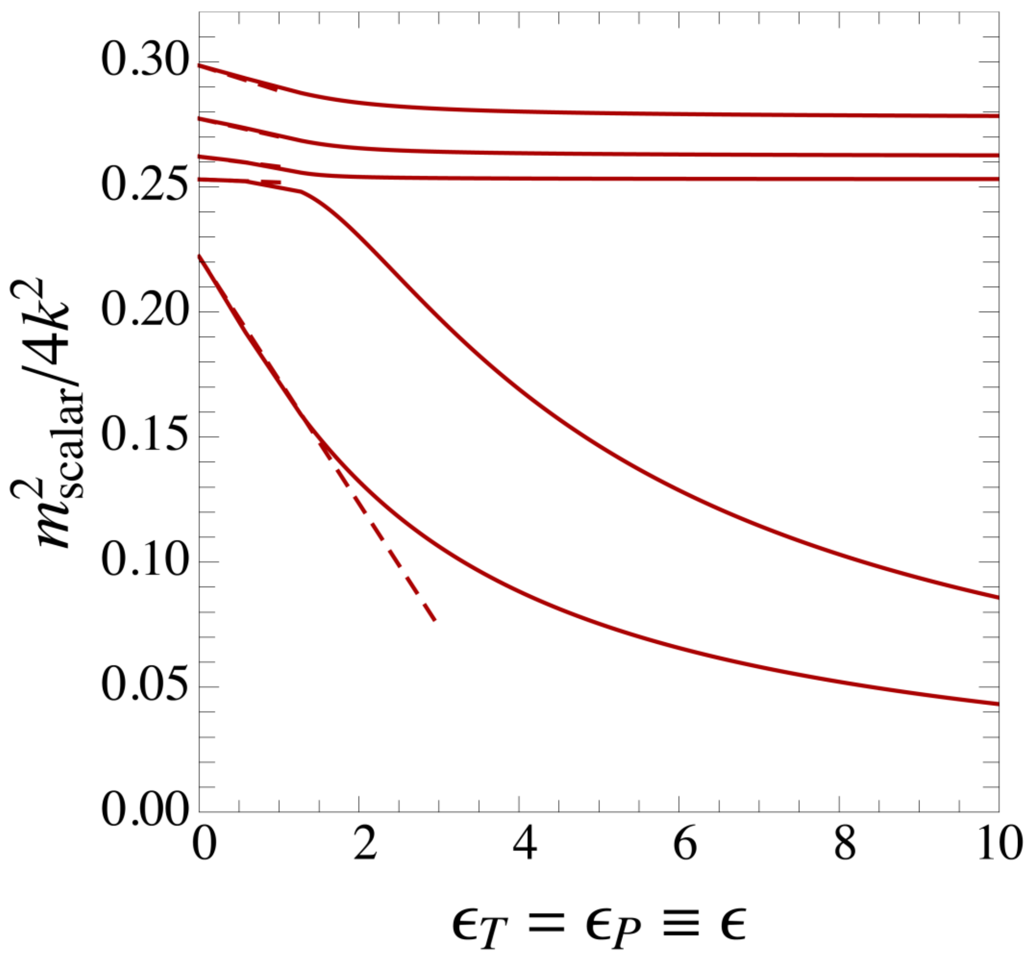
$$\Gamma_{f\bar{f}} = N_{c,f}\left(\frac{m_f}{m_n}\right)^2 \Gamma_0$$

where for $n \gg kR$

$$\Gamma_0 \simeq \frac{1}{54\pi^2 kR}\left(\frac{k}{M_5}\right)^3 m_n$$

Dependence on stabilization mechanism

KK dilaton/radion mass spectrum



Dependence on stabilization mechanism

KK dilaton/radion couplings

$$\mathcal{L}_n \supset -\left(\frac{\Phi_n(0)}{2}\, T^{\mu}_{\mu} + \frac{\varphi_n(0)}{3}\, \mathcal{L}_{\mathrm{SM}}\right) S^{(n)}$$