Multi-Higgs doublet models with symmetry

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Minima of multi-Higgs potentials

Multi-Higgs models: well motivated BSM scenario
Great to improve our understanding of CP violation (CPV)

1. Method to test for Spontaneous CP Violation (SCPV)
2. Method to find the vacuum expectation values (VEVs)
3. For each potential, find the minima and find if there is SCPV

Based on

IdMV, King, Luhn, Neder

http://arxiv.org/abs/1603.06942

http://arxiv.org/abs/1704.06322

http://arxiv.org/abs/1706.07606

See also:

Branco, Rebelo, Silva-Marcos

http://arxiv.org/abs/hep-ph/0502118

Davidson, Haber

http://arxiv.org/abs/hep-ph/0504050

Gunion, Haber

http://arxiv.org/abs/hep-ph/0506227

CP in the scalar sector

Ogreid, Osland, Rebelo
http://arxiv.org/abs/1701.04768
(uses Higgs basis)

Standard form

$$V = \phi^{*a} Y_a^b \phi_b + \phi^{*a}\phi^{*c} Z_{ac}^{bd} \phi_b\phi_d$$

For n Higgs doublets $H_{i\alpha} = (h_{i,1}, h_{i,2})$, where $\alpha = 1, 2$ denotes the $SU(2)_L$ index and i goes from 1 to n

$$\phi = (\varphi_1, \varphi_2, \ldots, \varphi_{2n-1}, \varphi_{2n}) = (h_{1,1}, h_{1,2}, \ldots, h_{n,1}, h_{n,2})$$

Invariance under symmetries is encoded in the Y and Z tensors
For example, $SU(2)_L$ invariance, $-m^2 \sum_\alpha h_\alpha h^{*\alpha}$:
$Y_1^1 = Y_2^2 = -m^2$, $Y_1^2 = Y_2^1 = 0$

Complex conjugation

Complex conjugation raises/lower indices:

$$\phi_a \mapsto (\phi_a)^* \equiv \phi^{*a}$$
$$\phi^{*a} \mapsto (\phi^{*a})^* \equiv \phi_a$$

$V = V^*$ therefore

$$(Y^a_b)^* = Y^b_a$$

and

$$(Z^{ac}_{bd})^* = Z^{bd}_{ac}$$

Basis change (V)

$$\phi_a \mapsto V_a^{a'} \phi_{a'}$$
$$\phi^{*a} \mapsto \phi^{*a'} {V^\dagger}^{a}_{a'}$$

$$Y_a^b \mapsto V_a^{a'} \; Y_{a'}^{b'} \; {V^\dagger}^{b}_{b'}$$
$$Z_{ac}^{bd} \mapsto V_a^{a'} \; V_c^{c'} \; Z_{a'c'}^{b'd'} \; {V^\dagger}^{b}_{b'} \; {V^\dagger}^{d}_{d'}$$

General CP transformation (X)

$$\phi_a \mapsto \phi^{*a'} X^{a}_{a'}$$
$$\phi^{*a} \mapsto X^{\dagger}{}^{a'}_{a} \phi_{a'}$$

E.g. for 3 scalars (ϕ_1, ϕ_2, ϕ_3) one may have:

$$X_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$(\phi_1, \phi_2, \phi_3) \mapsto X_{23}(\phi^{*1}, \phi^{*2}, \phi^{*3}) = (\phi^{*1}, \phi^{*3}, \phi^{*2})$$

Potential $\Delta(6n^2)$, $n > 3$

$$V_{\Delta(6n^2)}(\varphi) = V_0(\varphi) \equiv -\, m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

V_0 is invariant under both CP_0 and CP with X_{23} ($i = 1, 2, 3$)

A_4 potential, CP conserving with X_{23}

$$V_{A_4}(\varphi) = V_0(\varphi) + c\left(\varphi_1\varphi_1\varphi^{*3}\varphi^{*3} + \varphi_2\varphi_2\varphi^{*1}\varphi^{*1} + \varphi_3\varphi_3\varphi^{*2}\varphi^{*2}\right) + c^*\left(\varphi^{*1}\varphi^{*1}\varphi_3\varphi_3 + \varphi^{*2}\varphi^{*2}\varphi_1\varphi_1 + \varphi^{*3}\varphi^{*3}\varphi_2\varphi_2\right) ,$$

$V_{A_4}(\varphi)$ is invariant under CP with X_{23} for any c

$(\phi_1, \phi_2, \phi_3) \mapsto X_{23}(\phi^{*1}, \phi^{*2}, \phi^{*3}) = (\phi^{*1}, \phi^{*3}, \phi^{*2})$

Potential $\Delta(27)$

$$V_{\Delta(27)}(\varphi) = V_0(\varphi) + d\left(\varphi_1\varphi_1\varphi^{*2}\varphi^{*3} + \text{cycl.}\right) + d^*\left(\varphi^{*1}\varphi^{*1}\varphi_3\varphi_2 + \text{cycl.}\right)$$

$V_{\Delta(27)}(\varphi)$ is in general CP Violating (CPV)
Invariant under both CP_0 and CP with X_{23} only if d is real.

Basis invariants (no Vs, absolutely no Xs)

Basis invariants: contract all the indices
(this cancels V and X dependence)

Examples

$$Z^{ab}_{ab} \text{ and } Z^{ab}_{ba}$$

Of 24 possibilities with two Z, just two independent ones, e.g.:

$$Z^{ab}_{bd} Z^{cd}_{ac} \text{ and } Z^{ab}_{cd} Z^{cd}_{ab}$$

Many invariants are equivalent, or products of smaller invariants

From basis invariants to CPIs

Many basis invariants are CP-even (all examples so far)

Construct CP-odd invariant (CPI)

$$\mathcal{I} = I - I^*$$

Complex conjugation swaps upper and lower indices: diagrams and other technique help find cases with $I \neq I^*$

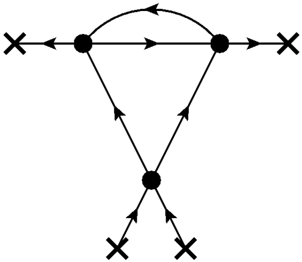
These CPIs can test if a potential has explicit CPV (without worrying about Xs)

SCPIs

$$\mathcal{J} = J - J^*$$

Involves the VEVs $v_a \equiv \langle \phi_a \rangle$
These SCPIs can test if a VEV has SCPV
(in a specific CP conserving potential)

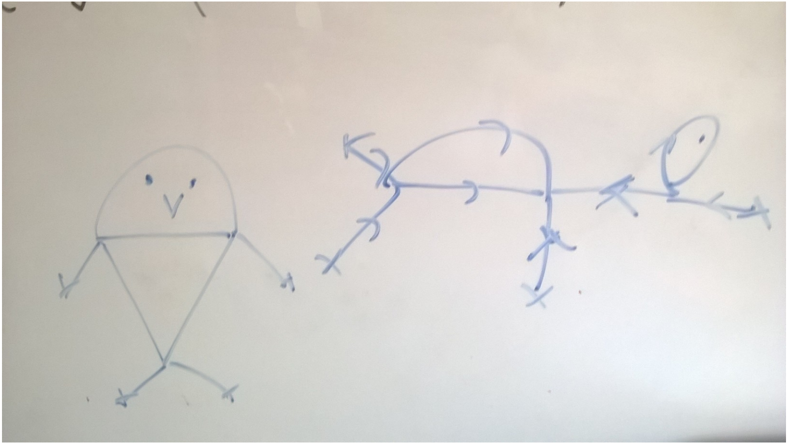
$$J^{(3,2)} \equiv Z^{a_1 a_2}_{a_4 a_5} Z^{a_3 a_4}_{a_2 a_6} Z^{a_5 a_6}_{a_7 a_8} v_{a_1} v_{a_3} v^{* a_7} v^{* a_8} =$$



The real (well, complex) Penguin diagram!

$$J^{(3,2)} = Z^{a_1 a_2}_{a_4 a_5} Z^{a_3 a_4}_{a_2 a_6} Z^{a_5 a_6}_{a_7 a_8} v_{a_1} v_{a_3} v^{*a_7} v^{*a_8} =$$

Whiteboard...



SCPI for $V(H_1, H_2)$ with CP_0

SCPV makes sense if potential is CP conserving

$$\mathcal{J}^{(3,2)}_{CP_0} = F(a_1, a_2, b, b', c_1, c_2, d, \langle h_{i,j} \rangle)[\langle h_{2,1} \rangle \langle h_{1,1} \rangle^* + \langle h_{2,2} \rangle \langle h_{1,2} \rangle^* - h.c.]$$

SCPV depends on relative phase between $\langle H_1 \rangle$ and $\langle H_2 \rangle$
(note $\langle h_{1,1} \rangle = \langle h_{2,1} \rangle = 0$ for charge preserving VEVs)

Potential $\Delta(27)$ (reminder)

$$V_{\Delta(27)}(\varphi) = V_0(\varphi) + d\left(\varphi_1\varphi_1\varphi^{*2}\varphi^{*3} + \text{cycl.}\right)$$
$$+ d^*\left(\varphi^{*1}\varphi^{*1}\varphi_3\varphi_2 + \text{cycl.}\right)$$

$V_0(\varphi)$ has two quartics, with coefficients r and s

$V_{\Delta(27)}(\varphi)$ has CPV in general, invariant under CP_0 if d is real

SCPI for $V_{\Delta(27)}(\varphi)$ (general structure)

$$\mathcal{J}^{(3,2)} = A(d)Q(|v_i|) + B(d,s)R(v_i)$$

$A(d) = 0$ for some CP syms.,
$R(v_i) \neq 0$ for VEVs that violate those

$B(d,s) = 0$ for some other CP syms.,
$Q(|v_i|) \neq 0$ for VEVs that violate those

SCPI for $V_{\Delta(27)}(\varphi)$

$$\mathcal{J}^{(3,2)} = \frac{1}{4}(d^{*3} - d^3)Q(|v_i|)$$
$$+\frac{1}{2}(dd^{*2} - 2d^* s^2 + d^2 s)(v_2 v_3 v_1^{*2} + v_1 v_3 v_2^{*2} + v_1 v_2 v_3^{*2})$$
$$-\frac{1}{2}(d^2 d^* - 2ds^2 + d^{*2} s)(v_2^* v_3^* v_1^2 + v_1^* v_3^* v_2^2 + v_1^* v_2^* v_3^2) \ .$$

Impose trivial CP, CP_0: forces $\mathrm{Arg}(d) = 0$

SCPI for $V_{\Delta(27)}(\varphi)$ with CP_0

$$\mathcal{J}^{(3,2)}_{CP_0} = \frac{1}{2}(d^3 - 2ds^2 + d^2 s)$$

$$\left[(v_2 v_3 v_1^{*2} + v_1 v_3 v_2^{*2} + v_1 v_2 v_3^{*2}) - (v_2^* v_3^* v_1^2 + v_1^* v_3^* v_2^2 + v_1^* v_2^* v_3^2)\right]$$

VEVs: $\omega \equiv e^{i2\pi/3}$ is a geometrical phase in the following VEVs

$\langle\varphi\rangle = (1, \omega, \omega^2)$ conserves CP

$\langle\varphi\rangle = (\omega, 1, 1)$ violates CP with a geometrical phase:
Spontaneous Geometrical CP Violation (SGCPV)

Decreasing symmetry

Using a method of decreasing symmetry, found minima for potentials with two triplets of $\Delta(3n^2)$ and $\Delta(6n^2)$ (with $n = 2, 3$ and $n > 3$)

For one triplet of $A_4, S_4, \Delta(27), \Delta(54)$,
our simple method reproduces results in
I.P. Ivanov, C.C. Nishi
`https://arxiv.org/abs/1410.6139`

For $\Delta(6n^2)$ with $n > 3$ we get

$$v_1(1,0,0), \quad v_2(1,1,0), \quad v_3(1,1,1)$$

as follows...

Minima for one triplet of $\Delta(6n^2)$, $n > 3$

$$V_0(\varphi) \equiv -\, m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$$V_0 = V_{U(3)} + V_{\Delta(6\infty^2)\times U(1)}$$

$U(3)$ - single orbit connected by the continuous symmetry

Orbit split

$$V_{\Delta(6\infty^2)\times U(1)} = s(\varphi_1\varphi^{*1}\varphi_1\varphi^{*1} + \varphi_2\varphi^{*2}\varphi_2\varphi^{*2} + \varphi_3\varphi^{*3}\varphi_3\varphi^{*3})$$

splits the single $U(3)$ orbit into 3 classes of directions

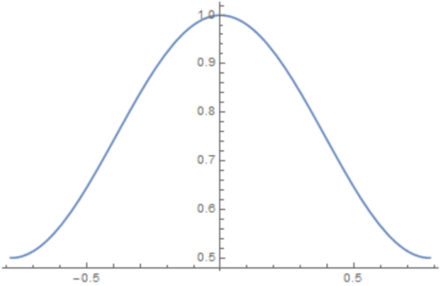
$$\{\begin{pmatrix} e^{i\eta} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ e^{i\eta} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ e^{i\eta} \end{pmatrix}\}, \{\begin{pmatrix} e^{i\eta} \\ e^{i\zeta} \\ 0 \end{pmatrix}, \text{permut.}\}, \{\begin{pmatrix} e^{i\eta} \\ e^{i\zeta} \\ e^{i\theta} \end{pmatrix}, \text{permut.}\}.$$

2 generation example

Extrema of $|\varphi_1|^4 + |\varphi_2|^4$ for fixed magnitude v:
Positive: $\propto (1,1)/\sqrt{2} \to V \sim +2v^4/4$
Negative: $\propto (0,1) \to V \sim -v^4$

2 generation example plot

```
Plot[Cos[θ]^4 + Sin[θ]^4, {θ, -π/4, π/4}]
```




```
Minimize[Cos[θ]^4 + Sin[θ]^4, θ]
```

$\{\mathrm{Cos}[2\,\mathrm{ArcTan}[1-\sqrt{2}]]^4 + \mathrm{Sin}[2\,\mathrm{ArcTan}[1-\sqrt{2}]]^4, \{\theta \to -2\,\mathrm{ArcTan}[1-\sqrt{2}]\}\}$

$\mathrm{FullSimplify}[\mathrm{Pi}/4 == -2\,\mathrm{ArcTan}[1-\sqrt{2}]]$

```
True
```

Minima for two triplets of $\Delta(3n^2)$, $n > 3$

For two triplets, method reveals some candidate VEVs like

$$(e^{i\eta}, 0, 0), (e^{i\eta'}, 0, 0) \rightarrow (1, 0, 0), (1, 0, 0)$$
$$(e^{i\eta}, e^{i\zeta}, e^{i\theta}), (e^{i\eta'}, e^{i\zeta'}, e^{i\theta'}) \rightarrow (1, 1, 1), (1, e^{i\zeta'}, e^{i\theta'}).$$

For last case, phases are physical.
Minimize phase-dependent part of potential to fix the phases.

Minima with geometrical phases

Found new VEVs with geometrical phases

$$(1, 1, 1), (1, \omega, \omega^2) \text{ and } (1, 1, 1), (1, \omega^2, \omega).$$

Do they SCPV?

Use SCPI to test for SCPV

$$\mathcal{J}^{(3,2)} = -\frac{1}{16}\tilde{s}_2[\tilde{r}_2(-4s - 4s' + 2\tilde{s}_1 - \tilde{s}_2 + 3\tilde{r}_2) - \tilde{s}_3^2]W_{CP_0}$$
$$-\frac{1}{8}i\tilde{s}_3\left[\tilde{s}_3^2 - 3\tilde{r}_2^2\right]W_{CP_{23}}$$
$$-\frac{1}{16}i\tilde{s}_2\tilde{s}_3[-4s - 4s' + 2\tilde{s}_1 - \tilde{s}_2](...)$$

where

$$W_{CP_0} \equiv [(v_1 {v'}_1^* {v'}_2 v_2^* + v_2 {v'}_2^* {v'}_3 v_3^* + v_3 {v'}_3^* {v'}_1 v_1^*) - h.c.]$$

and CP_0 forces $\tilde{s}_3 = 0$

New SGCPV

$$W_{CP_0} \equiv [(v_1 v'^*_1 v'_2 v^*_2 + v_2 v'^*_2 v'_3 v^*_3 + v_3 v'^*_3 v'_1 v^*_1) - h.c.]$$

$$W^{(3,2)}_{CP_0}[v(1,1,1), v'(1,\omega,\omega^2)] = 3(\omega-\omega^2) v^2 v'^2 \neq 0$$

Found new cases of SGCPV in $\Delta(3n^2) \times CP_0$, for 6HDM

Summary

- Developed formalism for CPIs and SCPIs
- Methods for explicit and spontaneous CP violation are valid for any potential when brought to standard form
- Verified the CP properties of 3HDM and 6HDM symmetric under $\Delta(3n^2)$ and $\Delta(6n^2)$ groups, for new minima and found new cases with SGCPV

Potential 2HDM

$$\begin{aligned} V(H_1, H_2) \quad = \quad & m_1^2\ H_1^\dagger H_1 + m_{12}^2\ e^{i\theta_0}\ H_1^\dagger H_2 + m_{12}^2\ e^{-i\theta_0}\ H_2^\dagger H_1 + m_2^2\ H_2^\dagger H_2 + \\ & + a_1\ \left(H_1^\dagger H_1\right)^2 + a_2\ \left(H_2^\dagger H_2\right)^2 \\ & + b\ \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + b'\ \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) + \\ & + c_1\ e^{i\theta_1}\ \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_1\right) + c_1\ e^{-i\theta_1}\ \left(H_1^\dagger H_1\right)\left(H_1^\dagger H_2\right) + \\ & + c_2\ e^{i\theta_2}\ \left(H_2^\dagger H_2\right)\left(H_2^\dagger H_1\right) + c_2\ e^{-i\theta_2}\ \left(H_2^\dagger H_2\right)\left(H_1^\dagger H_2\right) + \\ & + d\ e^{i\theta_3}\ \left(H_1^\dagger H_2\right)^2 + d\ e^{-i\theta_3}\ \left(H_2^\dagger H_1\right)^2 . \end{aligned}$$

Potentials $\Delta(3n^2)$, A_4, $\Delta(27)$

$$V_0(\varphi) = V_{\Delta(3n^2)}(\varphi) = - m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$$\begin{aligned} V_{A_4}(\varphi) = V_0(\varphi) &+ c \left(\varphi_1 \varphi_1 \varphi^{*3} \varphi^{*3} + \varphi_2 \varphi_2 \varphi^{*1} \varphi^{*1} + \varphi_3 \varphi_3 \varphi^{*2} \varphi^{*2} \right) \\ &+ c^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_3 + \varphi^{*2} \varphi^{*2} \varphi_1 \varphi_1 + \varphi^{*3} \varphi^{*3} \varphi_2 \varphi_2 \right) \;, \end{aligned}$$

$$\begin{aligned} V_{\Delta(27)}(\varphi) = V_0(\varphi) &+ d \left(\varphi_1 \varphi_1 \varphi^{*2} \varphi^{*3} + \text{cycl.} \right) \\ &+ d^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_2 + \text{cycl.} \right) \end{aligned}$$

Potential $\Delta(3n^2)$

$$V_{\Delta(3n^2)}(\varphi) = -m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$V_{\Delta(3n^2)}(\varphi)$ is invariant under both CP_0 and CP with X_{23}

Potential $\Delta(27)$, doublets

$$
\begin{aligned}
V_0(H) \quad &= \quad -m_h^2 \sum_{i,\alpha} h_{i\alpha} h^{*i\alpha} + \sum_{i,j,\alpha,\beta} \left[r_1 (h_{i\alpha} h^{*i\alpha})(h_{j\beta} h^{*j\beta}) + r_2 (h_{i\alpha} h^{*i\beta})(h_{j\beta} h^{*j\alpha}) \right] \\
&\quad + s \sum_{i,\alpha,\beta} (h_{i\alpha} h^{*i\alpha})(h_{i\beta} h^{*i\beta})
\end{aligned}
$$

$$
V_{\Delta(27)}(H) = V_0(H) \; + \sum_{\alpha,\beta} \left[d \left(h_{1\alpha} h_{1\beta} h^{*2\alpha} h^{*3\beta} + \text{cycl.} \right) + \text{h.c.} \right]
$$

Potential $\Delta(3n^2)$, $n > 3$ with two triplets

$$
\begin{aligned}
V_1(\varphi,\varphi') \quad = \quad & + \tilde{r}_1 \left(\sum_i \varphi_i \varphi^{*i}\right)\left(\sum_j \varphi'_j \varphi'^{*j}\right) + \tilde{r}_2 \left(\sum_i \varphi_i \varphi'^{*i}\right)\left(\sum_j \varphi'_j \varphi^{*j}\right) \\
& + \tilde{s}_1 \sum_i \left(\varphi_i \varphi^{*i} \varphi'_i \varphi'^{*i}\right) \\
& + \tilde{s}_2 \left(\varphi_1 \varphi^{*1} \varphi'_2 \varphi'^{*2} + \varphi_2 \varphi^{*2} \varphi'_3 \varphi'^{*3} + \varphi_3 \varphi^{*3} \varphi'_1 \varphi'^{*1}\right) \\
& + i\, \tilde{s}_3 \left[\left(\varphi_1 \varphi'^{*1} \varphi'_2 \varphi^{*2} + \text{cycl.}\right) - \left(\varphi^{*1} \varphi'_1 \varphi'^{*2} \varphi_2 + \text{cycl.}\right)\right].
\end{aligned}
$$

$$
V_{\Delta(3n^2)}(\varphi,\varphi') \quad = \quad V_0(\varphi) + V'_0(\varphi') + V_1(\varphi,\varphi'),
$$