

Precise predictions for the trilinear Higgs self-coupling in the Standard Model and beyond

predicting κ_λ in *any* model at the one-loop order [2305.03015]

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Scalars, Sep 2023

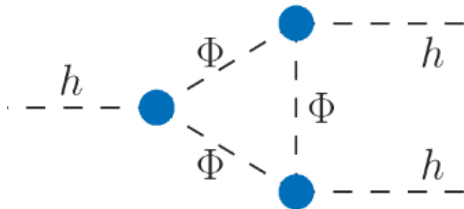


Why the trilinear self-coupling?

- > probes electroweak symmetry breaking mechanism
- > influences shape of the potential
- > important for electroweak phase transition
- > important input for di-Higgs production
- > very sensitive to BSM

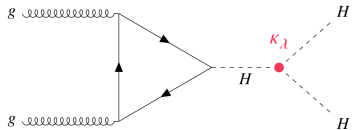
$$V_{\text{SM}} \supset \frac{m_h^2}{2} h^2 + \frac{1}{3!} \frac{3m_h^2}{v} \kappa_\lambda h^3 + \frac{1}{4!} \frac{3m_h^2}{v^2} \kappa_{2\lambda} h^4$$

$$\kappa_\lambda = \frac{\lambda_{hhh}^{\text{BSM}}}{\lambda_{hhh}^{\text{SM}}}$$

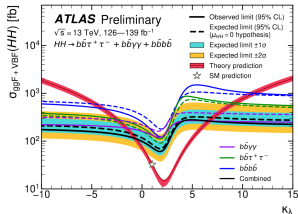


Constraining $\lambda_{hhh} / \kappa_\lambda$ using Higgs pair production

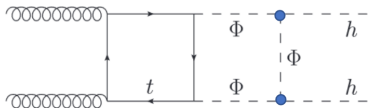
Leading-order parametrization used by experiment:



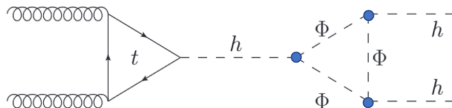
$$pp \rightarrow hh: -0.4 < \kappa_\lambda < 6.3 \text{ [Phys. Lett. B 843 (2023) 137745]}$$



Possible next-to-leading order BSM contributions in concrete models:



$$\propto \mathcal{O}(y_t^2 g_{hh\Phi\Phi}^2) \text{ (not included)}$$



$$\propto \mathcal{O}(y_t g_{hh\Phi\Phi}^3) \text{ (included)}$$

When to apply the κ_λ -constraint to BSM models?

- > only κ_λ is *significantly* modified by BSM physics
- > no additional resonance in s -channel
- > all other couplings SM-like

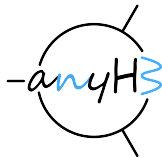
→ a scenario often enforced by experimental constraints

λ_{hhh} beyond the SM

- > Many studies for λ_{hhh} already exist
 - additional singlets [Kanemura et al. '16][Basler et al. '19],
 - doublets [Kanemura et al. '04][Basler et al. '17][Braathen et al. '19],
 - triplets [Aoki et al. '18][Chiang et al. '18],
 - SUSY [Hollik et al. '02][Brucherseifer et al. '13][Dao et al. '13][Dao et al. '15][Borschensky et al. '22]
- > Higher-order corrections can be significant
- > Many more models to explore!

→ anyH3  [Bahl, Braathen, MG, Weiglein '23]:
automated tool to calculate λ_{hhh} in *any* model

Feature list (so far) of anyH3



- > import/convert arbitrary UFO models
- > (semi)automatic renormalisation
 - OS or MS mass renormalization
 - OS or MS electroweak VEV
 - provide custom renormalization *conditions* (no need to compute diagrams)
→ estimate size of missing higher-orders
- > optional: full p^2 dependence
- > numerical / analytical / $\text{\LaTeX}$ outputs
- > Python-library with command-line- and Mathematica-interface
- > ...

```
pip install anyBSM
```

```
from anyBSM import anyH3

myfancymodel = anyH3('path/to/UFO/model')

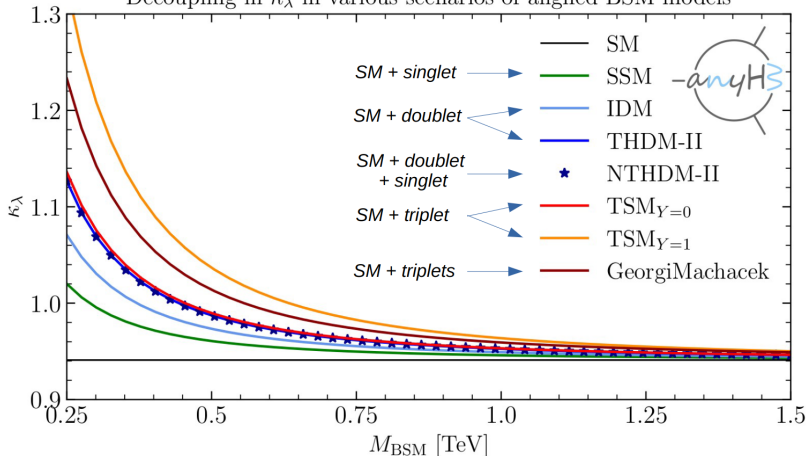
result = myfancymodel.lambdahhh()
```

more examples at anybsm.gitlab.io

Results

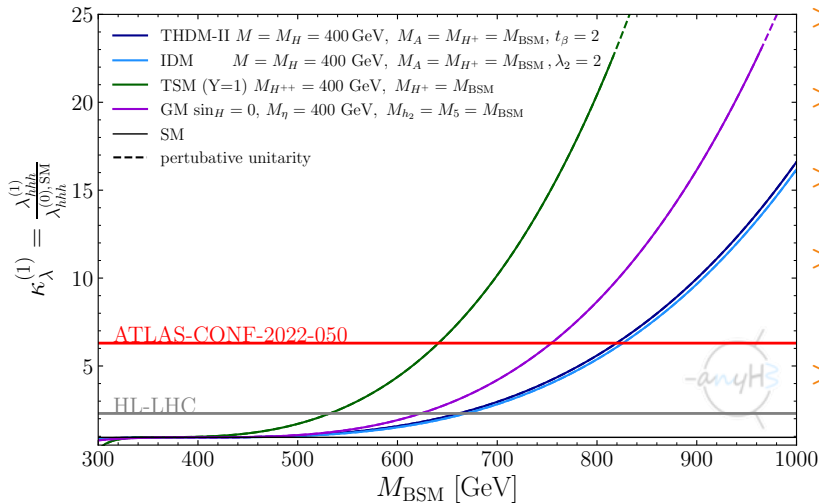
Cross-check: Decoupling w/ alignment

Decoupling in κ_λ in various scenarios of aligned BSM models



- > many models built-in and cross-checked
- > ensure *appropriate* decoupling behaviour
- > recover SM result for $M_{\text{BSM}} \rightarrow \infty$
- > easy to implement new models (UFO)
- > further checks
 - literature (if available, e.g. MSSM)
 - UV-finiteness
 - FeynArts/FormCalc

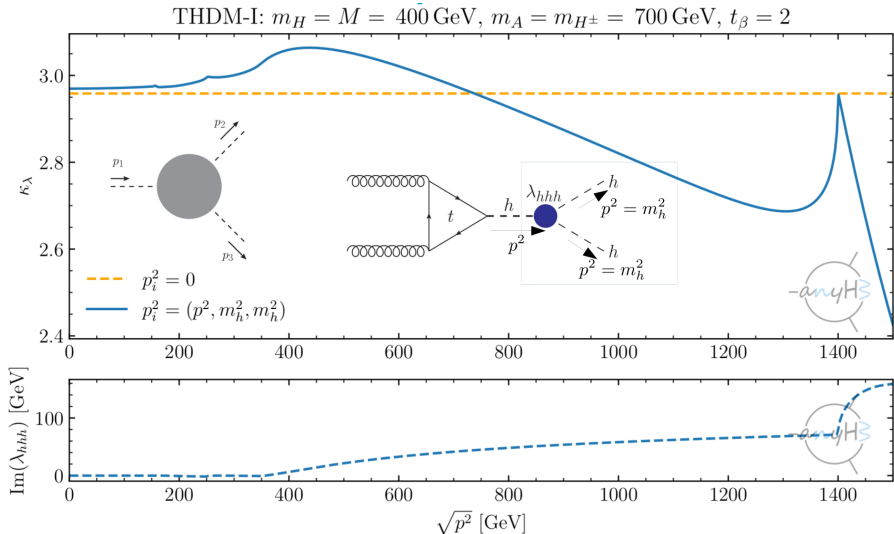
Alignment w/o decoupling



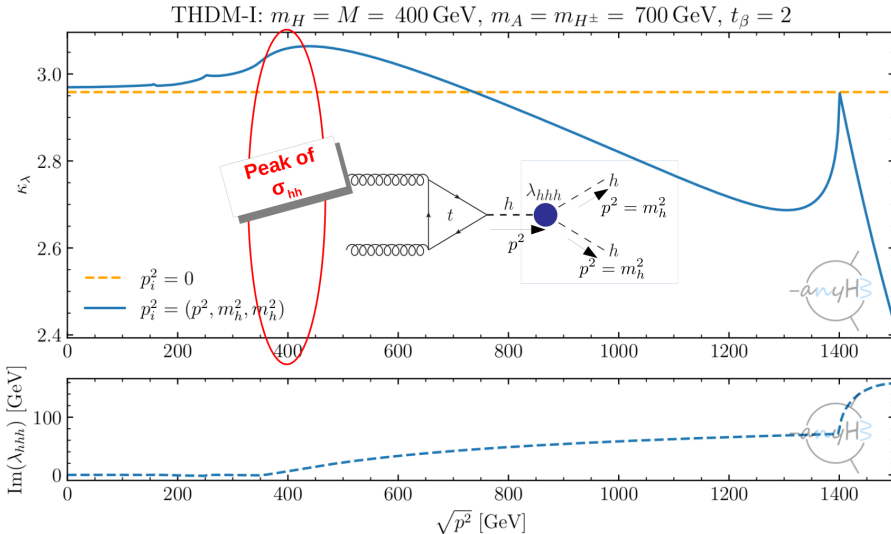
- > alignment means $\kappa_{h_{\text{SM}} h_{\text{SM}}}^{\text{tree-level}} = 1$
- > mass splitting within the same multiplet
- > induces large couplings for $M_{\text{BSM}} \rightarrow \infty$
- > corrections large-enough to exclude parameter space
- > see [Bahl, Braathen, Weiglein '22] for two-loop results in the THDM

Simplest case: $V_{H\Phi_{\text{BSM}}} \propto \lambda_{H\Phi_{\text{BSM}}} H^2 \Phi_{\text{BSM}}^2 + \mu^2 \Phi_{\text{BSM}}^2 \Rightarrow \lambda_{H\Phi_{\text{BSM}}} \propto (M_{\text{BSM}}^2 - \mu^2)/v^2$

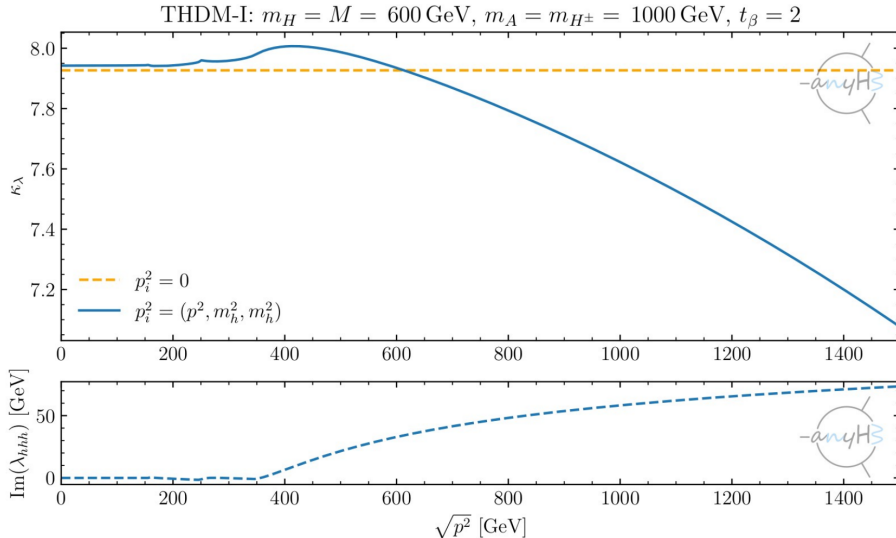
Momentum dependence in the THDM



Momentum dependence in the THDM

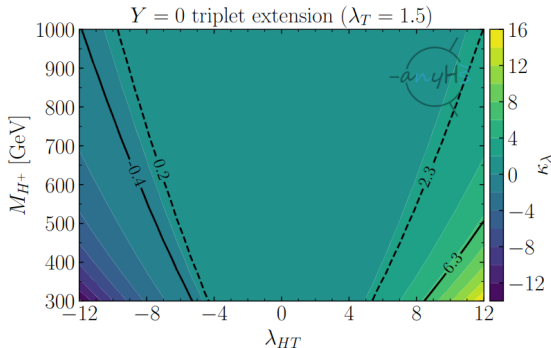
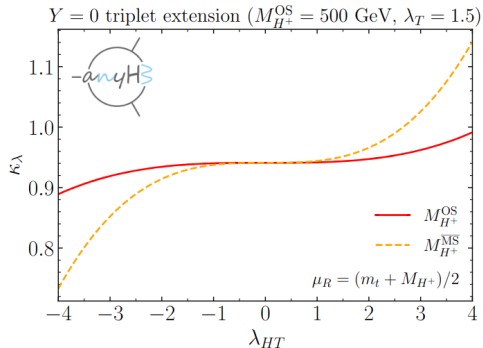


Momentum dependence in the THDM



Renormalization scheme comparison for a real Triplet

$$V(\Phi, T) = \mu^2 |\Phi|^2 + \frac{\lambda}{2} |\Phi|^4 + \frac{M_T^2}{2} |T|^2 + \frac{\lambda_T}{2} |T|^4 + \frac{\lambda_{HT}}{2} |T|^2 |\Phi|^2, \quad \langle T \rangle = 0, \quad \langle \Phi \rangle = v_{\text{SM}}$$



estimate of two-loop corrections generated by triplet self-coupling

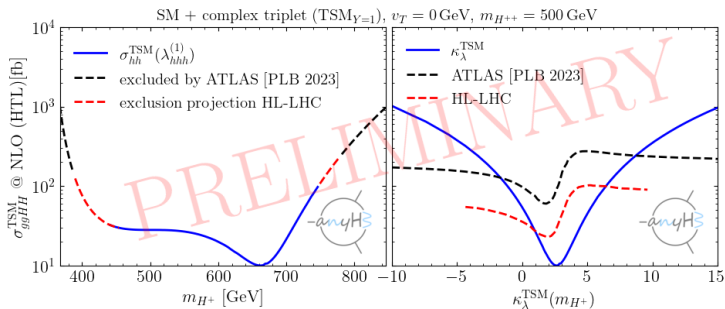
Outlook / Summary

> Outlook:

- influence on σ_{hh}
- non-SM self-couplings, $\kappa_{\lambda_{hhH}}$
- more models
- go beyond one-loop
- κ_t , κ_V etc.

> Summary:

- anyH3: λ_{hhh} in arbitrary ren. QFTs
 - at the full one-loop order
 - optional momentum dependence
 - with arbitrary choice of renormalization schemes
- uses UFO input
- analytical results; fast numerical results $\mathcal{O}(\text{ms})$
- already studied many models:
SM, SM+**singlets, doublets, triplets, SUSY**, ...
- found large mass-splitting effects



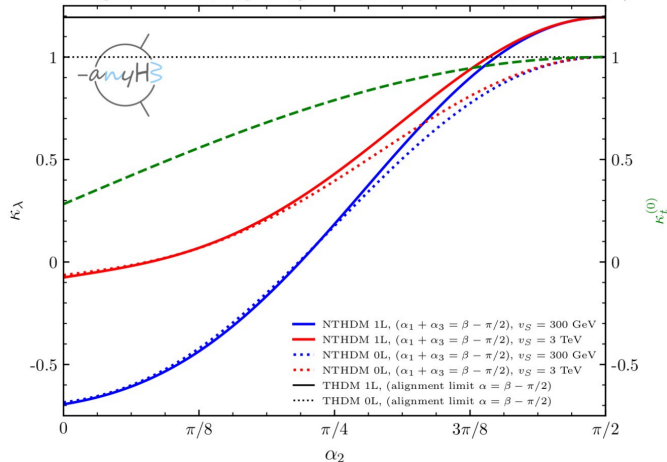
More info

- > pip install anyBSM
- > anyBSM --help
- > documentation, tutorials and examples: anybsm.gitlab.io

Backup

The sign of κ_λ in the NTHDM

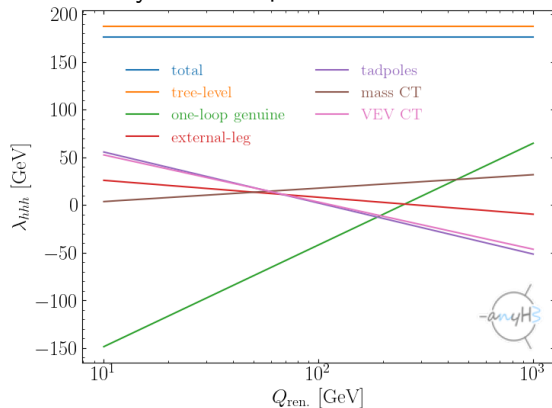
NTHDM: $m_{h_2} = 125.1$ GeV, $m_{h_1} = m_{h_3} = m_A = m_{H^\pm} = 300$ GeV, $\tilde{\mu} = 100$ GeV, $t_\beta = 2$



- > NTHDM=THDM+ real singlet
- > 3 CP-even scalars $h_{1,2,3}$, 3 mixing angles $\alpha_{1,2,3}$
- > $\alpha_2 \rightarrow \pi/2$: decoupling of singlet + alignment
- > **attention:** from ggHH we only get $\text{sgn}(\kappa_t/\kappa_\lambda)$, the relative sign of top- and Higgs modifiers!
- > $\kappa_t = \frac{y_t^{\text{BSM}}}{y_t^{\text{SM}}}$ strongly constrained

Simple cross-check: UV-finiteness in the SM

Numerically: scale independent result



Analytically: cancellation of $1/\epsilon$ poles

```
<< anyBSM`  
LoadModel["SM"]  
lam = lambdahhh[];  
(lam["total"] - lam["treelevel"] /. UVparts // Simplify) == 0  
True
```


Higher-order corrections to λ_{hhh} in any renormalisable theory

$$\begin{aligned}
 (\lambda_{hhh}^{\text{BSM}})^{\text{one-loop}} = & \underbrace{\text{tree-level: } \lambda_{hhh}^{(0)}}_{\text{tree-level: } \lambda_{hhh}^{(0)}} + \underbrace{\text{one-particle irreducible: } \delta\lambda_{hhh}^{\text{genuine}}}_{\text{one-particle irreducible: } \delta\lambda_{hhh}^{\text{genuine}}} + \underbrace{\text{tadpoles: } \delta\lambda_{hhh}^{\text{tad.}}}_{\text{tadpoles: } \delta\lambda_{hhh}^{\text{tad.}}} \\
 & + \underbrace{\text{external leg corrections: } \delta\lambda_{hhh}^{\text{WFR}}}_{\text{external leg corrections: } \delta\lambda_{hhh}^{\text{WFR}}} + \underbrace{\text{renormalisation: } \delta\lambda_{hhh}^{\text{CT}}}_{\text{renormalisation: } \delta\lambda_{hhh}^{\text{CT}}}
 \end{aligned}$$

- > Solid lines:
 - scalars,
 - fermions,
 - gauge/vector bosons,
 - ghosts
- > possibility to exclude/restrict certain particles and/or topologies
- > automatic non-trivial renormalization
 - OS or $\overline{\text{MS}}$ masses
 - $\sim$ size of two-loop

Renormalisation of λ_{hhh}

$$\delta\lambda_{hhh}^{\text{CT}} = \text{---} \bigotimes \text{---} = ?$$

- > one-loop $\rightarrow$ renormalisation of all parameters entering $\lambda_{hhh}^{(0),\text{BSM}}$ at tree-level
- > In the SM $\lambda_{hhh}^{(0),\text{SM}} = \frac{3m_h^2}{v}$
- > In general:

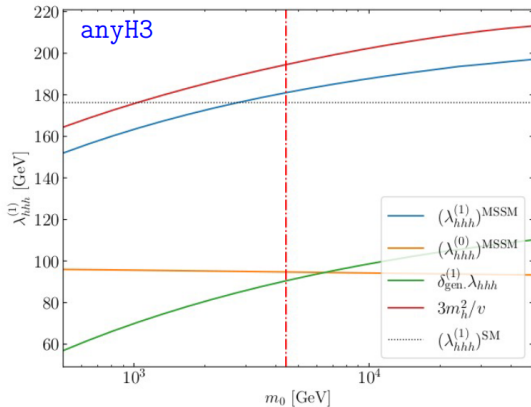
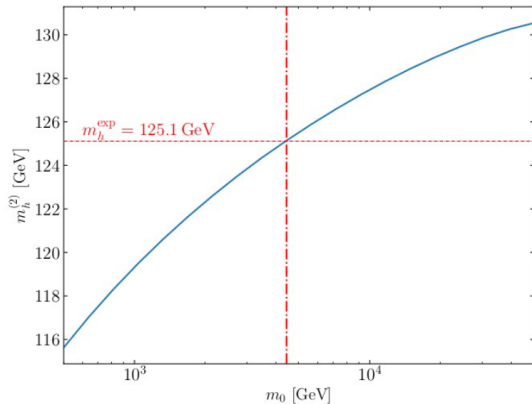
$$\lambda_{hhh}^{(0),\text{BSM}} \equiv \lambda_{hhh}^{(0),\text{BSM}} \left(\underbrace{v^{\text{SM}}, m_h^{\text{SM}}}_{\text{SM Higgs sector}}, \underbrace{m_{\chi_i}}_{\text{further (OS) masses}}, \underbrace{v_j}_{\text{BSM VEVs}}, \underbrace{\alpha_k}_{\text{mixing angles}}, \underbrace{g_l}_{\text{indep. couplings}} \right)$$

- > user's choice:
 - **SM sector:** fully OS or $\overline{\text{MS}}/\overline{\text{DR}}$
 - **BSM masses** (scalars/vectors/fermions): OS or $\overline{\text{MS}}/\overline{\text{DR}}$
 - **Additional couplings/vevs/mixings:** $\overline{\text{MS}}/\overline{\text{DR}}$ by default. **Custom ren. conditions possible!**

$$\delta_{\text{CT}}^{(1)} \lambda_{hhh} = \sum_p \left(\frac{\partial \lambda_{hhh}^{(0),\text{BSM}}}{\partial p} \right) \delta^{\text{CT}} p, \text{ with } p = \{m_h^{\text{SM}}, v^{\text{SM}}, m_{\chi_i}, \alpha_j, \dots\}^{\overline{\text{MS}}/\text{OS}/\text{custom}}$$

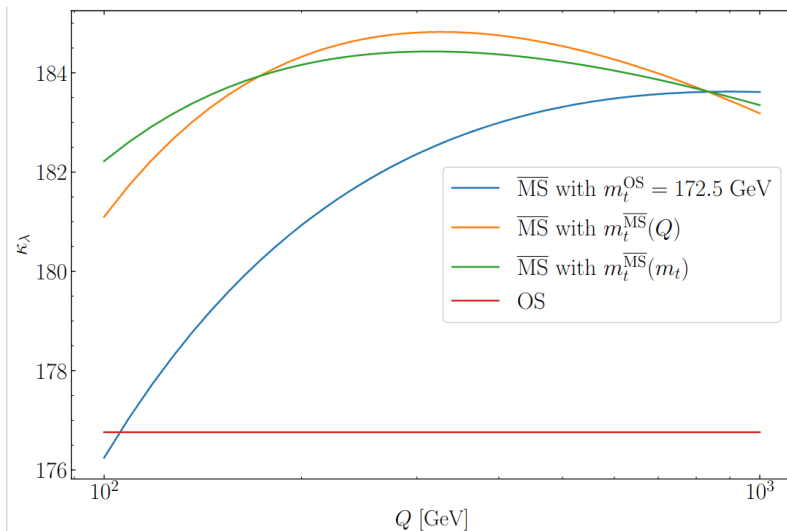
Full MSSM result: interface to SPheno

CMSSM, $m_0 = m_{1/2} = -A_0$, $\tan\beta = 10$, $\text{sgn}(\mu) = 1$, with m_h computed at 2L in SPheno



- Example for a very simple version of the constrained MSSM → BSM parameters m_0 , $m_{1/2}$, A_0 , $\text{sgn}(\mu)$, $\tan\beta$
- For each point, M_h computed at 2L with SPheno, and SLHA output of SPheno used as input of anyH3

Scheme- and top-mass- uncertainty



(Default) Renormalization choice of $(v^{\text{SM}})^{\text{OS}}$ and $(m_i^2)^{\text{OS}}$

- > $v^{\text{OS}} \equiv \frac{2M_W}{e} \sqrt{1 - \frac{M_W^2}{M_Z^2}}$ with (remember: $\lambda_{hhh}^{(0)} \approx 3m_h^2/v$)
 - $\delta^{(1)} M_V^2 = \frac{\text{Re}\Pi_V^{(1),T}}{M_V^{\text{OS}2}}, V = W, Z$
 - $\delta^{(1)} e = \frac{1}{2}\Pi_\gamma + \text{sign}(\sin\theta_W) \frac{\sin\theta_W}{M_Z^2 \cos\theta_W} \Pi_{\gamma Z}$
- > attention (i): $\rho^{\text{tree-level}} \neq 1 \rightarrow$ further CTs needed (depends on the model)
 $\rightarrow$ ability to define *custom* renormalisation conditions
- > scalar masses: $m_i^{\text{OS}} = m_i^{\text{pole}}$
 - $\delta^{\text{OS}} m_i^2 = -\text{Re}\Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$
- > attention (ii): scalar mixing may also require further CTs/tree-level relations

All bosonic one- & two-point functions and their derivatives for general QFTs are required for flexible OS renormalisation.

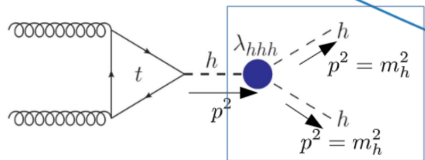
Treatment of external leg corrections

default treatment of external legs:

$$\delta^{(1), \text{ext.-legs}} \lambda_{hhh} = - \sum_i \left[\frac{1}{2} \Sigma'_{hh}(p_i^2) \lambda_{hhh}^{(0)} + \underbrace{\sum_{j, h_j \neq h} \frac{\Sigma_{hh_j}(p_i^2)}{p_i^2 - m_{h_j}^2} \lambda_{h_j hh}^{(0)}}_{=0, \text{ for alignment}} \right]$$

- > Attention: insert into di-Higgs production: need one off- and two on-shell Higgses:

$$\delta^{(1), \text{ext.-legs}} \lambda_{hhh} = - \left(\frac{1}{2} + \frac{1}{2} \right) \Sigma'_{hh}(m_h^2) \lambda_{hhh}^{(0)}$$



- > possible to turn-off default behaviour and specify ext.-leg contributions in terms of selfenergies

Treatment of tadpoles: many possibilities

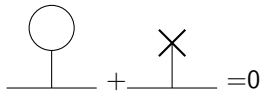
At tree-level:

- > define $t_h = \left. \frac{\partial V}{\partial h} \right|_{h=0}$ and $m_h^2 = \left. \frac{\partial^2 V}{\partial h^2} \right|_{h=0}$
- > then $V_{\text{SM}} \supset t_h h + \frac{1}{2} m_h^2 h^2 + \frac{m_h^2 - t_h/v}{2v} h^3 + \frac{m_h^2 - t_h/v}{8v^2} h^4$
- > popular choice $t_h = 0$ (but not the only choice!)

At one-loop: in general the renormalized tadpole consists of $\hat{t}_h = t_h + t_h^{(1)} + \delta t_h^{(1)}$

> "OS" tadpoles

- demand $\hat{t}_h = t_h = 0$ at one-loop such that $t_h^{(1)} = -\delta t_h^{(1)}$
- effectively no need to "attach" tadpoles to any diagrams



> "Fleischer-Jegerlehner (FJ)" tadpoles

- demand $t_h = 0$ at one-loop but let $\delta t_h^{(1)}$ cancel only divergent pieces
- need to consider finite contributions of *all* 1PI diagrams

> "tadpole-free $\overline{\text{MS}}$ scheme"

- set $\delta t_h^{(1)} = 0$ and demand $\hat{t}_h = 0 \Rightarrow t_h = -t_h^{(1)}$

Treatment of tadpole corrections for λ_{hhh}

w/o specifying a concrete scheme, nor the vacuum (in the alignment limit):

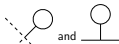
$$\lambda_{hhh}^{\text{tadpoles}} = \underbrace{-\frac{3t_h}{v^2}}_{\text{tree-level}} - \underbrace{\frac{6}{v^2}\delta_{\text{CT}}^{(1)}t_h}_{\text{CT-inserted diagrams}} + \underbrace{\delta_{\text{tadpoles}}^{(1)}\lambda_{hhh}}_{\text{tadpole diagrams}} + \underbrace{\frac{3}{v}\delta_{\text{CT, tadpoles}}^{(1)}m_h^2 - \frac{3m_h^2}{v^2}\delta_{\text{CT, tadpoles}}^{(1)}}_{\text{tad. contr. to input parameters}}v$$

- > In the SM (and BSM+alignment): once λ_{hhh} is expressed in terms of *physical* input parameters, its result is independent of the treatment (OS, FJ, ...) of the tadpoles (up to higher orders):

$$\delta^{(1)}\lambda_{hhh} \supset \frac{3}{v^2}\delta^{(1)}t_h|_{\text{finite}}$$

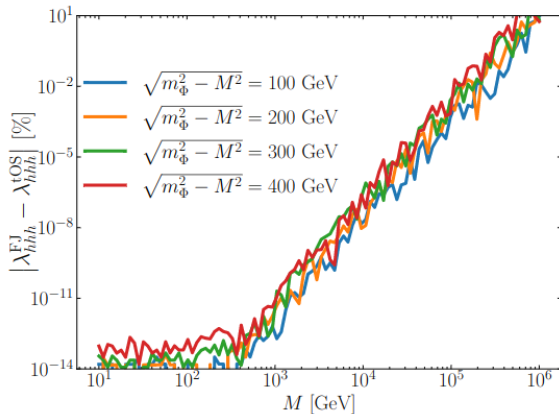
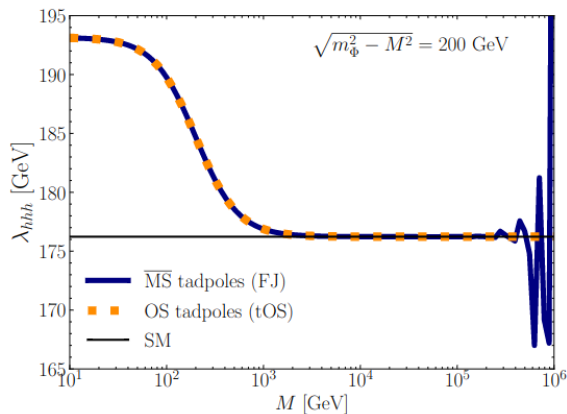
- > However: UFO models do (often) **not** contain the explicit dependence on the tree-level/one-loop/one-loop-CT tadpoles.
- > Thus: we choose the Fleischer-Jegerlehner treatment $t_h^{\text{tree-level}} = 0$ and renormalize $\delta^{(1)}t_h^{\text{CT}}|_{\text{finite}} = 0$ in the $\overline{\text{MS}}$ scheme per default (can also turn-off automatic tadpoles and implement own scheme).
- > only need to take into account tadpole contributions

to all two- and three-point functions:



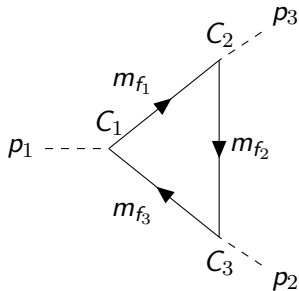
OS vs FJ tadpole treatment

THDM type-II, $s_{\beta-\alpha} = 1$, $t_\beta = 2$, $m_{h_2} = m_A = m_{H^\pm} = m_\Phi$



Example: generic fermion triangle

Idea: compute *generic* diagrams i.e. assume most generic



> couplings $C_i = P_L C_i^L + P_R C_i^R$, $P_{R/L} = \frac{1 \pm \gamma_5}{2}$

> as well as loop-masses m_{f_i} and

> external momenta p_i , $i = 1, 2, 3$.

$$\begin{aligned}
 &= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + \\
 &C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1} \mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + \\
 &C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R)m_{f_2} m_{f_3} + \\
 &2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + \\
 &C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + \\
 &C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)m_{f_1} + \\
 &(C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R)m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - \\
 &p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + \\
 &C_1^R C_2^R C_3^L)m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R)m_{f_3}))
 \end{aligned}$$

> insert concrete BSM model
(UFO [Degrande et al. '11])

> evaluate with the help of
(py)COLLIER [Denner et al. '16]

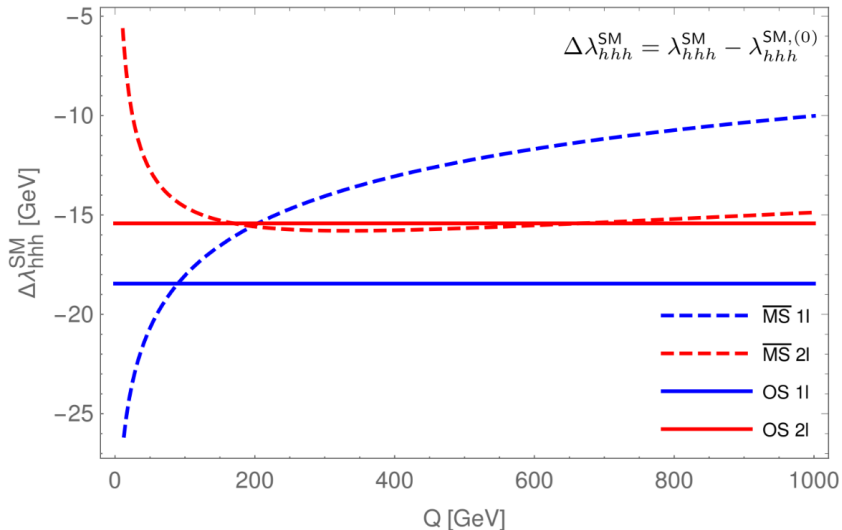
> public code anyH3

[Bahl, Braathen, MG, Weiglein '23]

Beyond anyH3

Two-loop effects in the SM

[Braathen, Kaneura '19]



W mass prediction

- > start with HO corrections to muon decay: $M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi \alpha_{\text{em}}}{\sqrt{2} G_F} [1 + \Delta r]$
- > and solve for: $M_W^2 = M_Z^2 \left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi \alpha_{\text{em}}}{\sqrt{2} G_F M_Z^2} (1 + \Delta r)} \right]$
- > with: $\Delta r^{(1)} = 2\delta^{(1)} e + \frac{\Pi_W^{(1),T}(0) - \delta^{(1)} M_W^2}{M_W^2} - \frac{\delta^{(1)} \sin^2 \theta_W}{\sin^2 \theta_W} + \delta_{\text{vertex+box}}$
- > and: $\frac{\delta^{(1)} \sin^2 \theta_W}{\sin^2 \theta_W} = \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \left(\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2} - \frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2} \right)$

It's all there but:

- > $\delta_{\text{vertex+box}}^{\text{SM}} = -\frac{2 \text{sign}(\sin \theta_W)}{\cos \theta_W \sin \theta_W M_Z^2} \Pi_{Z\gamma}(p^2 = 0) + \frac{\alpha_{\text{QED}}}{4\pi \sin^2 \theta_W} \left(6 + \frac{7-4 \sin^2 \theta_W}{2 \sin^2 \theta_W} \right) \log(\cos^2 \theta_W)$
- > $\delta_{\text{vertex+box}}^{\text{BSM}} = \text{needs to be implemented}$

However:

- > in many models $\Delta r \supset \frac{\delta \sin^2 \theta_W}{\sin \theta_W} \approx \delta \rho$ is the dominant effect!

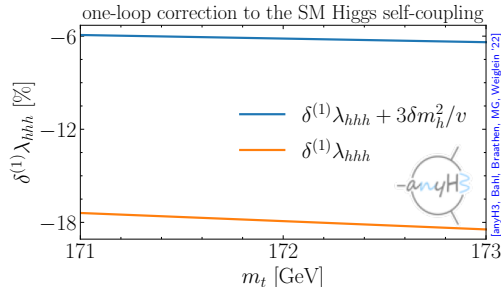
λ_{hhh} in the SM and in SUSY

In the SM at tree-level:

$$V(h) \supset \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} h^3 + \dots \quad \Rightarrow \quad \lambda_{hhh}^{\text{SM}} = \frac{\partial^3 V(h)}{\partial^3 h} = \frac{3m_h^2}{v}$$

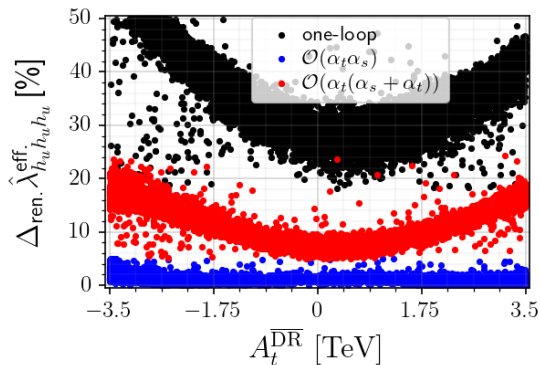
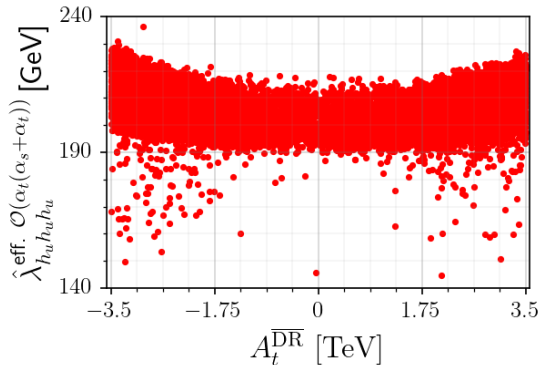
Thus $\lambda_{hhh}^{\text{SM}}$ can be predicted perturbatively as a function of the SM parameters.

- > corrections to λ_{hhh} are expected to behave similar to those of the Higgs boson mass
- > OS scheme for m_h allows to "absorb" large part of corrections
- > in SUSY:
 - $\lambda_{hhh} = 3m_h^2/v$ approximate [Dobado, Herrero, Hollik, Penaranda '02]
 - but m_h not free and $m_h \lesssim m_Z$ at tree-level!
 - requires loop corrections of about 40 GeV (15-30%)
 - can't stop at one-loop; need higher orders (→KUTS)



→ the precision of λ_{hhh} (order in perturbation theory) should match those of m_h !

Results: λ_{hhh} in the NMSSM at two-loops

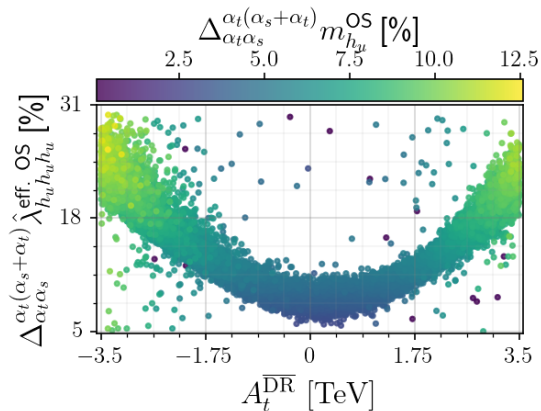
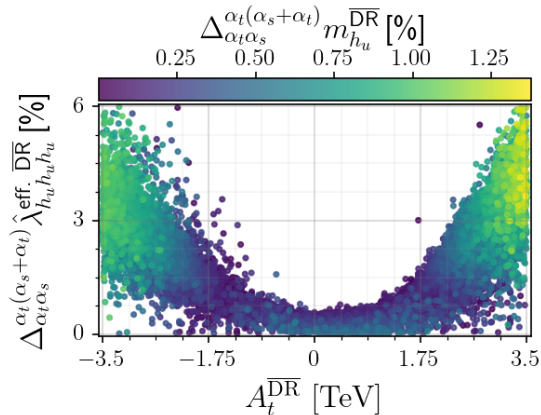


$$\Delta_{\text{ren.}} \lambda_{hhh} = \frac{\lambda_{hhh}(m_t^{\text{DR}}, A_t^{\text{DR}}) - \lambda_{hhh}(m_t^{\text{OS}}, A_t^{\text{OS}})}{\lambda_{hhh}(m_t^{\text{DR}}, A_t^{\text{DR}})} \sim \text{higher-orders} \rightarrow \text{estimates theory uncertainty}$$

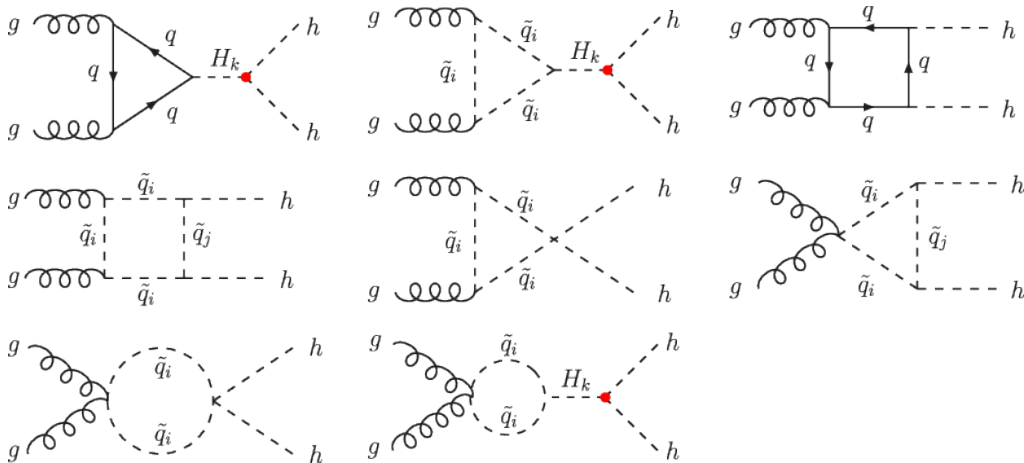
(Points checked against HiggsSignals 2.6.2 and HiggsBounds 5.10.2 as well as model-independent constraints on SUSY masses.)

Size of the $\mathcal{O}(\alpha_t^2)$ -corrections to λ_{hhh}

...and correlation to $\mathcal{O}(\alpha_t^2)$ m_h -corrections



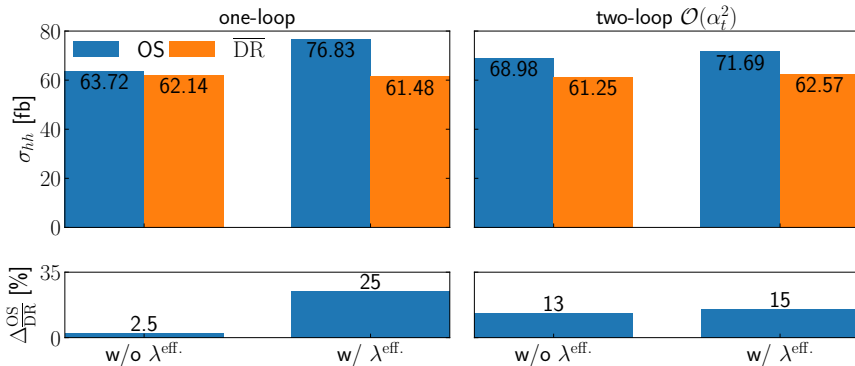
Double Higgs production



Use $\lambda_{hh}^{\alpha_t^2}$ as input in HPAIR [Spira] to estimate higher-order effects in σ_{hh} .

Double Higgs production

Parameter point with resonant contribution from intermediate BSM Higgs:



- > w/o $\lambda^{\text{eff.}}$: loop corrections to masses/mixing angles (and according LSZ-factors)
 - corrections to the input parameters
- > w/ $\lambda^{\text{eff.}}$: additionally use effective coupling at respective order
 - corrections to the di-Higgs process