# Introduction to neutrino mass models

# Lecture 2: Seesaw and radiative mass models

Igor Ivanov

CFTP, Instituto Superior Técnico, Lisbon

University of Warsaw

January 8-11, 2018

INVESTIGADOR FCT

GOVERNO DA REPÚBLICA PORTUGUESA

PROGRAMA OPERACIONAL POTENCIAL HUMANO

QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL

# Classic seesaw

## and some variations

# Just like quarks

Let's repeat the same Yukawa interactions for neutrinos.

We introduce the new field $\nu_R$:

| new fields | spin | $SU(2)_L$ irrep | $U(1)_Y$ charge |
|---|---|---|---|
| $\nu_R$ | $1/2$ | 1 | 0 |

Yukawa interactions → Dirac mass term:

$$y_\nu \bar{L}\tilde{\Phi}\nu_R + h.c. \to \frac{y_\nu v}{\sqrt{2}}(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L) = m_D \bar{\nu}\nu\,.$$

Formally, it is OK. However, two problems:

- $y_\nu \sim 10^{-13}$ is a ridiculously small number without explanation;
- Unlike quarks, Majorana term for $\nu_R$ is possible: $M_R[\overline{(\nu_R)^c}\nu_R + h.c.]/2$.
  There is no symmetry which would protect $M_R = 0$!

# Classic seesaw

So, let's allow for $\nu_R$ Majorana term:

$$m_D(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L) + \frac{1}{2}M_R\left[\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R\right]$$
$$= \frac{1}{2}\left[\overline{\nu_L}, \overline{(\nu_R)^c}\right]\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + h.c.$$

The initial $\nu_L$ Majorana term is forbidden by gauge interactions!

Mass matrix is diagonalized by rotation with angle $\alpha$:

$$\begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} = \begin{pmatrix} m_\nu & 0 \\ 0 & M \end{pmatrix},$$

with $\tan 2\alpha = 2m_D/M_R$.

# Classic seesaw

If $M_R$ is very large, $M_R \gg m_D$, we get $\alpha \approx m_D/M_R \ll 1$, and the masses

$$M \approx M_R\,, \quad m_\nu \approx -\frac{m_D^2}{M_R} \quad \xrightarrow{\text{rephasing}} \quad m_\nu = \frac{m_D^2}{M_R}\,.$$

Small $m_\nu$ does not require tiny Yukawa interactions!

$y_\nu = y_\tau \sim 0.01$ leads to meV neutrino masses for $M_R = 10^{13}$ GeV.

This offers an explanation of WHY neutrino masses are so tiny: not because of small $m_D$ but because the presence of huge $M_R$ drives two neutrino masses to opposite ends: one is huge, the other is tiny $\rightarrow$ seesaw [Minkowski, 1977; etc]

# Classic seesaw

For several generations, first block-diagonalization:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \quad \to \quad \begin{pmatrix} \mathcal{M}_\nu & 0 \\ 0 & M \end{pmatrix},$$

with $M \approx M_R$ and

$$\mathcal{M}_\nu = -m_D M_R^{-1} m_D^T ,$$

and then further diagonalization of the light active $3 \times 3$ neutrino mass matrix $\mathcal{M}_\nu$.

# Inverse seesaw

Classic seesaw is fine but boring! Just explains $m_\nu$, predicts nothing interesting up to the seesaw scale $\sim M_R \to$ hardly testable.

Inverse seesaw [Mohapatra, Valle, 1986]: a variation with $M \sim$ TeV

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $\nu_R$ | 1/2 | 1 | 0 |
| $X_L$ | 1/2 | 1 | 0 |

$$\mathcal{L} = \underbrace{y_\nu \overline{L}\tilde{\Phi}\nu_R}_{\text{Yukawa}} + \underbrace{\overline{\nu_R} M X_L}_{\text{new Dirac}} + \underbrace{\frac{1}{2}\mu_X \overline{X_L^c} X_L}_{\text{Majorana}} + h.c.$$

Lepton number is broken but it is meaningless to attribute this breaking to any individual term!

# Inverse seesaw

Classic seesaw is fine but boring! Just explains $m_\nu$, predicts nothing interesting up to the seesaw scale $\sim M_R \to$ hardly testable.

Inverse seesaw [Mohapatra, Valle, 1986]: a variation with $M \sim$ TeV

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $\nu_R$ | 1/2 | 1 | 0 |
| $X_L$ | 1/2 | 1 | 0 |

$$\mathcal{L} = \underbrace{y_\nu \overline{L}\tilde{\Phi}\nu_R}_{\text{Yukawa}} + \underbrace{\overline{\nu_R} M X_L}_{\text{new Dirac}} + \underbrace{\frac{1}{2}\mu_X \overline{X_L^c} X_L}_{\text{Majorana}} + h.c.$$

Lepton number is broken but it is meaningless to attribute this breaking to any individual term!

# Inverse seesaw

Classic seesaw is fine but boring! Just explains $m_\nu$, predicts nothing interesting up to the seesaw scale $\sim M_R \to$ hardly testable.

Inverse seesaw [Mohapatra, Valle, 1986]: a variation with $M \sim$ TeV

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $\nu_R$ | 1/2 | 1 | 0 |
| $X_L$ | 1/2 | 1 | 0 |

$$\mathcal{L} = \underbrace{y_\nu \bar{L}\tilde{\Phi}\nu_R}_{\text{Yukawa}} + \underbrace{\overline{\nu_R} M X_L}_{\text{new Dirac}} + \underbrace{\frac{1}{2}\mu_X \overline{X_L^c} X_L}_{\text{Majorana}} + h.c.$$

Lepton number is broken but it is meaningless to attribute this breaking to any individual term!

# Inverse seesaw

$3 \times 3$ mass matrix encodes interaction of three LH fields: $\nu_L$, $(\nu_R)^c$ and $X_L$.

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & \mu_X \end{pmatrix}\,, \quad |\det M_\nu| = m_D^2 \mu_X\,.$$

where $m_D = y_\mu v/\sqrt{2}$.

Suppose $\mu_X = 0$. Then the characteristic equation would be

$$\lambda^3 - \lambda(M^2 + m_D^2) = 0 \quad \Rightarrow \quad \lambda = 0,\ \pm\sqrt{M^2 + m_D^2}\,.$$

This implies one massless neutrino and one mass-degenerate pair (= Dirac neutrino) with mass $\sqrt{M^2 + m_D^2}$.

# Inverse seesaw

$3 \times 3$ mass matrix encodes interaction of three LH fields: $\nu_L$, $(\nu_R)^c$ and $X_L$.

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & \mu_X \end{pmatrix}\,, \qquad |\det M_\nu| = m_D^2 \mu_X\,.$$

where $m_D = y_\mu v/\sqrt{2}$.

Suppose $\mu_X = 0$. Then the characteristic equation would be

$$\lambda^3 - \lambda(M^2 + m_D^2) = 0 \quad \Rightarrow \quad \lambda = 0,\ \pm\sqrt{M^2 + m_D^2}\,.$$

This implies one massless neutrino and one mass-degenerate pair (= Dirac neutrino) with mass $\sqrt{M^2 + m_D^2}$.

# Inverse seesaw

For non-zero but small $\mu_X$, with $\mu_X \ll m_D \ll M$, the characteristic equation is

$$\lambda^3 - \lambda^2 \mu_X - \lambda(M^2 + m_D^2) + \mu_X m_D^2 = 0\,.$$

The eigenvalues are slightly shifted:

$$m_\nu \approx \frac{m_D^2}{M} \cdot \frac{\mu_X}{M}\,, \qquad M_{1,2} \approx M \pm \frac{1}{2}\mu_X\,.$$

One light neutrino and one quasi-Dirac pair.

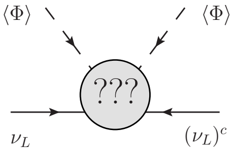
For $m_\nu$: extra suppression w.r.t. classic seesaw!

$$y_\nu \sim 0.01, \quad \mu_X \sim 10 \text{ keV} \quad \Rightarrow \quad M \sim 1 \text{ TeV}$$

Rich phenomenology at TeV scale!

# Three types of seesaw

# Opening up the Weinberg operator




- The Weinberg operator $Q_W$ is non-renormalizable → it is an effective operator of some New Physics bSM.
- Hundreds of neutrino mass models = various ways to "open up" the Weinberg operator
  - seesaw Type I, II, III: three ways to generate $Q_W$ at tree-level;
  - radiative neutrino mass models (1, 2, 3, 4 loops).
  - Recent reviews: [King, 2017; Cai et al, 2017]
- Be careful: not all mass models can be reduced to $Q_W$! Beware of light particles (sterile neutrinos, new scalars, etc).

# Master formula

Master formula for rough classification of Majorana mass models [Bonnet et al, 2012]:

$$m_\nu \propto \frac{v^2}{\Lambda} \times \epsilon \times \left(\frac{1}{16\pi^2}\right)^n \times \left(\frac{v}{\Lambda}\right)^{d-5} .$$

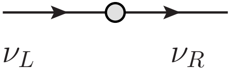
- $v^2/\Lambda$: minimal setting,
- $\epsilon$: possible suppression factors (symmetry related, small Yukawa, etc),
- $n$ is the number of loops,
- $d$ is the dimensionality of operators.

Minimal setting requires $\Lambda \sim 10^{15}$ GeV; but multi-loop models can easily bring it down to few TeV.

# Arrows on diagrams

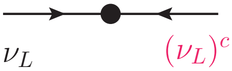
Dirac mass term

$$m_D \overline{\nu_L} \nu_R + h.c.$$

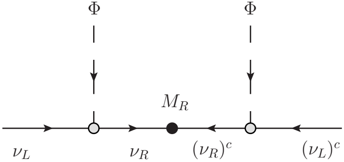



Majorana mass term

$$\frac{1}{2} M_L \nu_L^T \mathcal{C} \nu_L + h.c.$$

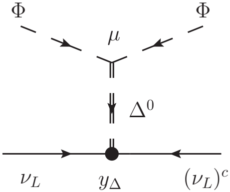


Type I (classical seesaw): add singlet $\nu_R$



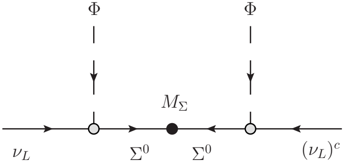

Type II: add new scalar triplet

$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$$



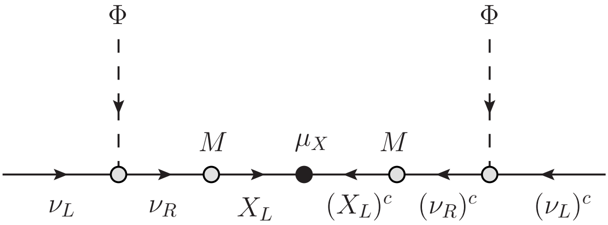

Type III: add triplet neutrinos

$$\Sigma = \begin{pmatrix} \Sigma^{+} \\ \Sigma^{0} \\ \Sigma^{-} \end{pmatrix}$$



# Inverse seesaw

Inverse seesaw: an elaborate version of seesaw type I




$$\mathcal{L} = \underbrace{y_\nu \bar{L}\tilde{\Phi}\nu_R}_{\text{Yukawa}} + \underbrace{\overline{\nu_R} {\color{blue}M} X_L}_{\text{new Dirac}} + \underbrace{\frac{1}{2}{\color{red}\mu_X}\overline{X_L^c}X_L}_{\text{Majorana}} + h.c.$$

# Type II seesaw

keep $\nu_L$ and renormalizability

but extend the Higgs sector

# Using scalar triplet

When constructing the Weinberg operator $(\overline{L^c}\tilde{\Phi}^*)(\tilde{\Phi}^\dagger L) + h.c.$, we used

$$\tilde{\Phi} = \epsilon\Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}, \qquad \tilde{\Phi}^\dagger = (\phi^0, -\phi^+)\,.$$

Similarly to $\tilde{\Phi}$, let's define $\tilde{L} \equiv -\epsilon L^c = (-e_L^c, \nu_L^c)^T$ and regroup it as

$$(\overline{L^c}\tilde{\Phi}^*)(\tilde{\Phi}^\dagger L) + h.c. = \overline{L^c}\cdot\epsilon\Phi\cdot\tilde{\Phi}^\dagger L + h.c. = \underbrace{\bar{\tilde{L}}_i}_{Y=-1}\ \underbrace{[\Phi\tilde{\Phi}^\dagger]_{ij}}_{Y=+2}\ \underbrace{L_j}_{Y=-1} + h.c.$$

# Using scalar triplet

Expanding explicitly:

$$\left(-\overline{(e_L)^c}, \overline{(\nu_L)^c}\right) \begin{pmatrix} \phi^0\phi^+ & -\phi^+\phi^+ \\ \phi^0\phi^0 & -\phi^0\phi^+ \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + h.c.$$

$$\to \quad \left(-\overline{(e_L)^c}, \overline{(\nu_L)^c}\right) \begin{pmatrix} 0 & 0 \\ v^2/2 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + h.c. = \frac{v^2}{2}\left[\overline{(\nu_L)^c}\nu_L + h.c.\right]$$

But we used two Higgs fields $\to$ $\dim(Q_W) = 5$ $\to$ non-renormalizable operator.

# Using scalar triplet

Suppose that we have, in addition to $\Phi$, a new Higgs field $\Delta_{ij}(x)$. Then a new renormalizable term is possible:

$$y_\Delta \bar{\tilde{L}}_i \Delta_{ij} L_j + h.c. = y_\Delta(-\overline{(e_L)^c}, \overline{(\nu_L)^c}) \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + h.c.$$

with dimensionless $y_\Delta$. This is a complex EW triplet with $Y = 2$, $\Delta = \vec{\Delta}\vec{\sigma}$

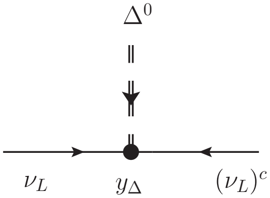
$$\vec{\Delta} = \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \end{pmatrix}.$$

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $\Delta$ | 0 | 3 | +2 |

# Using scalar triplet

Simplest idea: $\Delta^0$ acquires a non-zero vev:

$$\vec{\Delta} = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}, \qquad \langle\vec{\Delta}\rangle = \begin{pmatrix} 0 \\ 0 \\ v_\Delta \end{pmatrix},$$




Non-zero vev $\to$ Majorana mass $m_\nu = 2y_\Delta v_\Delta$.

Two problems emerge, though.

# Using scalar triplet: problem 1

First, $\Delta$ participates in gauge interactions:

$$\mathcal{L}_\Delta = \mathrm{Tr}\left[(D_\mu \Delta)^\dagger (D^\mu \Delta)\right] - V(\Delta)\,,$$

with

$$D_\mu = \partial_\mu - ig' \frac{Y}{2} B_\mu - igT_i W^i_\mu\,,$$

where $T^i$ are $SU(2)$ generators in the triplet representation.

Both $v_\Phi$ and $v_\Delta$ affect $m_W$ and $m_Z$, but in a different way! As a result,

$$\rho = \frac{m_W^2}{m_Z^2}\frac{g^2 + g'^2}{g^2} = \frac{v_\Phi^2 + 4v_\Delta^2}{v_\Phi^2 + 8v_\Delta^2} \neq 1.$$

Experimental measurements of $\rho$ push $v_\Delta \lesssim$ few GeV.

An explanation is needed for the small vev scale $v_\Delta$.

# Using scalar triplet: problem 1

First, $\Delta$ participates in gauge interactions:

$$\mathcal{L}_\Delta = \mathrm{Tr}\left[(D_\mu\Delta)^\dagger(D^\mu\Delta)\right] - V(\Delta)\,,$$

with

$$D_\mu = \partial_\mu - ig'\frac{Y}{2}B_\mu - igT_i W^i_\mu\,,$$

where $T^i$ are $SU(2)$ generators in the triplet representation.

Both $v_\Phi$ and $v_\Delta$ affect $m_W$ and $m_Z$, but in a different way! As a result,

$$\rho = \frac{m_W^2}{m_Z^2}\frac{g^2+g'^2}{g^2} = \frac{v_\Phi^2 + 4v_\Delta^2}{v_\Phi^2 + 8v_\Delta^2} \neq 1.$$

Experimental measurements of $\rho$ push $v_\Delta \lesssim$ few GeV.

An explanation is needed for the small vev scale $v_\Delta$.

# Using scalar triplet: problem 2

Recall that $\tilde{\bar{L}}\Delta L$ means that $\Delta$ carries the lepton number: $L(\Delta) = -2$.

- initial langrangian including conserves $L$;
- when generating a non-zero $v_\Delta$ from

$$V(\Delta) = -m^2 \mathrm{Tr}(\Delta^\dagger \Delta) + \lambda [\mathrm{Tr}(\Delta^\dagger \Delta)]^2 ,$$

  we spontaneously break $L$ $\rightarrow$ that's how Majorana mass terms appears here.

- Spontaneously broken global symmetry produces a massless Goldstone boson, Majoron $J$, which is not absorbed by gauge bosons!

$$\Delta^0 \rightarrow v_\Delta + \delta^0 + iJ .$$

- Majoron participates in gauge interactions and modifies the $Z$ decay width! Spontaneously broken lepton number is ruled out.

# Using scalar triplet: problem 2

Recall that $\bar{\tilde{L}}\Delta L$ means that $\Delta$ carries the lepton number: $L(\Delta) = -2$.

- initial langrangian including conserves $L$;
- when generating a non-zero $v_\Delta$ from

$$V(\Delta) = -m^2 \mathrm{Tr}(\Delta^\dagger \Delta) + \lambda[\mathrm{Tr}(\Delta^\dagger \Delta)]^2\,,$$

  we spontaneously break $L$ $\to$ that's how Majorana mass terms appears here.
- Spontaneously broken global symmetry produces a massless Goldstone boson, Majoron $J$, which is not absorbed by gauge bosons!

$$\Delta^0 \to v_\Delta + \delta^0 + iJ\,.$$

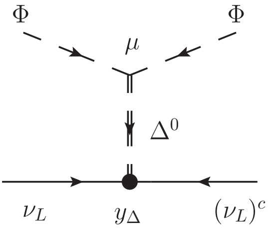
- Majoron participates in gauge interactions and modifies the $Z$ decay width! Spontaneously broken lepton number is ruled out.

# Type II seesaw

The Higgs doublet $\Phi$ rescues this idea $\to$ type II seesaw [Magg, Wetterich, 1980; Schechter, Valle, 1980; etc].

Don't break $L$ spontaneously, do it explicitly via $\Delta$-Higgs interactions:

$$V(\Delta,\Phi) = +m^2\mathrm{Tr}(\Delta^\dagger\Delta) + \mu\left(\Phi^\dagger\Delta\tilde{\Phi} + h.c.\right)$$
$$= m^2|\Delta^0|^2 + \mu[(\phi^{0*})^2\Delta^0 + h.c.] + \dots$$




Then, non-zero $v_\Phi$ forces $\Delta^0$ to acquire a small vev $\Delta^0 \to v_\Delta + \delta^0$:

$$m^2\cdot 2v_\Delta\delta^0 + \mu v_\Phi^2\delta^0 + \dots = 0 \quad\to\quad v_\Delta \approx -\frac{\mu v_\Phi^2}{2m^2}.$$

- no massless Goldstone boson appears,
- $m_\nu = 2y_\Delta v_\Delta$ can naturally be very small because of large $m^2$.

# Scotogenic model

Neutrino masses through the dark sector

"*skotos*" = "dark" in Greek

# Inert doublet model

Classic seesaw:

add $\nu_R$, use $\tilde{\Phi}$ for Dirac mass term, and add Majorana mass term for $\nu_R$.

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $\nu_R$ | 1/2 | 1 | 0 |

$$y_\nu \left(\bar{L}\tilde{\Phi}\nu_R + \overline{\nu_R}L\tilde{\Phi}^\dagger\right) + \frac{1}{2}M_R\left[\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R\right] .$$

Suppose there exists a second Higgs doublet $\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$, which is

- odd under a new "parity" ($\mathbb{Z}_2$ symmetry) transformation: $\Phi_2 \to -\Phi_2$,
- this $\mathbb{Z}_2$ symmetry remains unbroken after electroweak symmetry breaking.

# Inert doublet model

Classic seesaw:

add $\nu_R$, use $\tilde{\Phi}$ for Dirac mass term, and add Majorana mass term for $\nu_R$.

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $\nu_R$ | 1/2 | 1 | 0 |

$$y_\nu \left(\bar{L}\tilde{\Phi}\nu_R + \overline{\nu_R} L \tilde{\Phi}^\dagger\right) + \frac{1}{2} M_R \left[\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R\right] .$$

Suppose there exists a second Higgs doublet $\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$, which is

- odd under a new "parity" ($\mathbb{Z}_2$ symmetry) transformation: $\Phi_2 \to -\Phi_2$,
- this $\mathbb{Z}_2$ symmetry remains unbroken after electroweak symmetry breaking.

# Inert doublet model

Then

- $\Phi_2$ does not interact with the SM fermions: $\overline{Q}_L d_R \Phi_2$ are forbidden by $\mathbb{Z}_2$ parity $\to$ does not contribute to fermion masses;
- $\Phi_2$ does not acquire vev: non-zero $\langle\Phi_2\rangle$ would break $\mathbb{Z}_2$ parity $\to$ does not contribute to $W$, $Z$ masses;
- the lightest scalar from $\Phi_2$ is stable $\to$ natural dark matter candidate.

However, $\Phi_2$ interacts with gauge bosons and $\Phi$ via $|D_\mu \Phi_2|^2 - V(\Phi, \Phi_2)$, where

$$\begin{aligned} V(\Phi,\Phi_2) \;=\; & -m^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2 \\ & -m_2^2(\Phi_2^\dagger\Phi_2) + \lambda_2(\Phi_2^\dagger\Phi_2)^2 \\ & +\lambda_3(\Phi^\dagger\Phi)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi^\dagger\Phi_2)(\Phi_2^\dagger\Phi) + \frac{\lambda_5}{2}\left[(\Phi^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi)^2\right]. \end{aligned}$$

$\Rightarrow$ interesting and testable astrophysical and collider phenomenology.

# Inert doublet model

Then

- $\Phi_2$ does not interact with the SM fermions: $\overline{Q}_L d_R \Phi_2$ are forbidden by $\mathbb{Z}_2$ parity → does not contribute to fermion masses;
- $\Phi_2$ does not acquire vev: non-zero $\langle\Phi_2\rangle$ would break $\mathbb{Z}_2$ parity → does not contribute to $W$, $Z$ masses;
- the lightest scalar from $\Phi_2$ is stable → natural dark matter candidate.

However, $\Phi_2$ interacts with gauge bosons and $\Phi$ via $|D_\mu \Phi_2|^2 - V(\Phi, \Phi_2)$, where

$$\begin{aligned} V(\Phi,\Phi_2) &= -m^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2 \\ &\quad -m_2^2(\Phi_2^\dagger\Phi_2) + \lambda_2(\Phi_2^\dagger\Phi_2)^2 \\ &\quad +\lambda_3(\Phi^\dagger\Phi)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi^\dagger\Phi_2)(\Phi_2^\dagger\Phi) + \frac{\lambda_5}{2}\left[(\Phi^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi)^2\right]. \end{aligned}$$

⇒ interesting and testable astrophysical and collider phenomenology.

# Inert doublet + neutrino

How can it help neutrinos? Suppose $\nu_R$ exists but it is also odd under the same $\mathbb{Z}_2$ parity → we better call it $N$.

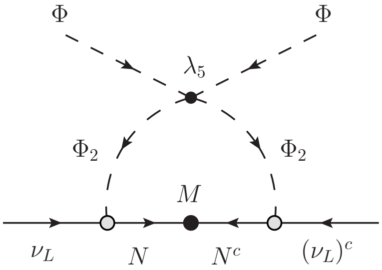
| | spin | $SU(2)_L$ | $U(1)_Y$ | $\mathbb{Z}_2$ parity |
|---|---|---|---|---|
| $\Phi_2$ | 0 | 2 | 1 | $-$ |
| $N$ | 1/2 | 1 | 0 | $-$ |

Then, Dirac mass terms linking $\nu_L$ and $N$ must use $\Phi_2$, not $\Phi$:

$$y_\nu \left(\overline{L}\tilde{\Phi}_2 N + \overline{N}\tilde{\Phi}_2^\dagger L\right) + \frac{1}{2}M(\overline{N}N^c + \overline{N^c}N)\,.$$

The usual seesaw does not work: $\langle\Phi_2\rangle = 0 \to m_\nu = 0$.

# Scotogenic model




But since $\nu N \leftrightarrow \Phi_2 \leftrightarrow \Phi$, it links $\langle\Phi\rangle$ and $\nu_L$ at one loop.

Dark-matter-assisted neutrino masses = scotogenic model [E. Ma, 2006]

# Scotogenic model

Not a tree-level mechanism $\to$ cannot use matrices as before.

To see it, pick up the most relevant terms (after EWSB)

$$y_\nu(\overline{\nu_L}\phi_2^{0*}N + \overline{N}\phi_2^0\nu_L) + \frac{1}{2}M(\overline{N}N^c + \overline{N^c}N) + \lambda_5[(\langle\phi^{0*}\rangle\phi_2^0)^2 + (\phi_2^{0*}\langle\phi^0\rangle)^2)$$

and track the neutrino line:

$$y_\nu\overline{\nu_L}\phi_2^{0*}N \cdot M\overline{N}N^c \cdot y_\nu\overline{N^c}\phi_2^{0*}(\nu_L)^c$$

and the two $\phi_2^{0*}$ will couple to $(\langle\phi^0\rangle)^2$ via $\lambda_5$ term. Effectively we get

$$\overline{\nu_L} \cdot \left[\lambda_5 v^2 y_\nu^2 \times \text{loop integral}\right] \cdot (\nu_L)^c$$

which is exactly the Majorana mass term fo $\nu_L$.

# Scotogenic model

The key role is played by $\lambda_5$:

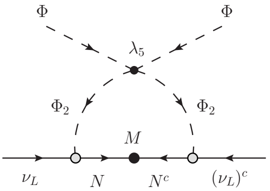
$$V(\Phi, \Phi_2) = -m^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2 - m_2^2(\Phi_2^\dagger\Phi_2) + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi^\dagger\Phi)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi^\dagger\Phi_2)(\Phi_2^\dagger\Phi) + \frac{\lambda_5}{2}\left[(\Phi^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi)^2\right].$$

The same $\lambda_5$ also determines scalar mass splitting:

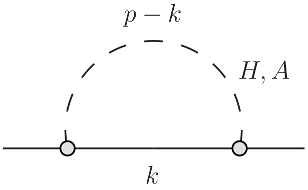
$$\phi_2^0 = \frac{1}{\sqrt{2}}(H + iA), \quad m_H^2 - m_A^2 = 2\lambda_5 v^2.$$

# Loop integral

The diagram shown usually




The actual diagram to calculate



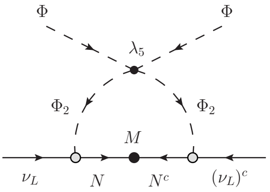

The relative sign comes from $i^2$ in the fermion line:

$$\cdots \phi_2^{0*} \cdots \phi_2^{0*} \cdots \to \cdots \frac{H - iA}{\sqrt{2}} \cdots \frac{H - iA}{\sqrt{2}} \cdots = [H\text{-loop}] - [A\text{-loop}]$$

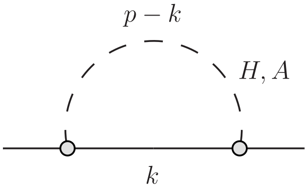
The divergent parts of the loop integrals cancel completely, but the finite parts do not due to $m_H \neq m_A$.

# Loop integral

The diagram shown usually




The actual diagram to calculate




The relative sign comes from $i^2$ in the fermion line:

$$\cdots \phi_2^{0*} \cdots \phi_2^{0*} \cdots \to \cdots \frac{H - iA}{\sqrt{2}} \cdots \frac{H - iA}{\sqrt{2}} \cdots = [H\text{-loop}] - [A\text{-loop}]$$

The divergent parts of the loop integrals cancel completely, but the finite parts do not due to $m_H \neq m_A$.

# Loop integral

Denoting $m_H \equiv m_1$, $m_A \equiv m_2$, we get $J = J_1 - J_2$, where

$$J_1 = i\int \frac{d^4k}{(2\pi)^4}\frac{(\gamma k)+M}{k^2-M^2}\cdot\frac{1}{(p-k)^2-m_1^2} = iM\int\frac{d^4k}{(2\pi)^4}\frac{1}{[k^2-M^2][(p-k)^2-m_1^2]}\,.$$

We pick up only $m_1$-dependent finite part of $J_1$. Using the "Feynman trick"

$$\frac{1}{AB} = \int_0^1 dx\,\frac{1}{[Ax+B(1-x)]^2}\,,$$

we get

$$\begin{aligned}[J_1]_{fin} &= iM\int_0^1 dx\int\frac{d^4k}{(2\pi)^4}\frac{1}{[(k^2-M^2)x+[(p-k)^2-m_1^2]]^2}\\ &= iM\int_0^1 dx\int\frac{d^4k}{(2\pi)^4}\frac{1}{[(k-p(1-x))^2-D_1]^2}\,,\end{aligned}$$

where

$$D_1 = M^2x + m_1^2(1-x) - \underbrace{p^2x(1-x)}_{=0}\,.$$

# Loop integral

Denoting $m_H \equiv m_1$, $m_A \equiv m_2$, we get $J = J_1 - J_2$, where

$$J_1 = i \int \frac{d^4k}{(2\pi)^4} \frac{(\gamma k) + M}{k^2 - M^2} \cdot \frac{1}{(p-k)^2 - m_1^2} = iM \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - M^2][(p-k)^2 - m_1^2]} \,.$$

We pick up only $m_1$-dependent finite part of $J_1$. Using the "Feynman trick"

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2} \,,$$

we get

$$\begin{aligned} [J_1]_{fin} &= iM \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k^2 - M^2)x + [(p-k)^2 - m_1^2]]^2} \\ &= iM \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k - p(1-x))^2 - D_1]^2} \,, \end{aligned}$$

where

$$D_1 = M^2 x + m_1^2(1-x) - \underbrace{p^2 x(1-x)}_{=0} \,.$$

# Loop integral

Shifting integration variable,

$$[J_1]_{fin} = iM \int_0^1 dx \int \frac{d^4\tilde{k}}{(2\pi)^4} \frac{1}{[\tilde{k}^2 - D_1]^2} = \frac{M}{16\pi^2} \int_0^1 dx \log D_1 \,.$$

Then,

$$\begin{aligned} J_1 - J_2 &= \frac{M}{16\pi^2} \int_0^1 dx \log \left[ \frac{M^2 x + m_1^2(1-x)}{M^2 x + m_2^2(1-x)} \right] \\ &= \frac{M^2 \log M^2 - m_1^2 \log m_1^2}{M^2 - m_1^2} - \frac{M^2 \log M^2 - m_2^2 \log m_2^2}{M^2 - m_2^2} \\ &= \frac{M}{16\pi^2} \left( \frac{m_1^2}{M^2 - m_1^2} \log \frac{M^2}{m_1^2} - \frac{m_2^2}{M^2 - m_2^2} \log \frac{M^2}{m_2^2} \right) . \end{aligned}$$

# Scotogenic model: three generations

Three generations: $\nu_{Li}$ and $N_i$ $\rightarrow$ Dirac couplings become $3 \times 3$ matrices $y_{ij}$.

The final result for Majorana mass matrix is

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{y_{ik} y_{jk} M_k}{32\pi^2} \left( \frac{m_H^2}{M_k^2 - m_H^2} \log \frac{M_k^2}{m_H^2} - \frac{m_A^2}{M_k^2 - m_A^2} \log \frac{M_k^2}{m_A^2} \right) .$$

# Scotogenic model

Small $\lambda_5$ → small mass splitting between $H$ and $A$ → extra suppression for $m_\nu$!

Assuming

$$2\lambda_5 v^2 = m_H^2 - m_A^2 \ll \frac{m_H^2 + m_A^2}{2} \equiv m_0^2 \,,$$

and $M \gg m_H, m_A$, we get

$$(\mathcal{M}_\nu)_{ij} \approx \frac{\lambda_5 v^2}{16\pi^2} \sum_k \frac{y_{ik} y_{jk}}{M_k} \left( \log \frac{M_k^2}{m_0^2} - 1 \right) .$$

With respect to the classic seesaw, it has an extra suppression $\lambda_5/(16\pi^2)$.

If $\lambda_5 \sim y_\nu \sim 10^{-4}$, then $M \sim$ few TeV → testable at colliders!

# Zee model

# Majorana-like electron-neutrino coupling

We have seen two mechanisms for $m_\nu \neq 0$ without adding $\nu_R$:

- Weinberg operator:

$$\underbrace{\overline{\tilde{L}}}_{Y=-1}\ \underbrace{[\Phi\tilde{\Phi}^\dagger]}_{Y=+2}\ \underbrace{L}_{Y=-1} + h.c.$$

- seesaw type II:

$$\overline{\tilde{L}} \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} L + h.c.$$

Here, $\overline{\tilde{L}} = \overline{L^c} \cdot \epsilon = (-\overline{(e_L)^c}, \overline{(\nu_L)^c}) = (-e_L^T, \nu_L^T)\,\mathcal{C}$.

Trying a simpler combination $\overline{\tilde{L}}L$ does not work:

$$(-e_L^T, \nu_L^T)\,\mathcal{C} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = -e_L^T\,\mathcal{C}\,\nu_L + \nu_L^T\,\mathcal{C}\,e_L = -(\nu_L^T\,\mathcal{C}\,e_L)^T + \nu_L^T\,\mathcal{C}\,e_L = 0\,.$$

# Majorana-like electron-neutrino coupling

But wait, we have three lepton generations!

$$\overline{\tilde{L}}_i f_{ij} L_j \equiv -e_{Li}^T \mathcal{C}\, f_{ij} \nu_{Lj} + \nu_{Li}^T \mathcal{C}\, f_{ij} e_{Lj} = -\nu_{Li}^T \mathcal{C}\, (f^T)_{ij} e_{Lj} + \nu_{Li}^T \mathcal{C}\, f_{ij} e_{Lj}$$
$$= \nu_{Li}^T \mathcal{C}\, (f - f^T)_{ij} e_{Lj}\,.$$

An antisymmetric coupling matrix $f^T = -f$ is perfectly fine!

Since $\overline{\tilde{L}}L$ has $Y = -2$, we need to couple it with a gauge-singlet charged scalar $h^+$ with $Y = +2$:

$$\overline{\tilde{L}}_i f_{ij} L_j h^+ + h.c.$$

This is a Majorana-like coupling between $e_{Li}$ and $\nu_{Lj}$ $\to$ no RH neutrinos needed!

# Majorana-like electron-neutrino coupling

But wait, we have three lepton generations!

$$\overline{\tilde{L}}_i f_{ij} L_j \equiv -e_{Li}^T \mathcal{C}\, f_{ij} \nu_{Lj} + \nu_{Li}^T \mathcal{C}\, f_{ij} e_{Lj} = -\nu_{Li}^T \mathcal{C}\, (f^T)_{ij} e_{Lj} + \nu_{Li}^T \mathcal{C}\, f_{ij} e_{Lj}$$

$$= \nu_{Li}^T \mathcal{C}\, (f - f^T)_{ij} e_{Lj}\,.$$

An antisymmetric coupling matrix $f^T = -f$ is perfectly fine!

Since $\overline{\tilde{L}}L$ has $Y = -2$, we need to couple it with a gauge-singlet charged scalar $h^+$ with $Y = +2$:

$$\overline{\tilde{L}}_i f_{ij} L_j h^+ + h.c.$$

This is a Majorana-like coupling between $e_{Li}$ and $\nu_{Lj}$ $\to$ no RH neutrinos needed!

# Zee model

Two extra scalars: second Higgs doublet $\Phi_2$ and a leptophilic charged singlet $h^+$.

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $\Phi_2$ | 0 | 2 | 1 |
| $h^+$ | 0 | 1 | 2 |

$$\mathcal{L} = \overline{L}(Y_1\Phi_1 + Y_2\Phi_2)e_R + \overline{\tilde{L}} \cdot f \cdot L h^+ + \Phi_2^\dagger \tilde{\Phi}_1 h^+ + h.c.$$

with $Y_{1,2}$ and $f$ being $3 \times 3$ matrices [Zee, 1980].

Both doublets acquire vevs: $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$, $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$, and produce charged lepton mass matrix:

$$M_\ell = \frac{1}{\sqrt{2}}(v_1 Y_1 + v_2 Y_2)\,,$$

but neutrinos remain massless at tree level.

However, lepton number is violated $\to$ Majorana masses for $\nu_L$ must appear!

# Zee model

Two extra scalars: second Higgs doublet $\Phi_2$ and a leptophilic charged singlet $h^+$.

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $\Phi_2$ | 0 | 2 | 1 |
| $h^+$ | 0 | 1 | 2 |

$$\mathcal{L} = \bar{L}(Y_1\Phi_1 + Y_2\Phi_2)e_R + \bar{\tilde{L}} \cdot f \cdot Lh^+ + \Phi_2^\dagger\tilde{\Phi}_1 h^+ + h.c.$$

with $Y_{1,2}$ and $f$ being $3 \times 3$ matrices [Zee, 1980].

Both doublets acquire vevs: $\langle\phi_1^0\rangle = v_1/\sqrt{2}$, $\langle\phi_2^0\rangle = v_2/\sqrt{2}$, and produce charged lepton mass matrix:

$$M_\ell = \frac{1}{\sqrt{2}}(v_1 Y_1 + v_2 Y_2)\,,$$

but neutrinos remain massless at tree level.

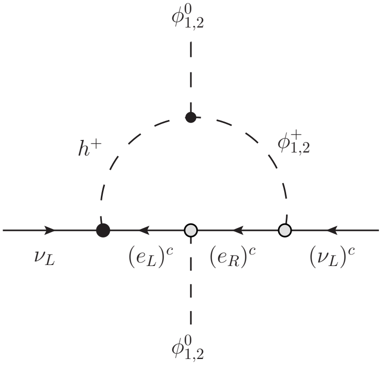
However, lepton number is violated $\to$ Majorana masses for $\nu_L$ must appear!

# Zee model

$$\mathcal{L} = \bar{L}(Y_1\Phi_1 + Y_2\Phi_2)\ell_R + \bar{\tilde{L}} \cdot f \cdot L h^+ + \Phi_2^\dagger \tilde{\Phi}_1 h^+ + h.c.$$

The key role is played by charged scalars:

- initial fields: $\phi_1^+$, $\phi_2^+$, $h^+$;
- after EWSB: a would-be Goldstone $G^+$ absorbed in $W^+$, two physical charged scalars remain: $h_1^+$ and $h_2^+$.
- their loops do not cancel completely due to $m_{h_1^+} \neq m_{h_2^+}$.



# Zee model

In the original Zee model, $Y_2 = 0$, yielding

$$\mathcal{M}_\nu \propto (fM_\ell^2 + M_\ell^2 f^T) \log \frac{m^2_{h_2^+}}{m^2_{h_1^+}}\,.$$

Its diagonal elements are zero $\to$ neutrino properties imcompatible with data.

But for $Y_2 \neq 0$, a good fit can be achieved.

# Zee-Babu model

# Zee-Babu model

Lepton number violating Majorana-like terms:

- neutrino × neutrino: $\overline{\nu^c}\nu$
- electron × neutrino: $\overline{\nu_L^c} e_L - \overline{e_L^c}\nu_L \to$ Zee model
- electron × electron: $\overline{(e_R)^c} e_R$ → Zee-Babu model

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $h^+$ | 0 | 1 | 2 |
| $k^{++}$ | 0 | 1 | 4 |

$$\mathcal{L} = \overline{L} Y \Phi \ell_R + \overline{\tilde{L}} \cdot f \cdot L h^+ + \overline{(\ell_R)^c} \cdot g \cdot \ell_R k^{++} + \mu h^+ h^+ k^{--} + h.c.$$

with antisymmetric $f$ and symmetric $g$.

# Zee-Babu model

Lepton number violating Majorana-like terms:

- neutrino × neutrino: $\overline{\nu^c}\nu$
- electron × neutrino: $\overline{\nu^c_L} e_L - \overline{e^c_L}\nu_L \to$ Zee model
- electron × electron: $\overline{(e_R)^c} e_R$ $\to$ Zee-Babu model

| | spin | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $h^+$ | 0 | 1 | 2 |
| $k^{++}$ | 0 | 1 | 4 |

$$\mathcal{L} = \overline{L} Y \Phi \ell_R + \overline{\tilde{L}} \cdot f \cdot L h^+ + \overline{(\ell_R)^c} \cdot g \cdot \ell_R k^{++} + \mu h^+ h^+ k^{--} + h.c.$$

with antisymmetric $f$ and symmetric $g$.

# Zee-Babu model

$$\bar{\tilde{L}} \cdot f \cdot L h^+ + \overline{(\ell_R)^c} \cdot g \cdot \ell_R k^{++} + \mu h^+ h^+ k^{--}$$

Lepton number breaking is a combined effect of all three terms.

- keep only $\bar{\tilde{L}} \cdot f \cdot L h^+ + \overline{(\ell_R)^c} \cdot g \cdot \ell_R k^{++}$

  $\Rightarrow L(h^+) = -2$, $L(k^{++}) = -2$ $\Rightarrow$ $L$ is conserved

- keep only $\bar{\tilde{L}} \cdot f \cdot L h^+ \quad + \quad \mu h^+ h^+ k^{--}$

  $\Rightarrow L(h^+) = -2$, $L(k^{++}) = -4$ $\Rightarrow$ $L$ is conserved

- keep only $\overline{(\ell_R)^c} \cdot g \cdot \ell_R k^{++} + \mu h^+ h^+ k^{--}$

  $\Rightarrow L(h^+) = -1$, $L(k^{++}) = -2$ $\Rightarrow$ $L$ is conserved

But if all three vertices are present, no conserved $L$ can be assigned!

# Zee-Babu model

$$\bar{\tilde{L}} \cdot f \cdot L h^+ + \overline{(\ell_R)^c} \cdot g \cdot \ell_R k^{++} + \mu h^+ h^+ k^{--}$$

Lepton number breaking is a combined effect of all three terms.

- keep only $\bar{\tilde{L}} \cdot f \cdot L h^+ + \overline{(\ell_R)^c} \cdot g \cdot \ell_R k^{++}$

  $\Rightarrow L(h^+) = -2$, $L(k^{++}) = -2 \Rightarrow L$ is conserved

- keep only $\bar{\tilde{L}} \cdot f \cdot L h^+ \quad + \quad \mu h^+ h^+ k^{--}$

  $\Rightarrow L(h^+) = -2$, $L(k^{++}) = -4 \Rightarrow L$ is conserved

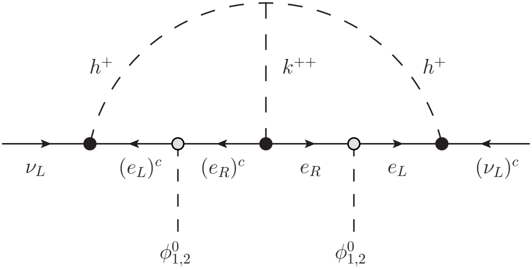
- keep only $\overline{(\ell_R)^c} \cdot g \cdot \ell_R k^{++} + \mu h^+ h^+ k^{--}$

  $\Rightarrow L(h^+) = -1$, $L(k^{++}) = -2 \Rightarrow L$ is conserved

But if all three vertices are present, no conserved $L$ can be assigned!

# Zee-Babu model

The minimal diagram requires 2 loops [Zee, Babu, 1986; Cheng, Li, 1980]




$$\mathcal{L} = \overline{L} Y \Phi \ell_R + \overline{\tilde{L}} \cdot f \cdot L h^+ + \overline{(\ell_R)^c} g \ell_R k^{++} + \mu h^+ h^+ k^{--} + h.c.$$

# Zee-Babu model

The light neutrino mass matrix:

$$\mathcal{M}_\nu \approx \frac{v^2\mu}{96\pi^2 M^2} f\, Y^\dagger g^\dagger Y^* f^T\,.$$

Since $f$ is antisymmetric, $\det f = 0 \to \det \mathcal{M}_\nu = 0 \to$ the lightest neutrino is exactly massless at this order.

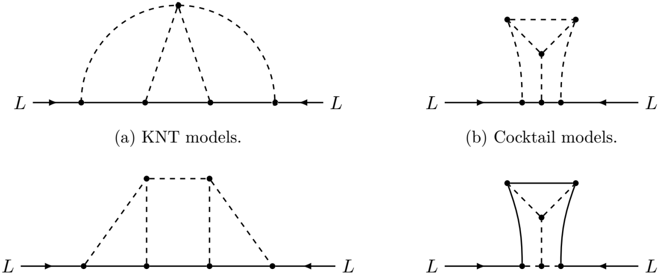
# Summary: new field content

New field content in models we studied

| model | new fermions | new scalars |
|---|---|---|
| classical seesaw | $\nu_R$ | — |
| inverse seesaw | $\nu_R$, $X_L$ | — |
| type II seesaw | — | $\Delta$ |
| scotogenic | $\nu_R$ | $\Phi_2$ |
| Zee | — | $\Phi_2$, $h^+$ |
| Zee-Babu | — | $h^+$, $k^{++}$ |

# Beyond two loops

There exist models in which $\mathcal{M}_\nu$ is generated at 3 loops.




(a) KNT models.

(b) Cocktail models.

(c) Trapezoid models. Cross diagrams may exist.

(d) Fermionic Cocktail models.

- Main goal: produce very light neutrinos from $[y^2/(16\pi^2)]^3$ suppression, keeping the new particle masses within TeV scale;
- Main tools: play with new fields and their quantum numbers.
- Recent review: [Cai et al, 1706.08524]