Super-Excited Initial States of Inflation

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Based on

A. Ashoorioon, R. Casadio, G. Geshnizjani, H. J. Kim, arXiv:1702.06101
A. Ashoorioon, K. Dimopoulos, G. Shiu, M. Sheikh-Jabbari, JCAP 1402 (2014) 025
A. Ashoorioon, T. Koivisto, R. Casadio, JCAP 1612 (2016) no.12, 002
A. Ashoorioon, Tomi Koivisto, Phys.Rev. D94 (2016) no.4, 043009

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• Introduction

- Much has been said about the effect of new physics, M on the inflationary predictions.

- The effect is usually *claimed* to be subdominant and dependent on $\left(\frac{H}{M}\right)^n$ where *H* is the Hubble parameter during inflation, $M \gg H$ and $n \geq 1$

Easther, Greene, Kinney, Shiu (2001,2002)
Ashoorioon, Mann (2004, 2005)

Kempf, Niemeyer (2000, 2001)
Kaloper, Kleban, Lawrence, et. al. (2002)

- In some cases, it was shown the effect in the power spectrum can be quite large.

Martin & Brandenberger (2003)

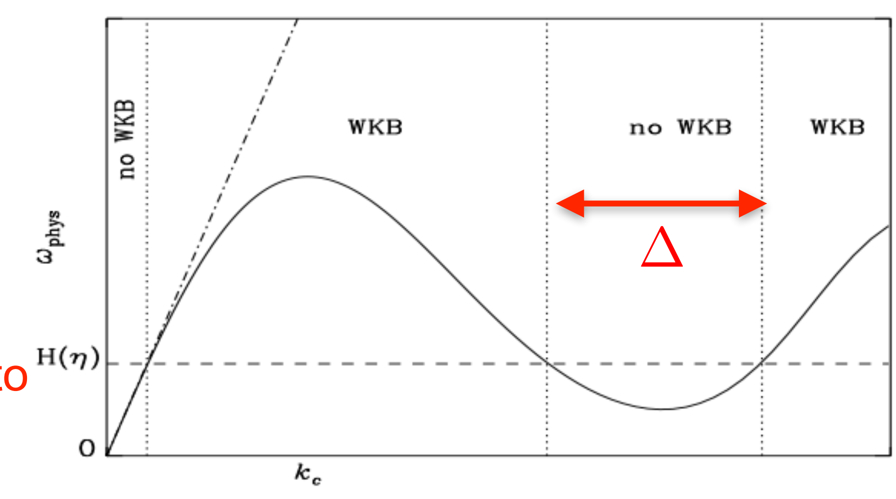
In particular, motivated by some condensed-matter studies, they assumed

$$\omega^2 = k^2 \quad \Longrightarrow \quad \omega^2 = k^2 - \alpha_0 k^4 + \beta_0 k^6$$

Gluing of the solution in disjoint regions

⇓

Oscillatory power spectrum where the amplitude of oscillations and the second Bog. coeff. is proportional to Δ



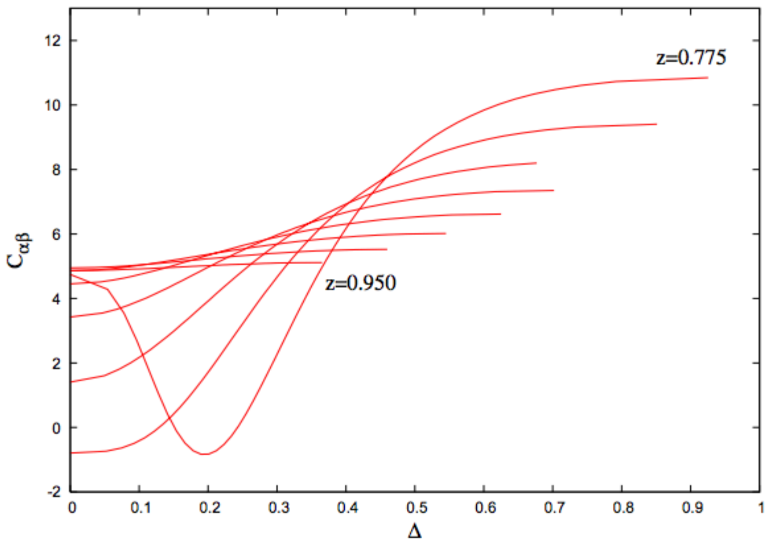
• Introduction

- At the time, backreaction was assumed to constrain the excited states considerably.

$$|\beta|^2 \propto \Delta^2 \ll 1$$

Tanaka (2001)

- Solving the mode equation numerically from WKB positive freq. mode



$$z \equiv \frac{3\beta_0}{\alpha_0^2}$$

$$C_{\alpha\beta} = P_S/P_{\mathrm{B.D.}} - 1$$

The correction to the power spectrum in this model could be quite large!

Marozzi & Joras (2008)

- Later it was shown that the backreaction effect is not that constraining

Greene, Shiu, Schalm & van der Schaar (2004)

$$|\beta| \lesssim \frac{\sqrt{\epsilon\eta} H M_P}{M^2}$$

M is the scale of new physics when the modes get excited.

- **Outline**

- Cosmological Perturbation Theory & Highly Excited Initial State

- Modified Dispersion Relation from the Effective Field Theory of Inflation (EFToI)

- Estimation of Bogoliubov coefficients

- Conclusion & Plans for Future Work

• Cosmological Perturbations and Excited Initial States

- The equation for gauge-invariant scalar perturbations

$$u''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right) u_{\vec{k}} = 0 \qquad z \equiv \frac{a\phi'}{\mathcal{H}}, \quad \mathcal{H} \equiv \frac{a'}{a}$$

$$u = -z\left(\frac{a'}{a}\frac{\delta\phi}{\phi'} + \Psi\right)$$

- In a quasi-de-Sitter background $a(\tau) \simeq -\frac{1}{H\tau}$

the most generic solution to the E.O.M. in the leading order in slow-roll parameters

$$u_k(\tau) = \frac{\sqrt{-\pi\tau}}{2}\left[\alpha_k^S H_{3/2}^{(1)}(-k\tau) + \beta_k^S H_{3/2}^{(2)}(-k\tau)\right]$$

where the Bogoliubov coefficients satisfy the Wronskian condition

$$|\alpha_k^S|^2 - |\beta_k^S|^2 = 1.$$

$\alpha_k^S = 1$ and $\beta_k^S = 0$ → Bunch-Davies vacuum

• Cosmological Perturbations and Excited Initial States

- Any excited state contains massless quanta whose positive pressure can tamper the slow-roll Inflation

Derailing can be avoided if

$$\delta\rho_{\text{non-BD}} \ll \epsilon\,\rho_0$$

$$\delta p'_{\text{non-BD}} \ll \mathcal{H}\,\eta\,\epsilon\,\rho_0$$

The second equation, which is the stronger one, can be written as

$$\int_{\mathcal{H}}^{\infty} \frac{d^3k}{(2\pi)^3} k|\beta_k^S|^2 \ll \epsilon\eta H^2 M_{\text{pl}}^2$$

As a specific example, let us consider the crude model in which the modes get excited when $k/a(\tau) = M$

$$\beta_k^S = \beta_0$$

Greene, Shiu, Schalm & van der Schaar (2004)

$$\beta_0 \lesssim \sqrt{\epsilon\eta}\,\frac{H\,M_{\text{Pl}}}{M^2}$$

Cosmological Perturbations and Excited Initial States

Scalar power spectrum

Ashoorioon, Dimopoulos, Sheikh-Jabbari & Shiu (2013)

$$\mathcal{P}_S = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2_{k/\mathcal{H}\to 0} \qquad \mathcal{P}_S = \mathcal{P}_{\rm BD}\,\gamma_S$$

$$\mathcal{P}_{BD} = \frac{1}{8\pi^2\epsilon}\left(\frac{H}{M_{\rm pl}}\right)^2, \qquad \gamma_s = |\alpha_k^S - \beta_k^S|^2_{k=\mathcal{H}}.$$

Parameterization of the Parameter Space

$$\alpha_{\vec{k}}^S = e^{i\varphi_S}\cosh\chi_S\ , \qquad \beta_{\vec{k}}^S = e^{-i\varphi_S}\sinh\chi_S\ ,$$

Let us focus on $V(\phi) = \frac{1}{2}m^2\phi^2$

Using the Planck normalization for the amplitude of density perturbations:

$$\frac{H}{M_{pl}} \simeq \frac{1}{\sqrt{\gamma_S}} 3.78\times 10^{-5}$$

that with the help of backreation condition, $\beta_0^S \leq \frac{\epsilon H M_{pl}}{M^2}$, yields

$$\frac{M^2}{H^2} \lesssim 220\,\frac{\sqrt{\gamma_S}}{\sinh\chi_S}$$

• Cosmological Perturbations and Excited Initial States

❑ **Quasi-BD region, $\chi_S \ll 1$ and general φ_S:**

- M can be arbitrary large
- H is very close to its Bunch-Davies value

❑ **Typical or large values of χ_S, $\chi_S \gtrsim 1$:**

- $\sqrt{\gamma_S} \simeq e^{\chi_S} \sin(\varphi_S)$
- $\sinh \chi_S \simeq \frac{e^{\chi_S}}{2}$
- generic values of φ_S

⟹

- $M \lesssim 21H$
- $H \lesssim H_{BD}$

- Desirable value of $M \simeq 21\, H$ is obtained if $\varphi_S \simeq \frac{\pi}{2}$.
- Very large values of χ_S (β) are phenomenologically allowed.
- The same could be said about tensor perturbations:

$$v_k^{\pm}(\tau) = \frac{\sqrt{-\pi\tau}}{2}\left[\alpha_k^T H_{3/2}^{(1)}(-k\tau) + \beta_k^T H_{3/2}^{(2)}(-k\tau)\right]$$

$$\beta_0^T \lesssim \frac{\sqrt{\epsilon\eta}\, H M_{pl}}{M^2} \simeq \frac{\epsilon H M_{pl}}{M^2}$$

- **Cosmological Perturbations and Excited Initial States**

$$P_T = P^T_{BD}\gamma_T \qquad P^T_{BD} = \frac{2}{\pi^2}\left(\frac{H}{M_{pl}}\right)^2 \qquad \gamma_T = |\alpha_T - \beta_T|^2_{k=\mathcal{H}}$$

$$r \equiv \frac{P_T}{P_S} = 16\gamma\epsilon = -8\gamma n_T \qquad \gamma \equiv \frac{\gamma_T}{\gamma_S} = \left.\frac{|\alpha^T_k - \beta^T_k|^2}{|\alpha^S_k - \beta^S_k|^2}\right|_{k=\mathcal{H}}$$

Violation of the consistency relation

- Using the same type of parameterisation $\alpha^T_k = \cosh\chi_T\, e^{i\varphi_T} \quad \beta^T_k = \sin\chi_T\, e^{-i\varphi_T}$

- χ_T can be either in the quasi-BD range or typical and large range.

$$\gamma \simeq \begin{cases} e^{-2\chi_S}, & \chi_T \ll 1 \\ e^{2(\chi_T-\chi_S)}\sin^2\varphi_T, & \chi_T \gtrsim 1, \text{ typical } \varphi_T \\ e^{-2(\chi_T+\chi_S)} & \chi_T \gtrsim 1, \tan\varphi_T \lesssim e^{-2\chi_T} \end{cases}$$

as long as the bounds from backreaction are respected → $\gamma \lesssim 1$

• Hemispherical Anomaly from Asymmetric Excited States

• Hemispherical Asymmetry by position-dependent excitations

Ashoorioon & Koivisto (2015)

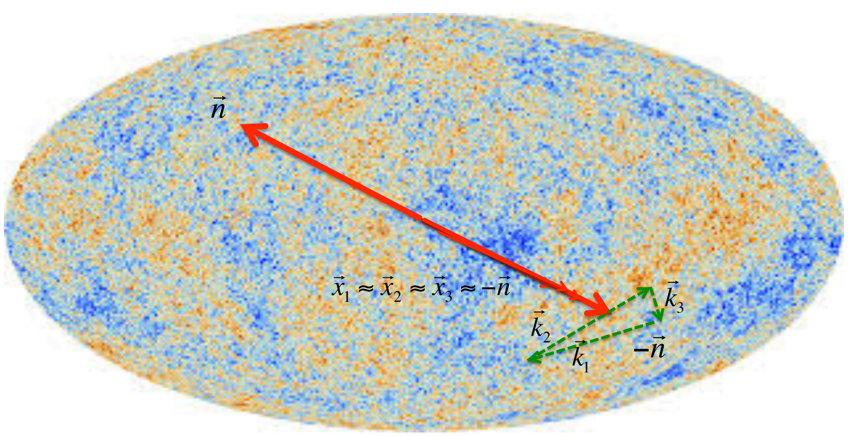
$$\Delta T(\hat{x}) = \Delta T_{\rm iso}(1 + 2A(\hat{x}.\hat{n})) \qquad A \simeq 6 - 7\%$$

$$\beta_0^S = \sinh\chi_S(1+\varepsilon\hat{x}.\hat{n})e^{i\varphi_S} \Longrightarrow \mathcal{P}_S = \mathcal{P}_{\rm iso}\left(1 + 2A(\hat{x}\cdot\hat{n}) + B(\hat{x}\cdot\hat{n})^2\right) \qquad A \simeq \varepsilon \gtrsim 0.07$$

Quadrupolar modulation in position space proportional to $B \simeq \varepsilon^2$

$|\vec{k}_3| << |\vec{k}_1| \approx |\vec{k}_2| \approx |\vec{k}_{l=2500}|$

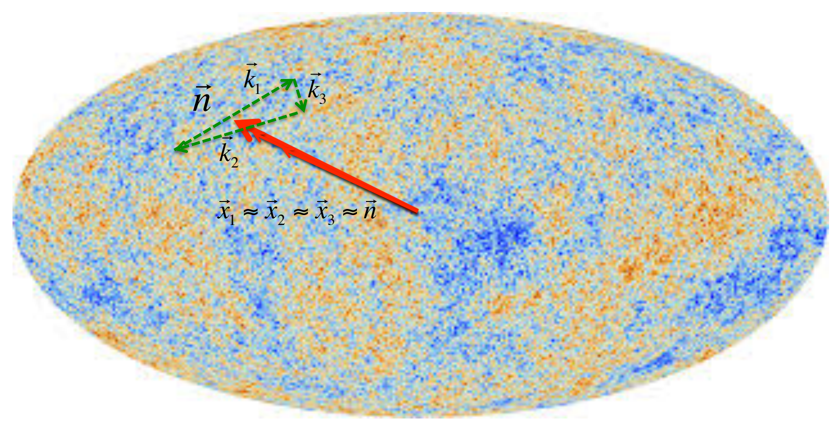
$|\vec{k}_3| \approx |\vec{k}_{l=10}|$



$$f_{\rm NL}^{\rm max} \simeq f_{NL}^{(0)}(1 + 2\varepsilon + 3\varepsilon^2) \approx 4.81$$

$|\vec{k}_3| << |\vec{k}_1| \approx |\vec{k}_2| \approx |\vec{k}_{l=2500}|$

$|\vec{k}_3| \approx |\vec{k}_{l=10}|$



$$f_{\rm NL}^{\rm min} \simeq f_{NL}^{(0)}(1 - 2\varepsilon + 3\varepsilon^2) \approx 3.64$$

$$\Delta f_{\rm NL} \simeq 1.17$$

Statistical Anisotropy from SO(3) non-invariant Excited States

Ashoorioon, Koivisto, Casadio (2016)

$$\Delta T(\hat{k}) = \Delta T_{\rm iso}(\hat{k})\left[1+M(\hat{k})\right] \ .$$

$$\mathcal{P}_S = \mathcal{P}_{\rm iso}\left[1+M(\hat{k})\right] \qquad M(\hat{k}) = A\,\hat{k}\cdot\hat{n} + B\,(\hat{k}\cdot\hat{n})^2 + C\,(\hat{k}\cdot\hat{n})^3 + \dots\ ,$$

A, C, … (odd multipoles) have to be pure imaginary numbers

dipole, quadrupole, Octupole

- Kim & Komatsu (2013), doing data analysis on the Planck 2013 data

$$-0.03 < B < 0.033 \ (95\% \ \text{C.L.})$$

We use the following parameterization:

$$\beta_0(\hat{k}) = \sinh\left(\chi_s + \varepsilon_2\, c_{\hat{k}}^2\right) e^{-i(\varphi_S+\delta_2\, c_{\hat{k}}^2)} \qquad \alpha_0(\hat{k}) = \cosh\left(\chi_s + \varepsilon_2\, c_{\hat{k}}^2\right) e^{i(\varphi_S+\delta_2\, c_{\hat{k}}^2)} \qquad \hat{k}\cdot\hat{n} \equiv \cos\psi_{\bar{k}} \equiv c_{\hat{k}}$$

- In the $\chi_S \gg 1$ where $\varphi_S \simeq \frac{\pi}{2}$

$$A = 0 \qquad B \simeq 2\varepsilon_2 \qquad C = 0$$

- Now from the observation constraint on *B,* the following constraint is obtained on ε_2

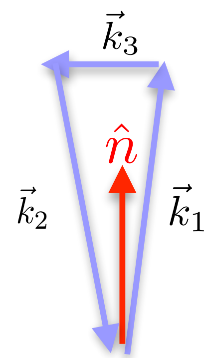
$$-0.015 < \varepsilon_2 < 0.0165 \ (95\% \ \text{C.L.})$$

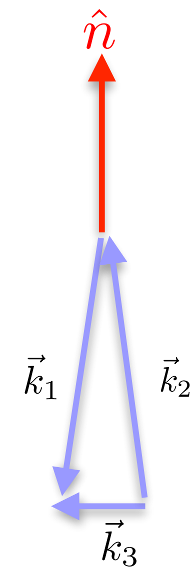
δ_2 remains indefinite in this regime from the constrains on the quadrupole moment.

• Statistical Anisotropy from SO(3) non-invariant Excited States

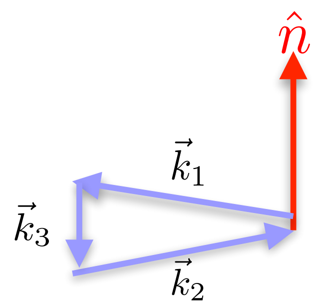
$\vec{k}_1$ corresponding to shortest scales probed by Planck and $\vec{k}_3$ corresponding to largest scale at which the cosmic variance is negligible, $l \simeq 10$. For $\epsilon \simeq 0.01$ and $\varepsilon_2 \simeq 0.0165$

$$f_{\rm NL}^{\rm max} \simeq 4.3$$





$$f_{\rm NL}^{\rm min} \simeq 4.03$$



$$\Delta f_{\rm NL} \simeq 0.27$$

Dispersion Relation from the Effective Field Theory of Inflation (EFToI)

- In "unitary gauge" where the inflaton fluct. are eaten by the perturbation of the metric, the time diffeomorphism is broken.

- In this gauge, the most general action that respects the remaining spatial diffeomorphism is

$$
\begin{aligned}
L_n = & \frac{M_2^4}{2!}(g^{00}+1)^2 - \frac{\bar{M}_2^2}{2}(\delta K^{\mu}{}_{\mu})^2 - \frac{\bar{M}_3^2}{2}\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} \\
& -\frac{\delta_1}{2}(\nabla_{\mu}\delta K^{\nu\gamma})(\nabla^{\mu}\delta K_{\nu\gamma}) - \frac{\delta_2}{2}(\nabla_{\mu}\delta K^{\nu}{}_{\nu})^2 - \frac{\delta_3}{2}(\nabla_{\mu}\delta K^{\mu}{}_{\nu})(\nabla_{\gamma}\delta K^{\gamma\nu}) \\
& -\frac{\delta_4}{2}\nabla^{\mu}\delta K_{\nu\mu}\nabla^{\nu}\delta K^{\sigma}_{\sigma} \ .
\end{aligned}
$$

- The time-diffeomorphism which is non-linearly realized can be restored using the Stueckelberg procedure

$$t \to t + \xi^0(x^\mu) \xrightarrow{\xi^0(x^\mu) \to -\pi(x^\mu)}$$ then we demand that under $t \to t + \xi^0(x^\mu)$

$$\pi \to \pi - \xi^0$$

Dispersion Relation from the Effective Field Theory of Inflation (EFToI)

$$\delta K_{ij} \supset (\partial_i \partial_j \pi + \partial_i g_{0j}), \quad \xrightarrow{u = a\pi} \quad u'' + \left(\gamma_0 k^2 + \alpha_0 k^4 \tau^2 + \beta_0 k^6 \tau^4 - \frac{2}{\tau^2}\right) u = 0$$

In fact implementing the stueckelberg mechanism to the spatially invariant action, yields

$$\begin{aligned}
\mathcal{L}_n^{(2nd)} = & -\frac{1}{2}\delta_1 \left(\frac{k^6\pi^2}{a^6} - \frac{13H^2k^4\pi^2}{a^4} + \frac{k^4\dot{\pi}^2}{a^4} + \frac{24H^4k^2\pi^2}{a^2} + \frac{2H^2k^2\dot{\pi}^2}{a^2} - 6H^4\dot{\pi}^2 - 3H^2\ddot{\pi}^2 \right) \\
& -\frac{1}{2}\delta_2 \left(\frac{k^6\pi^2}{a^6} + \frac{H^2k^4\pi^2}{a^4} - \frac{k^4\dot{\pi}^2}{a^4} + \frac{6H^4k^2\pi^2}{a^2} - 9H^2\ddot{\pi}^2 \right) \\
& -\frac{1}{2}\delta_3 \left(\frac{k^6\pi^2}{a^6} - \frac{10H^2k^4\pi^2}{a^4} + \frac{15H^4k^2\pi^2}{a^2} - 9H^4\dot{\pi}^2 \right) \\
& -\frac{1}{2}\delta_4 \left(\frac{k^6\pi^2}{a^6} - \frac{7H^2k^4\pi^2}{a^4} + \frac{Hk^4\pi^2}{2a^4} + \frac{21H^4k^2\pi^2}{2a^2} - \frac{9H^2k^2\dot{\pi}^2}{a^2} - \frac{9}{2}H^4\dot{\pi}^2 \right).
\end{aligned}$$

- As expected terms proportional to $k^6\pi^2$ appears.
- However terms proportional to $\ddot{\pi}$ appears too which leads to Ostrogradski ghosts.

Dispersion Relation from the Effective Field Theory of Inflation (EFToI)

- Also there will be correction of the dispersion relation from the k^6 at high momenta from the presence of $k^4\dot{\pi}^2$ and $k^2\dot{\pi}^2$

$$\delta_1 = \delta_2 = \delta_4 = 0$$

- With $\delta_3 > 0$, we can achieve our desired scenario where

$$\gamma_0 > 0, \ \alpha_0 < 0 \quad \text{and} \quad \beta_0 > 0$$

- γ_0 and respectively the speed of sound could be always set to one by a reparameterization $d\tau \to c_s d\tau$

- We also assume that the dispersion relation never becomes tachyonic on sub-Hubble scales

$$z \equiv \frac{\beta_0}{\alpha_0^2} \geq \frac{1}{4}$$

- We also assume there is one horizon-crossing event corresponding to $\omega^2(k) = 2H^2$
  - For $z > \frac{1}{3}$, there is only one turning point automatically.
  - For $\frac{1}{4} \leq z \leq \frac{1}{3}$

$$\alpha_0 \leq \frac{9z - 2 - 2(1-3z)^{3/2}}{54z^2}$$
$$\text{or} \quad \alpha_0 \geq \frac{9z - 2 + 2(1-3z)^{3/2}}{54z^2}.$$

- **Estimation of Bogoliubov coefficients**

- Sixth Order Polynomial with an intermediate negative group velocity

In terms of $x \equiv k\tau$

$$u_k'' + \left(\beta_0\, x^4 - \alpha_0\, x^2 + 1 - \frac{2}{x^2}\right) u_k = 0$$

Let us estimate the number density of particles

$$-\infty < x \lesssim x_1(\alpha_0, \beta_0) \qquad \text{region I}$$
$$x_1(\alpha_0, \beta_0) < x < 0 \qquad \text{region II}$$

$$\alpha_0 x_1^4 - \beta_0 x_1^2 = 1 - \frac{2}{x_1^2}$$

In region I:

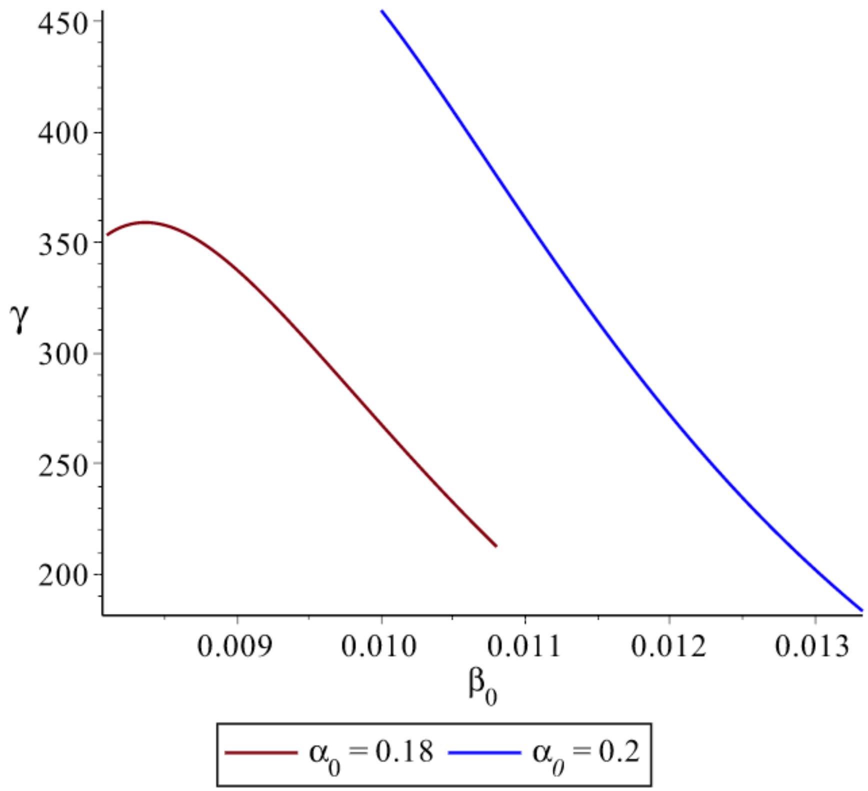
$$u_{k\,\mathrm{I}}(x) = \frac{\beta_0^{1/4}}{k^{1/2}\,\alpha_0^{1/2}} \mathrm{HeunT}(\mathscr{A}, 0, \mathscr{B}, -\mathscr{C}\; x)\; \exp(-y)$$

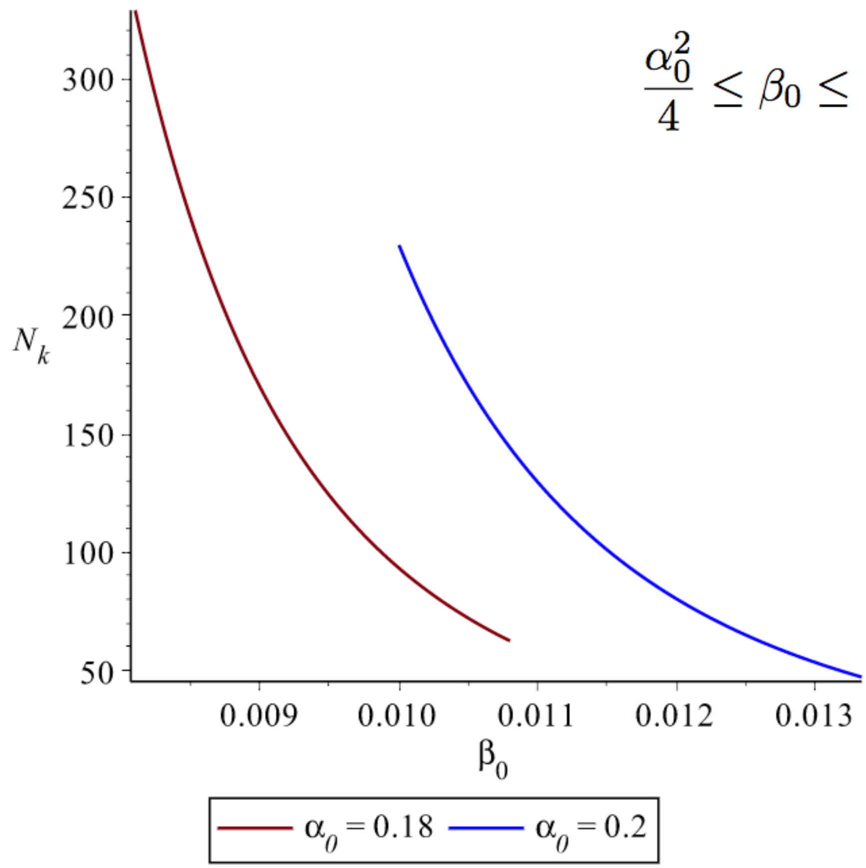
In region II:

$$u_{k\,\mathrm{II}} = \frac{\sqrt{-x\pi}}{2\sqrt{k}} \left[\xi\, H_{3/2}^{(1)}(-x) + \rho\, H_{3/2}^{(2)}(-x)\right]$$

- **Estimation of Bogoliubov coefficients**

- Sixth Order Polynomial with an intermediate negative group velocity





$$\frac{\alpha_0^2}{4} \leq \beta_0 \leq \frac{\alpha_0^2}{3}$$

For $\alpha_0 = 0.2$

$$183.35 \leq \gamma \leq 454.89, \qquad 47.31 \leq N_k \leq 229.63$$

- **Estimation of Bogoliubov coefficients**

  - Sixth Order Polynomial with an intermediate negative group velocity

Introducing the variable $x \equiv k\eta$

$$u_k'' + \left(\beta_0 x^4 - \alpha_0 x^2 + 1 - \frac{2}{x^2}\right) u_k = 0\,.$$

The positive frequency WKB mode in infinite past as the initial condition

$$\frac{1}{2}\sqrt{\frac{\pi}{3}}\sqrt{-x}H^{(1)}_{\frac{1}{6}}\left(-\frac{\sqrt{\beta_0}x^3}{3}\right)$$

one can integrate the mode equation for specific values of α_0 and β_0 .

Largest enhancement in the power spectrum is obtained for $\alpha_0 \simeq 0.2$ and $\beta_0 \simeq \frac{\alpha_0^2}{4}$

$P_S = \gamma_S P_{B.D.}$ where for these values of parameters $\gamma_S \simeq 14.738$

In order to determine the corresponding excited state we proceed as follows:
Mathematica can find an implicit solution for the e.o.m.

$$u_k(x) = c_1 u_k^{(1)}(x) + c_2 u_k^{(2)}(x) \qquad c_1\bar{c_2} - c_2\bar{c_1} = i.$$

$u_k^{(i)}(1) = 1$ and $u_k^{(i)'}(1) = 1$

- **Estimation of Bogoliubov coefficients**

  - Sixth Order Polynomial with an intermediate negative group velocity

$$c_1\bar{c_2} - c_2\bar{c_1} = i. \longrightarrow c_1 = \frac{1}{\sqrt{2a}} \text{ and } c_2 = -i\frac{a}{\sqrt{2}} \text{ where } a \in \mathbb{R}$$

a is determined such that the power spectrum from $u_k(\eta)$ is the same as the numerical result.

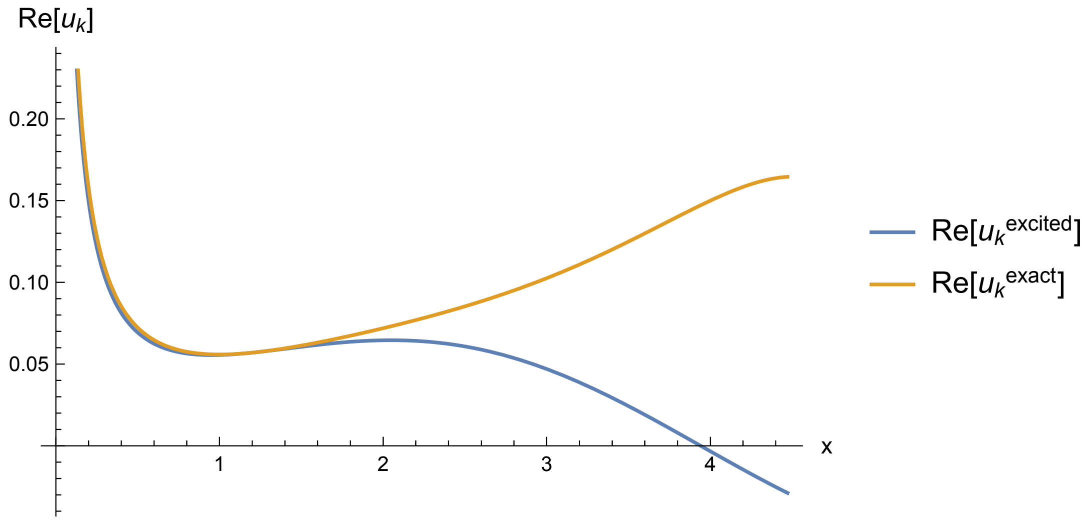
There will be four solutions where, two by two, they are negative of the other ones.

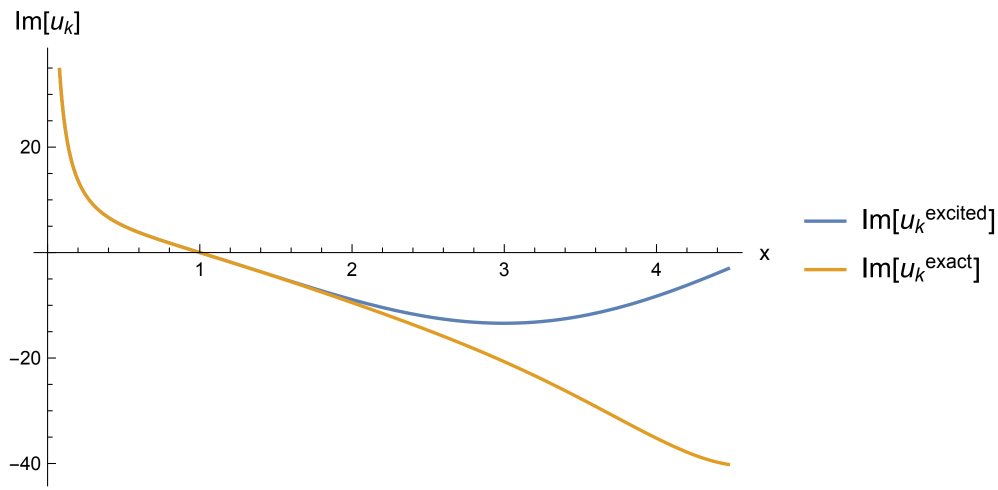
We look for a solution $u_k^X(\eta) = \frac{\sqrt{-\pi\eta}}{2}\left[\alpha_k H_{3/2}^{(1)}(-k\eta) + \beta_k H_{3/2}^{(2)(-k\eta)}\right]$ that produces such

value for power spectrum, it is continuous at an earlier point and it's derivative is also continuous at that point. From all 4 solutions for a, only has such a characteristic that $u_k(\eta)$ and $u_k^X(\eta)$ reconcile after the point of integration.

- **Estimation of Bogoliubov coefficients**

  - Sixth Order Polynomial with an intermediate negative group velocity





$$\beta(x_{\text{cross}}) = -1.88359 - 8.7681\ i$$

$$\alpha(x_{\text{cross}}) = 1.95519 - 8.80935\ i$$

$$N_k \equiv |\beta_k|^2 = 80.4275$$

• Conclusion

- Not always true that the effect of evolution of the modes when they have high momenta is sub-dominant.
- If there is modified dispersion relation with an interim phase with negative group velocity, the corrections to the power spectrum could be quite large.
- I also showed how one can realize these dispersion relations in the EFToI.
- I provided a method to Bogoliubov coefficients such that they lead to the exact estimate of the power spectrum.
- With the help of dispersion relation $\omega^2 = k^2 - \alpha_0 k^4 + \beta_0 k^6$ excited states with $|\beta_k| \gtrsim 9$ were built.
- If perturbations start from such super-excited states, $M \lesssim \text{few} \times 10H$

• Conclusion

- Generally, we will have enhancement of the non-gaussianity in the local shape for such super-excited states.

- Bispectrum in such modified dispersion relations is what we are investigating.

- Effect of $(\nabla K)^2$ on tensor perturbations EOM is under investigation too.

- The term proportional to $\nabla^{\mu}\delta K_{\nu\mu}\nabla^{\nu}\delta K^{\sigma}{}_{\sigma}$ leads to scale-dependent speed of sound. The effect on the spectrum and bispectrum with such a speed of sound is the other thing I am looking at.

Thank you!