Multi-component dark matter scenarii: vector and fermion case

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Why multi-component dark matter scenarios?

Standard practice in dark matter model building

- A stabilizing (discrete) symmetry in the dark sector, e.g. $\mathbb{Z}_2$.
- Most models have one particle which is charged under this stabilizing symmetry and hence is dark matter candidate.
- If many particle are charged under this stabilizing symmetry then the 'lightest stable particle' (LSP) is dark matter candidate.

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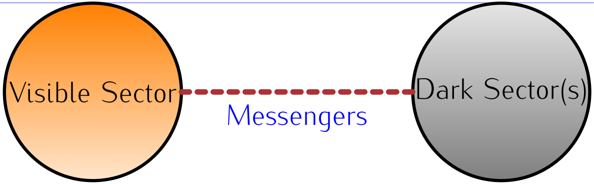
- If many particle are charged under this stabilizing symmetry then the 'lightest stable particle' (LSP) is dark matter candidate.

Going beyond the single component dark matter



- It is plausible to have dark sector(s) with many stable particles.
- What properties to expect from multi-component dark matter?

MCDM: a generic example with $\mathbb{Z}_2 \times \mathbb{Z}_2'$

- Let us consider a minimal example of MCDM with a $\mathbb{Z}_2 \times \mathbb{Z}_2'$ discrete symmetry involving χ, $\tilde{\chi}$, $\tilde{\phi}$ (dark-sector) and ϕ (messenger) .

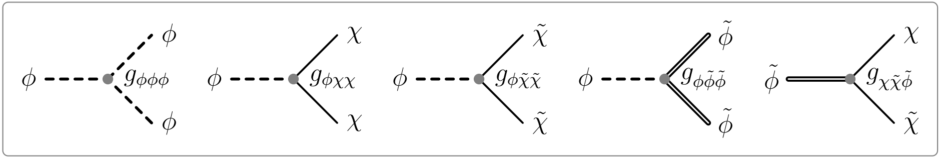
Symmetry	χ	$\tilde{\chi}$	$\tilde{\phi}$	ϕ
$\mathbb{Z}_2$	$+$	$-$	$-$	$+$
$\mathbb{Z}_2'$	$-$	$+$	$-$	$+$

MCDM: a generic example with $\mathbb{Z}_2 \times \mathbb{Z}'_2$

- Let us consider a minimal example of MCDM with a $\mathbb{Z}_2 \times \mathbb{Z}'_2$ discrete symmetry involving χ, $\tilde{\chi}$, $\tilde{\phi}$ (dark-sector) and ϕ (messenger) .

Symmetry	χ	$\tilde{\chi}$	$\tilde{\phi}$	ϕ
$\mathbb{Z}_2$	$+$	$-$	$-$	$+$
$\mathbb{Z}'_2$	$-$	$+$	$-$	$+$

- Possible couplings allowed under a $\mathbb{Z}_2 \times \mathbb{Z}'_2$ symmetry



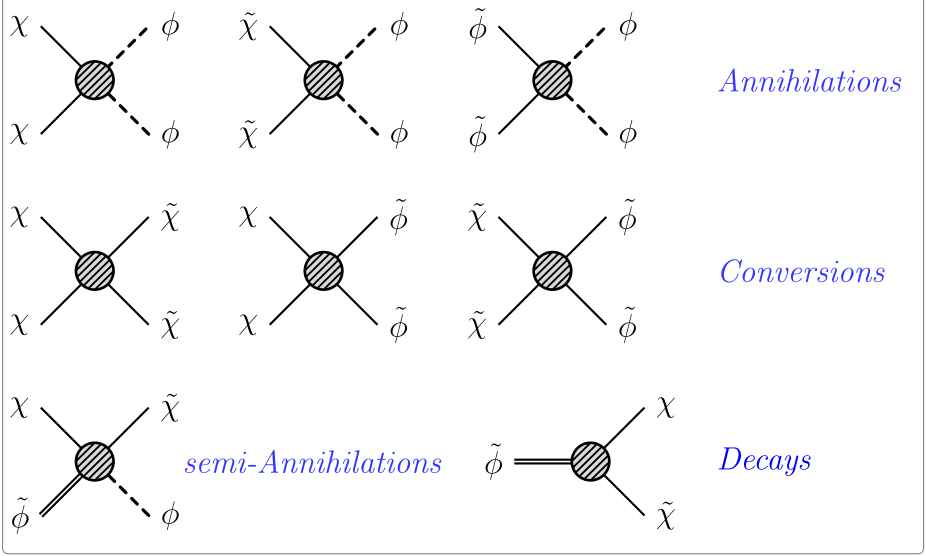
where we assume

$$\alpha \equiv \frac{g_{\phi\phi\phi}}{g_*} = \frac{g_{\phi\chi\chi}}{g_*} = \frac{g_{\phi\tilde{\chi}\tilde{\chi}}}{g_*}, \qquad \beta \equiv \frac{g_{\phi\tilde{\phi}\tilde{\phi}}}{g_*}, \qquad \xi \equiv \frac{g_{\chi\tilde{\chi}\tilde{\phi}}}{g_*}$$

where g_* is electroweak coupling.

MCDM: a generic example

- Four classes of processes happen among χ, $\tilde{\chi}$, $\tilde{\phi}$ (dark-sector) and ϕ.



MCDM: Boltzmann equations for minimal example

$$\frac{dn_\chi}{dt} = -3Hn_\chi - \langle\sigma^{\chi\chi\phi\phi}v_{\text{Møl}}\rangle\left(n_\chi^2 - \bar{n}_\chi^2\right)$$ annihilation

$$- \left[\langle\sigma^{\chi\chi\tilde{\chi}\tilde{\chi}}v_{\text{Møl}}\rangle\left(n_\chi^2 - n_{\tilde{\chi}}^2\frac{\bar{n}_\chi^2}{\bar{n}_{\tilde{\chi}}^2}\right) + \{\tilde{\chi}\to\tilde{\phi}\}\right]$$ conversion

$$- \left[\langle\sigma^{\chi\tilde{\phi}\tilde{\chi}\phi}v_{\text{Møl}}\rangle\left(n_\chi n_{\tilde{\phi}} - \bar{n}_\chi\bar{n}_{\tilde{\phi}}\frac{n_{\tilde{\chi}}}{\bar{n}_{\tilde{\chi}}}\right) + \{\tilde{\phi}\leftrightarrow\tilde{\chi}\}\right]$$ semi-annihilation

$$+ \Gamma_{\tilde{\phi}\to\chi\tilde{\chi}}\left(n_{\tilde{\phi}} - \bar{n}_{\tilde{\phi}}\frac{n_\chi}{\bar{n}_\chi}\frac{n_{\tilde{\chi}}}{\bar{n}_{\tilde{\chi}}}\right),$$ semi-decay

$$\frac{dn_{\tilde{\chi}}}{dt} = \frac{dn_\chi}{dt}[\chi\leftrightarrow\tilde{\chi}],$$

$$\frac{dn_{\tilde{\phi}}}{dt} = -3Hn_{\tilde{\phi}} - \langle\sigma^{\tilde{\phi}\tilde{\phi}\phi\phi}v_{\text{Møl}}\rangle\left(n_{\tilde{\chi}}^2 - \bar{n}_{\tilde{\chi}}^2\right)$$ annihilation

$$- \left[\langle\sigma^{\tilde{\phi}\tilde{\phi}\chi\chi}v_{\text{Møl}}\rangle\left(n_{\tilde{\phi}}^2 - n_\chi^2\frac{\bar{n}_{\tilde{\phi}}^2}{\bar{n}_\chi^2}\right) + \{\chi\leftrightarrow\tilde{\chi}\}\right]$$ conversion

$$- \left[\langle\sigma^{\tilde{\chi}\tilde{\phi}\chi\phi}v_{\text{Møl}}\rangle\left(n_{\tilde{\chi}} n_{\tilde{\phi}} - \bar{n}_{\tilde{\chi}}\bar{n}_{\tilde{\phi}}\frac{n_\chi}{\bar{n}_\chi}\right) + \{\chi\leftrightarrow\tilde{\chi}\}\right]$$ semi-annihilation

$$- \Gamma_{\tilde{\phi}\to\chi\tilde{\chi}}\left(n_{\tilde{\phi}} - \bar{n}_{\tilde{\phi}}\frac{n_\chi}{\bar{n}_\chi}\frac{n_{\tilde{\chi}}}{\bar{n}_{\tilde{\chi}}}\right)$$ semi-decay

Solving the Boltzmann Eqs. for minimal example

- Thermally averaged cross sections of the order of the electroweak scale

$$\langle \sigma^{abcd} v_{\text{Møl}} \rangle \approx \underbrace{\frac{G_F^2}{2\pi} m^2}_{\sim 10^{-11}\ \text{GeV}^{-2}} \mathcal{M}^2_{abcd}(\alpha, \beta, \xi),$$

where m is the DM mass of order electroweak scale ~ 100 GeV.

- $\mathcal{M}_{abcd}(\alpha, \beta, \xi)$ are rescaled dimensionless matrix element

$$\begin{aligned}
\mathcal{M}_{\chi\chi\phi\phi} \sim \mathcal{M}_{\tilde{\chi}\tilde{\chi}\phi\phi} &\propto \alpha^2, && \chi,\ \tilde{\chi}\ \text{annihilation}\\
\mathcal{M}_{\tilde{\phi}\tilde{\phi}\phi\phi} &\propto (\alpha+\beta)\beta, && \tilde{\phi}\ \text{annihilation}\\
\mathcal{M}_{\chi\chi\tilde{\chi}\tilde{\chi}} \sim \mathcal{M}_{\tilde{\chi}\tilde{\chi}\chi\chi} &\propto (\alpha^2+\xi^2), && \text{conversion}\\
\mathcal{M}_{\tilde{\phi}\tilde{\phi}\chi\chi} \sim \mathcal{M}_{\tilde{\phi}\tilde{\phi}\tilde{\chi}\tilde{\chi}} &\propto (\alpha\beta+\xi^2), && \text{conversion}\\
\mathcal{M}_{\chi\tilde{\phi}\tilde{\chi}\phi} \sim \mathcal{M}_{\tilde{\chi}\tilde{\phi}\chi\phi} \sim \mathcal{M}_{\chi\tilde{\chi}\tilde{\phi}\phi} &\propto (\alpha+\beta)\xi, && \text{semi-annihilation}
\end{aligned}$$

- Decay width is $\Gamma_{\tilde{\phi}\to\chi\tilde{\chi}} \sim \xi^2 \times \mathcal{O}(1)$ GeV when kinematically allowed.

Solving the Boltzmann Eqs. for minimal example

- Let us imagine there is a very weak or no direct coupling between $\tilde{\phi}$ and visible sector ϕ, i.e $\beta \equiv \frac{g_{\phi\tilde{\phi}\tilde{\phi}}}{g_*} \approx 0$.
- This implies no direct annihilation of $\tilde{\phi}$

$$\mathcal{M}_{\tilde{\phi}\tilde{\phi}\phi\phi} \propto (\alpha + \beta)\beta \approx 0$$

$\tilde{\phi}$ annihilation

- However, $\tilde{\phi}$ annihilates through semi-annihilation processes

$$\mathcal{M}_{\chi\tilde{\phi}\tilde{\chi}\phi} \sim \mathcal{M}_{\tilde{\chi}\tilde{\phi}\chi\phi} \sim \mathcal{M}_{\chi\tilde{\chi}\tilde{\phi}\phi} \propto (\alpha + \beta)\xi$$

semi-annihilation

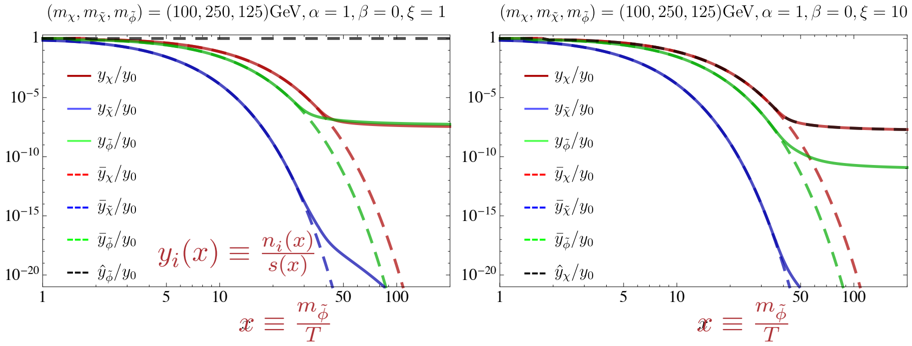
- Also it could decay $\Gamma_{\tilde{\phi}\to\chi\tilde{\chi}} \sim \xi^2$ GeV when kinematically allowed.
- Conversion processes with keep equilibrium within the dark sector

$$\mathcal{M}_{\tilde{\phi}\tilde{\phi}\chi\chi} \sim \mathcal{M}_{\tilde{\phi}\tilde{\phi}\tilde{\chi}\tilde{\chi}} \propto (\alpha\beta + \xi^2)$$

conversion

Two-component dark matter case

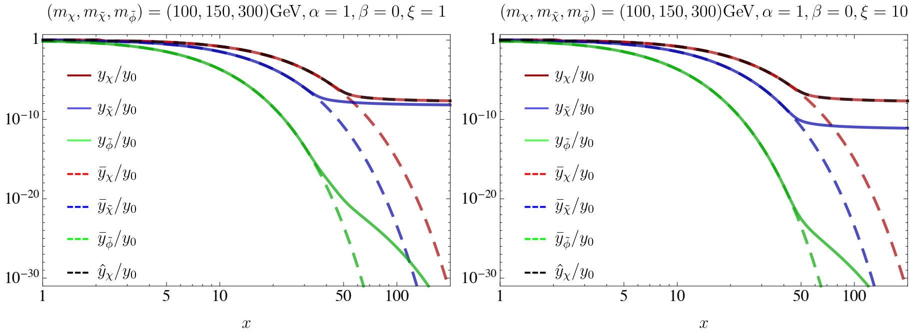
$m_\chi = 100$ GeV, $m_{\tilde{\chi}} = 250$ GeV and $m_{\tilde{\phi}} = 125$ GeV



- Note for $\beta = 0$ there is no direct annihilation of the $\tilde{\phi}\tilde{\phi}$ to visible sector.
- The only way the $\tilde{\phi}$ annihilates into the visible sector is through the semi-annihilation processes of the type $\tilde{\phi}\tilde{\chi} \leftrightarrow \chi\phi$.
- Hence when any of the two remaining state χ or $\tilde{\chi}$ decouples and goes out of the equilibrium, then the $\tilde{\phi}$ also decouples.

Two-component dark matter

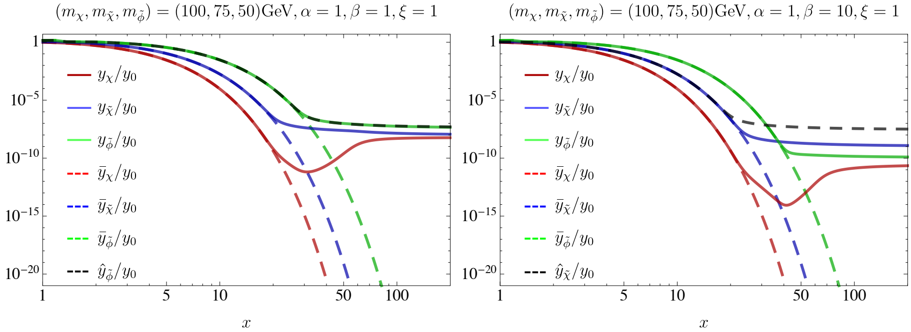
$m_\chi = 100$ GeV, $m_{\tilde{\chi}} = 150$ GeV and $m_{\tilde{\phi}} = 300$ GeV



- For of this point $m_{\tilde{\phi}} > m_\chi + m_{\tilde{\chi}}$, the decay $\Gamma_{\tilde{\phi}\to\chi\tilde{\chi}}$ is allowed.
- Note the contribution of $\tilde{\phi}$ to the relic is negligible however its presence is important for the dominant components through $\xi \sim g_{\chi\tilde{\chi}\tilde{\phi}}$ interactions.

Three-component dark matter case

$m_\chi = 100$ GeV, $m_{\tilde{\chi}} = 75$ GeV and $m_{\tilde{\phi}} = 50$ GeV



- There are parameter regions where all three stable particles contribute almost equally to the dark matter relic density.
- Note that conversion processes tend to bring equilibrium within the dark sector.

Take-home message from our generic MCDM

A minimal example

- We consider a $\mathbb{Z}_2 \times \mathbb{Z}_2'$ discrete symmetry such that two or three states in the dark sector $[\chi(-,+)$, $\tilde{\chi}(+,-)$ and $\tilde{\phi}(-,-)]$ are stable.

Take-home message from our generic MCDM

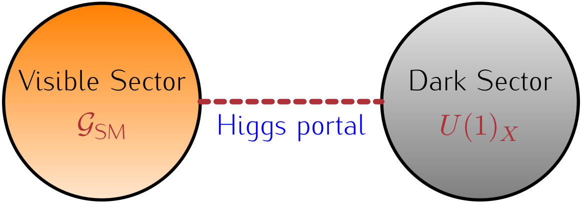
A minimal example

- We consider a $\mathbb{Z}_2 \times \mathbb{Z}'_2$ discrete symmetry such that two or three states in the dark sector $[\chi(-,+)$, $\tilde{\chi}(+,-)$ and $\tilde{\phi}(-,-)]$ are stable.

Take-home message

- When for a given dark matter species the standard annihilation channel is suppressed then its abundance might be very sensitive to the presence of other ingredients of the dark sector, even if their direct contributions to the total abundance are negligible.

VFDM: a vector and fermion dark matter



- Gauge symmetry: $\underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\mathcal{G}_{\text{SM}}} \times U(1)_X$
- Visible sector (SM) fields are not charged under $U(1)_X$.
- Matter content in the dark sector:
  a complex scalar S, a Dirac fermion χ and a gauge boson X_μ
- Charges under gauge symmetry: $S : (1, 1, 0, 2)$, $\quad \chi : (1, 1, 0, 1)$
- Higgs mechanism in the dark sector: $\langle S \rangle$ generates a mass for X_μ.

VFDM: a vector and fermion dark matter

- Lagrangian for our VFDM model:

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DS}} + \mathcal{L}_{\text{int}}}$$

- $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian.
- Dark sector Lagrangian is

$$\mathcal{L}_{\text{DS}} = -\frac{1}{4}\mathcal{F}^{X}_{\mu\nu}\mathcal{F}^{\mu\nu}_{X} + (\mathcal{D}_\mu S)^* \mathcal{D}^\mu S + \mu_S^2 |S|^2 - \lambda_S |S|^4$$
$$+ \bar{\chi}(i\not{\mathcal{D}} - m_D)\chi - \frac{1}{\sqrt{2}}(y S^* \chi^{\intercal} \mathcal{C} \chi + \text{h.c.}),$$

  where $\mathcal{D}_\mu = \partial_\mu + i g_X X_\mu$ with gauge coupling g_X.
- Interaction Lagrangian (Higgs portal):

$$\boxed{\mathcal{L}_{\text{int}} = -\kappa |S|^2 |H|^2}$$

- Note that the dark-sector has a charge conjugate symmetry $\mathcal{C}$:

$$\boxed{X_\mu \xrightarrow{\mathcal{C}} -X_\mu, \qquad S \xrightarrow{\mathcal{C}} S^*, \qquad \chi \xrightarrow{\mathcal{C}} \chi^c \equiv -i\gamma_2 \chi^*}$$

Higgs portal

- Scalar potential for our model:

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

- Gauge symmetry is broken spontaneously by the VEVs of Higgs doublet $H^\intercal = (0, v+h)/\sqrt{2}$ and scalar $S = (v_X + \phi)/\sqrt{2}$.

- Physical scalars h and ϕ mix and are diagonalized by the orthogonal rotational matrix $\mathcal{R}$:

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \mathcal{R}^{-1} \begin{pmatrix} h \\ \phi \end{pmatrix}, \quad \text{where} \quad \mathcal{R} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix},$$

  with masses $(m_{h_1}^2, m_{h_2}^2)$.

- The scalar h_1 as the SM-like Higgs with mass $m_{h_1} = 125$ GeV.

Fermion mixing and Majorana states

- After the SSB the dark sector Dirac fermion χ acquires mass yv_X through the Yukawa interaction along with its Dirac mass m_D.
- We diagonalize the Dirac fermion into mass eingenbases by defining the following two self-conjugate (Majorana) states:

$$\psi_+ \equiv \frac{1}{\sqrt{2}}(\chi + \chi^c), \qquad \psi_- \equiv \frac{1}{i\sqrt{2}}(\chi - \chi^c)$$

with the mass eigenvalues: $m_\pm = m_D \pm yv_X$.

Dark sector interactions and discrete symmetries

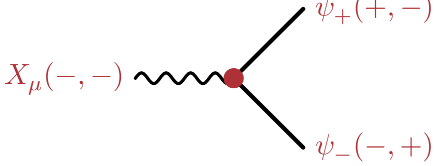
- Dark sector interaction Lagrangian:

$$\mathcal{L}_{\text{dark-int}} = \sqrt{2} v_X g_X^2 X_\mu X^\mu \phi + g_X^2 X_\mu X^\mu \phi^2 - \frac{i}{2} g_X \left(\bar{\psi}_+ \gamma^\mu \psi_- + \bar{\psi}_- \gamma^\mu \psi_+\right) X_\mu - \frac{y}{2} \left(\bar{\psi}_+ \psi_+ + \bar{\psi}_- \psi_-\right) \phi$$

Discrete symmetries: $\mathbb{Z}_2 \times \mathbb{Z}_2'$

Symmetry	X_μ	ψ_+	ψ_-	ϕ
$\mathbb{Z}_2$	$-$	$+$	$-$	$+$
$\mathbb{Z}_2'$	$-$	$-$	$+$	$+$

- Most important interaction under $\mathbb{Z}_2 \times \mathbb{Z}_2'$:



$$-\frac{i}{2} g_X \left(\bar{\psi}_+ \gamma^\mu \psi_- + \bar{\psi}_- \gamma^\mu \psi_+\right) X_\mu$$

VFDM: solving Boltzmann equations

- After solving the Boltzmann equations we calculate the present relic density of the dark species as,

$$\Omega_i h^2 = \frac{h^2 s_0}{\rho_{\text{cr}}} m_i Y_i = 2.742 \times 10^8 \left(\frac{m_i}{\text{GeV}}\right) Y_i$$

- s_0 is the total entropy density today, ρ_{cr} is the critical density, m_i is the mass of the dark particle and Y_i is the yield of the dark particle today.

- Total dark matter relic density is a sum of the individual relics, i.e.

$$\Omega_{\text{tot}} h^2 = \sum_i \Omega_i h^2,$$

$$\Omega_{\text{obs}} h^2 = 0.1197 \pm 0.0022 \text{ (PLANCK)}$$

- To calculate matrix element squared we employed `CalcHEP` [arXiv:1207.6082].

- For thermal averaging of cross-sections and solutions of the Boltzmann equations we have developed a dedicated `C++` code.

VFDM: solving Boltzmann equations

- We adopt the following two strategies:

Strategy-A: $y \gg 1 \ (m_+ \gg m_-)$

- In this case one expects fast $\psi_\pm \psi_\pm$ annihilation and so that X_μ may dominate the dark matter abundance.

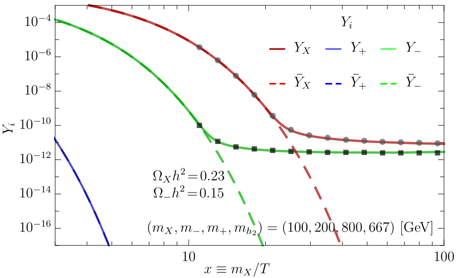
Strategy-B: $y \ll 1 \ (m_+ \simeq m_-)$

- Small y implies suppressed $\psi_\pm \psi_\pm$ annihilation, so $\psi_\pm$ dominates the dark matter abundance.

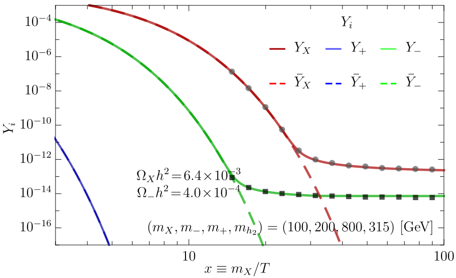
VFDM: Strategy-A

$m_X = 100$ GeV, $m_- = 200$ GeV, and $m_+ = 800$ GeV

$\kappa = 1,\ \sin\alpha = 0.3,\ g_X = 0.2,\ \lambda_H = 0.45,\ \lambda_S = 0.8,\ y = 0.6$



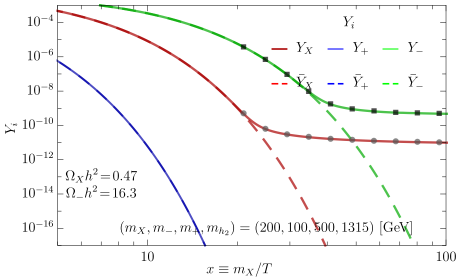
$\kappa = 1,\ \sin\alpha = 0.3,\ g_X = 1,\ \lambda_H = 0.2,\ \lambda_S = 4.7,\ y = 3$

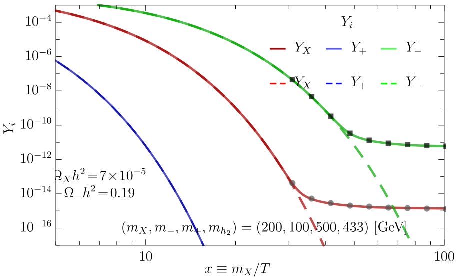


- A 2CDM case: X_μ and ψ_- are stable.
- Comparison with `micrOMEGAs 4.3` is $\mathcal{O}(10\%)$ or less.
- Note that for left (right) plot the coupling g_X is for $0.2(1)$ determines interesting aspects of dynamics of dark matter evolution.
  (in the generic setup g_X corresponds to ξ)

VFDM: Strategy-A

$m_X = 200$ GeV, $m_- = 100$ GeV, and $m_+ = 500$ GeV

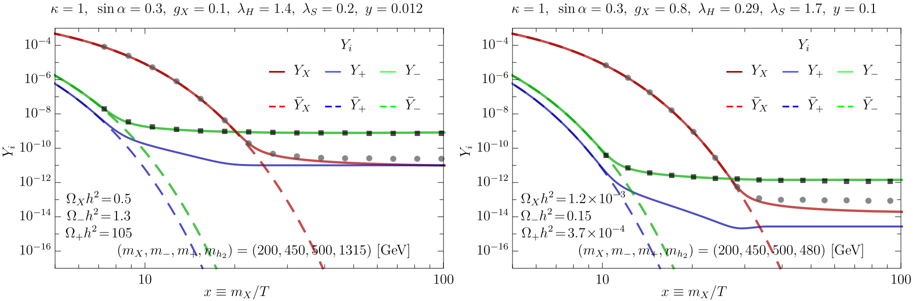




- A 2CDM case: X_μ and ψ_- are stable.
- Comparison with `micrOMEGAs 4.3` is $\mathcal{O}(10\%)$ or less.
- Note that for left (right) plot the coupling g_X is for $0.1(1)$ determines interesting aspects of dynamics of dark matter density evolution (in the generic setup g_X corresponds to $\xi \propto g_{\chi\tilde{\chi}\tilde{\phi}}$)

VFDM: Strategy-B

$m_X = 50$ GeV, $m_{\psi_-} = 40$ GeV, and $m_{\psi_+} = 40.1$ GeV



- A 3CDM case, X_μ, ψ_+ and ψ_- are stable.
- Comparison of the 3CDM with `micrOMEGAs 4.3` with the dominant component is $\mathcal{O}(10\%)$ but with the subdominant component it could be up to few times.

Summary

- We showed that presence of a second/third component could play a very crucial role in the dark matter evolution.
- A dark matter – with very very weak (or zero) coupling with the SM – could constitute the observed relic density through semi-annihilations.
- We consider a simple BSM model where a vector and Majorana fermion(s) are dark matter particles.
- MCDM scenarios have many interesting features and opens up interesting model building possibilities.

Summary

- We showed that presence of a second/third component could play a very crucial role in the dark matter evolution.
- A dark matter – with very very weak (or zero) coupling with the SM – could constitute the observed relic density through semi-annihilations.
- We consider a simple BSM model where a vector and Majorana fermion(s) are dark matter particles.
- MCDM scenarios have many interesting features and opens up interesting model building possibilities.

Thank You!

Backup slides

Theoretical and experimental constraints

- Scalar potential for our model:

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

- Tree-level positivity or stability of scalar potential implies

$$\lambda_H > 0, \qquad \lambda_S > 0, \qquad \kappa > -2\sqrt{\lambda_H\lambda_S}$$

- Perturbativity puts upper bounds

$$\lambda_H < 4\pi, \qquad \lambda_S < 4\pi, \qquad \kappa < 4\pi$$

- Higgs-gauge boson coupling: $\cos\alpha = g_{h_1VV}/g_{h_1VV}^{\text{SM}}$

$$0.85 < \cos\alpha < 1$$

- Invisible Higgs decays put bounds on decay processes

$$h_1 \to XX, \quad h_1 \to h_2h_2, \quad h_1 \to \psi_\pm\psi_\pm$$

Input Parameters

- Free parameters in the Lagrangian are 8:

$$\mu_H, \quad \mu_S, \quad \lambda_H, \quad \lambda_S, \quad \kappa, \quad g_X, \quad m_D, \quad y$$

- After fixing SM Higgs mass and Higgs vev to $m_{h_1} = 125$ GeV and $v_{\text{SM}} = 246$ GeV, we are left with 6 free parameters.
- We trade the remaining 6 parameters with more “physical parameters”, namely:

$$\boxed{m_X, \quad m_+, \quad m_-, \quad g_X, \quad \kappa, \quad \sin\alpha.}$$

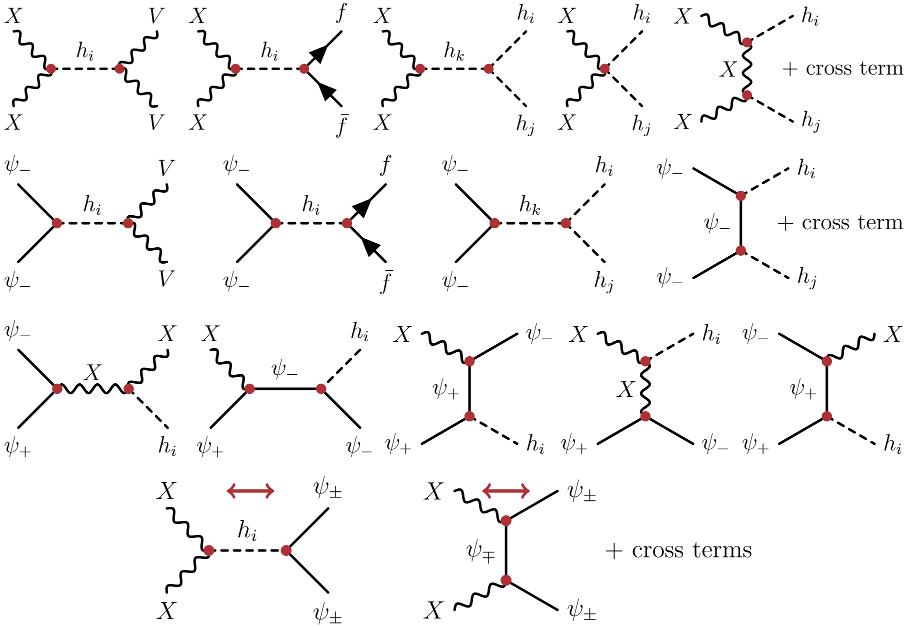
- Lagrangian parameters could be expressed in terms of physical ones as

$$m_{h_2}^2 = m_{h_1}^2 + \frac{2\kappa v_X v}{\sin(2\alpha)}, \qquad v_X = \frac{m_X}{g_X},$$

$$\lambda_H = \frac{m_{h_1}^2 \cos\alpha + \kappa v_X v \sin\alpha}{2v^2 \cos\alpha}, \qquad \lambda_S = \frac{m_{h_1}^2 \sin\alpha + \kappa v_X v \cos\alpha}{2v_X^2 \sin\alpha},$$

$$y = \frac{g_X(m_+ - m_-)}{2m_X}, \qquad m_D = \frac{m_+ + m_-}{2}$$

Annihilation, semi-annihilation and conversion



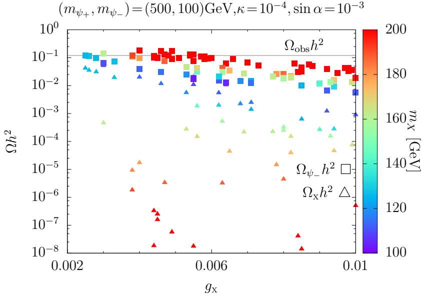
Boltzmann equations for VFDM model

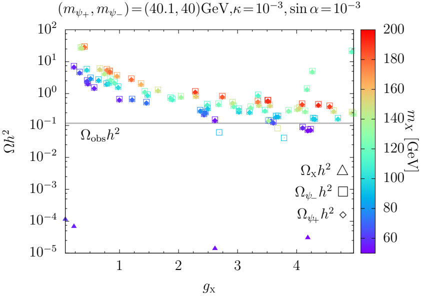
$$\frac{dn_X}{dt} = -3Hn_X - \langle\sigma_{v_{\text{Møl}}}^{XX\phi\phi}\rangle\left(n_X^2 - \bar{n}_X^2\right) - \langle\sigma_{v_{\text{Møl}}}^{X\psi_+\psi_- h_i}\rangle\left(n_X n_{\psi_+} - \bar{n}_X\bar{n}_{\psi_+}\frac{n_{\psi_-}}{\bar{n}_{\psi_-}}\right)$$
$$- \langle\sigma_{v_{\text{Møl}}}^{X\psi_-\psi_+ h_i}\rangle\left(n_X n_{\psi_-} - \bar{n}_X\bar{n}_{\psi_-}\frac{n_{\psi_+}}{\bar{n}_{\psi_+}}\right) - \langle\sigma_{v_{\text{Møl}}}^{Xh_i\psi_+\psi_-}\rangle\bar{n}_{h_i}\left(n_X - \bar{n}_X\frac{n_{\psi_+}n_{\psi_-}}{\bar{n}_{\psi_+}\bar{n}_{\psi_-}}\right)$$
$$- \langle\sigma_{v_{\text{Møl}}}^{XX\psi_+\psi_+}\rangle\left(n_X^2 - \bar{n}_X^2\frac{n_{\psi_+}^2}{\bar{n}_{\psi_+}^2}\right) - \langle\sigma_{v_{\text{Møl}}}^{XX\psi_-\psi_-}\rangle\left(n_X^2 - \bar{n}_X^2\frac{n_{\psi_-}^2}{\bar{n}_{\psi_-}^2}\right)$$
$$+ \Gamma_{\psi_+\to X\psi_-}\left(n_{\psi_+} - \bar{n}_{\psi_+}\frac{n_X}{\bar{n}_X}\frac{n_{\psi_-}}{\bar{n}_{\psi_-}}\right),$$

$$\frac{dn_{\psi_-}}{dt} = -3Hn_{\psi_-} - \langle\sigma_{v_{\text{Møl}}}^{\psi_-\psi_-\phi\phi}\rangle\left(n_{\psi_-}^2 - \bar{n}_{\psi_-}^2\right) - \langle\sigma_{v_{\text{Møl}}}^{\psi_-\psi_+ X h_i}\rangle\left(n_{\psi_-}n_{\psi_+} - \bar{n}_{\psi_-}\bar{n}_{\psi_+}\frac{n_X}{\bar{n}_X}\right)$$
$$- \langle\sigma_{v_{\text{Møl}}}^{X\psi_-\psi_+ h_i}\rangle\left(n_X n_{\psi_-} - \bar{n}_X\bar{n}_{\psi_-}\frac{n_{\psi_+}}{\bar{n}_{\psi_+}}\right) - \langle\sigma_{v_{\text{Møl}}}^{\psi_- h_i X\psi_+}\rangle\bar{n}_{h_i}\left(n_{\psi_-} - \bar{n}_{\psi_-}\frac{n_{\psi_+}}{\bar{n}_{\psi_+}}\frac{n_X}{\bar{n}_X}\right)$$
$$- \langle\sigma_{v_{\text{Møl}}}^{\psi_-\psi_- XX}\rangle\left(n_{\psi_-}^2 - \bar{n}_{\psi_-}^2\frac{n_X^2}{\bar{n}_X^2}\right) - \langle\sigma_{v_{\text{Møl}}}^{\psi_-\psi_-\psi_+\psi_+}\rangle\left(n_{\psi_-}^2 - \bar{n}_{\psi_-}^2\frac{n_{\psi_+}^2}{\bar{n}_{\psi_+}^2}\right)$$
$$+ \Gamma_{\psi_+\to X\psi_-}\left(n_{\psi_+} - \bar{n}_{\psi_+}\frac{n_{\psi_-}}{\bar{n}_{\psi_-}}\frac{n_X}{\bar{n}_X}\right),$$

$$\boxed{\frac{dn_{\psi_+}}{dt} = \frac{dn_{\psi_-}}{dt}[\psi_- \leftrightarrow \psi_+]}$$

VFDM: Scanning the parameter space





- The left plot shown scan of parameter space for our strategy-A, where X_μ, and ψ_- are stable.
- The right plot shows the same but for the strategy-B and we have 3CDM, i.e. X_μ, ψ_+ and ψ_- are stable.
- Horizontal gray line is the observed DM relic density by PLANCK satellite $\Omega_{\rm obs} h^2 = 0.1197 \pm 0.0022$.

PRELIMINARY!