A Pitfall in the Standard Way of Calculating Relic Density

Andrzej Hryczuk

University of Oslo

based on work with:

T. Binder, T. Bringmann and M. Gustafsson

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*on leave from National Centre for Nuclear Research, Warsaw, Poland

Thermal Relic Density

standard approach

$T \uparrow$

$\Gamma_{\text{ann}} > H$	DM in equilibrium	time $\downarrow$
$\Gamma_{\text{ann}} \sim H$	**chemical decoupling**	
$\Gamma_{\text{ann}} < H$	freeze-out	

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi] \quad \Longrightarrow \quad \frac{dn_\chi}{dt} + 3Hn_\chi = C$$

Liouville operator in FRW background

the collision term (integrated)

assumptions for using Boltzmann eq: classical limit, molecular chaos,...

…for derivation from thermal QFT see M. Beneke, F. Dighera, AH; JHEP 1410 (2014) 45

Thermal Relic Density

the collision term

for $2 \leftrightarrow 2$ CP invariant process:

$$C_{\mathrm{LO}} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3} \frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\, \sigma_{\chi\bar\chi\to ij} v_{\mathrm{rel}}\, \left[f_\chi f_{\bar\chi}(1 \pm f_i)(1 \pm f_j) - f_i f_j (1 \pm f_\chi)(1 \pm f_{\bar\chi})\right]$$

assuming kinetic equilibrium at chemical decoupling: $f_\chi \sim a(\mu) f_\chi^{\mathrm{eq}}$

$$C_{\mathrm{LO}} = -\langle \sigma_{\chi\bar\chi\to ij} v_{\mathrm{rel}} \rangle^{\mathrm{eq}} \left(n_\chi n_{\bar\chi} - n_\chi^{\mathrm{eq}} n_{\bar\chi}^{\mathrm{eq}}\right)$$

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar\chi\to ij} v_{\mathrm{rel}} \rangle^{\mathrm{eq}} = -\frac{h_\chi^2}{n_\chi^{\mathrm{eq}} n_{\bar\chi}^{\mathrm{eq}}} \int \frac{d^3\vec{p}_\chi}{(2\pi)^3} \frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\, \sigma_{\chi\bar\chi\to ij} v_{\mathrm{rel}}\, f_\chi^{\mathrm{eq}} f_{\bar\chi}^{\mathrm{eq}}$$

Thermal Relic Density

Boltzmann eq.

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

Re-written for the comoving number density:

$$\frac{dY}{dx} = \sqrt{\frac{g_* \pi m_\chi^2}{45G}}\, \frac{\langle\sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}}{x^2}\left(Y^2 - Y_{\rm eq}^2\right)$$

$$\lim_{x\to 0} Y = Y_{\rm eq} \qquad \lim_{x\to\infty} Y = {\rm const}$$

Recipe:
compute annihilation cross-section,
take a thermal bath average,
throw it into BE… and voilà

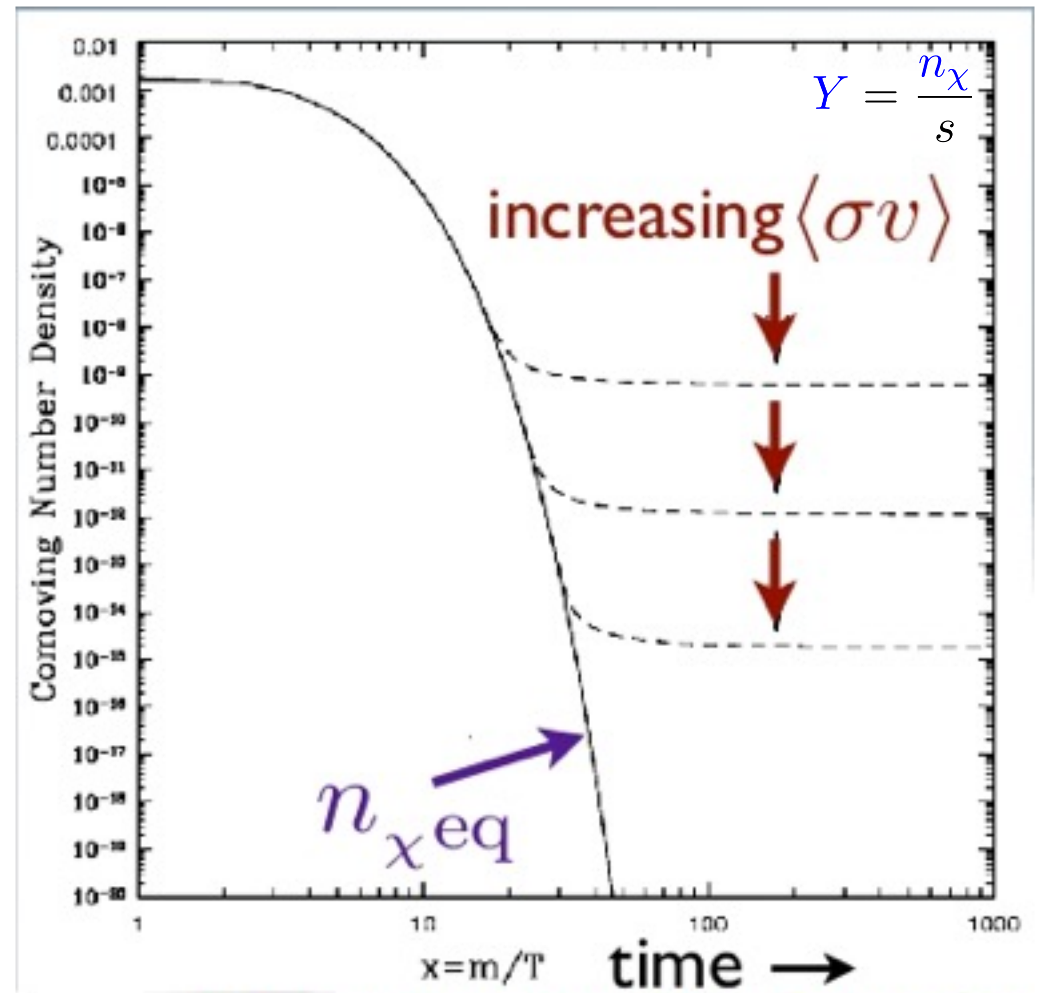


Fig.: Jungman, Kamionkowski & Griest, PR'96

THERMAL RELIC DENSITY
"EXCEPTIONS"

1. Co-annihilations, thresholds and poles

   Griest, Seckel '91

2. Bound State Formation

   recent e.g., Petraki at al. '15, '16; An et al. '15, '16; Cirelli et al. '16; …

3. $3 \rightarrow 2$ and $4 \rightarrow 2$ annihilation

   e.g., D'Agnolo, Ruderman '15; Cline at al. '17; Choi at al. '17; …

   see H. Min Lee talk

4. Semi-annihilation

   D'Eramo, Thaler '10

5. Finite temperature effects

   Wizansky '06; Beneke, Dighera, AH '14, '16

6. Non-standard cosmology

   many works… very recent e.g., D'Eramo, Fernandez, Profumo '17

7. …

Many of these "exceptions" appear for non-minimal scenarios and do have significant impact — but do not affect the foundations of modern calculations

Pitfall in a Nutshell

T ↑

$\Gamma_{\rm ann} > H$ — DM in equilibrium

$\Gamma_{\rm ann} \sim H$ — chemical decoupling

$\Gamma_{\rm ann} < H$ — freeze-out

time ↓

can it happen?

typically

kinetic decoupling

If KD happens before CD ⟶ what would be the relic density?

↓

how to even compute that? ⇒ need for refined treatment of solving the Boltzmann eq.

assuming kinetic equilibrium at chemical decoupling: $f_\chi \sim a(\mu) f_\chi^{\rm eq}$

$$C_{\rm LO} = -\langle \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel}\rangle^{\rm eq} \left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

What are the relevant Rates?

Around freeze-out, typically:

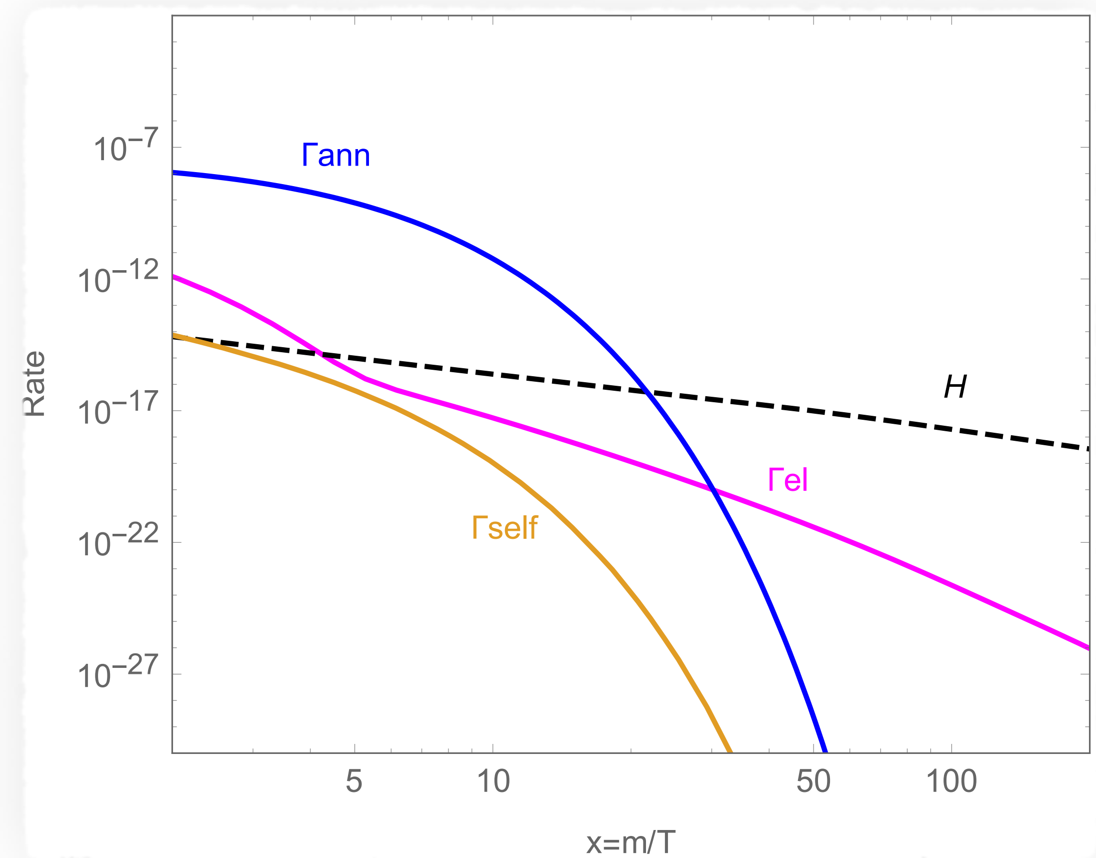
scattering — Hubble — annihilation

$$\Gamma_{\rm el} \gg H \sim \Gamma_{\rm ann}$$

what if

$$H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$$

instead?



- can the kinetic equilibrium be still maintained?
- what can be the size of departure from $f_\chi^{\rm eq}$?
- how does this impact $\langle \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel}\rangle$ and the DM density evolution?

KINETIC DECOUPLING 101

We start from full BE: $E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$

→ contains both scatterings and annihilation

First consider only temperature evolution - i.e. leave out feedback on number density, and *define*:

$$T_\chi \equiv \frac{g_\chi}{3m_\chi n_\chi}\int \frac{\mathrm{d}^3p}{(2\pi)^3} p^2 f_\chi(p) \qquad\qquad y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$$

then 2nd moment of full BE (up to terms p^2/m_χ^2) gives:

$$\frac{y'}{y} = \qquad - \left(1 - \frac{x}{3}\frac{g'_{*S}}{g_{*S}}\right)\frac{2m_\chi c(T)}{Hx}\left(1 - \frac{y_{\rm eq}}{y}\right)$$

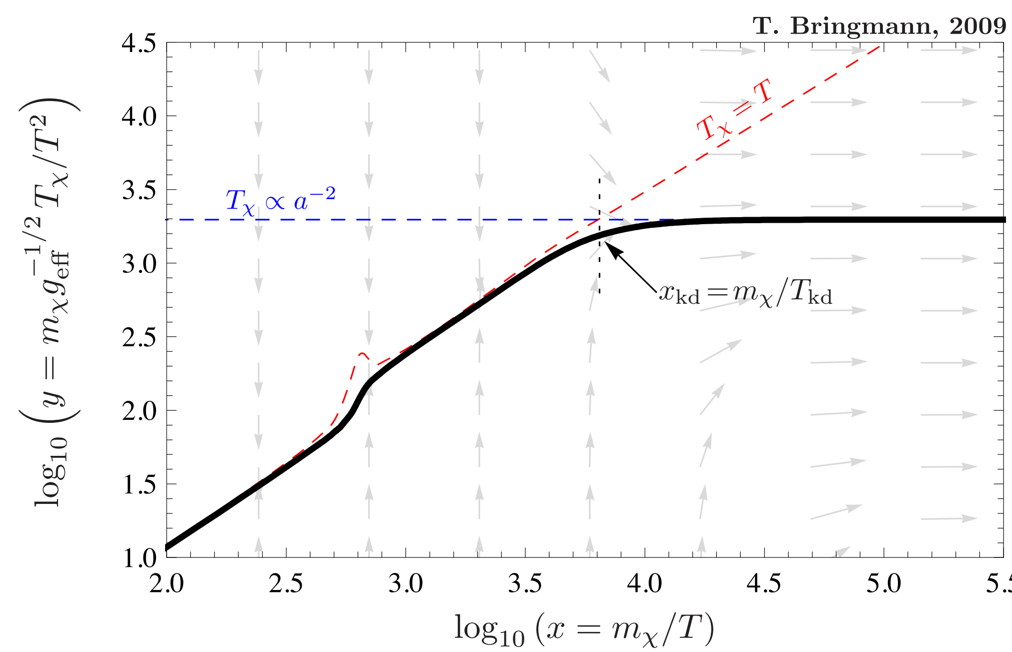
where:

$$\langle\sigma v_{\rm rel}\rangle_2 \equiv \frac{g_\chi^2}{3Tm_\chi n_\chi^2}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3\tilde{p}}{(2\pi)^3}\, p^2 v_{\rm rel}\sigma_{\bar{\chi}\chi\to\bar{X}X} f(E)f(\tilde{E})$$

impact of annihilation

$$c(T) = \frac{1}{12(2\pi)^3 m_\chi^4 T}\sum_X \int dk\, k^5 \omega^{-1} g^{\pm}\left(1\mp g^{\pm}\right)\int_{-4k^2}^{0}(-t)\frac{1}{8k^4}|\mathcal{M}_{\rm el}|^2$$

impact of elastic scatterings



Bringmann, Hofmann JCAP 0704 (2007) 016

ONE STEP FURTHER…

The full evolution of DM temperature and number density is governed by a coupled system of BEs for 0th and 2nd moments:

annihilation and production thermal averages done at different T — feedback of modified y evolution

see also talk by M. Duch

$$\frac{Y'}{Y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx} sY \left(\langle\sigma v_{\rm rel}\rangle|_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\rm eq}^2}{Y^2}\langle\sigma v_{\rm rel}\rangle|_x \right)$$

$$\frac{y'}{y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx}\left[2m_\chi c(T)\left(1-\frac{y_{\rm eq}}{y}\right) - sY\left(\left(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\right)_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\rm eq}^2}{Y^2}\left(\langle\sigma v_{\rm rel}\rangle - \frac{y_{\rm eq}}{y}\langle\sigma v_{\rm rel}\rangle_2\right)_x \right)\right]$$

$$-\frac{4H}{3\sqrt{\pi}x\tilde{H}}\sum_{m=1}^{\infty}(-1)^m\Gamma\left(m+\frac{5}{2}\right)\left(\frac{2}{x}\right)^m\frac{K_{m+2}(x)}{K_2(x)}$$

"relativistic" term

elastic scatterings term

impact of annihilation

$$T_\chi \equiv \frac{g_\chi}{3n_\chi}\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{E}f_\chi(p)$$

These equations still assume the equilibrium shape of $f_\chi(p)$ — but with variant temperature

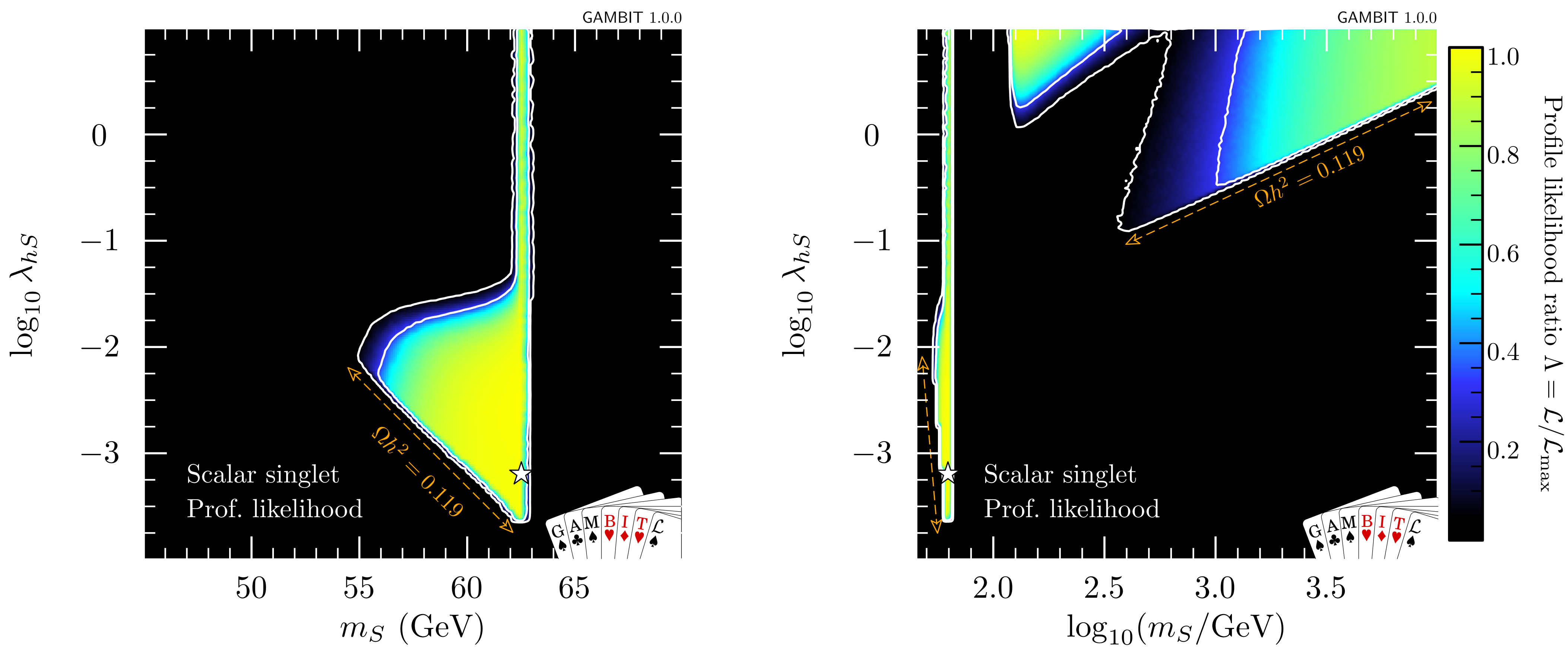
Example:
Scalar Signlet DM

SCALAR SINGLET DM
VERY SHORT INTRODUCTION

To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2$$

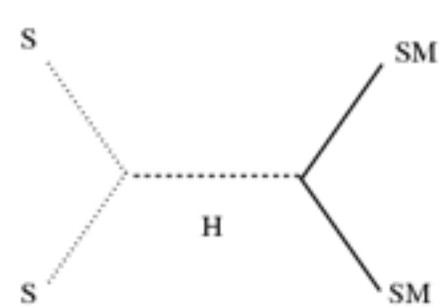
$$m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$



GAMBIT collaboration 1705.07931

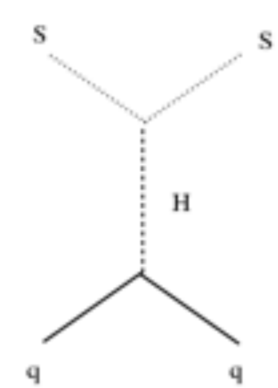
Annihilation processes:

resonant



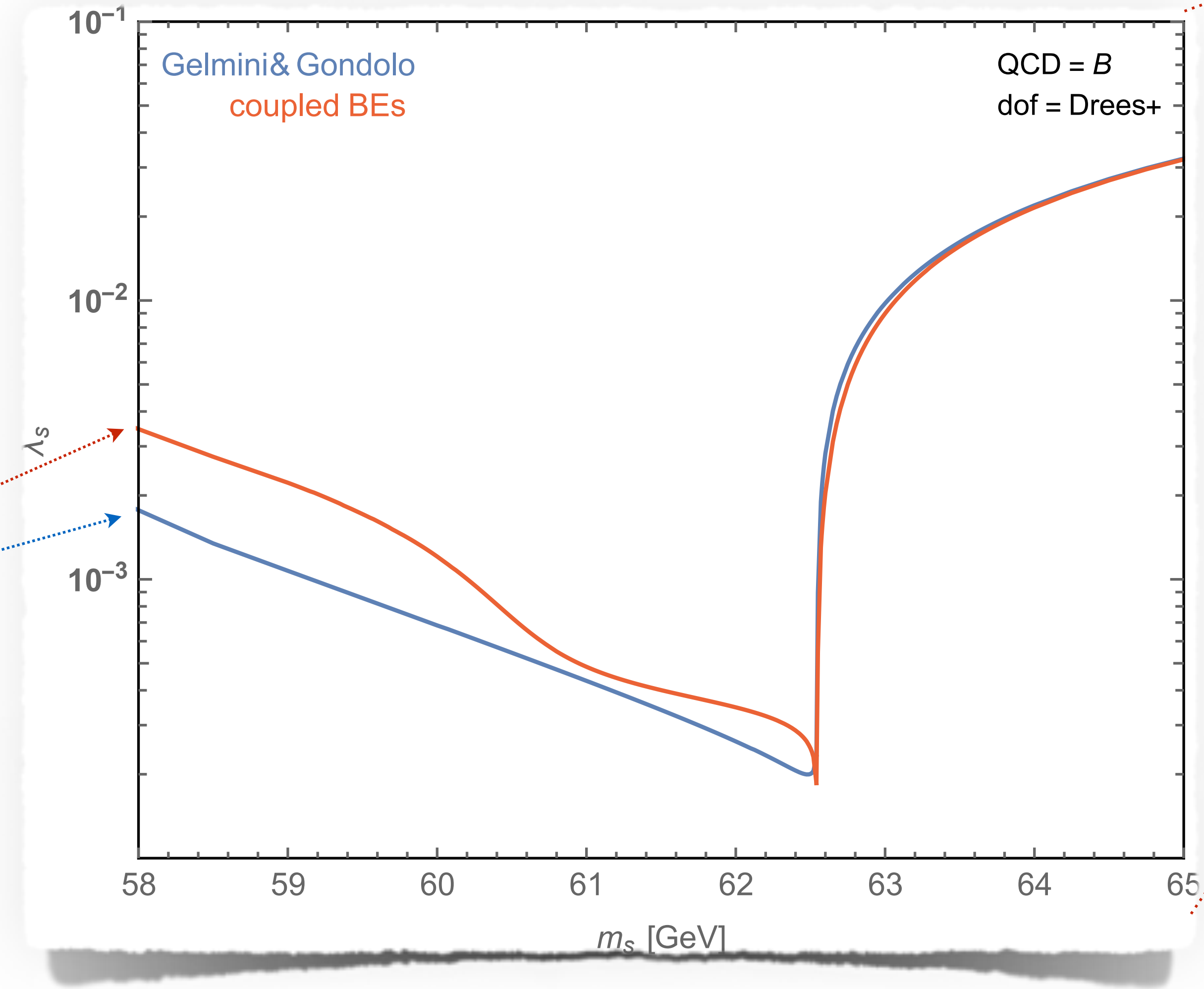
El. scattering processes:

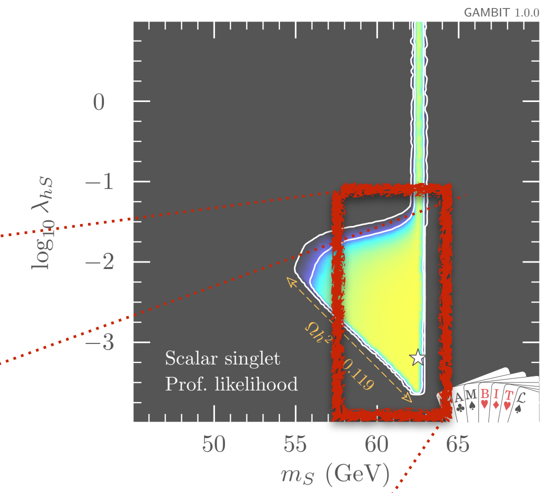
non-resonant



Results

RD contours





the contours converge for lower masses, when resonance is not relevant

essentially the only region left for this model

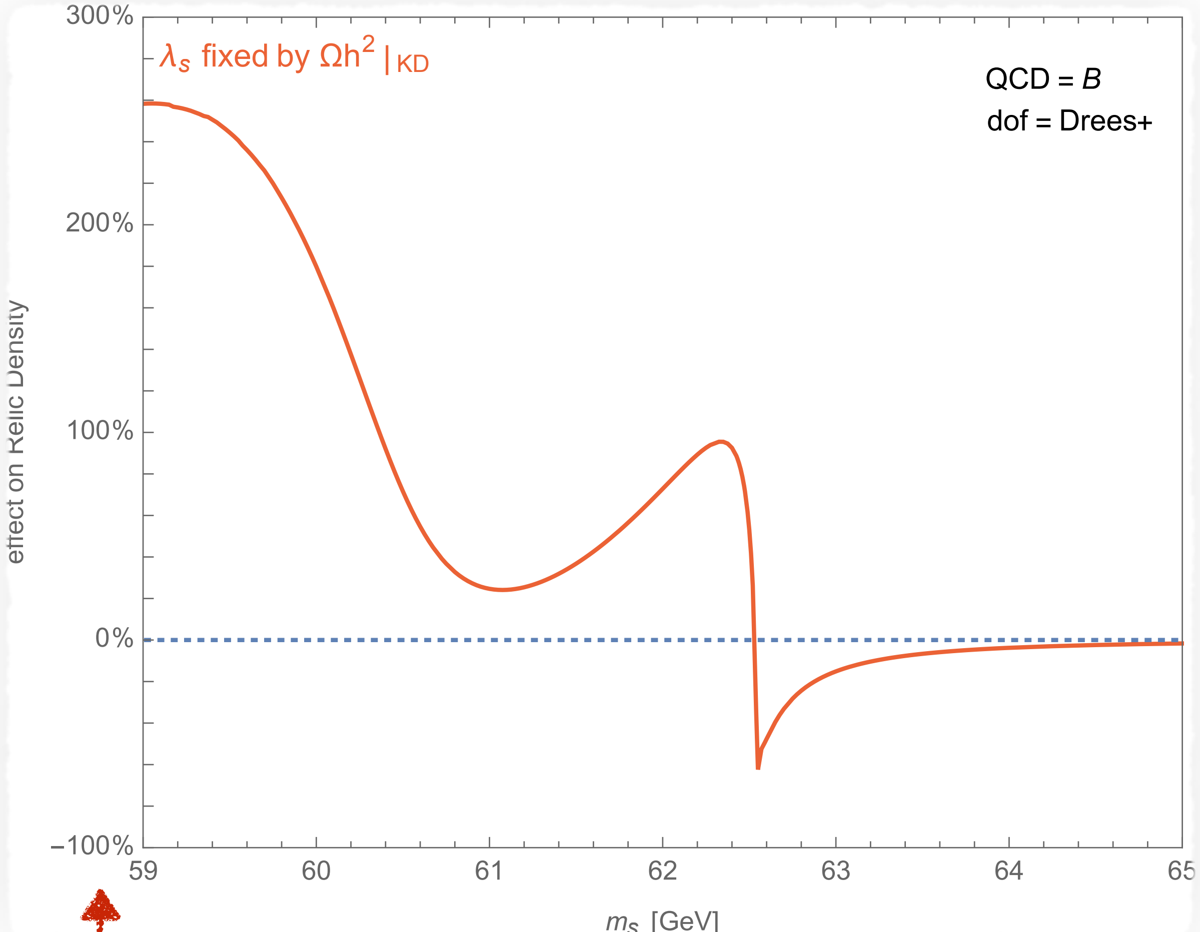
Significant <u>modification</u> of the observed relic density contour in the Scalar Singlet DM model

→ larger coupling needed → better chance for closing the last window

Results

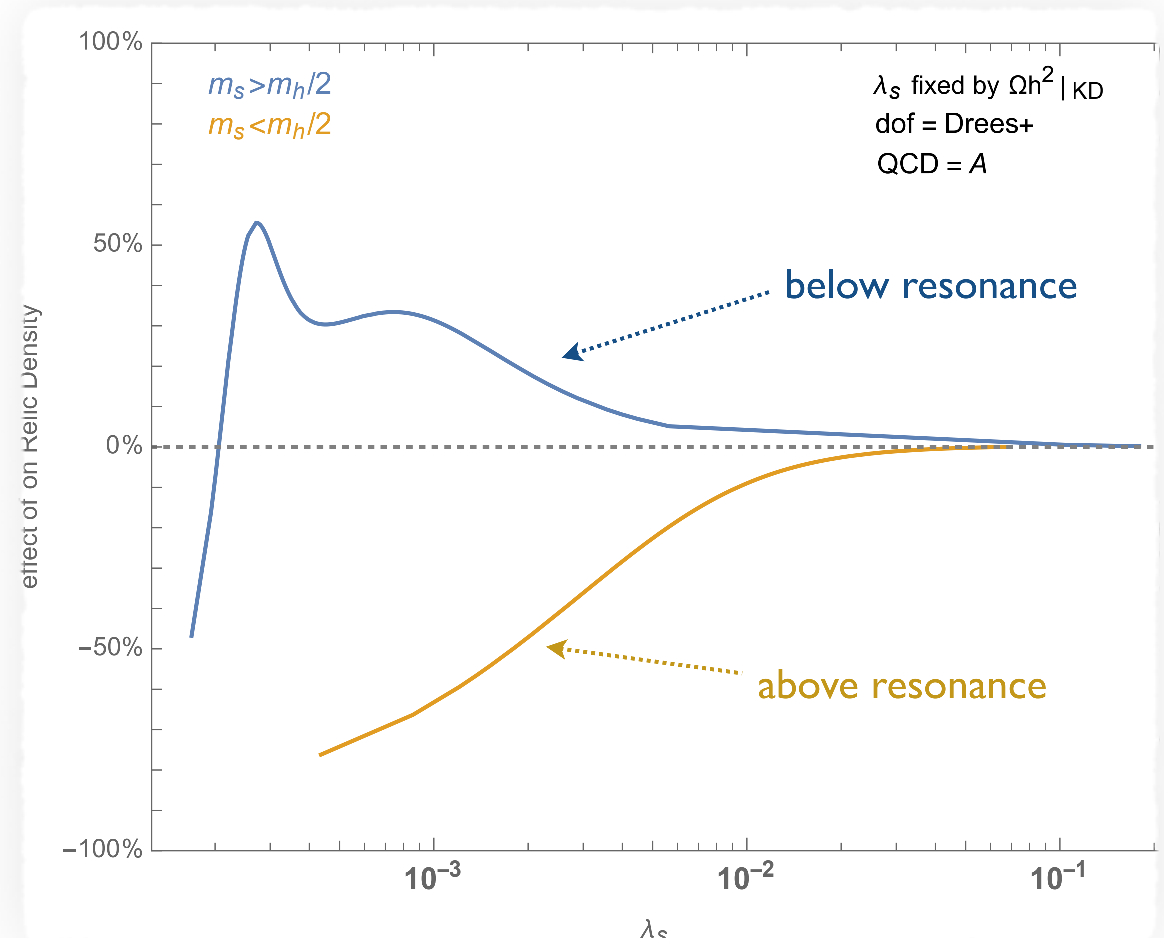
% Effect

scatterings only on **light** q's + leptons:



mass

scatterings on **all** q's + leptons:



coupling

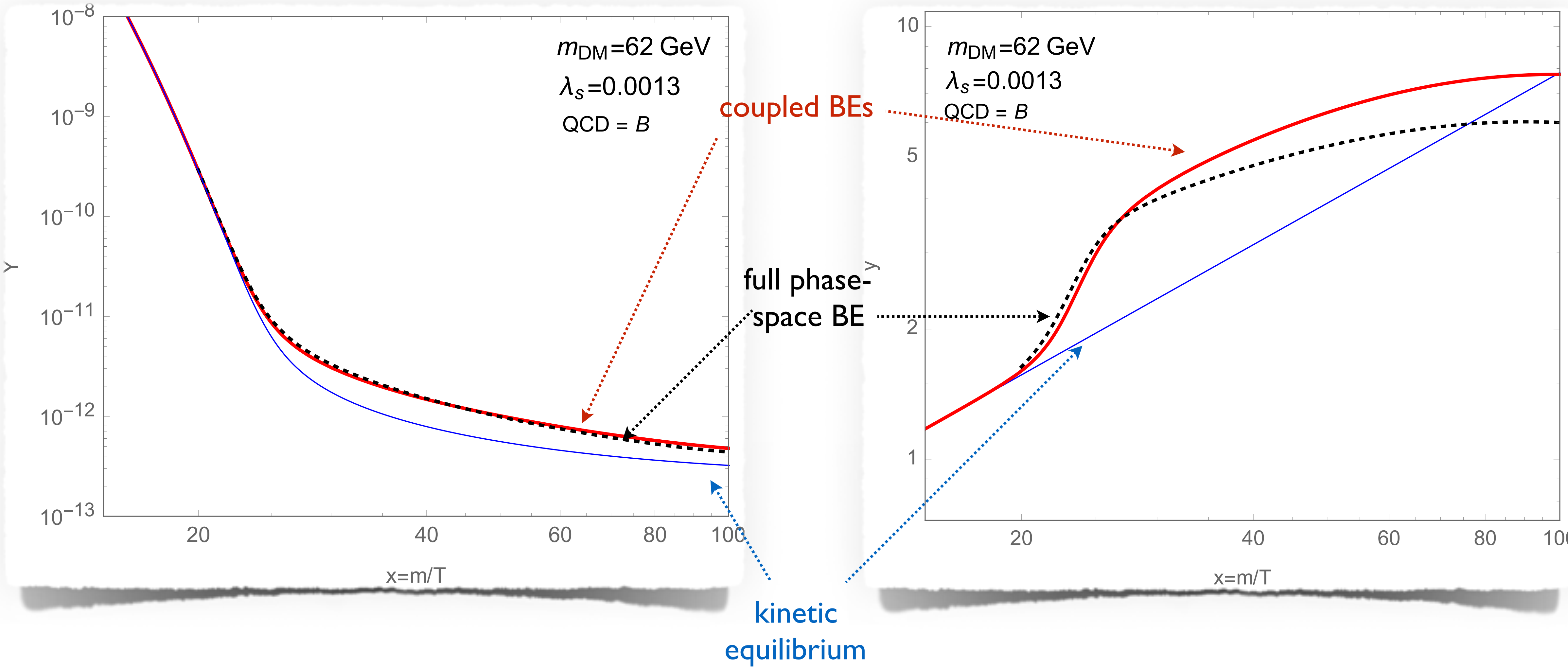
effect on relic density:
up to O(~2)

Why such non-trivial shape of the effect of early kinetic decoupling?

→ Let's inspect the *y* and *Y* evolution…

Density and T_{DM} evolution

for m_{DM} = 62 GeV, i.e. just below the resonance:

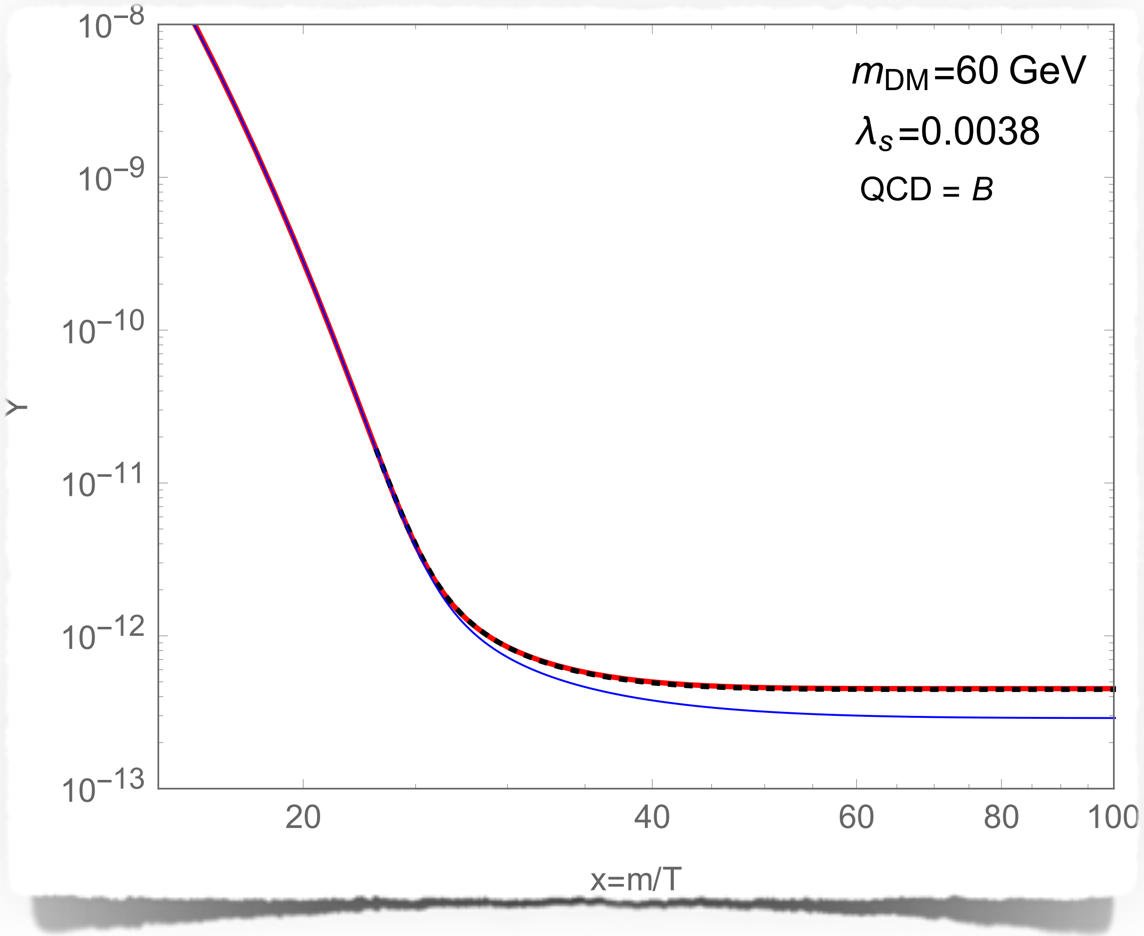


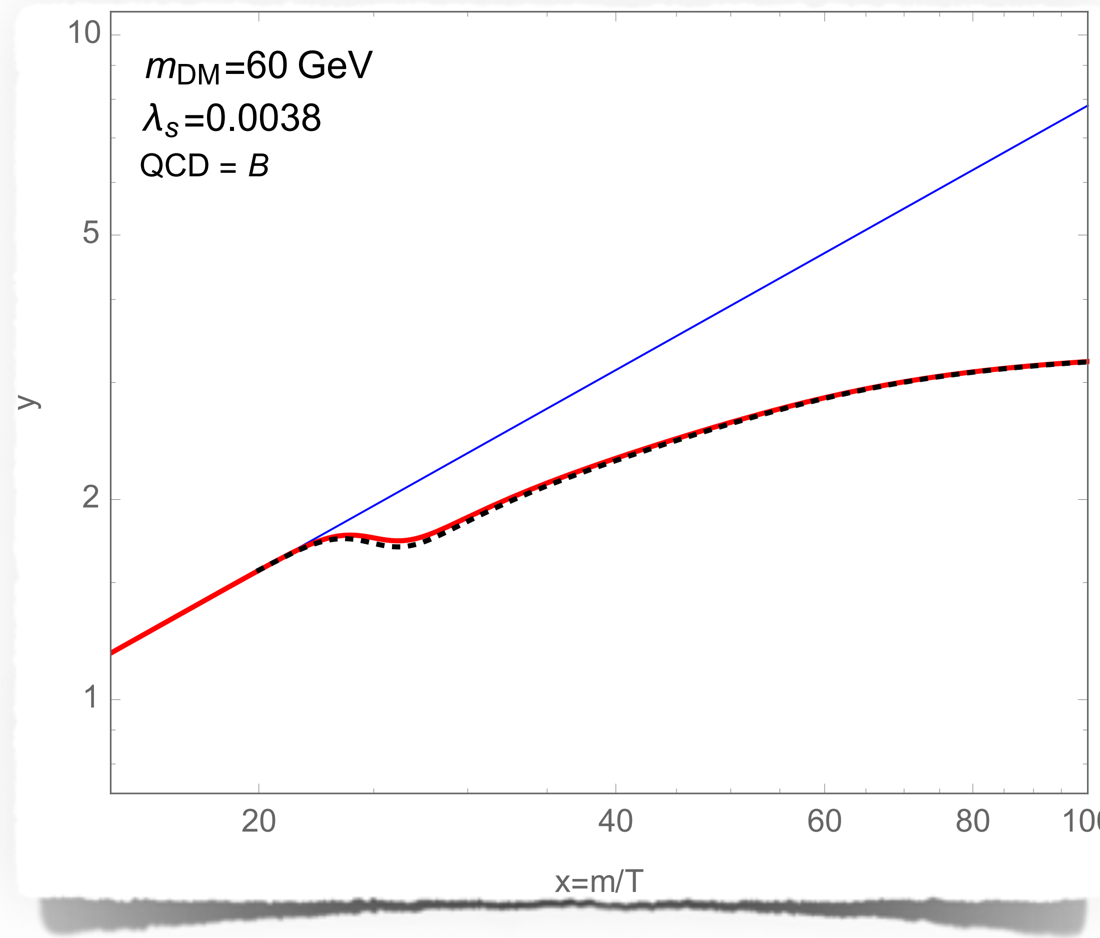
Resonant annihilation **most effective** for **low momenta**

⟶ DM fluid goes through "heating" phase before leaves kinetic equilibrium

DENSITY AND T_{DM} EVOLUTION

for m_{DM} = 60 GeV, i.e. <u>further away</u> from the resonance:





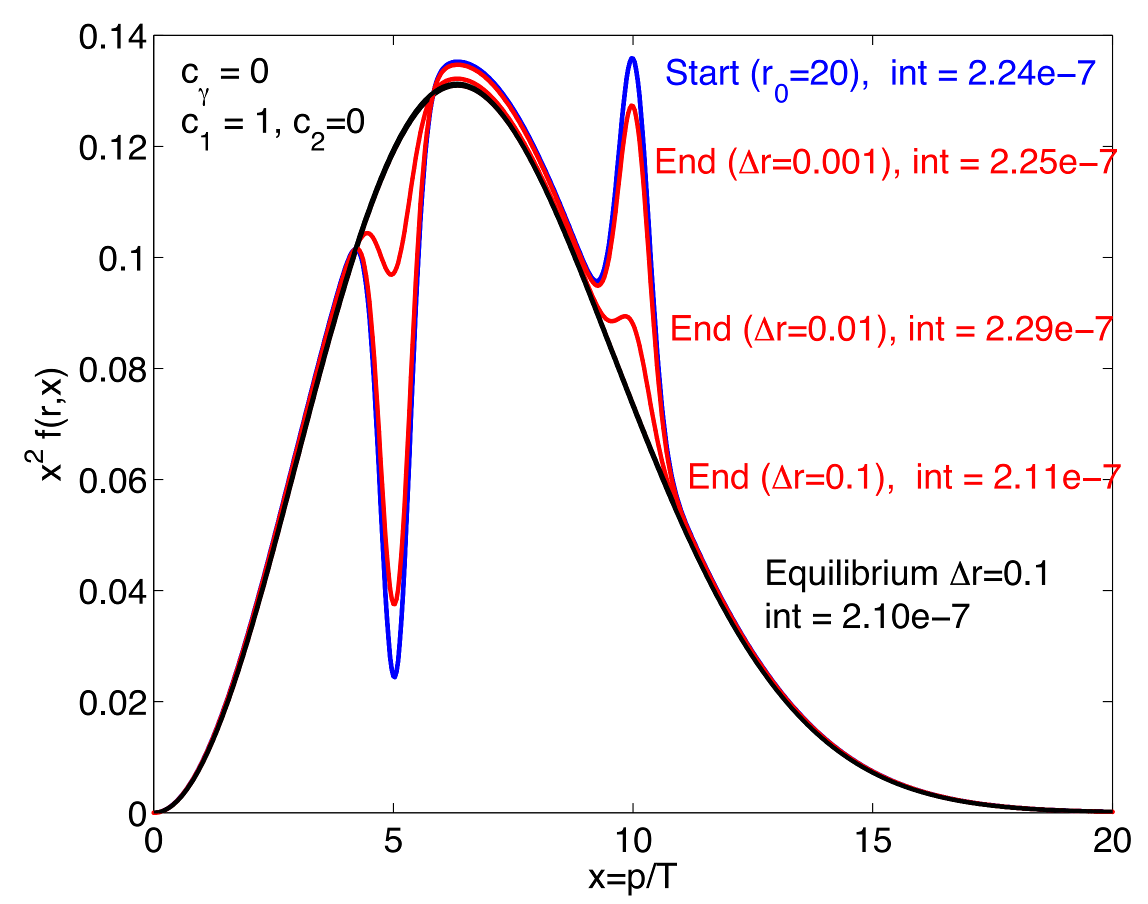
Resonant annihilation **most effective** for **high momenta**

→ DM fluid goes through fast "cooling" phase
after that when T_{DM} drops to much annihilation not effective anymore

Full phase-space BE solver

In order to check the assumption of equilibrium shape, we developed a code numerically solving full phase-space BE

example of how it deals with local disturbances →



The numerical approach based on discretization in momentum and solving system of coupled differential equations

Allows to study the evolution of $f_\chi(p)$ and the interplay between scatterings and annihilation!

CONLCUSIONS

1. One needs to remember that kinetic equilibrium is a necessary assumption for standard relic density calculations

2. Coupled system of Boltzmann equations for 0th and 2nd moments allow for a very accurate treatment of the kinetic decoupling and its effect on relic density

3. In special cases the full phase space Boltzmann equation can be necessary — especially if one wants to trace DM temperature as well