Recent progresses in Higgs mass calculations in BSM models

Florian Staub | Planck2017, Warsaw, 25th May 2017

KARLSRUHE INSTITUTE OF TECHNOLOGY, ITP & IKP

Summary of

M. D. Goodsell, (Kilian Nickel,) FS
2-Loop Higgs mass calculations in SUSY models with SARAH and SPheno
`1411.0675`, `1503.03098`, `1604.05335`

FS, W. Porod: "Improved predictions for intermediate and heavy SUSY in the MSSM and beyond", `1703.03267`

J. Braathen, M. D. Goodsell, FS: "Supersymmetric and non-supersymmetric models without catastrophic Goldstone bosons", `1705.XXXXX`

The Higgs mass as precision observable

The Higgs mass has been measured with an incredible precision:

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  - Missing three-loop corrections

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- An 1-loop eff. pot. calculation (still often done for new models) suffers from more than 10 GeV uncertainty!

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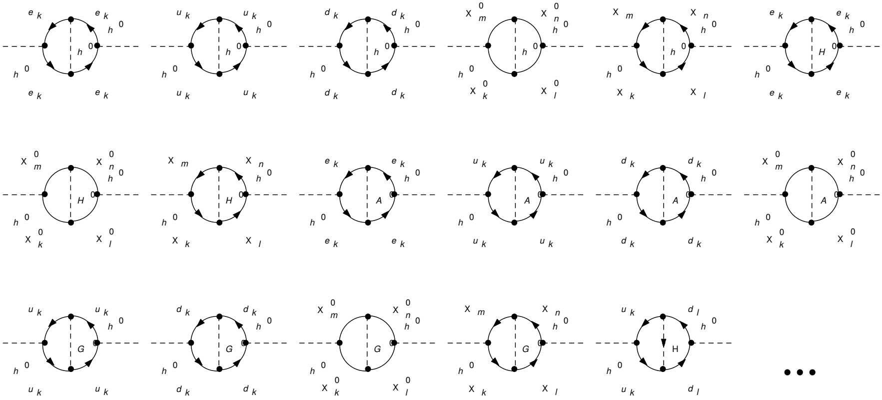
(At least) MSSM precision is necessary to be able to con-
front BSM models with the measurements

Generic Higgs mass calculations

Thousands of Feynman diagrams are needed to be calculated:



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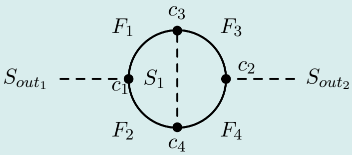
Generic expressions



Generic expression $f(m_{out_i}, m_S, m_{F_i}, c_i)$ are

- **Valid for any model** and **for any real scalar**

→ **Disentangle** the calculation of ...

... **loop amplitudes** (difficult) and **masses & couplings** (easy)

Fully automatised two-loop calculations

The combination SARAH/SPheno provides a **fully automatised two-loop calculation** of the Higgs mass in SUSY models.

Approach

[Goodsell,Nickel,FS,1411.0675,1503.03098]

- Generic one- and two-loop calculations which are matched on concrete models.
- Auto-generated Fortran code for numerical evaluation

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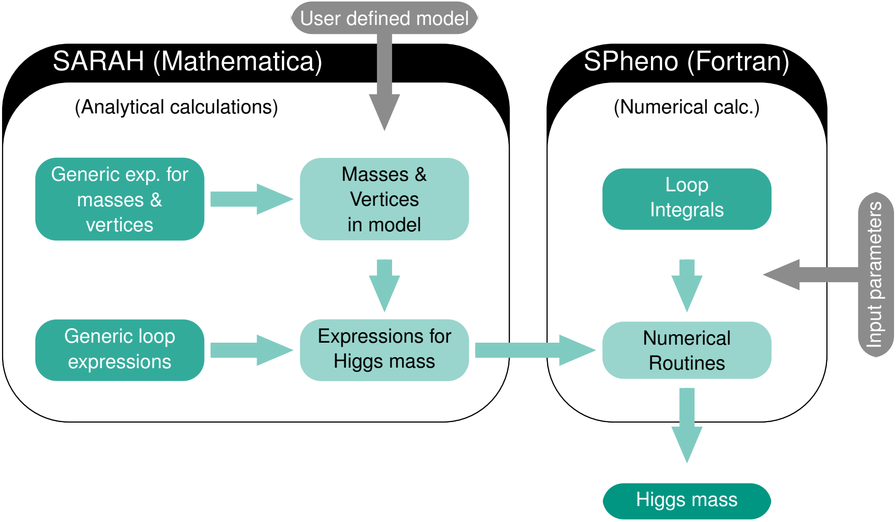
Approximations @2-loop: gaugeless limit ($g_1 = g_2 = 0$), $p^2 = 0$:

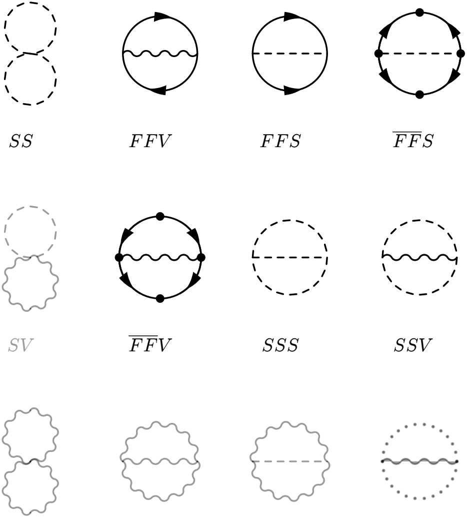
- similar precision as most public tools provide for MSSM
- All available ($\overline{\text{DR}}$) **two-loop results** (MSSM, NMSSM, NMSSM-CPV) are **exactly reproduced!**

The setup


User defined model
SARAH (Mathematica)
(Analytical calculations)
Generic exp. for masses & vertices
Masses & Vertices in model
Generic loop expressions
Expressions for Higgs mass
SPheno (Fortran)
(Numerical calc.)
Loop Integrals
Numerical Routines
Input parameters
Higgs mass

Effective potential

- **Generic expressions** for all two-loop diagrams are known [Martin,hep-ph/0111209]
- Expressions have been **translated into 4-component notation** [Goodsell,Nickel,FS,1411.0675]
- **ew gauge** contributions usually **neglected**



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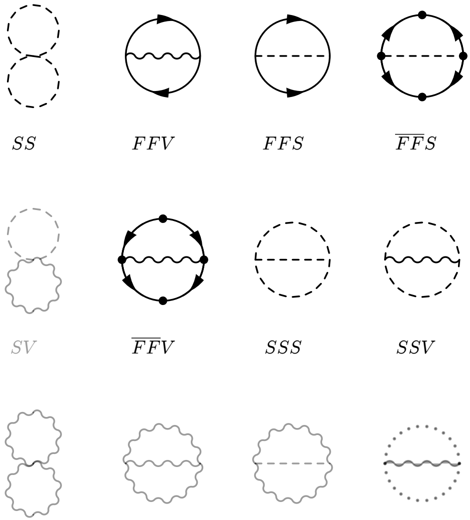
- Numerical derivation to get

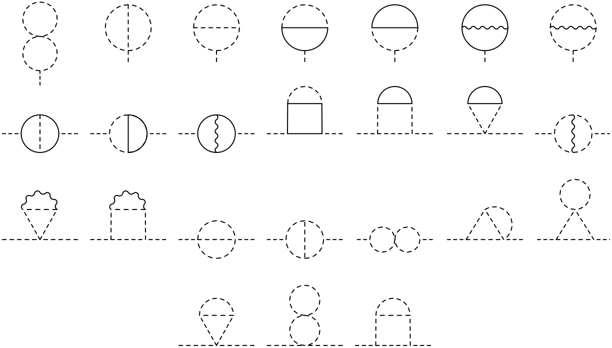
$$\delta t_i^{(2)} = \frac{\partial V^{(2)}}{\partial v_i}$$

$$\Pi_{ij}^{(2)} = \frac{\partial^2 V^{(2)}}{\partial v_i \partial v_j}$$



Generic approach

Diagrammatic calculation



- We **derived** a **new set of generic expressions**:

  → **fully numerical agreement** with S. Martin, but expressions often shorter for **2-point functions**; tadpoles available for the first time

- Advantages: (i) **No numerical derivation**; (ii) can be used for **CP-odd scalars and CPV** [Goodsell,FS,1604.05335]
- momentum dependence possible, but linking `TSIL` is very slow ☹

Further improvements

- **The two fully independent calculations can be use for double checks**
- These automatised two-loop calculations could be used to obtain **many new results for SUSY models**:
  - NMSSM beyond $O(\alpha_S\alpha_t)$ [Goodsell,Nickel,FS,1411.4665]
  - Two-loop results for vector-like states, R-parity violation, Dirac gauginos, Non-holomorphic soft-terms, ...

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New developments

1. EFT Higgs mass calculation
   → better precision for heavy new scales
2. Solution to the Goldstone boson catastrophe
   → two-loop results for non-SUSY models

The Legacy of the hope for light SUSY

All MSSM spectrum generators use(d) the same approach to get the SUSY and Higgs masses

Standard Matching

[Pierce, Bagger, Matchev, Zhang; hep-ph/9606211]

- **DR values** of g_i, Y_i at $Q = M_Z$ derived from $G_F, \alpha_{ew}, \alpha_S, M_Z, m_q, m_l$ at one-loop
- SUSY RGEs between M_Z and M_{SUSY}

→ only a good approximation for $M_{\text{SUSY}} \simeq M_Z$

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Higgs mass calculation

m_h is calculated at $Q = M_{SUSY}$ in full MSSM usually at two-loop level

→ uncertainty increases with increasing M_{SUSY}

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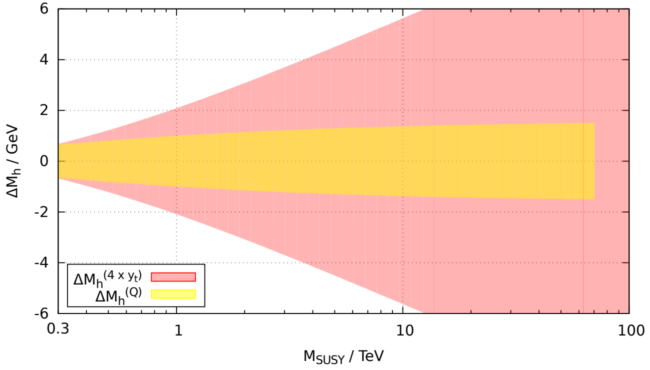

[Athron,Park,Steudtner,Stöckinger,Voigt;1609.00371]

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α_S, M_Z, m_q, m_l at

M_Z

o-loop level

SUSY

Two-Scale Matching (2SM) in SARAH

$\overline{\text{MS}}$ parameters at M_Z ($g_i^{\overline{\text{MS}}}(M_Z)$, $Y_i^{\overline{\text{MS}}}(M_Z)$, $v^{\overline{\text{MS}}}(M_Z)$):
full one-loop matching including higher order corrections

$\downarrow$

Running Up:
SM RGEs up to three-loop

$\downarrow$

**$\overline{\text{DR}}$ parameters at M_{SUSY} ($g_i^{\overline{\text{DR}}}(M_{\text{SUSY}})$, $Y_i^{\overline{\text{DR}}}(M_{\text{SUSY}})$,
$v^{\overline{\text{DR}}}(M_{\text{SUSY}})$:**
two-loop $\overline{\text{MS}}$–$\overline{\text{DR}}$ conversion; one-loop SUSY shifts

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- Fixed order calculation less accurate for heavy SUSY scales

  → Necessary to consider effective theory

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- Higgs mass calculation within effective SM:
  1. Match the SM and MSSM at M_{SUSY} to obtain $\lambda_{\text{SM}}(M_{\text{SUSY}})$
  2. Run λ_{SM} to m_t
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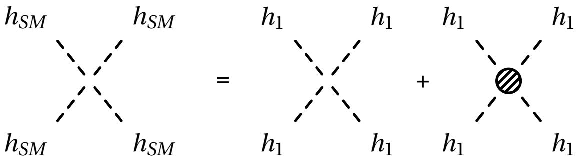
- Methods to obtain λ_{SM}:
  1. Matching of four-point function



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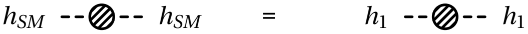
- Methods to obtain λ_{SM}:
  1. Matching of four-point function
  2. Matching of Higgs pole masses

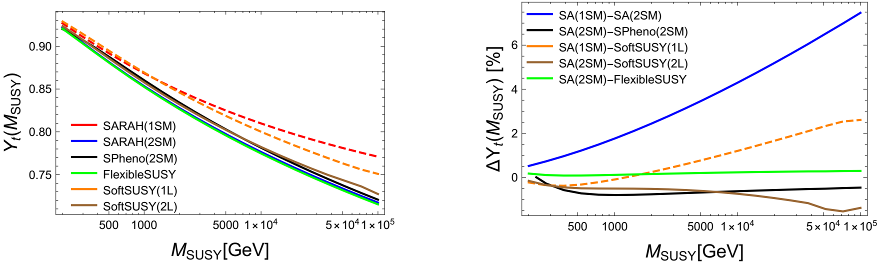
[Athron,Park,Steudtner,Stöckinger,Voigt;1609.00371]



- Terms v^2/M_{SUSY}^2 included
- Higher order corrections 'easily' included via m_h^{pole}
- Works also for other models

Impact on $Y_t^{\overline{\text{DR}}}(M_{\text{SUSY}})$

[Porod,FS,1703.03267]



- 1SM→2SM: Sizeable change in the top Yukawa coupling
- Good agreement in codes with 2SM even for huge M_{SUSY}
- Very good agreement with 2-loop matching of `SoftSUSY` up to 10 TeV

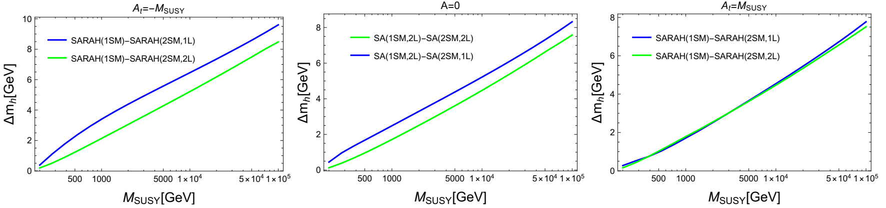
Changes in m_h

[Porod,FS,1703.03267]

We consider in the following a simplified model with

$$M_1 = M_2 = M_3 = M_A = \mu \equiv M_{\text{SUSY}}\,, \quad m^2_{\tilde{e}} = m^2_{\tilde{l}} = m^2_{\tilde{d}} = m^2_{\tilde{u}} = m^2_{\tilde{q}} = \mathbf{1} M^2_{\text{SUSY}}$$

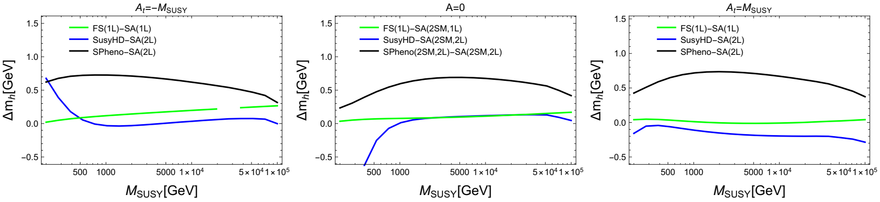
and all trilinear couplings vanish, but $A_t Y_t \tilde{t}_L \tilde{t}^*_R H_u$.



→ Large changes in m_h for heavy SUSY!

Comparison with other codes

[Porod,FS,1703.03267]



- Good agreement between the EFT codes for heavy SUSY scales
- Differences to `SPheno` because of different matching of Y_t
- `SusyHD` deviates for small M_{SUSY} because of missing v^2/M_{SUSY}^2 terms

Beyond the MSSM

Karlsruher Institut für Technologie

2SM and EFT Higgs mass calculation available for all models now in SARAH

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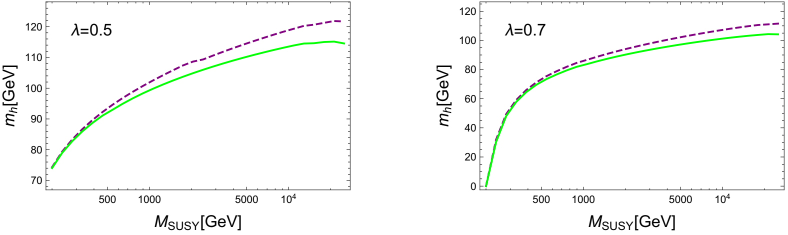

Condition for EFT Higgs mass calculation: all BSM scalars heavier than 125 GeV

The Goldstone Boson catastrophe

The effective potential in Landau gauge

$$V^{(1)} \sim m^2 \log(m/Q)$$
$$V^{(2)} \sim M \log(M/Q)\, m \log(m/Q)$$
$$V^{(3)} \sim (M \log(M/Q))^2 \log(m/Q)$$

For $m \to 0$:

→ Second derivative (self-energies) of $V^{(1)}$ diverges
⇒ is cured by including momentum dependence

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⇒ depend **not** on external momenta!

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- Always problematic are the **Goldstones of broken groups**
  → **ew corrections** are **not considered** in the **MSSM at 2-loop**
- In other BSM models also **other scalars** can cause **similar problems**

A SUSY solution

- The Goldstone problem can be avoided by
  - Dropping D-term contributions in the mass matrices ...
  - ...but keeping **tadpole equations unchanged** in gaugeless limit

→ generates finite Goldstone masses $O(M_{ew})$.

→ effect is of order of the neglected ew two-loop corrections

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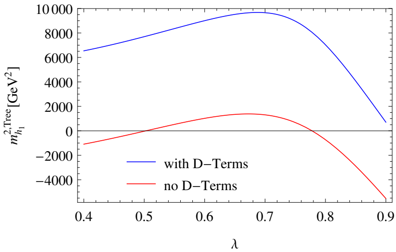
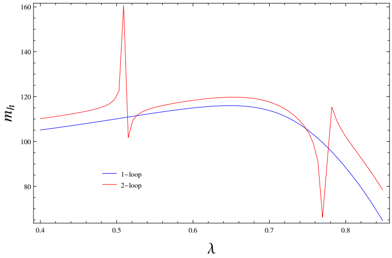
- **Doesn't work for non-SUSY models (no D-terms!)**
- Can cause new divergences, e.g.





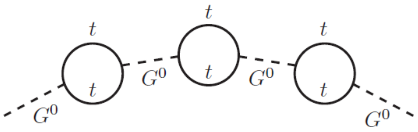
($\tan\beta = 3$, $\kappa = 0.6$, $A_\lambda = 200$ GeV, $A_\kappa = -200$ GeV, $\mu_{\text{eff}} = 150$ GeV)

General solutions

Exact methods:

1. Resummation of Goldstones contributions

[Martin,Kumar;Elias-Miro,Espinosa,Konstandin,...]



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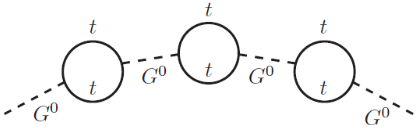

2. Taking Goldstone on-shell

[Braathen,Goodsell]

Replace Goldstone masses in loops:

$$m_G^{2,\text{run}} \to m_G^{2,\text{OS}} - \Pi_G(m_G^{2,\text{OS}}) = -\Pi_G(0)$$

→ **see talk by Johannes Braathen today at 15:35**

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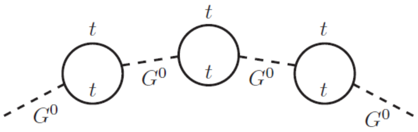

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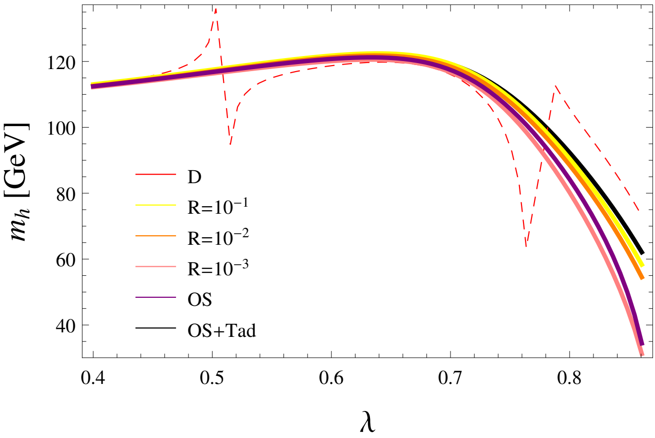
Alternatively, one can try to cheat:

1. Finding a renormalisation scale where the problem is absent
2. Introducing a regulator mass

We have implemented the on-Shell solution in SARAH/SPheno:

[Braathen, Goodsell,FS]

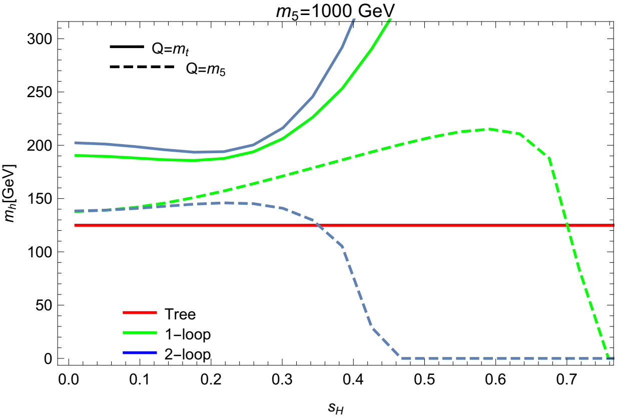


→ Poles in SUSY calculations under control

→ Results for several non-SUSY models available for the first time!

Example: Georgi-Machacek Model

[Braathen, Goodsell,FS]



→ corrections can be huge!

→ Obvious break-down of perturbation theory for $s_H > 0.35$

Summary

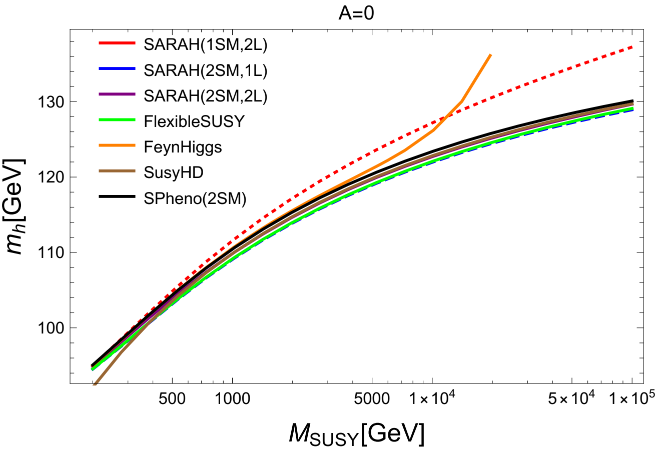
- I have given an overview of the **automatised Higgs mass calculations** with SARAH and SPheno
- The available two-loop calculations are now combined with
  1. An EFT calculation to handle high BSM scales
  2. A general solution to the Goldstone boson catastrophe

Robust and precise results for many BSM models are now available out-of-the-box

- MSSM, NMSSM, MRSSM, TNMSSM, UMSSM, DiracGauginos, ...
  → more than **50 models** delivered with SARAH
- SplitSUSY, THDM, Georgi-Machacek, SSM, TSM, SM+VL, ...
  → more than **30 models** delivered with SARAH

SARAH/SPheno is the only combination which provides MSSM-like accuracy for other BSM models!

Backup


A=0
SARAH(1SM,2L)
SARAH(2SM,1L)
SARAH(2SM,2L)
FlexibleSUSY
FeynHiggs
SusyHD
SPheno(2SM)
130
120
110
100
500
1000
5000
1 × 10^4
5 × 10^4
1 × 10^5
m_h[GeV]
M_{SUSY}[GeV]

Higgs mass in the NMSSM

We consider the NMSSM with

$$\mu_{\text{eff}} = M_{\text{SUSY}}\,, \quad A_\kappa = -\lambda M_{\text{SUSY}}\,, \quad A_\lambda = M_{\text{SUSY}}\left(\frac{\tan\beta}{(1+\tan\beta^2)} - \frac{\kappa}{\lambda}\right)\,, \quad \tan\beta = 4$$

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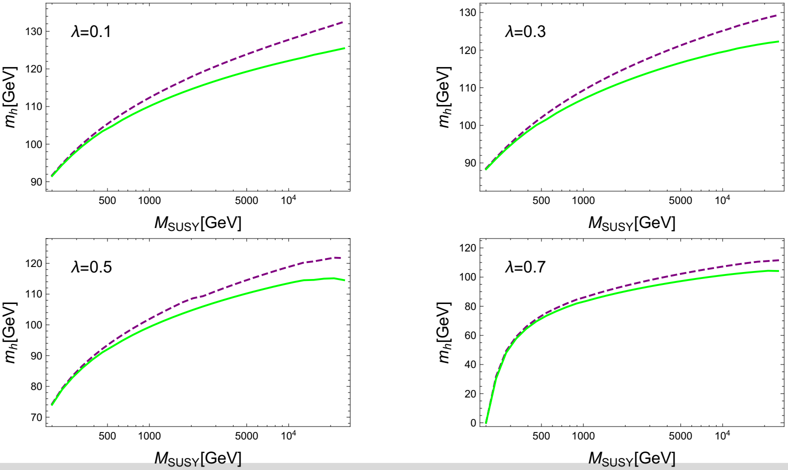

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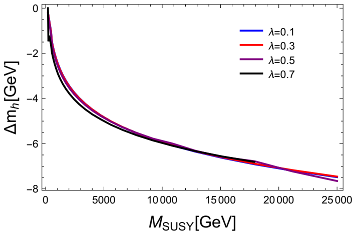
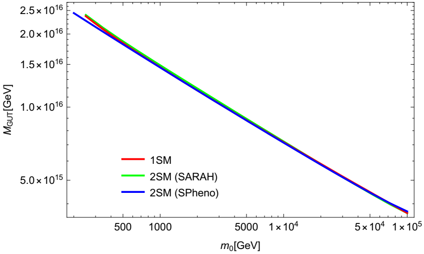
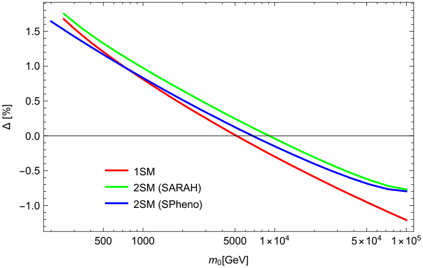


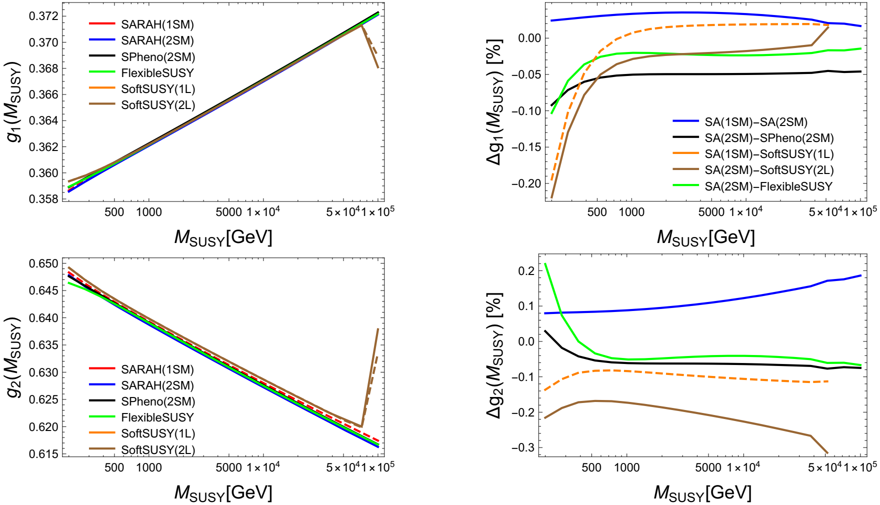
- Dominant corrections from (s)tops
- Small sensitivity on λ

Unification



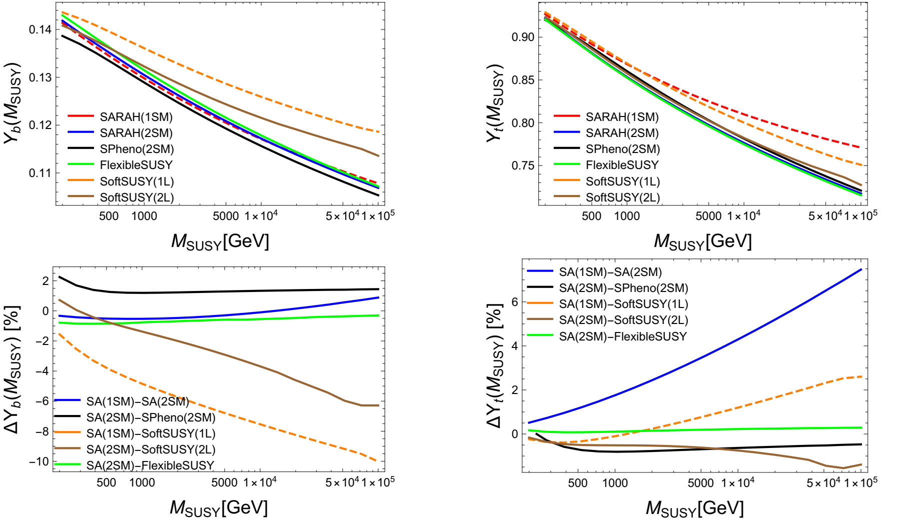


g_1, g_2



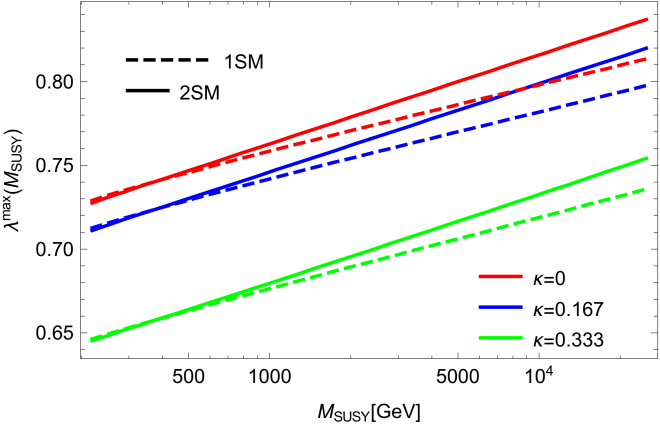
SARAH(1SM)
SARAH(2SM)
SPheno(2SM)
FlexibleSUSY
SoftSUSY(1L)
SoftSUSY(2L)
g1(MSUSY)
MSUSY[GeV]
0.372
0.370
0.368
0.366
0.364
0.362
0.360
0.358
500
1000
5000
1×10^4
5×10^4
1×10^5
SA(1SM)–SA(2SM)
SA(2SM)–SPheno(2SM)
SA(1SM)–SoftSUSY(1L)
SA(2SM)–SoftSUSY(2L)
SA(2SM)–FlexibleSUSY
Δg1(MSUSY) [%]
0.00
−0.05
−0.10
−0.15
−0.20
g2(MSUSY)
0.650
0.645
0.640
0.635
0.630
0.625
0.620
0.615
Δg2(MSUSY) [%]
0.2
0.1
0.0
−0.1
−0.2
−0.3

Y_b, Y_t

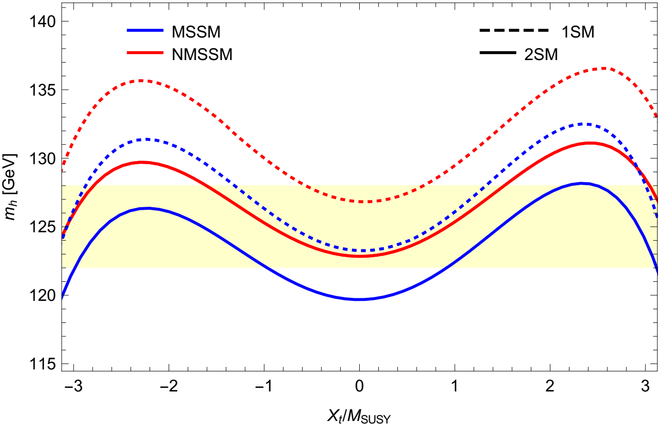


SARAH(1SM)
SARAH(2SM)
SPheno(2SM)
FlexibleSUSY
SoftSUSY(1L)
SoftSUSY(2L)
$Y_b(M_{SUSY})$
$Y_t(M_{SUSY})$
M_{SUSY}[GeV]
SA(1SM)–SA(2SM)
SA(2SM)–SPheno(2SM)
SA(1SM)–SoftSUSY(1L)
SA(2SM)–SoftSUSY(2L)
SA(2SM)–FlexibleSUSY
$\Delta Y_b(M_{SUSY})$ [%]
$\Delta Y_t(M_{SUSY})$ [%]

NMSSM: $\lambda_{\max}$



MSSM, NMSSM: X_t



The gaugeless limit

Self-energies and tadpoles are calculate as

$$t_i = \frac{\partial}{\partial v_i} V^{(eff)} = \frac{\partial m_j}{\partial v_i} \frac{\partial V^{(eff)}}{\partial m_j}$$

→ If $\frac{\partial m_j}{\partial v_i}$ vanishes, there is no Goldstone problem!

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- This **doesn't work any longer in the NMSSM** because of terms $\lambda^2 v_i^2$, $\kappa \lambda v_i v_S$, $T_\lambda v_i$, in the pseudo-scalar mass matrix

The gaugeless limit does **not** solve the Goldstone catastrophe in general for **extended Higgs sectors**!