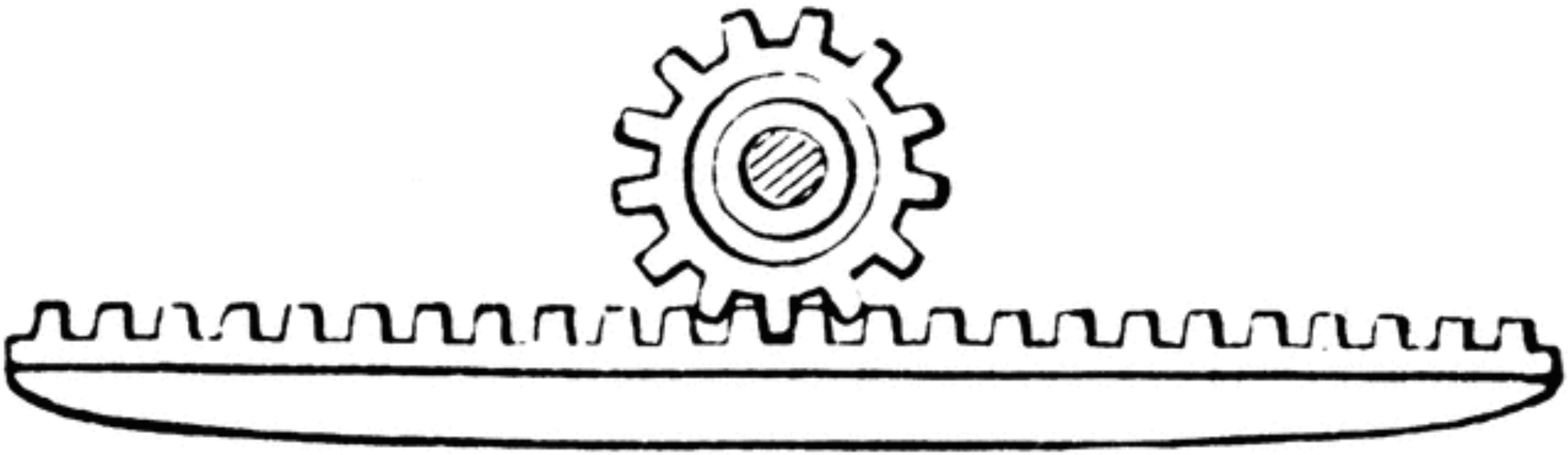
A Clockwork Theory



Planck 2017

May 23rd 2017

Based on: Giudice, MM, 2016

On Masses and Scales

There is a fundamental difference between masses and scales, sometimes overlooked.

Example: Global Symmetry Breaking

$$V = \lambda \left(\frac{f^2}{2} - |\phi|^2 \right)^2 \longleftrightarrow \phi = \frac{f + \rho}{\sqrt{2}} e^{i\pi/f}$$

Massless Goldstone boson, massive radial mode:

$$m_\pi = 0 \quad , \quad m_\rho = \sqrt{2\lambda} f$$

At low energies, Goldstone self-interactions

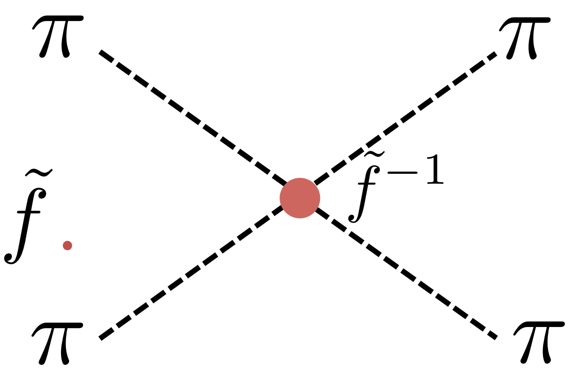
$$\mathcal{L} \supset \frac{1}{\tilde{f}^4} (\partial\pi)^4 \quad , \quad \tilde{f}^4 = 4\lambda f^4$$

On Masses and Scales

<u>Example: Global Symmetry Breaking</u>

If we could do Goldstone scattering at low energies, could measure this interaction scale:

May be tempted to think this points to UV-completion at $E \sim \tilde{f}$.



However, UV-completion enters at m_ρ which could be completely different:

$$\boxed{m_\rho^4 = \lambda \tilde{f}^4}$$

In fact, not necessarily anything important at $\tilde{f}$.

On Masses and Scales

<u>Example: Global Symmetry Breaking</u>

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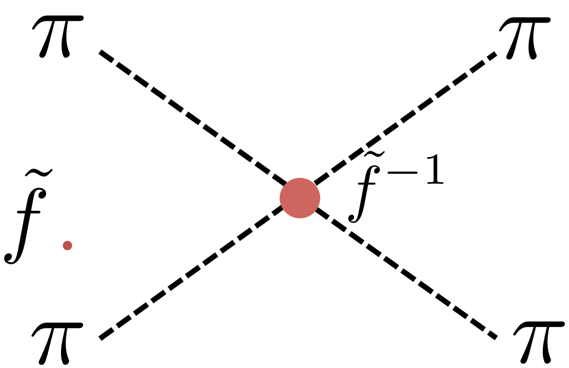

May be tempted to think this points to UV-completion at $E \sim \tilde{f}$.

...letion enters at m_ρ which

cou...

m_ρ ...

In fact, not necessarily anything important at ...

Although unitarity does provide an upper limit by which UV-completion must kick in...

A Clockwork Scalar

Choi & Im, Kaplan & Rattazzi

Take N+1 copies of original story, assume λ≈1, such that at low energies only have Goldstones:

$$\phi_j \sim \frac{f}{\sqrt{2}} e^{i\pi_j/f} \quad , \quad j = 0, .., N$$

Now explicitly break N of the U(1) symmetries explicitly with spurions,

$$\mathcal{L} = \mathcal{L}(\phi_j) - \sum_{j=0}^{N-1} \epsilon \phi_j^* \phi_{j+1}^3 + h.c.$$

This action is justified by symmetry assignments for spurions.

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Can take other "q"

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This action is justified by symmetry assignments for spurions.

A Clockwork Scalar

Action given by

“Interaction basis π”

$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N}(\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\left(e^{\frac{i}{f}(q\pi_{j+1}-\pi_j)} + h.c.\right)$$

Spontaneous symmetry breaking pattern:

$$\mathrm{U}(1)^{N+1} \to \emptyset$$

So expect $N+1$ Goldstones.

Explicit symmetry breaking:

$$\mathrm{U}(1)^{N+1} \to \mathrm{U}(1)$$

So expect N pseudo-Goldstones and one true Goldstone.

Can identify true Goldstone direction from remaining shift symmetry

$$\pi_j \to \pi_j + \kappa/q^j$$

A Clockwork Scalar

Identify Goldstone couplings by promoting shift parameter to a field:

$$\pi_j \to \pi_j + a(x)/q^j$$

Now, imagine we had some fields coupled to π_N.
Coupling to massless Goldstone becomes:

Cutoff

$$\frac{\pi_N}{f} \to \frac{a_0}{q^N f}$$

Coupling

Exponential separation between zero mode coupling and cutoff! This is generated entirely from the shift symmetry, not from the form of the interaction or potential.

A Clockwork Scalar

Peculiar spectrum, reminiscent of Condensed Matter...

Mass matrix

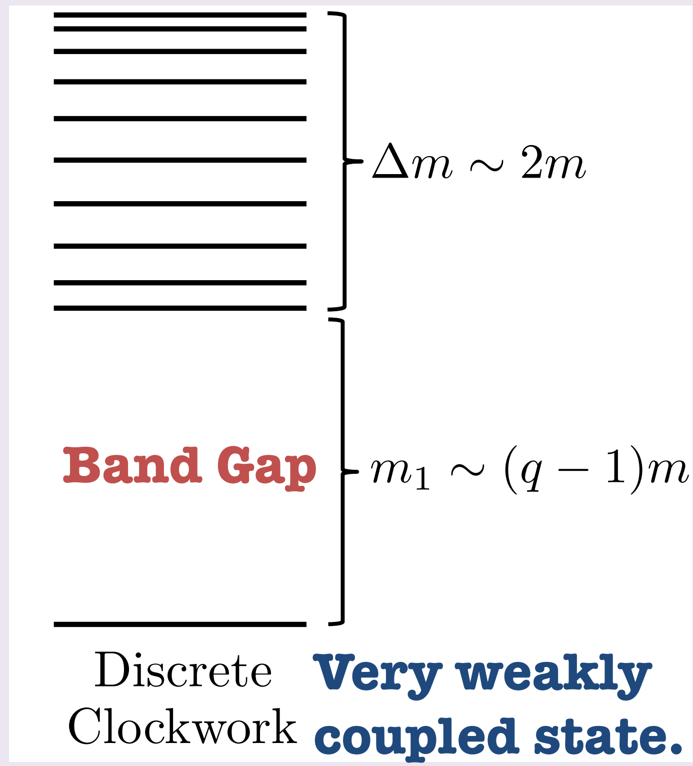
$$M_\pi^2 = m^2 \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 \end{pmatrix} .$$

Eigenvalues for “Clockwork Gears”

$$m_{a_k}^2 = \left(q^2 + 1 - 2q \cos \frac{k\pi}{N+1} \right) m^2$$

$$k = 1, .., N$$

Mass spectrum



How might this be useful in practice?

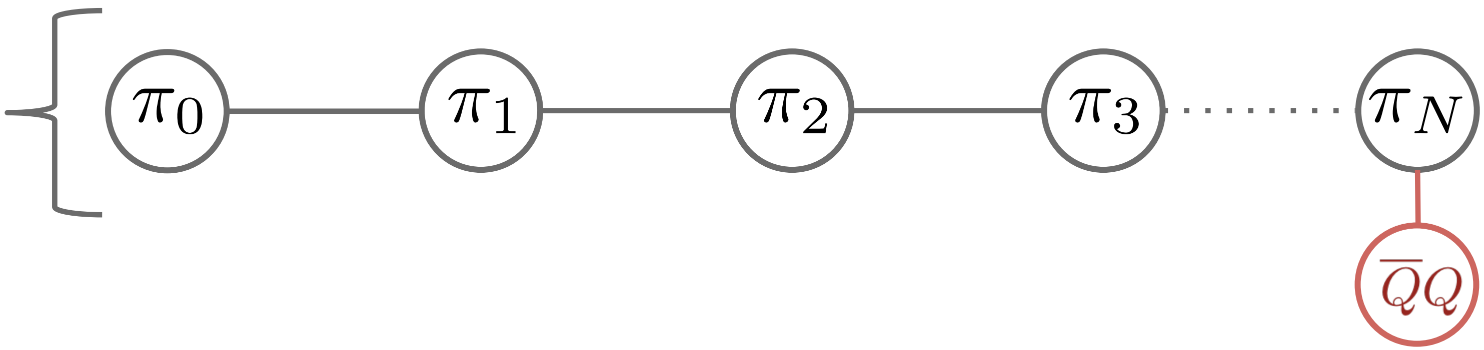
A Clockwork Axion

See also Farina et al 2016.

Imagine clockworking Peccei-Quinn at weak scale:

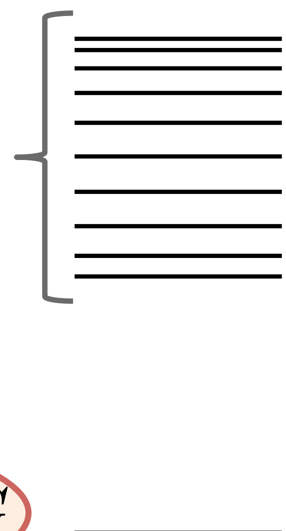
$$\langle \phi_j \rangle = \frac{f}{\sqrt{2}}$$

$$\sim m_H$$



An invisible axion and band of weak-scale “gears”:

$$m_{a_j} \sim m_H \;,\; \mathcal{L} \sim \frac{a_j}{f} \widetilde{G}G$$

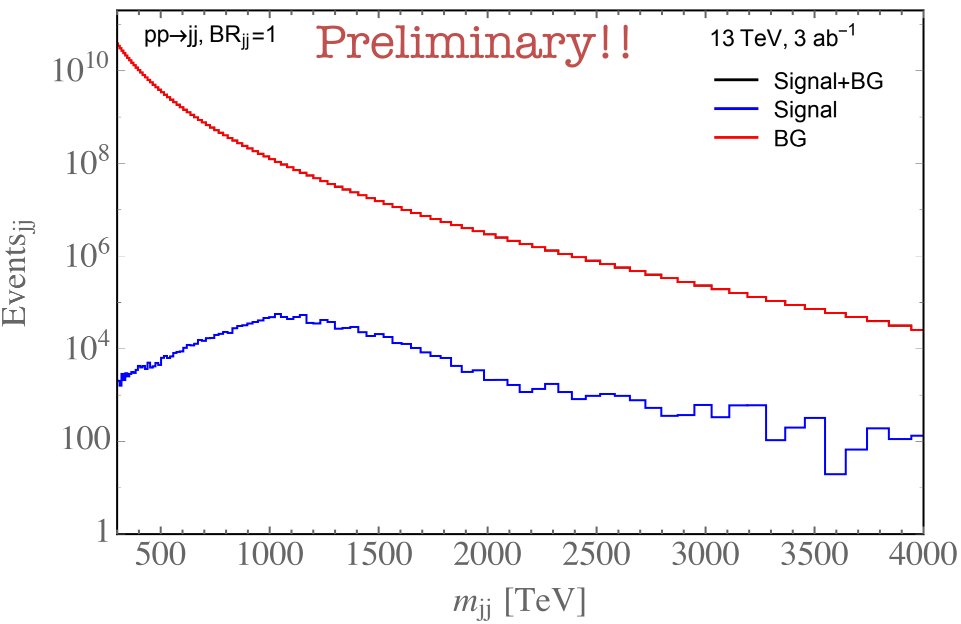


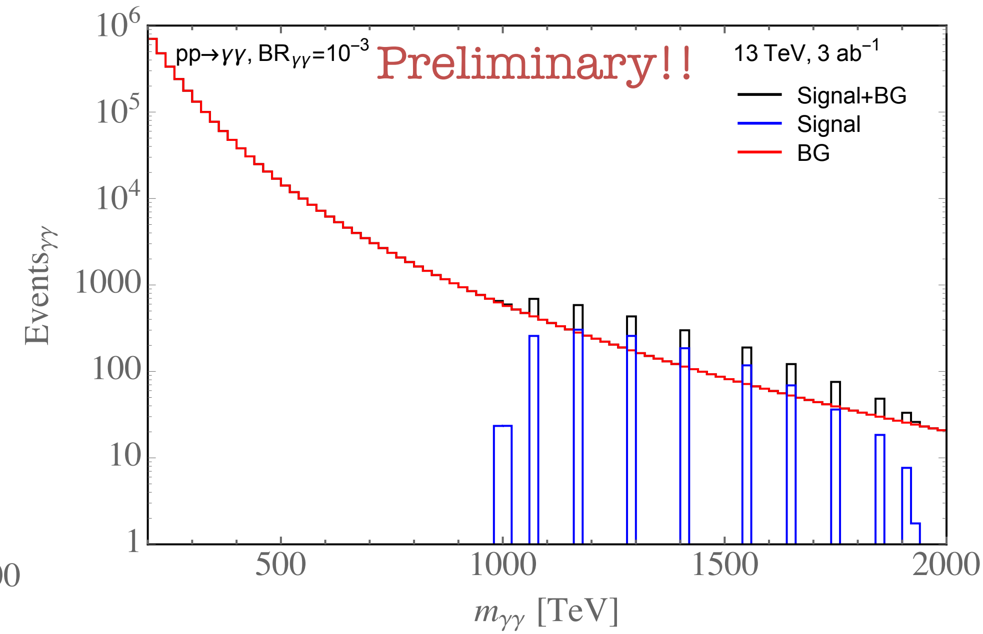
$$m_{a_j} \sim \frac{\Lambda_{QCD}^2}{q^N f} \;,\; \mathcal{L} \sim \frac{a_j}{q^N f} \widetilde{G}G$$

- Clockwork gears could show up as a band of states at colliders.
- Cosmology / thermal history of invisible axion radically altered: stays in thermal equilibrium to late times.

A Clockwork Axion

The phenomenology of the clockwork gears would be very exotic:





Dijet spectrum likely too smeared, and background too large, to reveal anything here. Perhaps diphotons could reveal gears.

A Clockwork Axion

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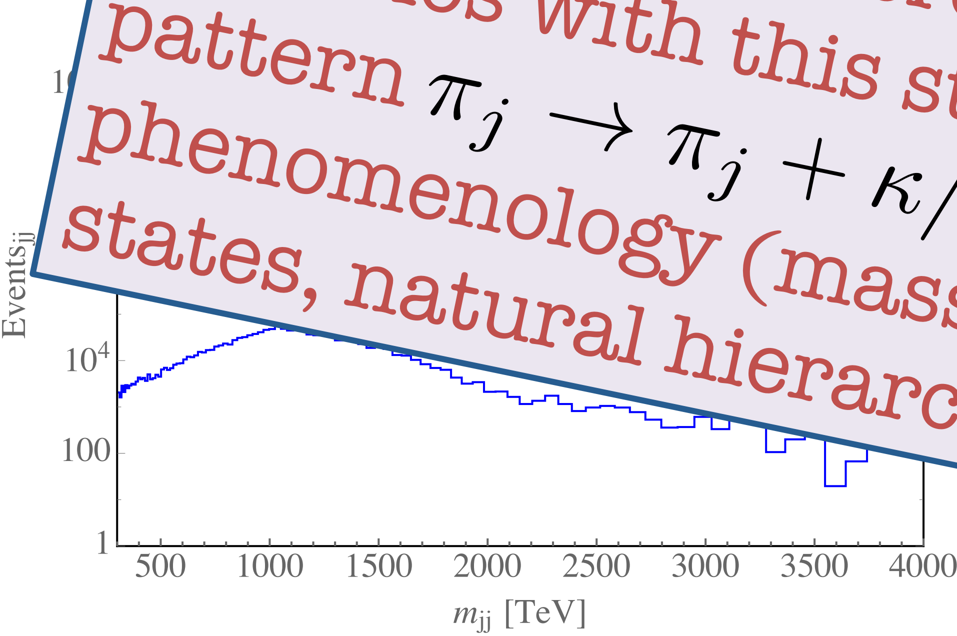
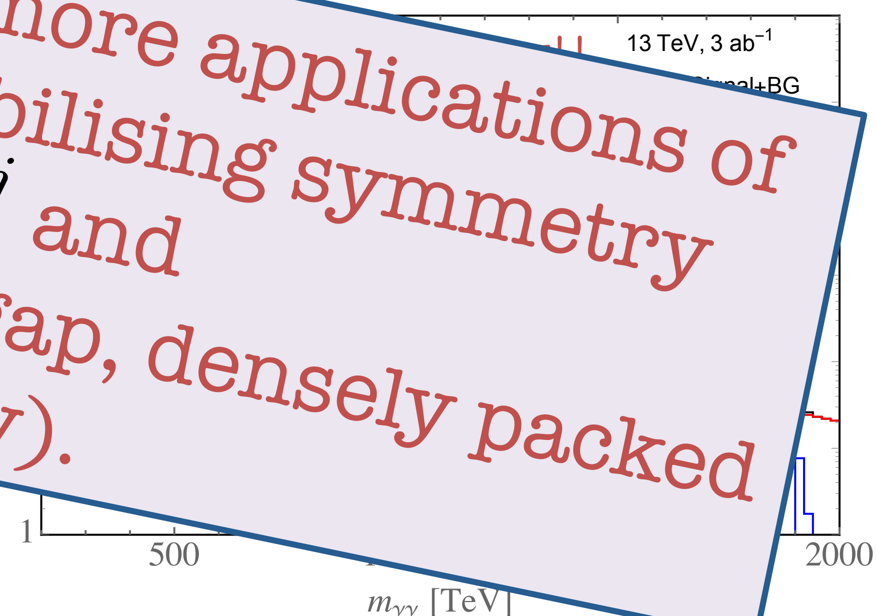


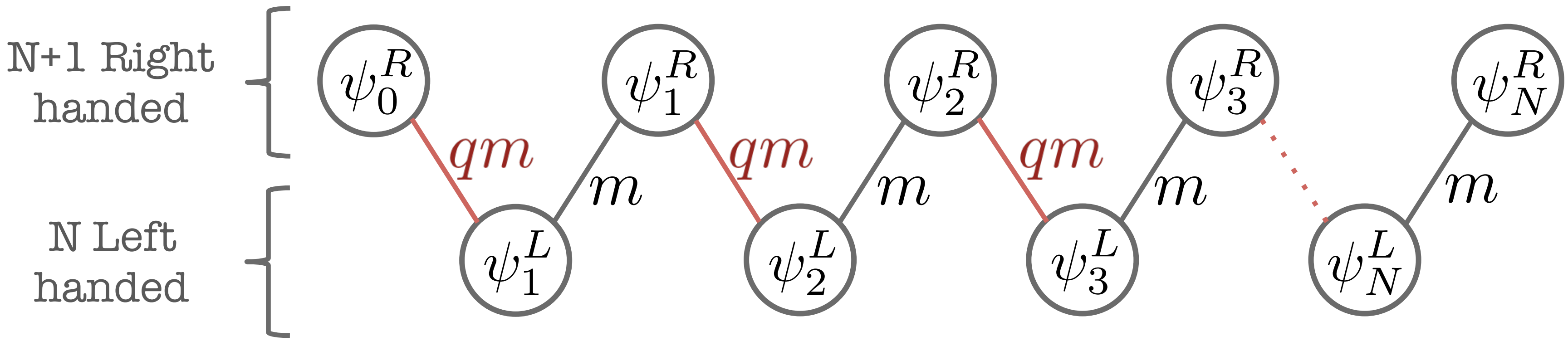


This work: Are there more applications of scenarios with this stabilising symmetry pattern $\pi_j \to \pi_j + \kappa/q^j$ and phenomenology (mass gap, densely packed states, natural hierarchy).

Dijet spectrum likely too smeared, and background too large, to reveal anything here. Perhaps diphotons could reveal gears.

Clockwork Fermion

Can also construct analogous fermion models:



One Weyl fermion left over to be massless. If last site is the RHD Neutrino, then clockworked interaction is:

$$\mathcal{L} = -\lambda H \bar{L}_L \psi_N^R \longrightarrow \mathcal{L} = -\frac{1}{q^N} \lambda H \bar{L}_L \tilde{\psi}$$

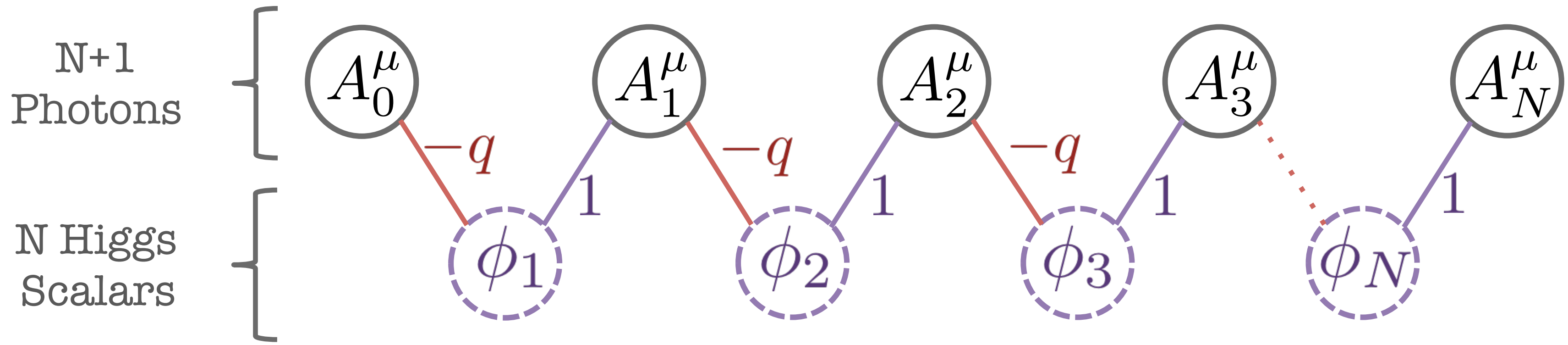
Tiny Dirac neutrino masses! Again, much interesting phenomenology to look into.

See Hambye, Teresi, Tytgat

Clockwork Photon

First proposed by Saraswat.

Can even have clockwork photons:



If all scalars get vevs $\langle \phi_j \rangle = \dfrac{f}{\sqrt{2}}$, vector action becomes

$$\mathcal{L} = -\sum_{j=0}^{N} \frac{1}{4} F^j_{\mu\nu} F^{j\,\mu\nu} + \sum_{j=0}^{N-1} \frac{g^2 f^2}{2} (A^j_\mu - q A^{j+1}_\mu)^2$$

Clockwork mass terms

Interesting applications: millicharges, dark forces, etc...

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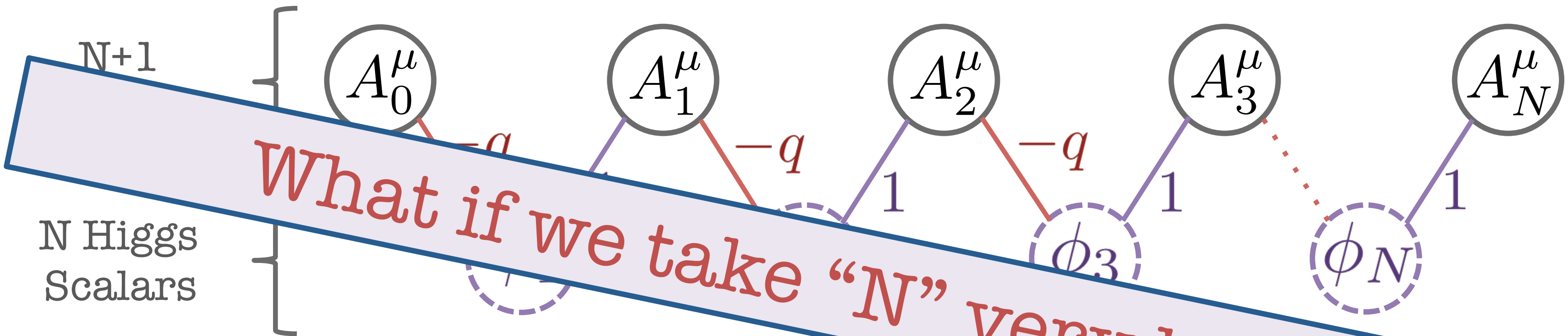

What if we take "N" very large?

If all scalars get vevs $\langle \phi_j \rangle = \frac{f}{\sqrt{2}}$, vector acti
becomes

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Clockwork mass terms

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Looking back to scalar

$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N}(\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\left(e^{\frac{i}{f}(q\pi_{j+1}-\pi_j)} + h.c.\right)$$

This action exhibits a single continuous $\mathrm{U}(1)_{CW}$ symmetry, under which the complex scalars have charge

$$\boxed{Q_{CW} = 1, 1/q, .., 1/q^N}$$

The "j'th" field of carries charge $Q_j^{CW} = q^{-j}$ under $\mathrm{U}(1)_{CW}$. Axion of spontaneously broken symmetry couples proportional to charge, thus

$$\Delta\mathcal{L} = \frac{\partial_\mu a_0}{f} Q^{CW} J_{CW} \rightarrow \frac{\partial_\mu a_0}{q^N f} J_{CW}$$

This sets discrete gauge symmetry of axion.

Continue to Continuum

$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N}(\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\left(e^{\frac{i}{f}(q\pi_{j+1}-\pi_j)} + h.c.\right)$$

This action exhibits a single continuous $\mathrm{U}(1)_{CW}$ symmetry, under which the complex scalars have charge

$$Q_{CW} = 1, 1/q, .., 1/q^N$$

Taking continuum limit, with q^N fixed.

Infinite number of states with charge between 1 and q^N. In other words, $\mathbb{R}$, not $\mathrm{U}(1) \cong \mathbb{R}/\mathbb{Z}$.

Continuum clockwork is non-compact, non-fun?

Continue to Continuum

$$m^2 f^2 \sum^{N-1} \left(e^{\frac{i}{f}(q\pi_{j+1}-\pi_j)} + h.c.\right)$$

T

So, if restricting to the original clockwork action, that's the end of the story. No gauged non-compact symmetries in string theory etc..

But let's keep looking anyway...

Taking continu

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Continue to Continuum

Take original clockwork model

$$\mathcal{L} = -\frac{f^2}{2}\sum_{j=0}^{N}(\partial_\mu \pi_j)^2 - m^2 f^2 \sum_{j=0}^{N-1}\cos\left[\frac{1}{f}(\pi_j - q\pi_{j+1})\right] + \frac{\pi_j}{f}G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

$$+\frac{1}{g^2}G^{\mu\nu}G_{\mu\nu}$$

Perform a field redefinition

$$\pi_j \to \pi_j / q^j$$

in a 5D interval of length πR. Scalar action is

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} q^{-2j}(\partial_\mu \pi_j)^2 - m^2 f^2 \sum_{j=0}^{N-1}\cos\left(\frac{q^{-j}}{f}(\pi_{j+1} - \pi_j)\right)$$

$$\frac{1}{g^2}G^{\mu\nu}G_{\mu\nu} + q^{-j}\frac{\pi_j}{f}G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Continue to Continuum

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Perform a field redefinition

$$\pi_j \to \pi_j / q^j$$

in a 5D interval of length πR. Scalar action is

Warping in kinetic terms.

No more "by hand" than this.

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} q^{-2j}(\partial_\mu \pi_j)^2 - m^2 f^2 \sum_{j=0}^{N-1} \cos\left(\frac{q^{-j}}{f}(\pi_{j+1} - \pi_j)\right)$$

Symmetry (axion) shared symmetrically among sites, flat wavefunction!

Position-dependent coupling now explicit

$$+q^{-j}\frac{\pi_j}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Continue to Continuum

Take original clockwork model

$$\mathcal{L} \ldots)^2 - m^2 f^2 \sum_{j=0}^{N-1} \cos\left[\frac{1}{f}(\pi_j - q\pi_{j+1})\right] + \frac{\pi_j}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Path integral is invariant under field redefinitions.

Perio…

π_j

in a 5D interval of length πR. Scala…

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Continue to Continuum

Ta[...] clockwork model

$$\mathcal{L} \ldots \Big] + \frac{\pi_j}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

This is a particularly illuminating frame to work in...

If uncomfortable, just reverse:

$$\pi_j \to q^j \pi_j$$

in a 5D inter[...]

Warping in kinetic terms.

No [...]

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} q^{-2j}(\partial_\mu \pi_j)^2 - m^2 f^2 \sum_{j=0}^{N-1} \cos\left(\frac{q^{-j}}{f}(\pi_{j+1} - \pi_j)\right)$$

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Continue to Continuum

Take continuum limit

$$m^2(a) = \frac{1}{a^2} \,, \qquad q(a) = e^{ka}$$

Including derivatives

$$\pi_{j+1} - \pi_j \to a\partial_y \pi$$

in a 5D interval of length πR

In continuum limit, only quadratic terms survive:

$$\left[\frac{1}{a^2}\cos\left(\frac{a}{\kappa}\partial_y \pi\right)\right]_{a\to 0} \to \left(\frac{1}{\kappa}\partial_y \pi\right)^2$$

Continue to Continuum

Ta... continuum limit

$$q(a) = e^{ka}$$

in a 5D interval of leng...

In continuum limit $q \to 1+\epsilon \;\;,\;\; q^N$ fixed. Thus in terms of complex scalars, non-local operators. Only EFT of pNGBs can be trusted.

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Continue to Continuum

Take continuum limit...

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} q^{-2j}(\partial_\mu \pi_j)^2 - m^2 f^2 \sum_{j=0}^{N-1} \cos\left(\frac{q^{-j}}{f}(\pi_{j+1} - \pi_j)\right)$$

$$+q^{-j}\frac{\pi_j}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

In continuum limit, only quadratic terms survive:

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu \pi\, \partial^\mu \pi + (\partial_y \pi)^2\right] + e^{-ky}\frac{\pi}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Interaction explicitly breaks discrete gauge symm.

Continue to Continuum

Take continuum limit

$$m^2(a) = \frac{1}{g^2}\,, \qquad q(a) = e^{ka}$$

Gravitational couplings remained the same thoughout this procedure.

Includ...

$$\pi_{j+1}$$

in a 5D interval of length πR

In continuum limit, only quadratic terms survive:

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu\pi\,\partial^\mu\pi + (\partial_y\pi)^2\right] + e^{-ky}\frac{\pi}{f}G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Interaction explicitly breaks discrete gauge symm.

...tinue to Continuum

Ta...

Deconstruct this action, with appropriate irrelevant operators, and you recover the original clockwork model.

If uncomfortable with basis, reverse field redefinition:

$$\pi \to e^{ky}\tilde{\pi} \quad , \quad \tilde{\pi} \to \tilde{\pi} + \kappa e^{-ky}$$

In continuum limit, ...

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu\pi\,\partial^\mu\pi + (\partial_y\pi)^2\right] + e^{...}\frac{...}{f} ... \tilde{F}_{\mu\nu}$$

Interaction explicitly breaks discrete gauge symm.

Continue to Continuum

Connection with the linear dilaton model:

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu \pi\, \partial^\mu \pi + (\partial_y \pi)^2\right] + e^{-ky}\frac{\pi}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Where $\langle S \rangle = \pm 2ky$. Interaction term arises from "k"-like parameter. This is the direct continuum limit of the original clockwork model.

See e.g. Antoniadis, Dimopoulos, Giveon, 2001.

What's the linear dilaton model?

$$\mathcal{S} = \int d^4x\, dy\, \sqrt{-g}\,\frac{M_5^3}{2} e^S \left(\mathcal{R} + g^{MN}\partial_M S\, \partial_N S + 4k^2\right)$$

Solution of Einstein's equations: $\langle S \rangle = \pm 2ky$

Continue to Continuum

Connection with the linear dilaton model:

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Where $\langle S\rangle = \pm 2ky$. Interaction term arises from "k"-like parameter. This is the direct continuum limit of the original clockwork model.

If coupled at different sites, for example,

$$y_a = 0\ ,\ \ y_b = \log(2)/2k$$

Then compact discrete shift symmetry explicitly broken by brane couplings (not bulk action). Symmetry is <u>non-compact</u>.

Continue to Continuum

Connection with the linear dilaton model:

$$\mathcal{L} \quad \frac{1}{\quad} \int dy\, e^{-2ky} \left[\partial_\mu \pi\, \partial^\mu \pi + (\partial_y \pi)^2\right] + e^{-ky} \frac{\pi}{f} G^{\mu\nu} \widetilde{G}_{\mu\nu}$$

Non-compact, as expected.

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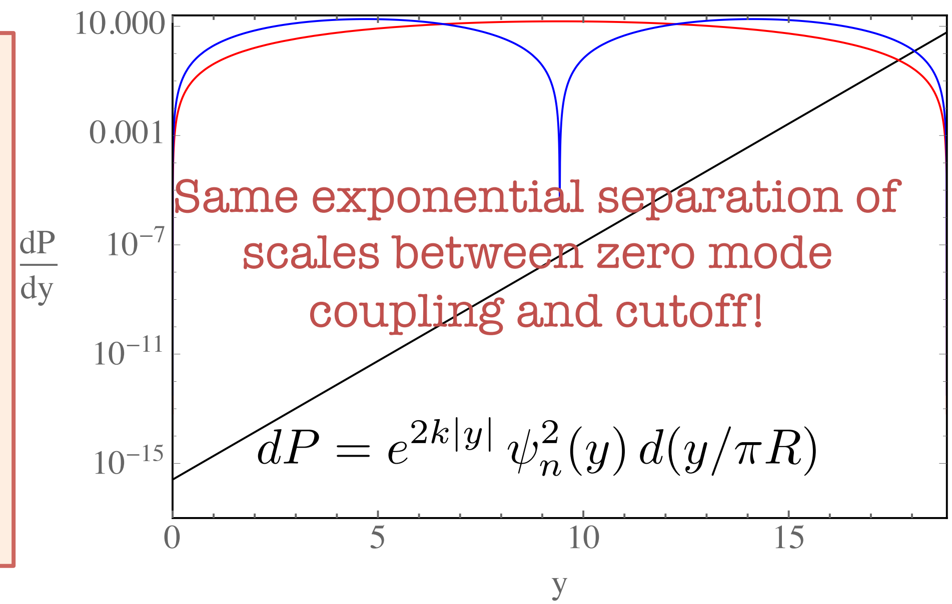
Where $\langle S \rangle = \pm 2ky$. This is the direct continuum limit of the original clockwork model. In GM, just coupled at $y = 0$, to preserve compact symmetry.

Mass: $m_0^2 = 0$

Wavefunction:

$$\psi_0(y) = \sqrt{\frac{k\pi R}{e^{2k\pi R} - 1}}$$



Continue to Continuum

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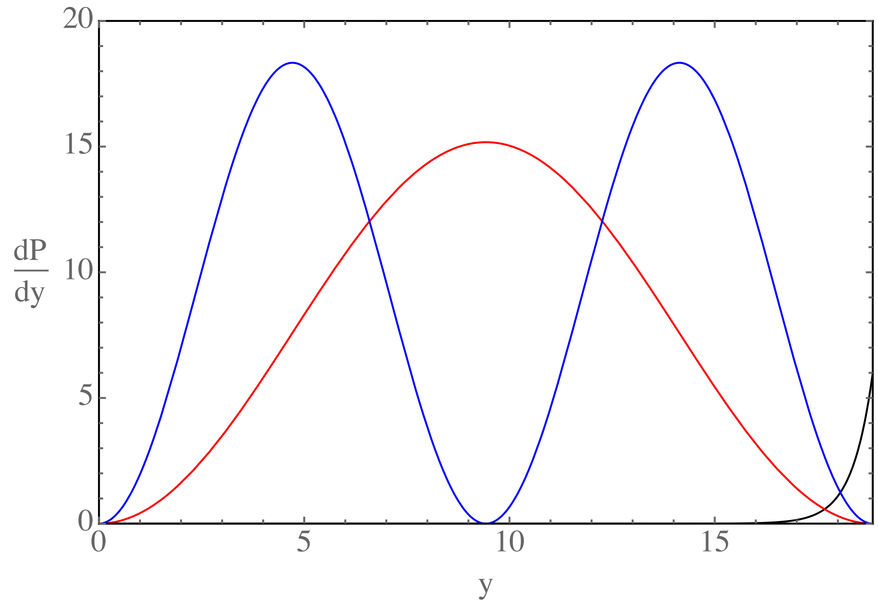
Mass: $m_n^2 = k^2 + \frac{n^2}{R^2}$

Wavefunction:

$$\psi_n(y) = \frac{n}{m_n R}\, e^{-k|y|}\left(\frac{kR}{n}\sin\frac{n|y|}{R} + \cos\frac{ny}{R}\right)$$



Continue to Continuum

Connection with the linear dilaton model:

$$\mathcal{L} \ldots \left[\partial_\mu \pi \partial^\mu \pi + (\partial_y \pi)^2\right] + e^{-ky}\frac{\pi}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

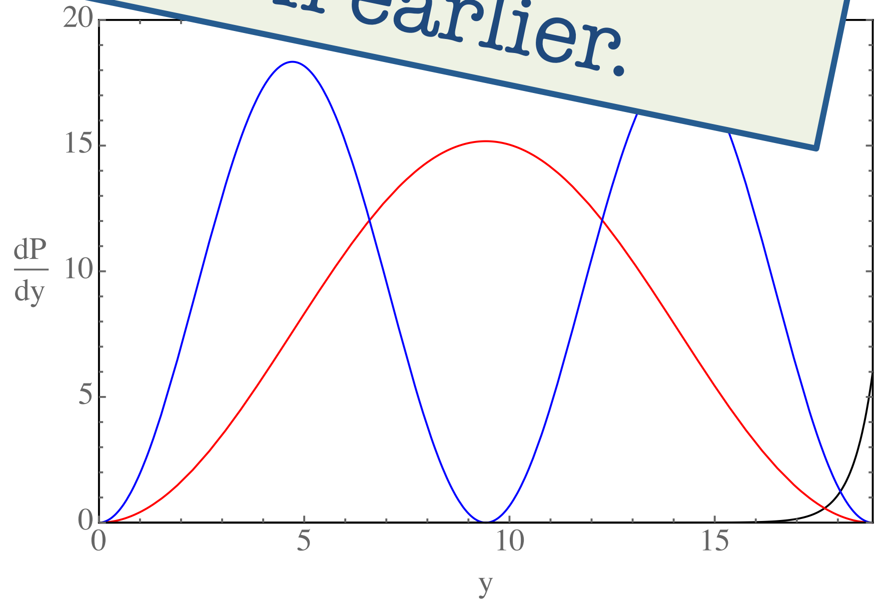
... tinuum limit of the ... coupled at $y = 0$, to pr...

Mass spectrum, wavefunction, coupling pattern all continuum limit of discrete model, see QCD application earlier.

Mass: $m_n^2 = k^2 + \dfrac{n^2}{R^2}$

Wavefunction:

$$\psi_n(y) = \frac{n}{m_n R}\, e^{-k|y|}\left(\frac{kR}{n}\sin\frac{n|y|}{R} + \cos\frac{ny}{R}\right)$$



Continue to Continuum

Connection with the linear dilaton model:

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu \pi\, \partial^\mu \pi + (\partial_y \pi)^2\right] + e^{-ky}\frac{\pi}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Where $\langle S \rangle = \pm 2ky$. If coupled only at $y = 0$ all features physically identical to being at the end of the clockwork chain, since this is the continuum of the clockwork.

Continue to Continuum

Connection with the linear dilaton model:

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu\pi\,\partial^\mu\pi + (\partial_y\pi)^2\right] + \cancel{e^{-ky}}\,\frac{\pi}{f}G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Where $\langle S\rangle = \pm 2ky$. If coupled only at $y = 0$ all features physically identical to being at the end of the clockwork chain, since this is the continuum of the clockwork.

Could also remove dilaton factor from topological term, but then zero mode couplings become position-independent: No longer a continuum limit of the clockwork.

Continue to Continuum

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$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu \pi\, \partial^\mu \pi + (\partial_y \pi)^2\right] + \cancel{e^{-ky}}\,\frac{\pi}{f} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Where $\langle S \rangle = \pm 2ky$. If coupled only at $y = 0$ all features physically identical to being at the end of the clockwork chain, since this is the continuum of the clockwork.

Localisation in terms of zero-mode coupling to gluons no longer varies exponentially with position. This is the central objection of Craig et al, with regard to the original clockwork model, and we are in complete agreement on this. But can we still use it for other purposes?

Continue to Continuum

Connection with the linear dilaton model:

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu\pi\,\partial^\mu\pi + (\partial_y\pi)^2\right] + \cancel{e^{-ky}}\,\frac{\pi}{f}G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Where $\langle S\rangle = \pm 2ky$. If coupled only at $y=0$ all features physically identical to being at the end of the clockwork chain, since this is the continuum of the clockwork.

Bulk unchanged, properties of continuum limit of:

$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N}(\partial_\mu\pi_j)^2 - \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\left(e^{\frac{i}{f}(q\pi_{j+1}-\pi_j)} + h.c.\right)$$

preserved, including clockworked shift symmetry.

Continue to Continuum

Connection with the linear dilaton model:

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu\pi\,\partial^\mu\pi + (\partial_y\pi)^2\right] + \cancel{e^{-ky}}\,\frac{\pi}{f}G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

Where $\langle S\rangle = \pm 2ky$. If coupled only at $y=0$ all features physically identical to being at the end of the clockwork chain, since this is the continuum of the clockwork.

Localisation and hierarchy of zero-mode coupling

$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N}(\partial_\mu\pi_j)^2 - \frac{m^2f^2}{2}\sum_{j=0}^{N-1}\left(e^{\frac{i}{f}(q\pi_{j+1}-\pi_j)} + h.c.\right)$$

to cutoff the same. Can now have compact symm.

Linear Dilaton Model

This means that any massless field (scalar, fermion, vector, graviton) placed in the linear dilaton background

$$\mathcal{L} = -\frac{1}{2}\int dy\, e^{-2ky}\left[\partial_\mu \pi\, \partial^\mu \pi + (\partial_y \pi)^2\right]$$

Has the same physical localisation, mass spectrum, and hierarchy between zero-mode coupling, all canonically normalized fields obey symmetry

$$\boxed{\pi_j \to \pi_j + \kappa/q^j}$$

Linear Dilaton Model

Th... massless field (scalar,
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For us, all of these features are clockworky enough, but, as pointed out by Craig, Garcia-Garcia, and Sutherland, now position-independent means charges not clockworked, but this allows to go from non-compact to compact.

Has the same ph...
spectrum, and hierarchy bet...
coupling, all canonically normalized field...
symmetry

$$\pi_j \to \pi_j + \kappa/q^j$$

Linear Dilaton Model

Things get really interesting when looking to the phenomenology...

See: Work in progress with Giudice, Kats, Torre, Urbano.

Previous related studies:

- Antoniadis, Arvanitaki, Dimopoulos, Giveon, 2011. (Large-k)
- Baryakhtar, 2012. (All-k)
- Cox, Gherghetta, 2012. (Dilatons)
- Giudice, Plehn, Strumia, 2004. Franceschini, Giardino, Giudice, Lodone, Strumia, 2011. (Large extra dimensions, pheno similar.)

Linear Dilaton Model

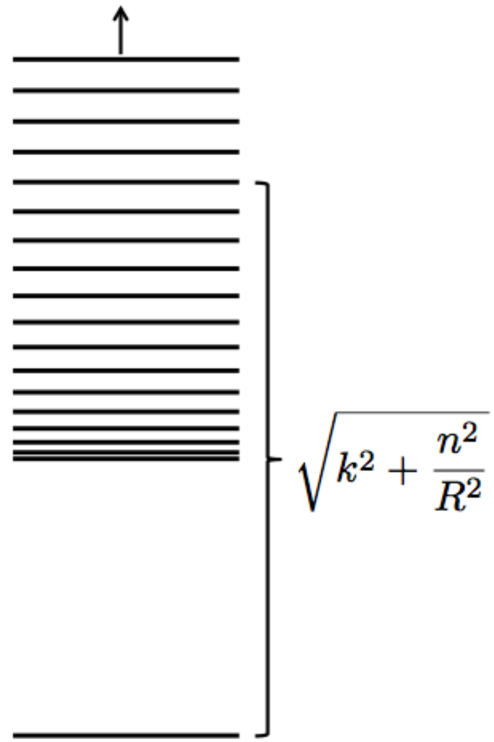
Irreducible prediction:

In this theory Planck scale is:

$$M_P \sim \sqrt{\frac{M_5^3}{k}}\, e^{k\pi R}$$

So if all other parameters at the weak scale, require:

$$kR \sim 11$$



Continuum Clockwork

But the mass spectrum is given by:

$$m_n \sim k\left(1 + \frac{n^2}{2(kR)^2}\right)$$

Thus the first few states will always be split by %'s, with the relative splitting decreasing for heavier modes.

This splitting is thus a key prediction of the theory.

Linear Dilaton Model

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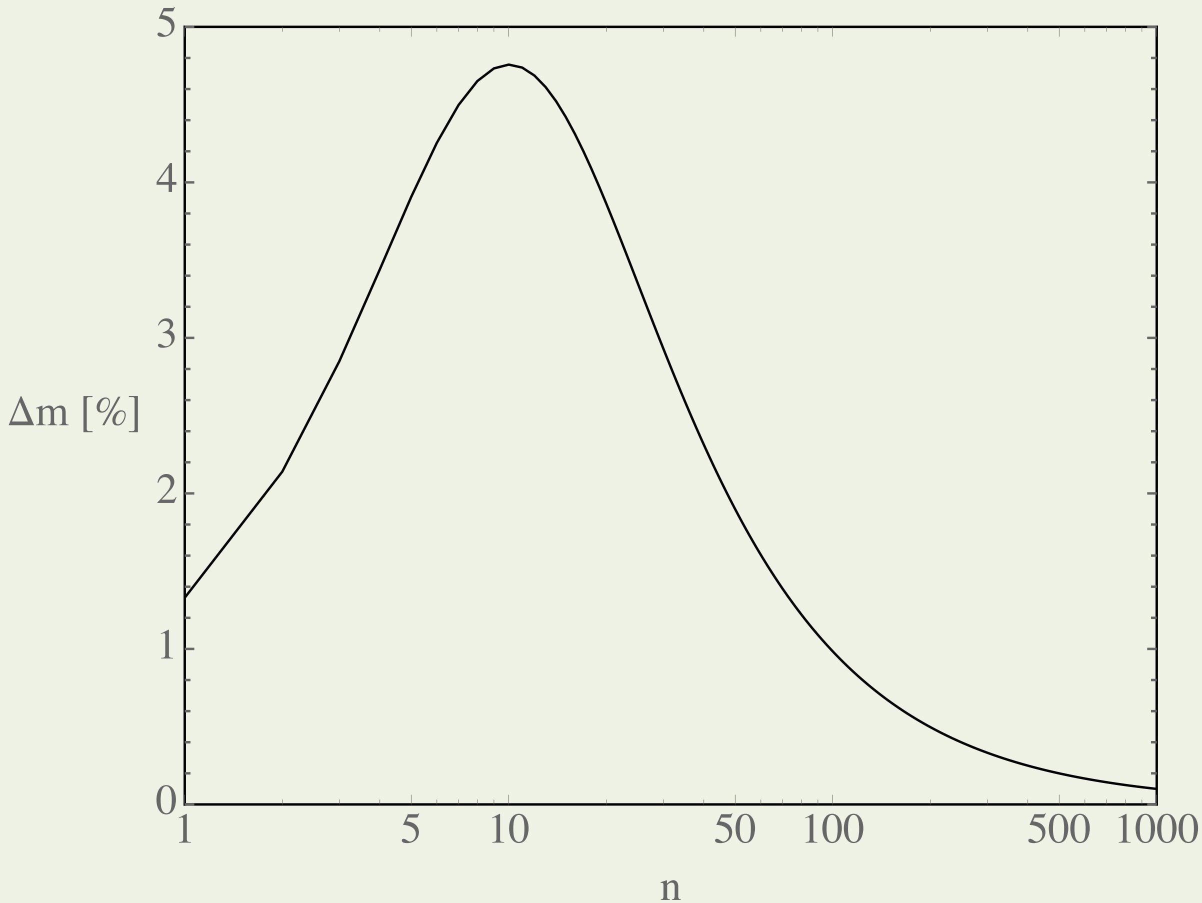
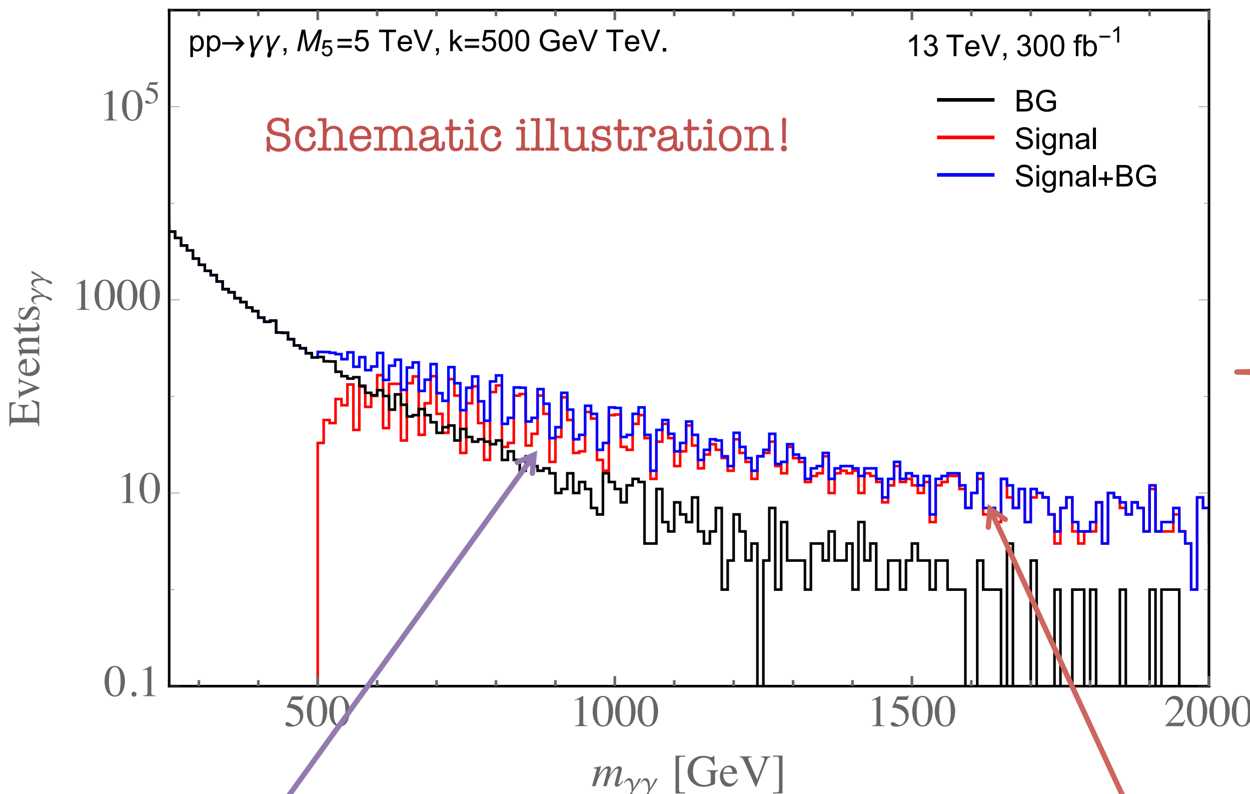
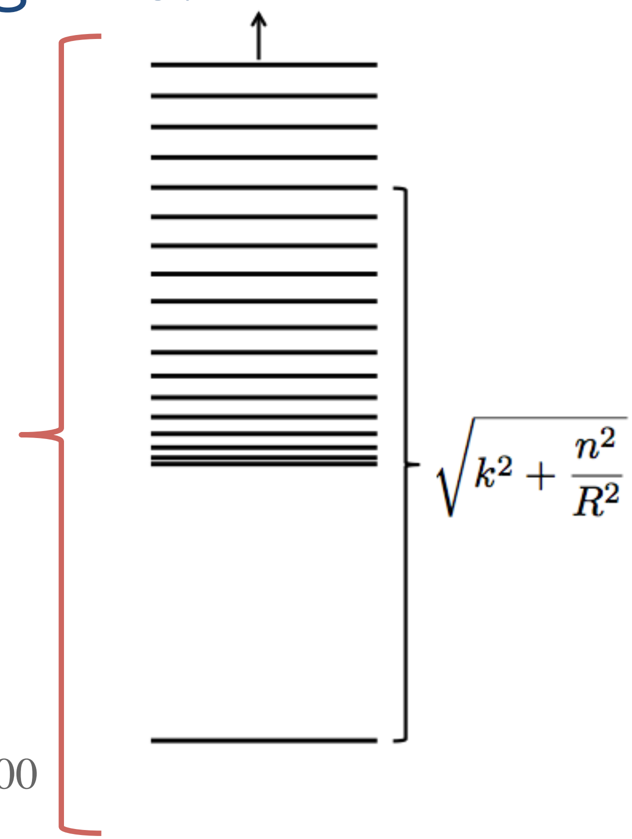
Clockwork mass splitting:



Linear Dilaton Model

At colliders would look something like:





Most interestingly, due to splittings, signal appears to "oscillate". Thus get extra sensitivity by doing spectral analysis… The "power spectrum" of LHC data!

Can search for continuum spectrum at high energies. BG modelling essential…

Fin