A new twist on multifield inflation

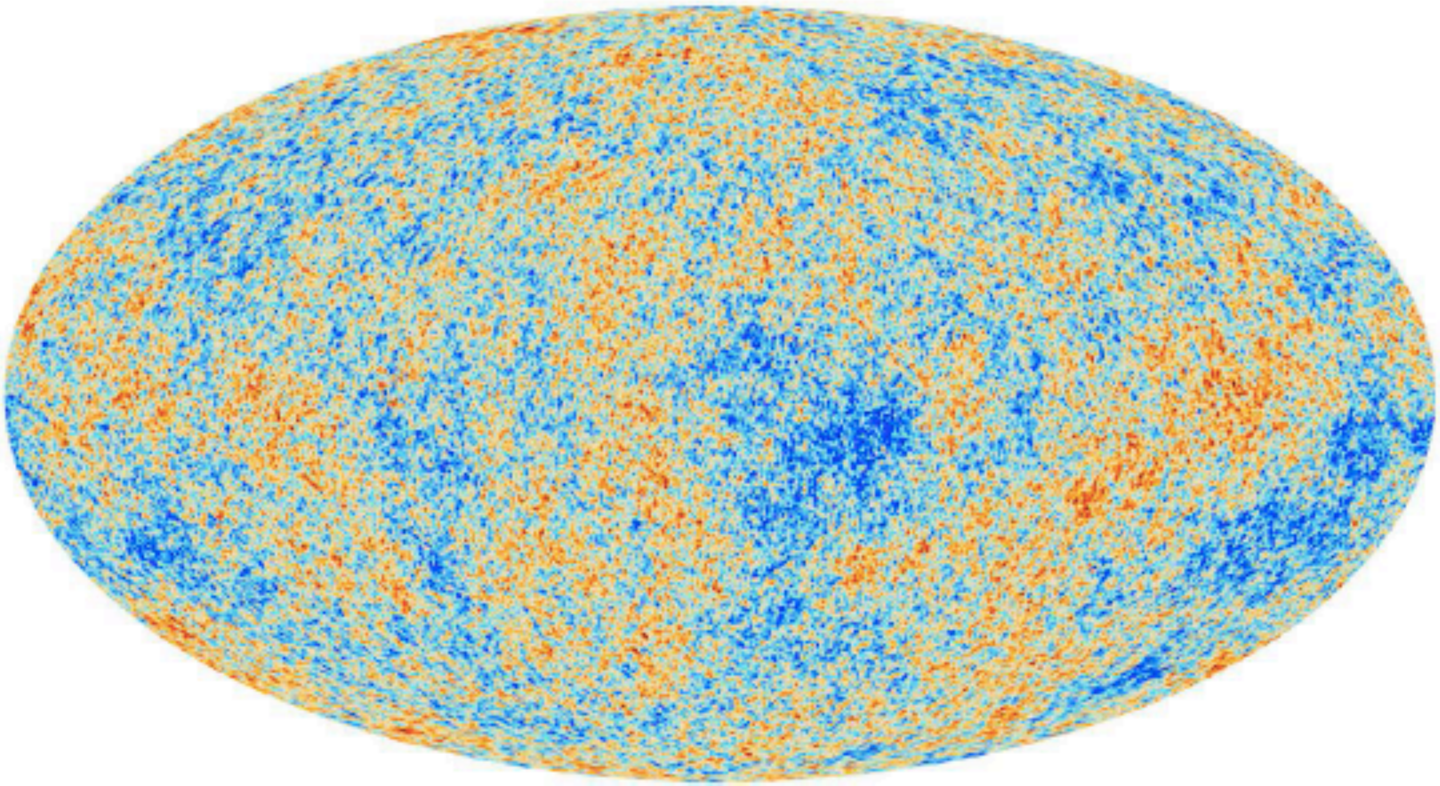
with **V. Atal, C. Germani, G. Palma** JCAP 2017 (1607.08609)

(building on earlier work also with
J-O. Gong, S. Hardeman, B. Hu, P. Ortiz, S. Patil, J. Torrado, Y. Welling)

Ana Achúcarro (U. Leiden/UPV-EHU Bilbao) **Planck 2017**, Warsaw, 23/5/17

the other PLANCK 2017…

THE CMB AS SEEN BY PLANCK



Next (final) data release in a few months

Inflation

(accelerated expansion)

-dilutes massive relics (e.g. monopoles)

-solves horizon problem

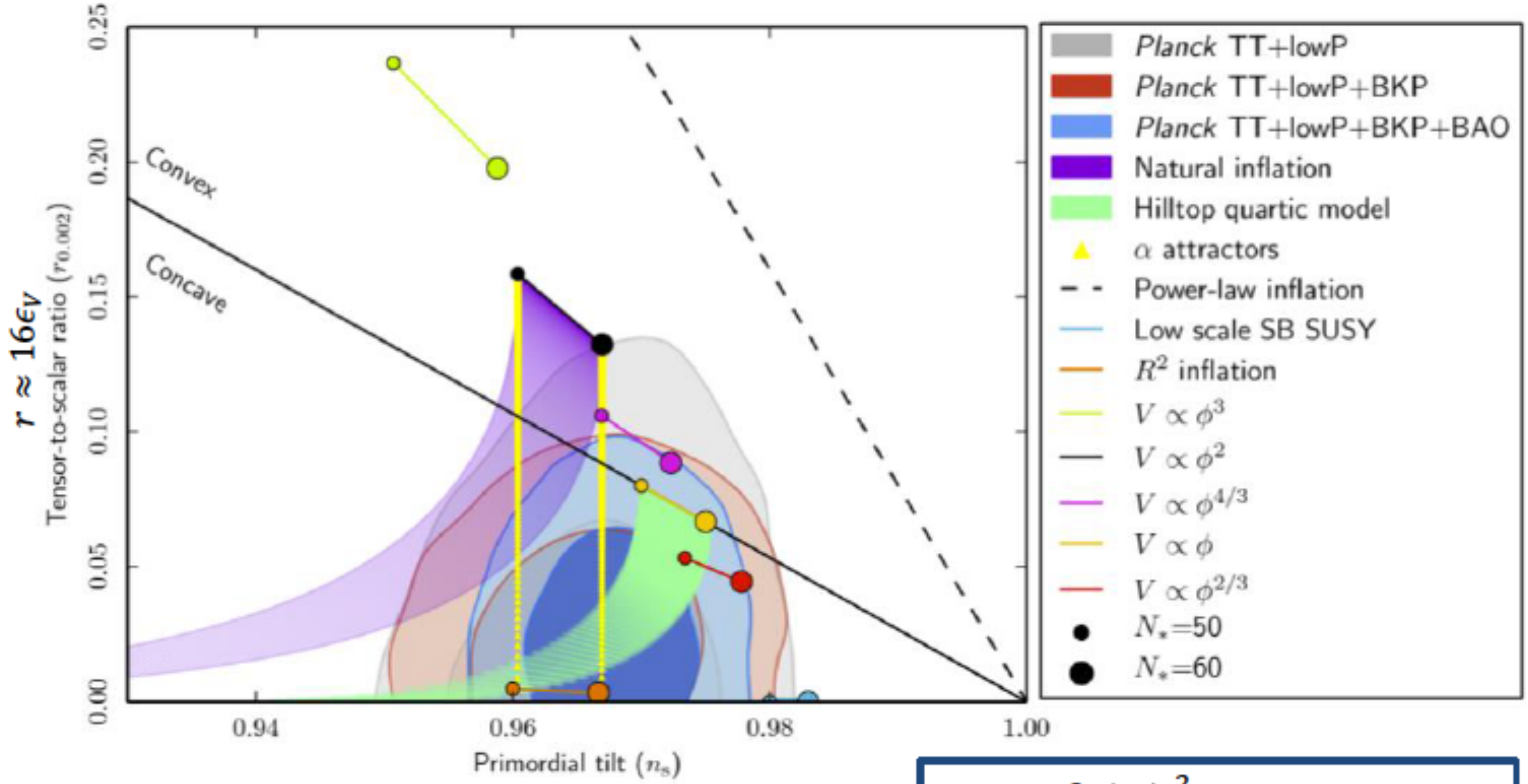
-solves flatness problem if it lasts long enough (~ 55 e-folds)

-gives mechanism for approximately scale invariant primordial inhomogeneities (from quantum fluctuations)

-produces a background of gravitational waves

Planck 2015

Ade et al 1502.01589



$$n_s \approx 1 - 6\epsilon_V + 2\eta_V$$

$$\boxed{\epsilon_V = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta_V = M_P^2 \frac{V''}{V}}$$

A **single scalar field**, minimal coupling to gravity,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{pl}}^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Canonical kinetic terms

Bunch-Davies vacuum for the fluctuations ("Minkowski on short scales")

Slow roll: $\epsilon \equiv -\frac{\dot{H}}{H^2}$ **<< 1** **"approximately de Sitter expansion"**

sustained for at least O (50-60) e-folds of expansion: $\frac{\dot{\epsilon}}{H\epsilon} << 1$

Evolution is adiabatic

Single field slow roll inflation, vanilla

Single-field slow-roll inflation with **canonical** kinetic terms
predicts perturbations that are

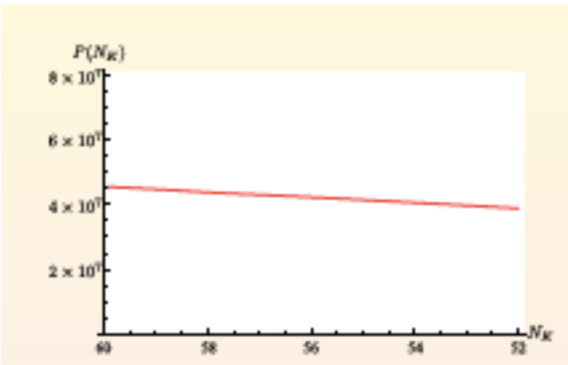
and Bunch Davies vacuum,
minimal coupling to gravity

adiabatic
near scale-invariant
almost gaussian

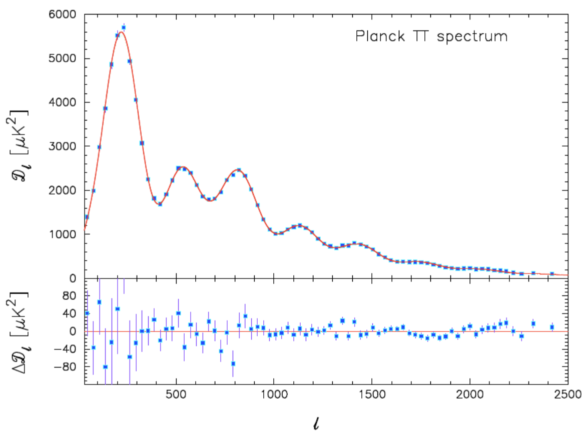
self-interactions (in the potential) are limited by the slow roll condition

Bispectrum is negligible, O(slow roll)

Primordial power spectrum

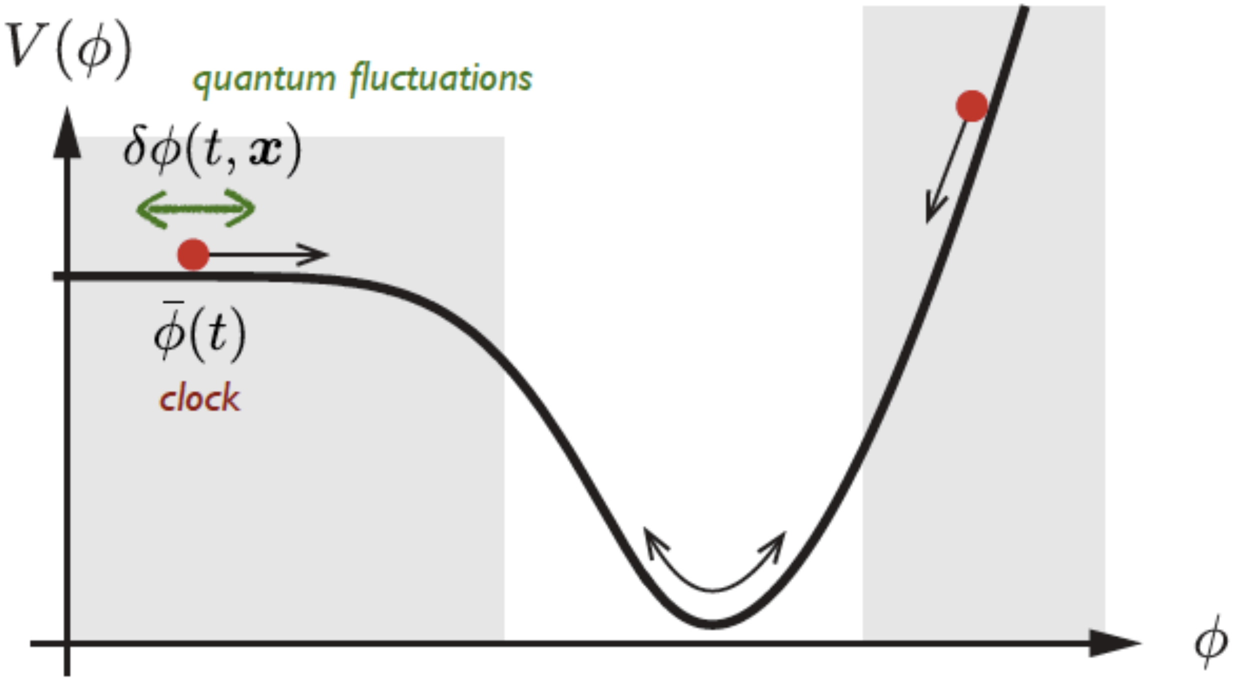


CMB power spectrum



Baumann's 2013 lectures @ Varenna

The quantum origin of density perturbations is quite intuitive:



vacuum fluctuations spread the inflaton vev ...

... which translates into density fluctuations after inflation

$$\delta\phi(\boldsymbol{x}) \longrightarrow \delta t(\boldsymbol{x}) \longrightarrow \delta\rho(\boldsymbol{x}) \longrightarrow \delta T(\boldsymbol{x})$$

... which induces a local time delay for the end of inflation

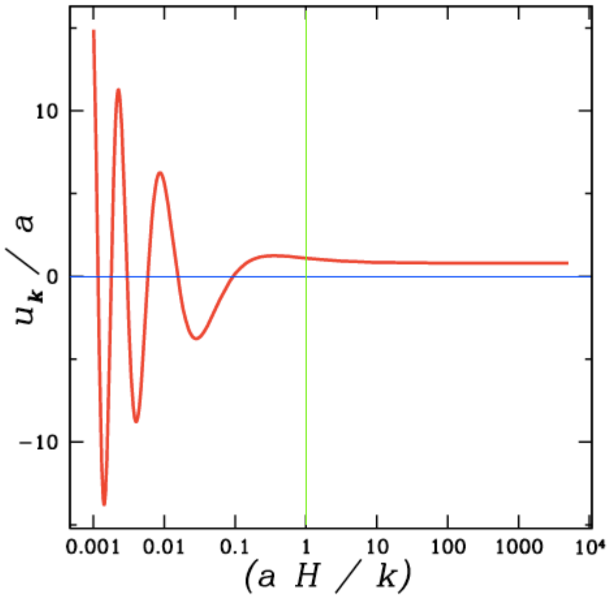
... which become the CMB anisotropies.

Fluctuation equations are like a simple harmonic oscillator
with friction from the expansion

$$\delta\ddot{\phi}_k + 3H\,\delta\dot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k + \ldots = 0$$

large k "inside the horizon" : friction negligible

small k "outside the horizon" : friction dominates,



fluctuations freeze out at

$$k/a_\star = H_\star$$

Can trade fluctuations in scalar field for fluctuations in spatial curvature

$$\delta\phi = 0\,, \qquad g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]\,, \qquad \partial_i h_{ij} = h^i_i = 0$$

In single-clock inflation these are **conserved on superhorizon scales**
(regardless of the details of reheating − conservation of energy momentum)

Inflation in multi-scalar theories

single-field or multi-field ?

The problem:

Multifield inflation with light fields (e.g. moduli) has at least two undesirable properties for model building

-potentially large **isocurvature** perturbations

-curvature perturbation is **not conserved** on superhorizon scales

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Inflation in BSM scenarios includes the challenge to find

a "single field"- looking needle

(almost scale invariant, adiabatic, gaussian perturbations)

in a "multifield" haystack.

NB: opens the possibility to detect other fields interacting with the inflaton

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in a "multifield" haystack.

NB: opens the possibility to detect other fields interacting with the inflaton

Usual approach: stabilize spectators and **integrate them out**

(more on this in a moment)

But maybe there is another possibility

The problem:

Multifield inflation with light fields (e.g. moduli) has at least two undesirable properties for model building

-potentially large **isocurvature** perturbations

-curvature perturbation is **not conserved** on superhorizon scales

The message of this talk:

There is a regime in which **these two "problems" cancel each other** on observable scales: "massless"**(*)** isocurvature perturbation freezes and sources curvature. Sustained over many efolds. We get

-scale invariant spectra

-suppressed isocurvature

-suppressed tensor-to-scalar ratio

Requires sustained turns **(**)**

(*) there is a new "Stueckelberg-like" shift symmetry involving both R and σ

(**) can be inherited from symmetries of the background

(e.g. "axion-dilaton" hyperbolic coset spaces)

First, a comment about integrating out heavy fields.

If there is a large separation of scales we can
integrate out heavy modes to get
effectively single field inflation

However, to get the right observables, one has to
pay attention to derivative interactions

HEAVY vs LIGHT – what is the “right” definition ?

1) Calculate the mass matrix from V

2) Calculate the mass matrix of fluctuations about the classical solution

3) Calculate natural frequencies of fluctuations - fast vs slow

All three agree on a static background -- otherwise, not, in general

On a turning trajectory:

1) If the heavy field has mass^2 = M^2 on a straight trajectory

2) The heavy fluctuation has mass^2 = $$M_{\rm eff}^2 = M^2 - \dot{\theta}^2$$

3) The fast mode has frequency $$\omega_{heavy}^2 = M_{eff}^2 c_s^{-2} = M^2 + 3\dot{\theta}^2$$

(long wavelengths)

AA Atal Cespedes Gong Palma Patil 1205.0710
Castillo Koch Palma 1312.3338

REVIEWS:
Chluba Hamann Patil 1505.01834
Welling MSc Thesis arxiv 2015

If $M^2 \gg H^2$ **,**
a sufficiently heavy field can **still** be integrated out − but…

get an effective single-field theory with a **reduced speed of sound** for the adiabatic mode

$$c_s^{-2} = 1 + \frac{4\dot{\theta}^2}{M_{\rm eff}^2}$$

effective mass of heavy field at turn

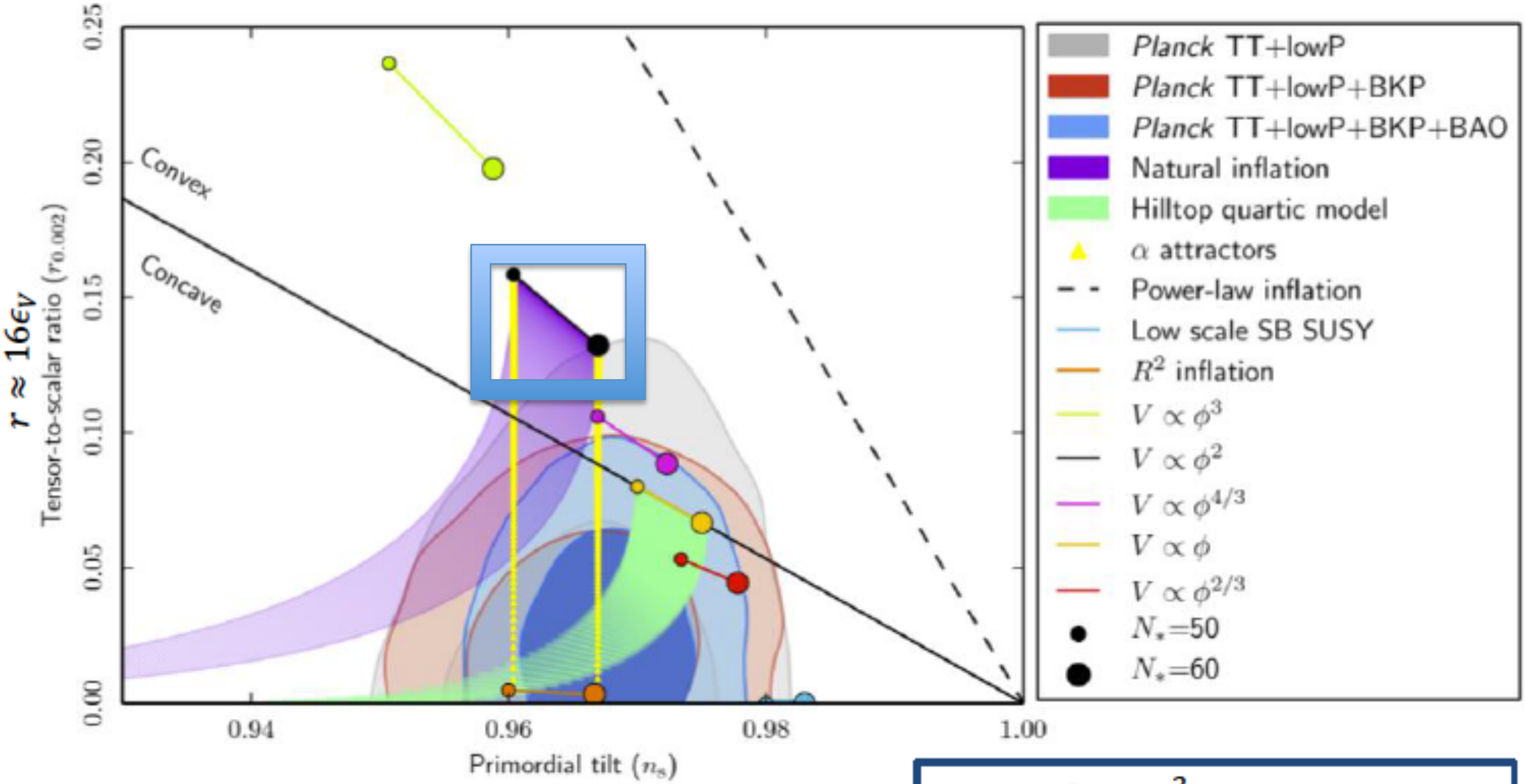
$$M_{\rm eff}^2 = M^2 - \dot{\theta}^2$$

mass of heavy field on straight trajectory
(including effect of curvature of field manifold)

and this affects the predictions for the inflationary perturbations

$m^2\phi^2$ inflation is practically ruled out

Ade et al 1502.01589



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Evolution is adiabatic

Single field slow roll inflation, vanilla

The flatness of the Inflaton potential / the mass of the inflaton should be protected by some approx. **shift symmetry**

Suppose the shift symmetry is **U(1)**, inherited from the background.

Think of the simplest two-field completion

$$\Phi(t,\vec{x}) = \rho(t,\vec{x})e^{i\theta(t,\vec{x})}$$

(pseudo) Goldstone boson
(inflaton)

very heavy field
(stabilized at its adiabatic vacuum)

(the inflaton is the phase of some complex field)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\Phi^\dagger\partial^\mu\Phi - V(|\Phi|)\right] \quad \mathbf{+ \ ...}$$

soft U(1) breaking
(inflaton potential)

Note: Lorentz invariance,
canonical kinetic terms

Single field slow roll inflation, pistachio

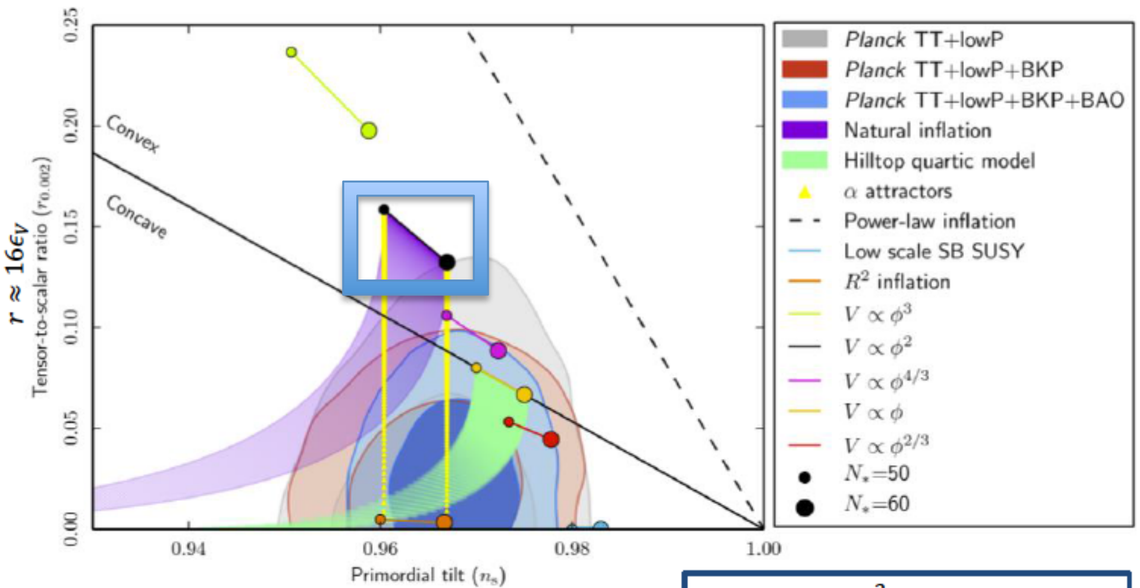
$$\mathcal{L} = \frac{1}{2}\partial_\nu \rho \partial^\nu \rho + \frac{1}{2}\rho^2 \partial_\nu \theta \partial^\nu \theta - \frac{m_\rho^2}{2}(\rho - \rho_0)^2 - V(\theta).$$

$$\Lambda^4 \theta^2$$

Very large mass (>> H)

$$\mathcal{L} = \frac{1}{2}\rho_0^2 \partial_\nu \theta \partial^\nu \theta - V(\theta)$$

$$\phi = \rho_0 \theta \qquad m_\phi^2 = 2\Lambda^4/\rho_0^2$$



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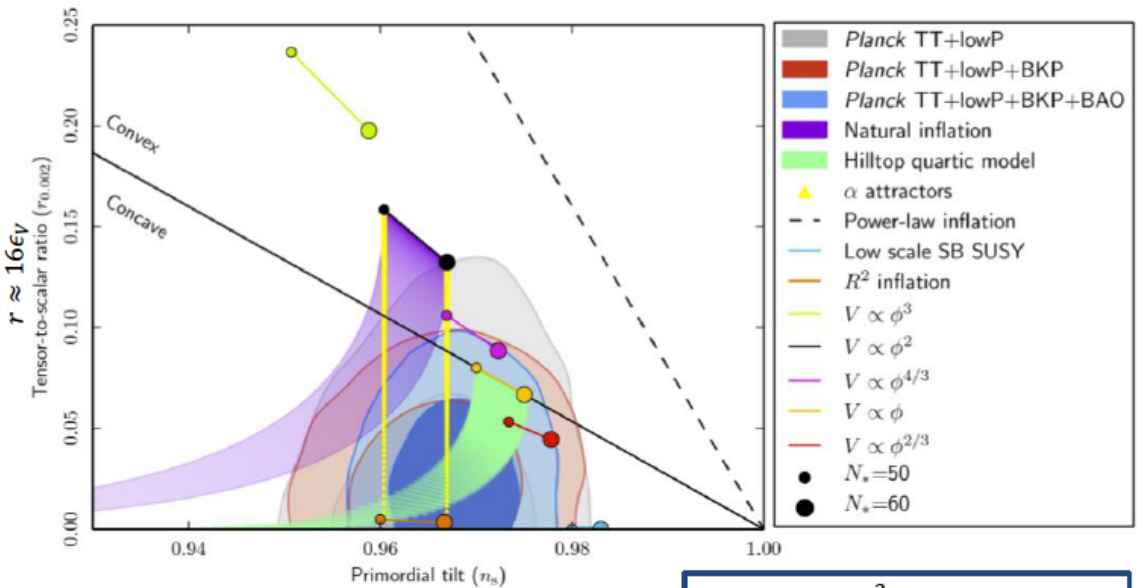
this model is consistent with the observations
If the vev of the heavy field is subPlanckian

$C_s < 1$

Planck 2015



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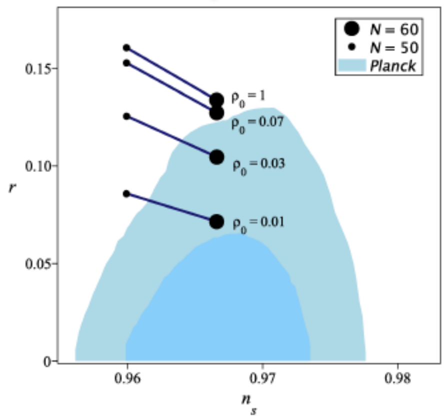

Cs = 1

Cs ~ 0.55

AA Atal Welling 1503.07486

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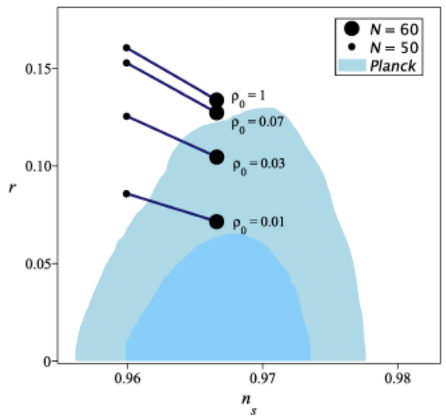
Integrating out the heavy mode produces
a (mild) reduction in the speed of sound



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from AA Atal Welling 1503.07486

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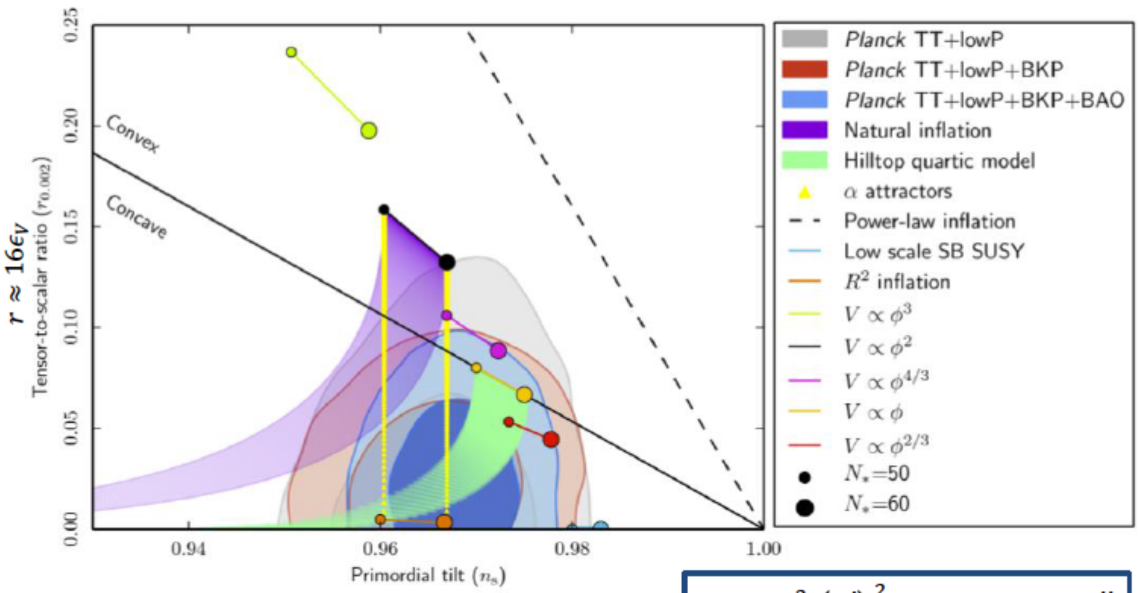
(inconsistent truncation)

$$\mathcal{L} = \frac{1}{2}\rho_0^2 \partial_\nu \theta \partial^\nu \theta - V(\theta)$$

truncating

vs

integrating out



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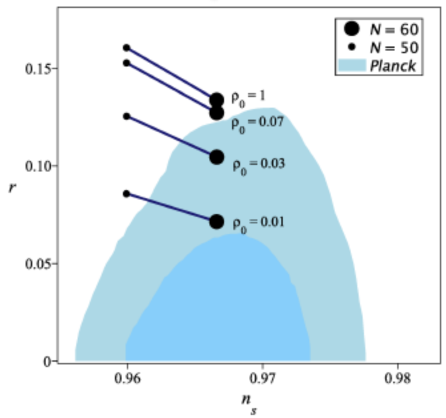


AA Atal Welling 1503.07486

The naïve truncation misses the effect of derivative interactions

The naïve truncation misses the effect of derivative interactions

NB − reduction in speed of sound can win over “flattening” of potential
Dong Hong Silverstein Westphal arxiv 1011.4521
(see discussion in **AA, Atal, Welling 1503.07486**)

In a Lorentz-invariant background we expect
derivative interactions to first enter at cubic order, with dimension five.

$$\phi(\partial\phi)^2$$

In inflation we are interested in the perturbations. Background is not Lorentz-inv
Derivative interactions enter at quadratic order, with dimension three

$$\dot{\mathcal{R}}\sigma$$

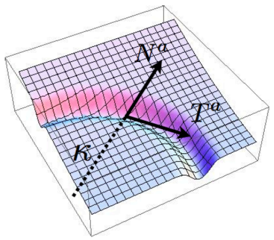
The coupling constant introduces an important mass scale in the problem
N.B.: It does **not** require non-canonical kinetic terms

Derivative interactions (at quadratic level) are **unavoidable** at tree level

unless the background inflationary trajectory **is geodesic**

--coupling “constant” is proportional to rate of turning **--**

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,R - \int d^4x\sqrt{-g}\left[\frac{1}{2}\gamma_{ab}g^{\mu\nu}\nabla_\mu\phi^a\nabla_\nu\phi^b + V\right]$$



$$S = \int d^4x a^3\left[\epsilon\dot{\mathcal{R}}^2 - \frac{\epsilon}{a^2}(\nabla\mathcal{R})^2 + 2\sqrt{2\epsilon}\Omega\dot{\mathcal{R}}\sigma + \frac{1}{2}\dot{\sigma}^2 - \frac{1}{a^2}(\nabla\sigma)^2 - \frac{1}{2}M_{\text{eff}}^2\sigma^2\right]$$

$$m_\sigma^2 \equiv N^a N^b(V_{ab} - \Gamma^c_{ab}V_c) + \epsilon H^2\mathbb{R} - \Omega^2$$

“effective mass”

not the mass of any physical mode !

Derivative interactions (at quadratic level) – effect on the spectrum (two fields)

$$\boxed{\begin{aligned} \ddot{\mathcal{R}} + \omega_{\mathcal{R}}^2 \mathcal{R} &= \alpha\dot{\sigma} \\ \ddot{\sigma} + \omega_\sigma^2\, \sigma &= -\alpha\dot{\mathcal{R}} \end{aligned}}$$

c.f.

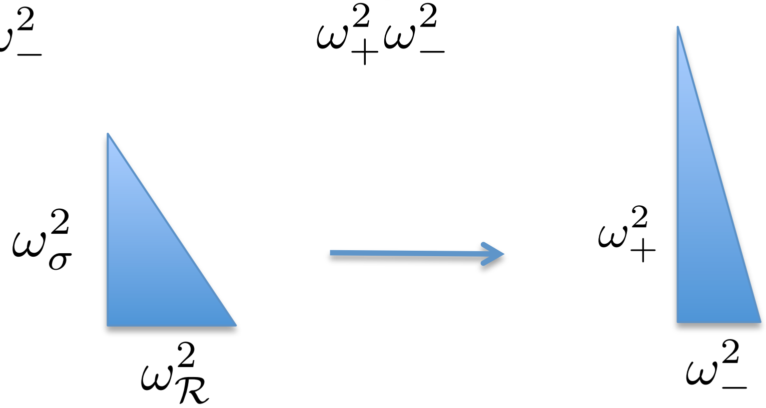
$$\begin{aligned} (\Box + m_{\mathcal{R}}^2)\ \mathcal{R} &= \alpha\dot{\sigma} \\ (\Box + m_\sigma^2)\ \sigma &= -\alpha\dot{\mathcal{R}} \end{aligned}$$

Eigenmodes?

$$\begin{pmatrix} \mathcal{R} \\ \sigma \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{i\omega t}$$

$$\omega^4 - \underbrace{[\omega_{\mathcal{R}}^2 + \omega_\sigma^2 + \alpha^2]}_{\omega_+^2 + \omega_-^2}\omega^2 + \underbrace{\omega_{\mathcal{R}}^2\omega_\sigma^2}_{\omega_+^2\omega_-^2} = 0$$

Frequency (mass) hierarchy **increases**



If $\omega_{\mathcal{R}} = 0, \quad \omega_\sigma^2 \to \omega_\sigma^2 + \alpha^2$

The action of the inflationary perturbations:

$$S = \int d^4x\, a^3 \left[\epsilon\dot{\mathcal{R}}^2 - 2\epsilon\alpha\dot{\mathcal{R}}\sigma - \frac{\epsilon}{a^2}(\nabla\mathcal{R})^2 + \frac{1}{2}\left(\dot{\sigma}^2 - \frac{1}{a^2}(\nabla\sigma)^2\right) - \frac{1}{2}m_\sigma^2\sigma^2\right]$$

Invariant under $\mathcal{R} \rightarrow \mathcal{R} + \delta C_2.$

$$S = \int d^4x\, a^3 \left[\epsilon(\dot{\mathcal{R}} - \alpha\sigma)^2 - \frac{\epsilon}{a^2}(\nabla\mathcal{R})^2 + \frac{1}{2}\left(\dot{\sigma}^2 - \frac{1}{a^2}(\nabla\sigma)^2\right) - \frac{1}{2}\mu^2\sigma^2\right]$$

$$\mu = \sqrt{m_\sigma^2 + 2\epsilon\alpha^2}$$

"entropy mass"

If $\mu = 0$ a second "shift" symmetry arises

$$\dot{\mathcal{R}} \rightarrow \dot{\mathcal{R}} + \alpha\,\delta C_1,$$
$$\sigma \rightarrow \sigma + \delta C_1.$$

σ behaves as a massless field, freezes outside the horizon --- "ultra-light"

N.B. effective mass "tachyonic", but this is **OK !**

(analytic approx, w constant ε, λ)

ultra-light: μ=0, Ω≠0 (massless, coupled=turning)

normal: μ>0, Ω=0 (massive, decoupled=geodesic)

curvature perturbation **isocurvature perturbation**

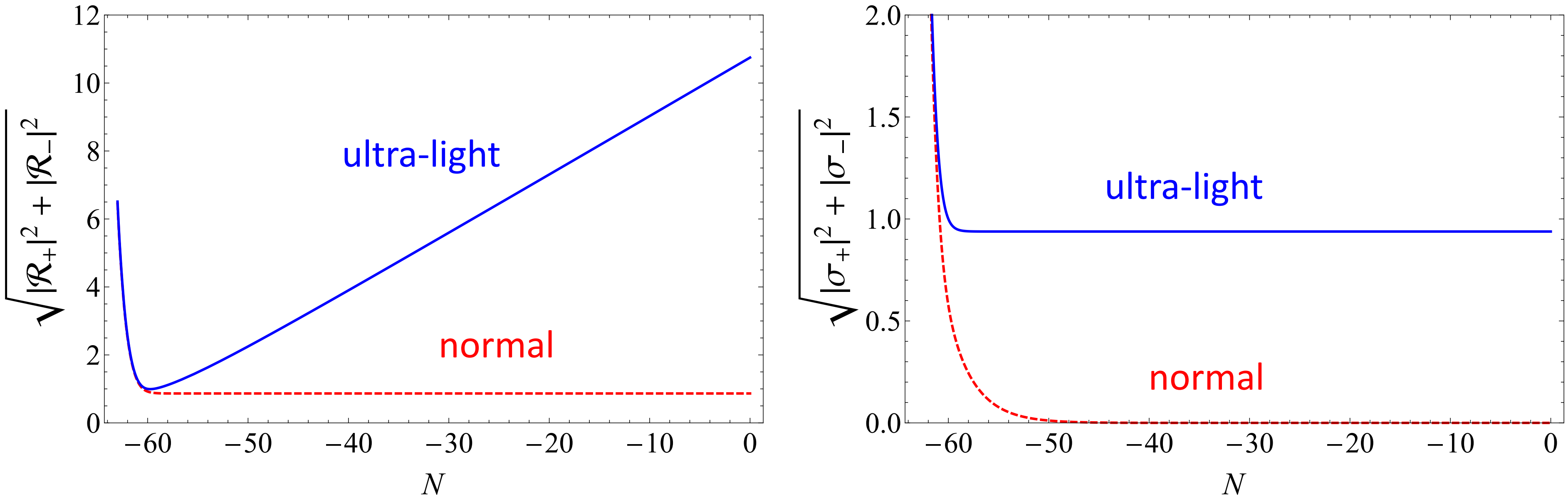


Figure 1: The figure shows the evolution of the amplitude of the fluctuations around the time of horizon crossing (at around $N = -60$). The left panel shows the amplitude of $\mathcal{R}$, whereas the right panel shows the amplitude of σ. The red dashed curves correspond to the case in which there is no coupling between $\mathcal{R}$ and σ (that is $\lambda = 0$), and σ has a nonzero entropy mass μ. It may be seen that $\mathcal{R}$ freezes whereas σ decays quickly once they cross the horizon. The blue solid lines show the case in which the two fields remain coupled, with $\lambda = 0.2$, and σ has zero entropy mass. In this case, $\mathcal{R}$ grows outside the horizon, and σ freezes.

AA Atal Germani Palma 1607.08609

A concrete example with $m_\sigma^2 = -4\Omega^2$ (axion-dilaton system)

Lalak Langlois Pokorsky Turzyńsky 0704.0212
Cremonini Lalak Turzyńsky 1005.4347, 1010.3021
Di Marco, Finelli 0505198 + Brandenberger 0211276

Kobayashi, Mukohyama 1003.0076
Renaux-Petel, Turzyńsky 1510.01281
Brown 1705.03023

$$S = \frac{1}{2}\int d^4x R - \int d^4x \left[\frac{1}{2}e^{2\mathcal{Y}/R_0}(\nabla\mathcal{X})^2 + \frac{1}{2}(\nabla\mathcal{Y})^2 + V(\mathcal{X})\right]$$

Hyperbolic manifold with constant negative Ricci scalar $\mathbb{R} = -2/R_0^2.$
and a symmetry

$$\mathcal{Y} \to \mathcal{Y}' = \mathcal{Y} + C \qquad \mathcal{X} \to \mathcal{X}' = e^{-C/R_0}\mathcal{X}.$$

$$\frac{\alpha}{H} = \frac{2}{R_0} \quad \text{constant}$$

The contribution to the mass from the curvature of the field metric exactly compensated by the contribution from turning trajectory

Monomial potential, n = 1/2

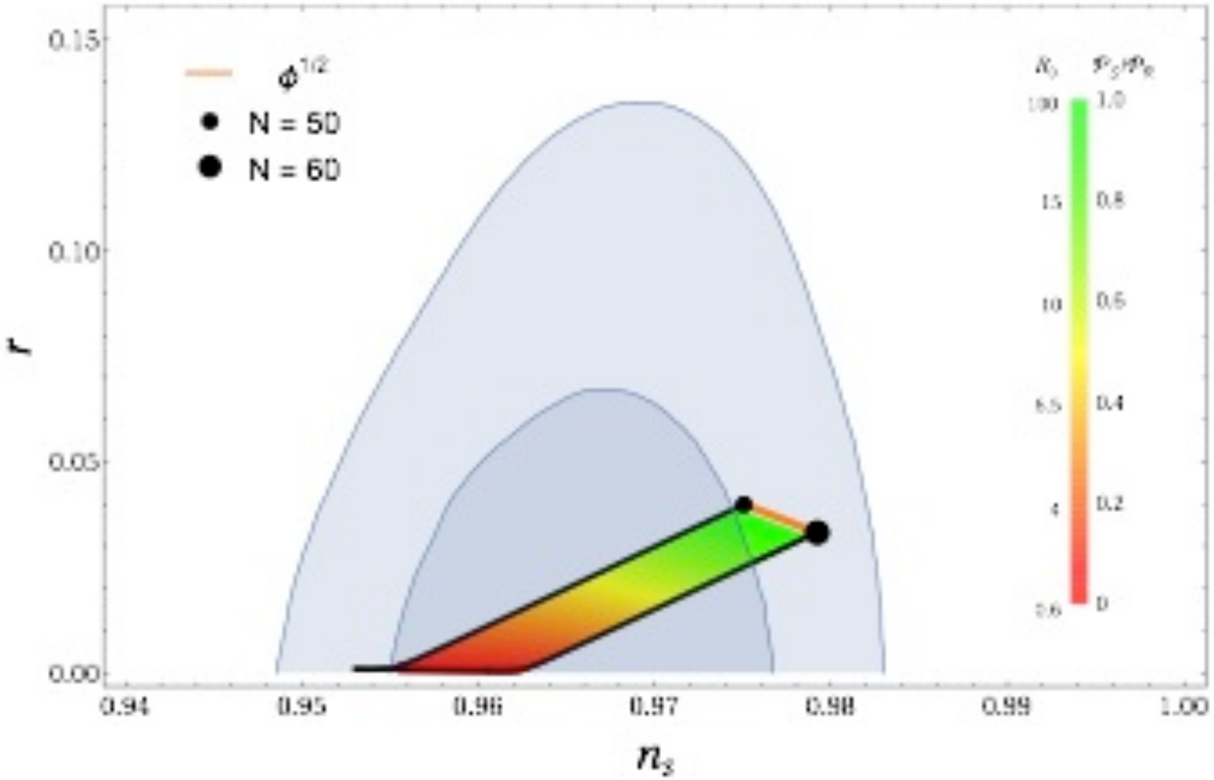


Figure 2: (n_s, r) plane as a function of R_0 for the exponential metric model with monomial potential $n = 1/2$. The predictions for (n_s, r) interpolate from the single field predictions (R_0 large) to the large coupling regime (R_0 small). In colors (red to green) we show the fraction of isocurvature to curvature perturbations. Isocurvature perturbations are suppresed by an order of magnitude or more for values of $R_0 < 1.5$. In blue we show the $1-\sigma$ and $2-\sigma$ contours from Planck [3].

Summary (I) – inflaton + heavy field

Tolley Wyman 0910.1853 **AA Gong Hardeman Palma Patil 1005.3848, 1010.3693**

If the inflaton trajectory is not straight, there is a **derivative coupling** between curvature perturbation and heavy field perturbation.

coupling constant = turning rate of trajectory

(straight = wrt sigma model metric of scalar fields)

(quadratic)

$$S = \int d^4x a^3 \left[\epsilon \dot{\mathcal{R}}^2 - \frac{\epsilon}{a^2}(\nabla \mathcal{R})^2 + 2\sqrt{2\epsilon}\Omega \dot{\mathcal{R}}\sigma + \frac{1}{2}\dot{\sigma}^2 - \frac{1}{a^2}(\nabla\sigma)^2 - \frac{1}{2}M_{\text{eff}}^2\sigma^2 \right]$$

modified dispersion relation for both modes

Integrating out the heavy field results in a reduced speed of sound for the adiabatic mode. But otherwise it is still **single field slow roll inflation.**

Heavy mode in its adiabatic vacuum, no interruption of slow roll

Summary (II) – inflaton + ultra-light field(s)

- Multifield inflation with light fields has at least two undesirable properties for model building

  -potentially large isocurvature perturbations

  -curvature perturbation is not conserved on superhorizon scales

- In the ultra-light regime these two "problems" cancel each other on observable scales: "massless"**(*)** isocurvature perturbation freezes and sources curvature. Sustained over many efolds. We get

  -scale invariant spectra

  -suppressed isocurvature

  -suppressed tensor-to-scalar ratio

Requires sustained turns, that can be provided by the geometry of the scalar manifold

(*) there is a "Stueckelberg-like" shift symmetry involving both R and σ

Inherited from symmetries of the background

(e.g. "axion-dilaton" hyperbolic coset spaces)

To be continued…