EFFECTIVE FIELD THEORY FOR MAGNETIC COMPACTIFICATIONS

Based on :
W. Buchmuller, M.Dierigl,E.D & J. Schweizer,
arXiv:1611.03798 [hep-th]

may 24, 2017
Planck 2017, Varsovie

Outline

1) Higher-dimensional completions of the Standard Model
2) Magnetic compactifications
3) Effective field theory
4) Quantum corrections, Wilson lines as goldstone bosons
5) Conclusions

1) Higher-dimensional embeddings of the Standard Model

Extra dims. address:

- Unification of all forces
- Holographic solutions to the hierarchy problem
- New ways to break SUSY
- New models of inflation

- Sometimes higher-dim. symmetries protect quantum corrections in a way invisible from 4d.

Ex: Internal comp. of a gauge field protected by gauge symmetry (gauge-Higgs unification)

$$\delta m_0^2 \sim (\text{loop}) \times \frac{1}{R^2}$$

(Antoniadis,Benakli,Quiros,2001…)

- Compactification scale $M_c = R^{-1}$ usually defines the GUT/unification scale.

- Scale of supersymmetry breaking M_{SUSY} usually much smaller.

2) Magnetic compactifications

(also talks H. Abe, W. Buchmuller)

Consider a 6-dim. theory : $x_0 x_1 x_2 x_3 x_5 x_6$

An internal magnetic field $F_{56} = B = f$

- break SUSY, due to the magnetic moment coupling

$$H = -\mu \mathbf{B} = -\frac{q}{m} \mathbf{S} \mathbf{B}$$

- Charged states: turns KK states k_1, k_2 into Landau levels n , mass

$$\delta M^2 = (2n+1)|qB| + 2qB\Sigma_{56}$$

where Σ_{45} is the internal helicity of particles.

- Uncharged states : standard KK masses

- An internal magnetic field is quantized

$$\int_{T^2} F = 2\pi N \quad \longrightarrow \quad f = \frac{N}{2\pi R_1 R_2} \sim M_{\rm GUT}^2$$; N = integer

- Each Landau level is N times degenerate.

- Precisely N chiral fermion zero modes (index theorem).

Magnetized models : Bachas (1995)….Cremades, Ibanez,Marchesano, Abe et al, Buchmuller et al.…

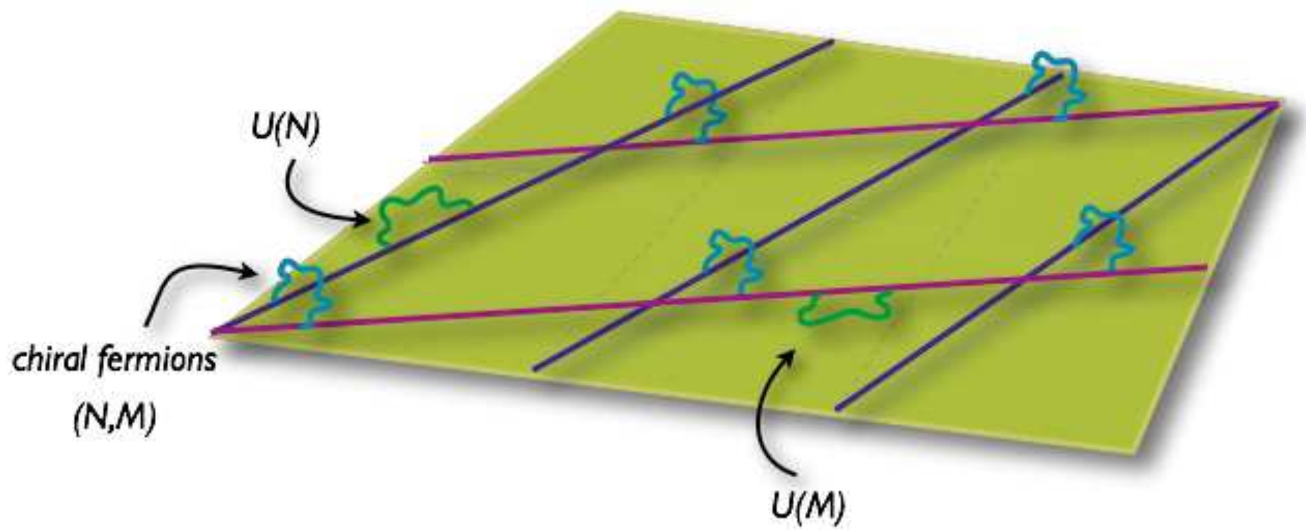
- Starting with a SUSY 6d theory, usually said that the effect of the magnetic field is to add a D-term Fayet-Iliopoulos (FI) term in 4d

$$D = f \quad \to \quad V = \frac{1}{2} D^2 = \frac{1}{2} f^2 \sim M_{\rm GUT}^4$$

Widely studied in string theory :

T-dual

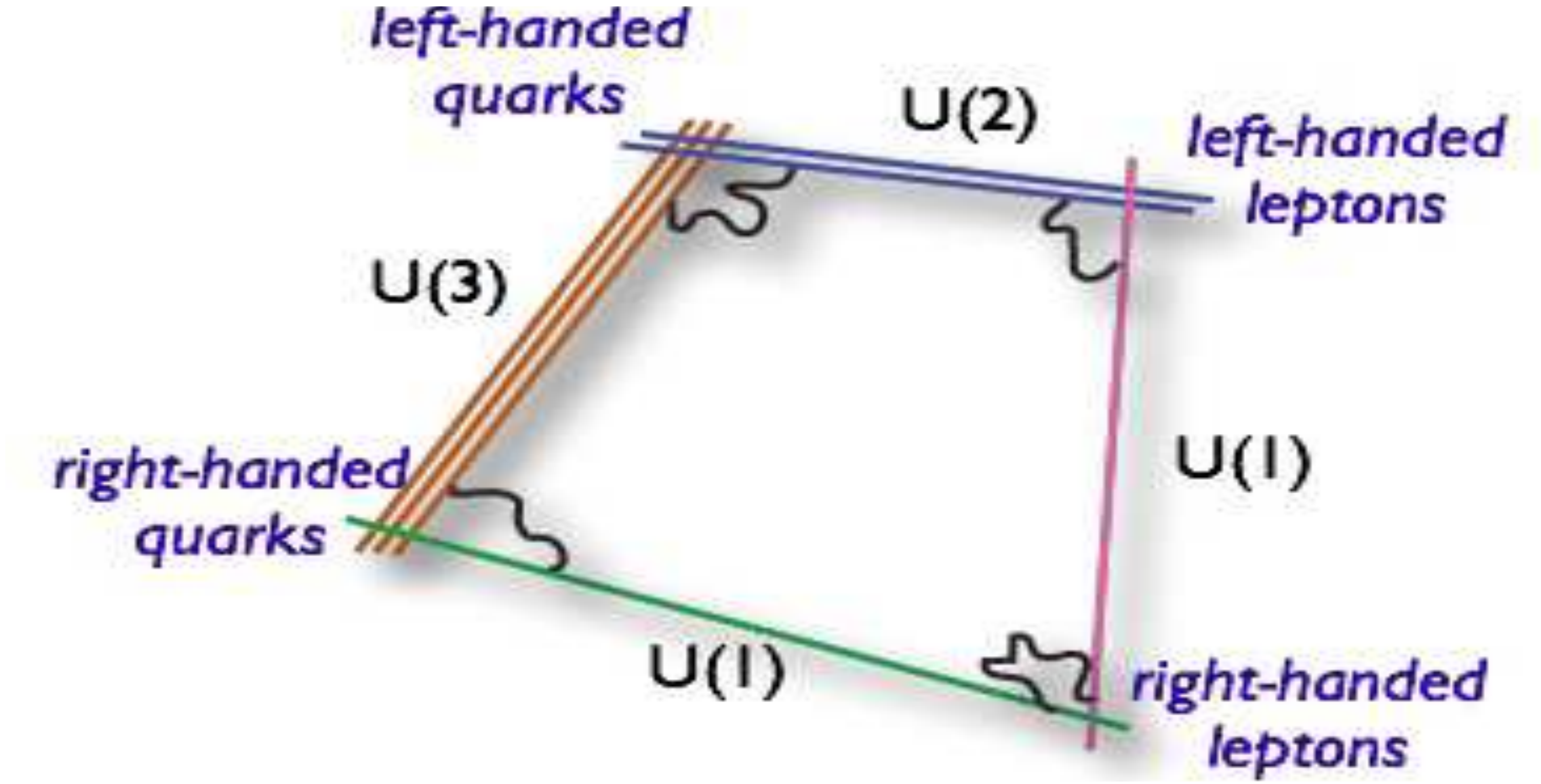
Internal magnetic fields ⟷ intersecting branes



Elegant geometrical intepretations :

- chiral fermions live at the intersection of branes
- Number of generations: intersection numbers
- Yukawa couplings : governed by areas

Among the most succesful quasi-realistic Standard Model realizations in String Theory



Why be interested in field theory approach to magnetic compactifications ? Several reasons:

- If broken SUSY, most of quantum corrections not calculable in string theory

- Subtlety: there is no mass gap in the spectrum : soft masses given by the FI term of the same order ($1/R$) as the masses of Landau levels

→

one needs an effective theory for the whole tower. Truncation to « zero modes » inconsistent.

3) Effective field theory

- Abelian 6d SUSY theory compactified on a torus.

N=2 SUSY in 4d before the magnetic flux;

4d multiplets: vector (V, ϕ)

charged hyper $(Q, \tilde{Q})$

- 6d effective action in superfields: (Marcus,Sagnotti,Siegel ; Arkani-Hamed,Gregoire,Wacker)

$$S_6 = \int d^6x \Big\{ \frac{1}{4} \int d^2\theta \, W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \left(\partial V \bar{\partial} V + \phi \bar{\phi} + \sqrt{2} V \left(\bar{\partial} \phi + \partial \bar{\phi} \right) \right)$$

$$+ \int d^2\theta \, \tilde{Q} (\partial + \sqrt{2} g q \phi) Q + \text{h.c.} + \int d^4\theta \left(\overline{Q} e^{2gqV} Q + \overline{\tilde{Q}} e^{-2gqV} \tilde{Q} \right) \Big\}$$

$$\partial = \partial_5 - i \partial_6 \, , \quad \phi|_{\theta = \bar{\theta} = 0} = \frac{1}{\sqrt{2}} (A_6 + i A_5)$$

ϕ are internal components of gauge fields = Wilson lines

Mode expansions with flux:

$$\phi_0\big|_{\theta=\overline{\theta}=0} = \frac{f}{2\sqrt{2}}\left(x_5 - i x_6\right) + \varphi\,, \qquad \varphi = \frac{1}{\sqrt{2}}(a_6 + i a_5)$$

$$\mathcal{Q}(x_M) = \sum_{n,j} \mathcal{Q}_{n,j}(x_\mu)\psi_{n,j}(x_m) = \sum_{n,j} \mathcal{Q}_{n,j}(x_\mu)\frac{1}{\sqrt{n!}}\left(a^\dagger\right)^n \psi_{0,j}(x_m)\,,$$

$$\overline{\mathcal{Q}}(x_M) = \sum_{n,j} \overline{\mathcal{Q}}_{n,j}(x_\mu)\overline{\psi}_{n,j}(x_m) = \sum_{n,j} \overline{\mathcal{Q}}_{n,j}(x_\mu)\frac{1}{\sqrt{n!}}\left(a\right)^n \overline{\psi}_{0,j}(x_m)\,.$$

where (harmonic oscillator algebra)

$$a = \sqrt{\frac{1}{-2qgf}}(iD_5 - D_6)$$

$$a^\dagger = \sqrt{\frac{1}{-2qgf}}(iD_5 + D_6)$$

The final 4d effective action for Landau levels is

FI term

$$S_4^* = \int d^4x \left[\int d^4\theta \left(\overline{\varphi}\varphi + \sum_{n,j} (\overline{Q}_{n,j} e^{2gqV_0} Q_{n,j} + \overline{\tilde{Q}}_{n,j} e^{-2gqV_0} \tilde{Q}_{n,j}) + 2fV_0 \right) \right.$$
$$+ \int d^2\theta \left(\frac{1}{4} \mathcal{W}_0^\alpha \mathcal{W}_{\alpha,0} \right.$$
$$\left. \left. + \sum_{n,j} \left(-i\sqrt{-2qgf(n+1)} \tilde{Q}_{n+1,j} Q_{n,j} + \sqrt{2} qg \tilde{Q}_{n,j} \, \varphi \, Q_{n,j} \right) \right) + \text{h.c.} \right]$$

Coupled mass terms

- SUSY broken like in the FI model, with an infinite number of fields. Truncation to a finite number inconsistent.

- We also worked out the non-abelian case: SU(2) gauge group in 6d with N=2 vector multiplet, flux in the generator T_3.

- In this case, there is always a tachyon (recombination mode) $\Phi_{+,0}$ which can restore SUSY by taking a vev (tachyon condensation)

  → Nielsen-Olesen instability

- The flux give mass to the $W^{\pm}$ gauge bosons and breaks $SU(2) \to U(1)$

- There is an induced Fayet-Iliopoulos term for the $U(1)$

- In the true vacuum $U(1)$ will also be broken

- Interesting subtleties with the Stueckelberg mechanism for Landau levels

$$\Phi_{n,j} = - \sqrt{\frac{n+1}{2n+3}} \bar{\phi}_{-,n,j} + \sqrt{\frac{n+2}{2n+3}} \phi_{+,n+2,j}$$ are absorbed by $A^{\mu}_{+,n+1,j}$

$\Phi_{+,1,j}$ absorbed by $A^{\mu}_{+,0,j}$

4) Quantum corrections, Wilson lines as goldstone bosons

Interested in Higgs = internal component of the gauge field. Without magnetic flux, 6d gauge symmetry could protect only partially its mass

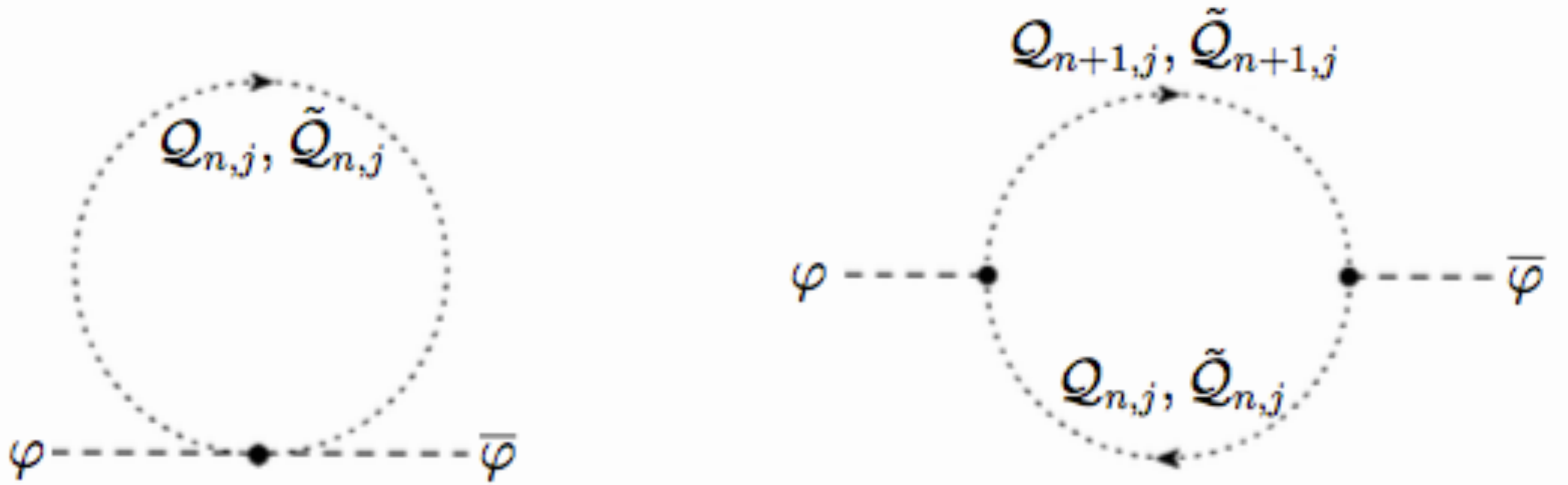


Figure 3: Bosonic contributions to the Wilson line mass with flux.

Each contribution quadratically divergent: sum over the whole charged tower of Landau levels is however exactly zero !

$$\delta m_b^2 = 2q^2 g^2 |N| \sum_n \int \frac{d^4k}{(2\pi)^4} \left(\frac{2}{k^2+\alpha(n+\frac{1}{2})} - \frac{2\alpha(n+1)}{\left(k^2+\alpha(n+\frac{3}{2})\right)\left(k^2+\alpha(n+\frac{1}{2})\right)} \right)$$

can be written in the form

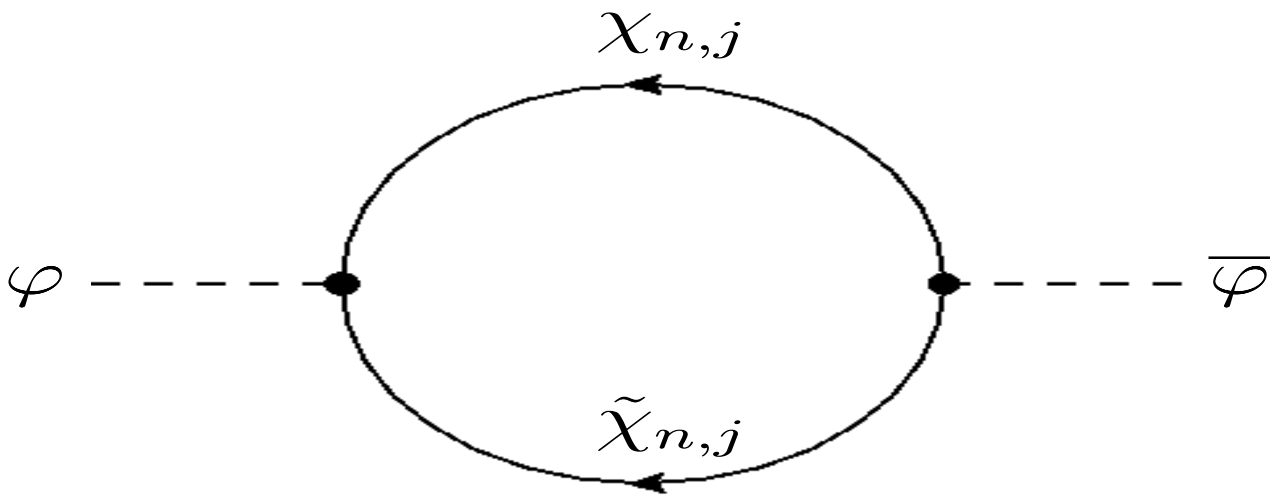
$$\delta m_b^2 = -4q^2 g^2 |N| \sum_n \int \frac{d^4k}{(2\pi)^4} \left(\frac{n}{k^2+\alpha(n+\frac{1}{2})} - \frac{n+1}{k^2+\alpha(n+\frac{3}{2})} \right)$$

$$= -\frac{q^2 g^2}{4\pi^2} |N| \sum_n \int_0^\infty dt \, \frac{1}{t^2} \left(n e^{-\alpha(n+\frac{1}{2})t} - (n+1) e^{-\alpha(n+\frac{3}{2})t} \right)$$

$$= -\frac{q^2 g^2}{4\pi^2} |N| \int_0^\infty dt \, \frac{1}{t^2} \left(\frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t}-1)^2} - \frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t}-1)^2} \right) = 0$$

Careful recent discussion of regularization: D. Ghilencea, H.M.Lee

The same is true for the fermionic contribution



Reminder: without the flux, scalar and fermion loops give separately

$$\delta m_0^2 \sim (\mathrm{loop}) \times \frac{1}{R^2}$$

We checked also that the quartic coupling is zero.

Is there's a symmetry reason ?

Action of charged matter fields invariant under translations

$$S_6 = \int d^6x \, \left(-D_M \bar{Q} D^M Q\right) \,, \qquad D_M Q = (\partial_M + iqgA_M) Q$$

$$\delta Q = \epsilon^m \partial_m Q \,, \qquad \delta A_n = \epsilon^m \partial_m A_n$$

Symmetries for constant Wilson line background

$$\delta Q = \epsilon^m \partial_m Q \,, \qquad \delta a_n = 0$$

Flux background breaks the symmetries spontaneously

$$D_m Q = \left(\partial_m + iqg\left(a_m + \frac{f}{2}\epsilon_{mn} x_n\right)\right) Q \,, \qquad \langle A_m \rangle = \frac{f}{2}\epsilon_{mn} x_n$$

Translational symmetries now non-linearly realized with Wilson lines as Goldstone bosons

$$\delta Q = \epsilon^m \partial_m Q \,, \qquad \delta a_n = \epsilon^m \frac{f}{2} \epsilon_{nm}$$

- Need realistic examples with pseudo-Goldstone bosons

$$\delta m_0^2 \ll \frac{1}{R^2}$$

maybe from gravitational or higher-loop corrections.

Conclusions, Perspectives

- Magnetized compactifications generate chirality and can break supersymmetry such that

$$M_{SUSY} \sim M_{GUT} \sim R^{-1}$$

- Magnetic fields can break spontaneously symmetries invisible from 4d → (pseudo) Goldstones from higher-dim. symmetries.
  Hope for a higher-dim. protection of scalar masses.

- Various applications possible: hierarchy problem, moduli stabilization, inflation, string and field theory orbifold GUT's.

Backup slides

Effective action non-abelian flux

$$
\begin{aligned}
S_4^* = \int d^4x \Bigg\{ & \int d^2\theta \left(\frac{1}{4} W_3^\alpha W_{\alpha,3} + \frac{1}{2} \sum_{n,j} W^\alpha_{+,n,j} W_{\alpha,-,n,j} \right) + \text{h.c.} \\
& + \int d^4\theta \Bigg[\overline{\varphi}_3 \varphi_3 + 2 f V_3 + \sum_{n,j} \left(\overline{\phi}_{+,n,j} e^{g V_3} \phi_{+,n,j} + \overline{\phi}_{-,n,j} e^{-g V_3} \phi_{-,n,j} \right) \\
& \quad + \sum_{n,j} \Big((2n+1)(-gf) V_{-,n,j} V_{+,n,j} + i \sqrt{2n(-gf)}\, g \varphi_3 V_{-,n-1,j} V_{+,n,j} \\
& \qquad - i \sqrt{2(n+1)(-gf)}\, g \overline{\varphi}_3 V_{-,n+1,j} V_{+,n,j} + g^2 \overline{\varphi}_3 \varphi_3 V_{-,n,j} V_{+,n,j} \Big) \\
& \quad + \sum_{n,j} \Bigg(\left(1 - \tfrac{g}{\sqrt{2}} V_3\right) \Big(-i \sqrt{2(n+1)(-gf)}\, V_{-,n+1,j} \overline{\phi}_{-,n,j} \\
& \qquad + i \sqrt{2n(-gf)}\, \phi_{-,n-1,j} V_{+,n,j} + g \varphi_3 V_{-,n,j} \overline{\phi}_{-,n,j} + g \overline{\varphi}_3 \phi_{-,n,j} V_{+,n,j} \Big) \\
& \qquad + \left(1 + \tfrac{g}{\sqrt{2}} V_3\right) \Big(i \sqrt{2(n+1)(-gf)}\, \overline{\phi}_{+,n+1,j} V_{+,n,j} \\
& \qquad - i \sqrt{2n(-gf)}\, V_{-,n-1,j} \phi_{+,n,j} - g \varphi_3 \overline{\phi}_{+,n,j} V_{+,n,j} - g \overline{\varphi}_3 V_{-,n,j} \phi_{+,n,j} \Big) \Bigg) \\
& \quad + \sum_I \frac{g^2}{2} C_I \left(V_{+,n,j} \phi_{-,\tilde{n},\tilde{j}} - V_{-,\tilde{n},\tilde{j}} \phi_{+,,n,j} \right) \left(V_{-,\tilde{m},\tilde{l}} \overline{\phi}_{-,m,l} - V_{+,m,l} \overline{\phi}_{+,\tilde{m},\tilde{l}} \right) \Bigg] \Bigg\} ,
\end{aligned}
\tag{4.8}
$$

with $I = \{n, j, \tilde{n}, \tilde{j}, m, l, \tilde{m}, \tilde{l}\}$ and

$$
C_I = \int_{T^2} d^2x \left(\psi_{n,j} \overline{\psi}_{\tilde{n},\tilde{j}} \psi_{m,l} \overline{\psi}_{\tilde{m},\tilde{l}} \right) . \tag{4.9}
$$