CP Violation in the Standard Model and Beyond

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Open Questions in CP Violation

- What is the Origin of CP Violation?
  - Explicitly broken as in the Standard Model through the Kobayashi-Maskawa mechanism (1973)

    or
  - Spontaneously broken as suggested by T.D. Lee (1973)
- Pure gauge interactions conserve CP

  W. Grimus, M.N. Rebelo

- How to conceive an experiment that could distinguish between spontaneous and explicit CP violation?

An important but very difficult question

Some interesting work was done using CP-odd invariants relevant for the scalar sector

G.C.B, M. N. Rebelo, J. Silva-Marcos

F. Gunion and H. Haber

B. Grzakoski, O. M. Ogreid, P. Osland

M. Krawczyk, D. Sokolowska

- What is the connection between CP Violation and Family Symmetries?

W. Grimus, G. Ecker

- Can one have Geometrical CP Violation?

GCB, J. M. Gérard, W. Grimus (1984)

Recent developments :

I. de Medeiros Varzielas
S. King
M. Lindner, M. Holthausen, M. Lindner, M. Schmidt
I. P. Ivanov, L. Lavoura
F. Feruglio
M. C. Chen, M. Fallbacher, K. Mahanthappa, M. Ratz
etc

- Are there New Sources of CP Violation beyond those present in the K.M. mechanism?

The answer is Yes!, since the SM cannot generate sufficient BAU.

But then the next question is:

- How to generate adequate BAU. Leptogenesis? Extended Higgs sector? Many scenarios have been proposed.
- How to test, experimentally, Leptogenesis?
- How relate CP violation relevant for Leptogenesis from leptonic CP violation, detectable at low energies through neutrino oscillations?

- Answer: In general, it is not possible to establish a connection. May be possible if leptonic flavour symmetries are introduced.
- How to solve the Strong CP problem?
  Peccei-Quinn provide an elegant solution
  … but Axions have not been found.
- Is it possible that all manifestations of CP violation have the same Origin?
  - CP violation in the quark sector
  - CP violation in the lepton sector
  - CP violation required to generate BAU

General Approach to the Study of CP Violation

J. Bernabéu, G.C.B, M. Gronau

The best way of studying the CP properties of any model/theory is to write the Lagrangian as:

$$\mathcal{L} = \mathcal{L}_{CP} + \mathcal{L}_{remaining}$$

$\mathcal{L}_{CP}$ → part of the Lagrangian which one knows that conserves CP. Typically $\mathcal{L}_{CP}$ contains the gauge interactions, since pure gauge interactions necessarily conserve CP

W. Grimus, M.N. Rebelo

- Then construct the most general CP transformation which leaves $\mathcal{L}_{CP}$ invariant.

- Investigate whether CP invariance implies restrictions on $\mathcal{L}$ remaining.

- If there are CP restrictions, then $\mathcal{L}$ can violate CP

  If there are no CP restrictions, $\mathcal{L}$ cannot violate CP.

How to apply this method to the Leptonic Sector with Majorana neutrinos?

In order to answer this question, one has to recall the conditions for CP invariance when one has n generations of left-handed Majorana neutrinos. Under CP the lepton fields transform as:

$$(CP)\, l_L\, (CP)^{\dagger} = U_L\, \gamma^0 C\, \bar{l}_L^{\,T}$$

$$(CP)\, \nu_L\, (CP)^{\dagger} = U_L\, \gamma^0 C\, \bar{\nu}_L^{\,T}$$

$$(CP)\, l_R\, (CP)^{\dagger} = U_R\, \gamma^0 C\, \bar{l}_R^{\,T}$$

where U_L, U_R are unitary matrices acting in flavour space.

The Leptonic Sector with Majorana Neutrinos

In order to have CP invariance in the leptonic sector, the mass matrices m_ν, m_ℓ have to satisfy the conditions:

$$U_L^T \, (m_\nu) \, U_L = - m_\nu^*$$

$$U_L^\dagger \, (m_\ell) \, U_R = m_\ell^*$$

$$\Downarrow$$

$$U_L^\dagger \, h_\ell \, U_L = h_\ell^*$$

These conditions are weak-basis independent !!

If there is a solution for U_L, U_R in one basis, there is a solution in any other WB

From these Eqs. one derives:

$$\boxed{\begin{aligned} U_L^\dagger\, h_\nu\, U_L &= h_\nu^* \,, \\ U_L^\dagger\, h_\ell\, U_L &= h_\ell^* \,, \end{aligned}}$$

where $h_\nu \equiv (m_\nu m_\nu^\dagger)^*$

$h_\ell = m_\ell m_\ell^\dagger$

$$\Downarrow$$

Since h_ν, h_ℓ are Hermitian matrices, $h_\nu^* = h_\nu^T$, $h_\ell^* = h_\ell^T$

Evaluating the commutator:

$$U_L^\dagger\, [h_\nu, h_\ell]\, U_L = [h_\nu^T, h_\ell^T] = -[h_\nu, h_\ell]^T$$

$$U_L^\dagger\, [h_\nu, h_\ell]^3\, U_L = -\left([h_\nu, h_\ell]^3\right)^T$$

CP invariance implies: $\boxed{\operatorname{tr}[h_\nu, h_\ell]^3 = 0}$

Therefore, one concludes that a necessary condition for CP invariance, for an arbitrary number of generations is :

$$\boxed{\operatorname{tr}\,[h_\nu, h_\ell]^3 = 0}$$

For 3 generations one has:

$$\operatorname{tr}\,[h_\nu, h_\ell]^3 = 6i\,(m_\mu^2 - m_e^2)(m_\tau^2 - m_\mu^2)(m_\tau^2 - m_e^2)\times$$
$$\times (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2)\,\mathrm{Im}\,(Q_L)$$

Dirac type CP violation

$$\mathrm{Im}\,Q_L = \mathrm{Im}\left(V_{e2} V_{\mu 3} V_{e3}^* V_{\mu 2}^*\right)$$

$V \rightarrow$ PMNS matrix

Leptonic mixing matrix

Can one have a WB invariant sensitive to Majorana-type CP Violation?

The best procedure to find such an invariant is to consider the case of two Majorana neutrinos, where one does know that there is no Dirac-type CP violation, but there is Majorana type CP-Violation

The simplest invariant of this type is:

$$\boxed{I_{\text{Majorana}} \equiv \operatorname{Im} \operatorname{tr}\left(h_\ell\, m^* m m^* h_\ell^* m\right)}$$

$m \equiv m_\nu$

For two generations:

$$\boxed{I_{\text{Maj}} = \frac{1}{2} m_1 m_2 \left(m_2^2 - m_1^2\right)\left(M_\mu^2 - M_e^2\right)^2 \times \sin^2(2\theta) \sin 2\gamma}$$

θ, γ mixing matrix and phase in leptonic mixing matrix

Invariants are very clever!

The invariant $I_{Maj.}$ vanishes in the limit of degenerate neutrinos even for 3 generations.

Question : Can one have a CP-odd WB invariant which does not vanish even in the limit of 3 exactly mass degenerate Majorana neutrinos?

Answer : Yes!

$$\boxed{I_{deg} \equiv \mathrm{tr}\left[(m h m^*), h^*\right]^3}$$

Question: At this stage, one may ask the following question: It is well known that in an extension of the SM with 3 left-handed Majorana neutrinos, there are 3 CP-violating phases in the PMNS matrix. Can one construct a set of 3 CP-odd Invariants, which together provide necessary and sufficient conditions for CP invariance in the Leptonic sector?

Answer: Yes! The 3 CP-odd invariants are:

$$I_1 \equiv \mathrm{tr}\left[h_\nu, h_\ell\right]^3 \ ; \quad I_2 \equiv \mathrm{Im}\,\mathrm{tr}\left(h_\ell m^* m m^* h_\ell^* m\right)$$

$$I_3 \equiv \mathrm{tr}\left[m h m^*, h^*\right]^3$$

Dirac type

The Question of Spontaneous CP Violation (SCPV)

Conditions to have SCPV:

(i) The Lagrangian has to be invariant under a CP transformation (There may be many choices !!)

(ii) There should be no CP transformation which leaves both the Lagrangian and the vacuum invariant.

These conditions become non-trivial in the presence of Family symmetries.

For definiteness, let us consider a model with an arbitrary number of Higgs doublets, invariant under some Family Symmetry S_F. Let us assume that under CP, the Higgs fields Φ_i, transform as:

$$CP \, \Phi_i \, CP^{-1} = U^{CP}_{ij} \, \Phi_j^*$$

where U^{CP} is a unitary matrix acting in Family Space. Note that in general, the Lagrangian allows for more than one CP transformation.

Derivation of a condition for the vacuum to be **CP invariant** : If the vacuum is CP invariant,

$$CP|0\rangle = |0\rangle \quad ,$$

one has

$$\langle 0|\phi_i^0|0\rangle = \langle 0|(CP)^{-1}CP\,\phi_i^0\,CP^{-1}CP|0\rangle$$

$$= \langle 0|CP\phi_i^0(CP)^{-1}|0\rangle = U_{ij}\langle 0|\phi_j^{*}|0\rangle$$

So **one obtains the condition** :

$$\boxed{\langle 0|\phi_i^0|0\rangle = U_{ij}^{CP}\langle 0|\phi_j^{0*}|0\rangle}$$

On the relationship between SCPV and Family Symmetries.

Let us consider the original Lee Model, with 3 quark generations: 2 Higgs doublets, no extra symmetry

As a result: down quarks (or up quarks) receive masses from both ϕ_1, ϕ_2. In the Higgs potential, there are terms like:

$$(\phi_i^\dagger \phi_i)(\phi_1^\dagger \phi_2) + \text{h.c.} \; ; \; (\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_2) + \text{h.c.} \; , \text{etc}$$

which ~~see~~ relative phases. There is a region of the parameter of the potential where the minimum is at

$$\langle \phi_1 \rangle = \begin{bmatrix} 0 \\ v_1 \end{bmatrix} \; ; \; \langle \phi_2 \rangle = \begin{bmatrix} 0 \\ v_2 e^{i\theta} \end{bmatrix} \Rightarrow \text{SCPV}$$

Actually the original T. D. Lee Model with 2 Higgs doublets generates a complex V_{CKM} :

$$M_d = Y_1^d \frac{v_1}{\sqrt{2}} + Y_2^d \frac{v_2}{\sqrt{2}} e^{i\theta}$$

$$M_d M_d^+ = \frac{1}{2}\Big\{ v_1^2 (Y_1 Y_1^+) + v_2^2 (Y_2 Y_2^+) + 2 v_1 v_2 (Y_1 Y_2^+ + Y_2^+ Y_1) c_\theta$$
$$+ 2i\, v_1 v_2 \sin\theta \,(Y_2 Y_1^+ - Y_1 Y_2^+) \Big\}$$

Similar for $M_u M_u^+$

In general this leads to a complex V^{CKM}

But too large FCNC !!

The simplest way of avoiding Higgs mediated FCNC consists of introducing a Z_2 symmetry under which:

$$\phi_1 \to -\phi_1 \;;\quad \phi_2 \to \phi_2 \;;\quad d_R \to -d_R$$

This is sufficient to guarantee Natural Flavour Conservation in the Higgs sector.

S. Glashow, and
S. Weinberg
M. Paschos

Due to the presence of the Z_2 symmetry, the only term in the scalar potential which is sensitive to relative phases is:

$$\lambda (\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_2) + h.c.$$

For λ positive, the minimum of the potential is:

$$\langle 0|\phi_1^0|0\rangle = v_1 \exp(i\,\pi/2)\,;\quad \langle 0|\phi_2^0|0\rangle = v_2$$

Does this lead to maximal SCPV?

No, the minimum is <u>CP invariant</u> and satisfies the condition:

$$\boxed{\langle 0|\phi_i^0|0\rangle = U_{ij}^{CP}\langle 0|\phi_j^*|0\rangle}$$

For the following choice of U^{CP}:

$$U^{CP} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

This is a "generic" phenomena, often (but not always!) the presence of "family symmetries" makes it impossible to generate SCPV in frameworks where, in principle, one can have SCPV. Can one conclude that all "geometrical" vacua are CP invariant?

No, an exception was found in 1984 (G.C.B., J.M. Gérard, W. Grimus)

and other examples were found in the past two years

I. de Medeiros Varzielas, S. King; M. Lindner, etc

see beginning of talk

Question : Is it possible to construct a realistic model, with SCPV, which avoids too large FCNC, while at the same time generating a complex CKM matrix?

Answer Yes, the simplest framework involves the introduction of vector-like quarks.

Vector-like quarks arise in many extensions of the SM: E_6 grand-unified theories

Extra-dimensions models

etc

Seven reasons to consider vector-like quarks

1. They provide a self-consistent framework with naturally small violations of 3×3 unitarity of V^{CKM}.

2. Lead to naturally small Flavour Changing Neutral currents (FCNC) mediated by Z_μ

3. Provide the simplest framework to have Spontaneous CP Violation, with a vacuum phase generating a non-trivial CKM phase.

4. Provide New Physics contributions to $B_d - \overline{B}_d$ and $B_s - \overline{B}_s$ mixings

5. Provide a simple solution to the Strong CP problem, which does not require Axions

6. May contribute to the understanding of the observed pattern of fermion masses and mixing.

7. Provide a framework where there is a common origin for all CP violations:

(i) CP violation in the Quark Sector

(ii) CP Violation in the Lepton Sector, detectable through neutrino oscillations

$U_{e3} \neq 0 \longrightarrow$ Great News

(iii) CP violation needed to generate the Baryon Asymmetry of the Universe (BAU) through Leptogenesis.

Comment :

There is nothing "strange" in having deviations of 3×3 unitarity. The PMNS matrix in the leptonic sector in the context of type-one seesaw (νSM) is not 3×3 unitary !!

Minimal realistic model with Spontaneous CP Violation

$$SM + 3\,\nu_R + \underbrace{D_L, D_R}, \quad S$$

vector like, singlet under SU(2).

complex scalar, singlet under SU(2)

Instead of a $Q = -1/3$ down-type vectorlike quark, one could have, of course, a $Q = 2/3$ up type vector like quark, or a combination of the two.

$\langle S \rangle = V e^{i\varphi}$ $\longrightarrow$ The phase φ is the only source of CP violation and generates a complex V_{CKM}, a complex V_{PMNS} and generates CP violation for Leptogenesis

Some features of the model:

- Naturally small violations of 3x3 unitarity in V_{CKM}
- Naturally suppressed Z-mediated FCNC
- Provides a possible solution to the Strong CP problem.
- Framework for a Common Origin of all CP violations.

- Since we want to have Spontaneous CP Violation we impose CP invariance at the Lagrangian level, so all couplings are real.

- Introduce a Z_4 symmetry on the Lagrangian, under which:

$$\psi_\ell^0 \to i\psi_\ell^0 \; ; \; e_{R_j}^0 \to i e_{R_j}^0 \; ; \; \nu_{R_j}^0 \to i \nu_{R_j}^0$$

lepton doublets

The Z_4 symmetry is crucial to obtain a solution to the Strong CP problem and Leptogenesis

Scalar Potential

The Scalar potential contains various terms which do have phase dependence, like:

$$\left(\mu^2 + \lambda_1 S^*S + \lambda_2 \phi^\dagger\phi\right)\left(S^2 + S^{*2}\right);$$

$$\lambda_3\left(S^4 + S^{*4}\right)$$

There is a range of the parameters of the scalar potential, where the minimum is at:

$$\langle\phi^0\rangle = \frac{v}{\sqrt{2}} \quad ; \quad \langle S\rangle = \frac{V}{\sqrt{2}}\, e^{i\varphi}$$

Most general $SU(2)_L \times U(1) \times SU(3)_c \times Z_4$ invariant Yukawa couplings in the quark sector:

$$\mathcal{L}_Y = -\left(\overline{u}^0\ \overline{d}^0\right)_{L_i} \left[g_{ij}\, d^0_{R_j}\, \Phi + h_{ij}\, \tilde{\Phi}\, u^0_{R_j}\right] - \overline{M}\left(\overline{D}^0_L D^0_R\right)$$

$$-\left(f_i S + f_i' S^*\right) \overline{D}^0 d^0_{R_i} + \text{h.c.}$$

Quark mass matrix for down-type quarks:

$$\left(\overline{d}^0_{1L}\ \overline{d}^0_{2L}\ \overline{d}^0_{3L}\ \overline{D}^0_L\right) \left[\begin{array}{c:c} \underset{\text{real}}{\overset{3\times 3}{m_d}} & 0 \\ \hdashline M_1\, M_2\, M_3 & \overline{M} \end{array}\right] \begin{bmatrix} d^0_{1R} \\ d^0_{2R} \\ d^0_{3R} \\ D^0_R \end{bmatrix}$$

"zero" due to the Z_4 symmetry

$$M_j = \left[f_j V e^{i\varphi} + f_j' V e^{-i\varphi}\right]$$

$$\mathcal{M} \equiv \left[\begin{array}{ccc|c} & m_d & & 0 \\ \hline M_1 & M_2 & M_3 & \bar{M} \end{array}\right]$$

$$\mathcal{U}_L^\dagger \mathcal{M}\mathcal{M}^\dagger \mathcal{U}_L = \mathrm{diag}\left(m_d^2, m_s^2, m_b^2, M_D^2\right)$$

$\mathcal{U} = \begin{bmatrix} K & R \\ S & T \end{bmatrix}$; One can can easily derive:

$$K^{-1} \underbrace{\left[m_d m_d^\dagger - \frac{m_d \overbrace{M^\dagger M}^{\text{complex}} m_d^\dagger}{(MM^\dagger + \bar{M}^2)} \right]}_{m_{eff}\, m_{eff}^\dagger} K = d^2$$

$$d^2 \equiv \mathrm{diag}\left(m_d^2\ m_s^2\ m_b^2\right)$$

$$M_j = f_j V e^{i\varphi} + f_j' V e^{-i\varphi}$$

A remarkable feature of the Model:

The phase φ arising from $\langle S \rangle$ generates a non-trivial CKM phase, provided $|M_j|$ and $\bar{M}$ are of the same order of magnitude (This is natural!!)

$$K^{-1}\, m_{eff}\, m_{eff}^{\dagger}\, K = \mathrm{diag.}\left(m_d^2, m_s^2, m_b^2\right)$$

$$M_j = \left(f_j V e^{i\varphi} + f_j' V e^{-i\varphi}\right)$$

$$m_{eff}\, m_{eff}^{\dagger} = m_d m_d^{\dagger} - \frac{m_d M^{\dagger} M m_d^{\dagger}}{M M^{\dagger} + \bar{M}^2}$$

Deviations of 3×3 unitarity in $V^{CKM} \equiv K$

are naturally small

$$\mathcal{U}_L^\dagger \mathcal{M} \mathcal{M}^\dagger \mathcal{U}_L = \text{diag.}\left(m_d^2, m_s^2, m_b^2, m_D^2\right)$$

$$\mathcal{U}_L = \begin{bmatrix} K & R \\ S & T \end{bmatrix} \quad ; \qquad K^\dagger K + S^\dagger S = 1$$

but $S \approx - \dfrac{M m_d^\dagger K}{\bar{M}^2} \rightarrow O(m/M)$,

so $$K^\dagger K = (\mathbb{1})_3 - O\left(m^2/\bar{M}^2\right)$$

Leptonic Sector

Recall that the leptonic fields transform under Z_4, as

$$\psi_L^0 \to i\,\psi_L^0 \;;\quad e_R^0 \to i\,e_R^0 \;;\quad \nu_R^0 \to i\,\nu_R^0$$

Leptonic Yukawa terms:

$$\mathcal{L}_\ell = \overline{\psi}_L^0\, G_\ell\, \phi\, e_R^0 + \overline{\psi}_0\, G_\nu\, \tilde{\phi}\, e_R + \frac{1}{2}\nu_R^{0T} C\left[f_\nu S + f' S^*\right]\nu_R^0 + h.c.$$

Leptonic mass matrices:

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & m \\ m^T & M \end{bmatrix} ;\quad m_\ell = \frac{v}{\sqrt{2}} G_\ell \qquad m = \frac{v}{\sqrt{2}} G_\nu$$

$$M = \frac{V}{\sqrt{2}} f_\nu^+ \cos\varphi + i f_\nu^- \operatorname{sen}\varphi$$

$$f^\pm = f_\nu \pm f_\nu'$$

Leptonic Mixing

In the weak-basis m_ℓ is diagonal, real, the light neutrino masses and low-energy leptonic mixing are obtained from

$$K^\dagger \left[m \frac{1}{M} m^T \right] K = d_\nu$$

m is real, but since M is a generic complex matrix, m_{eff} is also a generic complex matrix.

As a result: $K \rightarrow$ one Dirac type, two Majorana-type phases

There are also phases relevant for

<u>Leptogenesis</u>

Conclusions

- CP Violation is a central problem in Particle Physics. Apart from being a necessary condition for us to be here....
- CP Violation is closely related to the question of Flavour and a deep knowledge of the Origin of CP Violation requires a deeper knowledge of the Flavour Puzzle.
- Let experiment help us (LHC, linear collider) Future Accelerators...