A natural $S_4 \times SO(10)$ model of flavour

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invisibles neutrinos, dark matter & dark energy physics

Based on work in
[1705.01555 [hep-ph]]

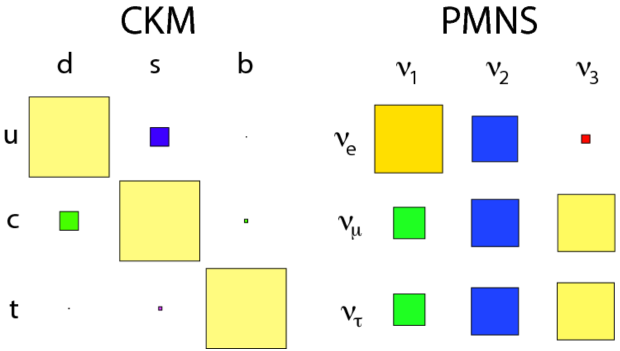
$SO(10)$ is great

- Unifies all fermions of a family into one **16** representation of $SO(10)$
- Predicts right-handed neutrinos and seesaw

But!

- Unifies all fermion Yukawa couplings also
- Naive $SO(10)$ like $\lambda_{ij} H \psi_i \psi_j$ suggests no mixing
- In reality, different hierarchies:
  - $y_u : y_d : y_t \sim 10^{-6} : 10^{-3} : 1$
  - $y_d : y_s : y_b \sim 10^{-5} : 10^{-4} : 10^{-1}$

Mixing patterns in PMNS, CKM completely different



[Stone, 1212.6374]

The model

1. Minimality

2. Naturalness

3. Completeness

1. Minimality
   - Smallest number of fields
   - Low-dimensional representations

2. Naturalness

3. Completeness

Guiding principles

1. Minimality
   - Smallest number of fields
   - Low-dimensional representations

2. Naturalness
   - Understanding fermion hierarchies
   - $\mathcal{O}(1)$ dimensionless parameters
   - "Universal sequential dominance"

3. Completeness

Guiding principles

1. Minimality
    - Smallest number of fields
    - Low-dimensional representations
2. Naturalness
    - Understanding fermion hierarchies
    - $\mathcal{O}(1)$ dimensionless parameters
    - "Universal sequential dominance"
3. Completeness
    - Successfully account for quark *and* lepton data
    - Address doublet-triplet splitting*
    - Generate a μ term*

Symmetries

- S_4
  - Ensures "horizontal" unification of SM fermions
  - Enforces CSD3 vacuum alignments
- $SO(10)$
  - Ensures "vertical" unification of SM fermions
  - Predicts right-handed neutrinos
- $\mathbb{Z}_4 \times \mathbb{Z}_4$
  - Prevents unwanted mixed terms
- $\mathbb{Z}_4^R$
  - Breaks to matter (or R) parity, protects LSP

Field content

- Matter
  - $\psi \sim (3', 16)$ contains all SM fields
- Higgs
  - H_{10}^u, H_{10}^d gives MSSM Higgs h_u, h_d
  - $H_{\overline{16}}$ breaks $SO(10)$, gives RH neutrinos mass
  - $3(+1) \times H_{45}$ break $SU(5)$, provide Clebsch-Gordan coefficients
- Flavons
  - $\phi \sim (3', 1)$
- Messengers
  - $\chi \sim (1, 16)$, $\bar{\chi} \sim (1, \overline{16})$, $\rho \sim (1, 1)$

Renormalisable at GUT scale

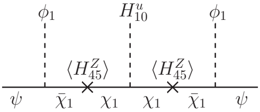
$$W_Y^{(\mathrm{GUT})} = \psi\phi_a\bar{\chi}_a + \bar{\chi}_a\chi_a H_{45}^Z + \chi_a\chi_a H_{10}^u + \rho\chi_3 H_{\overline{16}} + M_\rho\rho\rho + \bar{\chi}_b\chi_b'\left(H_{45}^X + H_{45}^Y\right) + \chi_b'\chi_b' H_{10}^d + \chi_1\chi_2 H_{10}^d$$

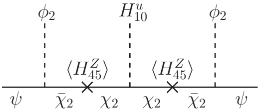
Planck-suppressed terms allowed by symmetries

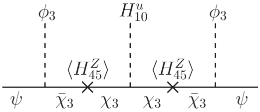
$$W_Y^{(\mathrm{Planck})} = \frac{\chi_a\chi_a H_{\overline{16}} H_{\overline{16}}}{M_P} + \frac{\psi\psi\phi_3 H_{10}^d}{M_P},$$

Mass structures

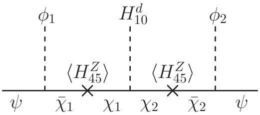
Up-type quarks and Dirac neutrinos

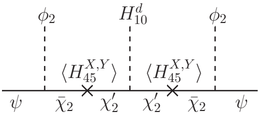


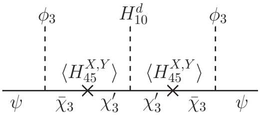




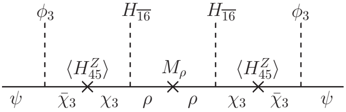
Down-type quarks and charged leptons

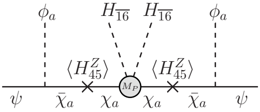






Right-handed neutrinos





Flavon VEVs which preserve SU generator of S_4 [King, Luhn '16]

$$\langle \phi_1 \rangle = v_1 \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \quad \langle \phi_2 \rangle = v_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \langle \phi_3 \rangle = v_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

VEVs driven to scales with the hierarchy

$$v_1 \ll v_2 \ll v_3 \sim M_{\mathrm{GUT}},$$

From product of triplets, like $\langle \phi \rangle \langle \phi \rangle^T$, construct a 3×3 numerical matrices!

We can then define numerical matrices

$$Y_{11} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix} \quad Y_{22} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad Y_{33} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Y_{12} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 6 \end{pmatrix} \quad Y_{P} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

- Y_{11}, Y_{22}, Y_{33} are **rank 1**
- Y_{12} comes from mixed $H^d_{10}(\psi\phi_1)(\psi\phi_2)$ term
- Y_P comes from S_4 triple product $\psi\psi\phi_3 H^d_{10}$

$$Y_{11} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix} \; Y_{22} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \; Y_{33} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \; Y_{12} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 6 \end{pmatrix} \; Y_P = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

The fermion mass/Yukawa matrices are then given by

$$\begin{aligned}
Y^u &= y_1^u e^{i\eta} Y_{11} + y_2^u Y_{22} + y_3^u e^{i\eta'} Y_{33} \\
Y^\nu &= y_1^\nu e^{i\eta} Y_{11} + y_2^\nu Y_{22} + y_3^\nu e^{i\eta'} Y_{33} \\
M^{\mathrm{R}} &= M_1^{\mathrm{R}} e^{i\eta} Y_{11} + M_2^{\mathrm{R}} Y_{22} + M_3^{\mathrm{R}} e^{i\eta'} Y_{33} \\
Y^d &= y_{12}^d e^{i\frac{\eta}{2}} Y_{12} + y_2^d e^{i\alpha_d} Y_{22} + y_3^d e^{i\beta_d} Y_{33} + y^P e^{i\gamma} Y_P \\
Y^e &= y_{12}^e e^{i\frac{\eta}{2}} Y_{12} + y_2^e e^{i\alpha_e} Y_{22} + y_3^e e^{i\beta_e} Y_{33} + y^P e^{i\gamma} Y_P
\end{aligned}$$

All hierarchies controlled by VEVs of flavons $\phi_{1,2,3} \to v_{1,2,3}$

- Universal sequential dominance

Choosing reasonable values of v_i:

$$v_1 \approx 0.002 M_{\mathrm{GUT}} \qquad v_2 \approx 0.05 M_{\mathrm{GUT}} \qquad v_3 \approx 0.5 M_{\mathrm{GUT}}$$

Estimated Yukawa parameters

$$\begin{aligned}
y_1^u \sim y_1^\nu \sim &\ v_1^2/M_{\mathrm{GUT}}^2 &&\approx 4 \times 10^{-6} \\
y_2^u \sim y_2^\nu \sim y_2^d \sim y_2^e \sim &\ v_2^2/M_{\mathrm{GUT}}^2 &&\approx 2.5 \times 10^{-3} \\
y_3^u \sim y_3^\nu \sim y_3^d \sim y_3^e \sim &\ v_3^2/M_{\mathrm{GUT}}^2 &&\approx 0.25 \\
y_{12}^d \sim y_{12}^e \sim &\ v_1 v_2/M_{\mathrm{GUT}}^2 &&\approx 1 \times 10^{-4} \\
y^P \sim &\ v_3/M_P &&\approx 5 \times 10^{-4}
\end{aligned}$$

RH neutrino parameters

$$\underset{(!)}{M_1^{\mathrm{R}} \sim 4 \times 10^7\ \mathrm{GeV}} \quad M_2^{\mathrm{R}} \sim 2.5 \times 10^{10}\ \mathrm{GeV} \quad M_3^{\mathrm{R}} \sim 10^{16}\ \mathrm{GeV}$$

Neutrino Majorana matrix after seesaw also has the CSD3 structure

$$
\begin{aligned}
m^\nu &= \mu_1 e^{i\eta} Y_{11} + \mu_2 Y_{22} + \mu_3 e^{i\eta'} Y_{33} \\
&= \mu_1 e^{i\eta} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix} + \mu_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mu_3 e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$

where the parameters μ_i are

$$\mu_i = v_u^2 \frac{(y_i^\nu)^2}{M_i^{\mathrm{R}}}$$

Almost all mixing in Y^d

- Hierarchy in Y^u very large ($m_u \ll m_c \ll m_t$)
- Texture (1,1) zero in Y^d gives GST relation [Gatto, Sartori, Tonin '68]

Keeping only important terms in Y^d (and ignoring phases)

$$Y^d \approx \begin{pmatrix} 0 & y^d_{12} & y^d_{12} - y^P \\ \cdot & y'_2 & y'_2 + 2(y^d_{12} - y^P) \\ \cdot & \cdot & y^d_3 \end{pmatrix}$$

Mixing angles approximated by

$$\theta^q_{12} \approx \frac{Y^d_{12}}{Y^d_{22}} = \frac{y^d_{12}}{y'_2} \quad \theta^q_{13} \approx \frac{Y^d_{13}}{Y^d_{33}} = \frac{y^d_{12} - y^P}{y^d_3} \quad \theta^q_{23} \approx \frac{Y^d_{23}}{Y^d_{33}} = \frac{y'_2 + 2(y^d_{12} - y^P)}{y^d_3}$$

Down-type quark Yukawa eigenvalues

$$y_d \approx (y^d_{12})^2 / y'_2 \qquad y_s \approx y'_2 \qquad y_b \approx y^d_3$$

Solving for y^d_{12}, y'_2 and y^d_3 and reintroducing into mixing angles gives

$$\theta^q_{12} \approx \sqrt{\frac{y_d}{y_s}} \quad \theta^q_{13} \approx \frac{\sqrt{y_d y_s} - y^P}{y_b} \quad \theta^q_{23} \approx \frac{y_s + 2(\sqrt{y_s y_d} - y^P)}{y_b}$$

With GUT-scale values* from observation (with $y^P = 0$)

$$\theta^q_{12} \approx 12.85° \qquad \theta^q_{13} \approx 0.23° \qquad \theta^q_{23} \approx 1.48°$$

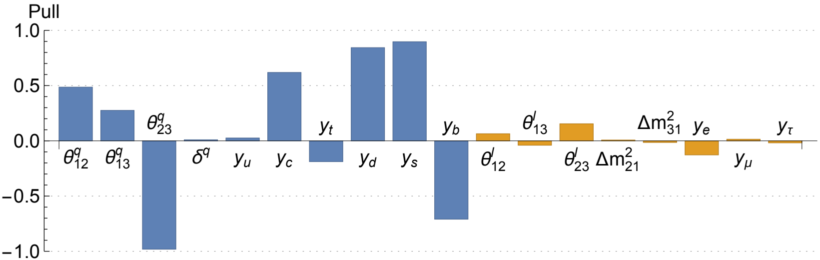
* assuming no SUSY threshold corrections

Numerical fit

- From MSSM to GUT scale
  - Quark, lepton masses and mixings need to be run up to M_{GUT}
  - This analysis was performed for MSSM [Antusch, Maurer '13]
  - SUSY threshold corrections parametrised by factors $\bar{\eta}_i$
- Fitting procedure
  - χ^2-minimisation to find best fit
  - MCMC to find Bayesian 95% credible intervals
- Lepton data from NuFit
  [Esteban, Gonzalez-Garcia, Maltoni, Martinez-Soler, Schwetz '16]

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ^ℓ_{12} $/^\circ$	33.57	$32.81 \to 34.32$	33.62	$31.69 \to 34.46$
θ^ℓ_{13} $/^\circ$	8.460	$8.310 \to 8.610$	8.455	$8.167 \to 8.804$
θ^ℓ_{23} $/^\circ$	41.75	$40.40 \to 43.10$	41.96	$39.47 \to 43.15$
δ^ℓ $/^\circ$	261.0	$202.0 \to 312.0$	300.9	$280.7 \to 308.4$
y_e $/10^{-5}$	1.017	$1.011 \to 1.023$	1.017	$1.005 \to 1.029$
y_μ $/10^{-3}$	2.147	$2.134 \to 2.160$	2.147	$2.121 \to 2.173$
y_τ $/10^{-2}$	3.654	$3.635 \to 3.673$	3.654	$3.616 \to 3.692$
$\Delta m^2_{21}/(10^{-5}\,\mathrm{eV}^2)$	7.510	$7.330 \to 7.690$	7.515	$7.108 \to 7.864$
$\Delta m^2_{31}/(10^{-3}\,\mathrm{eV}^2)$	2.524	$2.484 \to 2.564$	2.523	$2.443 \to 2.605$
m_1 /meV			0.441	$0.260 \to 0.550$
m_2 /meV			8.680	$8.435 \to 8.888$
m_3 /meV			50.24	$49.44 \to 51.05$
$\sum m_i$ /meV	< 230		59.36	$58.49 \to 60.19$

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ_{12}^q $/^\circ$	13.03	$12.99 \to 13.07$	13.02	$12.94 \to 13.10$
θ_{13}^q $/^\circ$	0.039	$0.037 \to 0.040$	0.039	$0.036 \to 0.041$
θ_{23}^q $/^\circ$	0.445	$0.438 \to 0.452$	0.439	$0.426 \to 0.450$
δ^q $/^\circ$	69.22	$66.12 \to 72.31$	69.21	$63.22 \to 73.94$
y_u $/10^{-6}$	2.988	$2.062 \to 3.915$	3.012	$1.039 \to 4.771$
y_c $/10^{-3}$	1.462	$1.411 \to 1.512$	1.493	$1.445 \to 1.596$
y_t	0.549	$0.542 \to 0.556$	0.547	$0.532 \to 0.562$
y_d $/10^{-5}$	2.485	$2.212 \to 2.758$	2.710	$2.501 \to 2.937$
y_s $/10^{-4}$	4.922	$4.656 \to 5.188$	5.168	$4.760 \to 5.472$
y_b	0.141	$0.136 \to 0.146$	0.137	$1.263 \to 1.429$


Pull
1.0
0.5
0.0
−0.5
−1.0
θ^q_{12}
θ^q_{13}
θ^q_{23}
δ^q
y_u
y_c
y_t
y_d
y_s
y_b
θ^l_{12}
θ^l_{13}
θ^l_{23}
Δm^2_{21}
Δm^2_{31}
y_e
y_μ
y_τ

Conclusions

Yes, but.

Yes

- Matter hierarchies fully explained* by flavon VEVs, including hierarchy *differences* between up-type and down-type quarks
- Large lepton mixing controlled by RH neutrinos (sequential dominance: $M_3^R \sim M_{\mathrm{GUT}}$) [King '98]

But

- Setting all λ couplings to 1, lightest RH neutrino predicted $\sim 10^7$ GeV, but fit prefers $\sim 10^5$ GeV
- Can be fixed by setting one coupling $\lambda_1^N \sim 0.01$

- Unifies all known fermions in single $(3', 16)$ of $S_4 \times SO(10)$
- Qualitatively explains mass hierarchies using low-rank matrix structures coming from flavon vacuum alignments
- Fits all known quark and lepton masses and mixings within 1σ
- Generates μ term of $\mathcal{O}(\mathrm{TeV})$ (not shown here)
- Splits Higgs doublets and triplets via DW mechanism [Dimopoulos, Wilczek '82] (not shown here)

Future work

- cosmology: leptogenesis & inflation

Thank you!

Matter, Higgs, flavons

	S_4	$SO(10)$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4^R$
ψ	$3'$	16	1	1	1
H_{10}^u	1	10	0	2	0
H_{10}^d	1	10	2	0	0
$H_{\overline{16}}$	1	$\overline{16}$	2	1	0
H_{16}	1	16	1	2	0
$H_{45}^{X,Y}$	1	45	2	1	0
H_{45}^Z	1	45	1	2	0
H_{45}^{B-L}	1	45	2	2	2
ξ	1	1	2	2	0
ϕ_1	$3'$	1	0	0	0
ϕ_2	$3'$	1	2	0	0
ϕ_3	$3'$	1	2	2	0

Messengers

	S_4	$SO(10)$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4^R$
$\bar{\chi}_1$	1	$\overline{16}$	3	3	1
χ_1	1	16	0	3	1
$\bar{\chi}_2$	1	$\overline{16}$	1	3	1
χ_2	1	16	2	3	1
$\bar{\chi}_3$	1	$\overline{16}$	1	1	1
χ_3	1	16	2	1	1
χ_3'	1	16	1	2	1
χ_2'	1	16	1	0	1
ρ	1	1	0	2	1