A few techniques in using SM EFT

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Xiaochuan Lu
UC Davis

with Brian Henning, Tom Melia, Hitoshi Murayama

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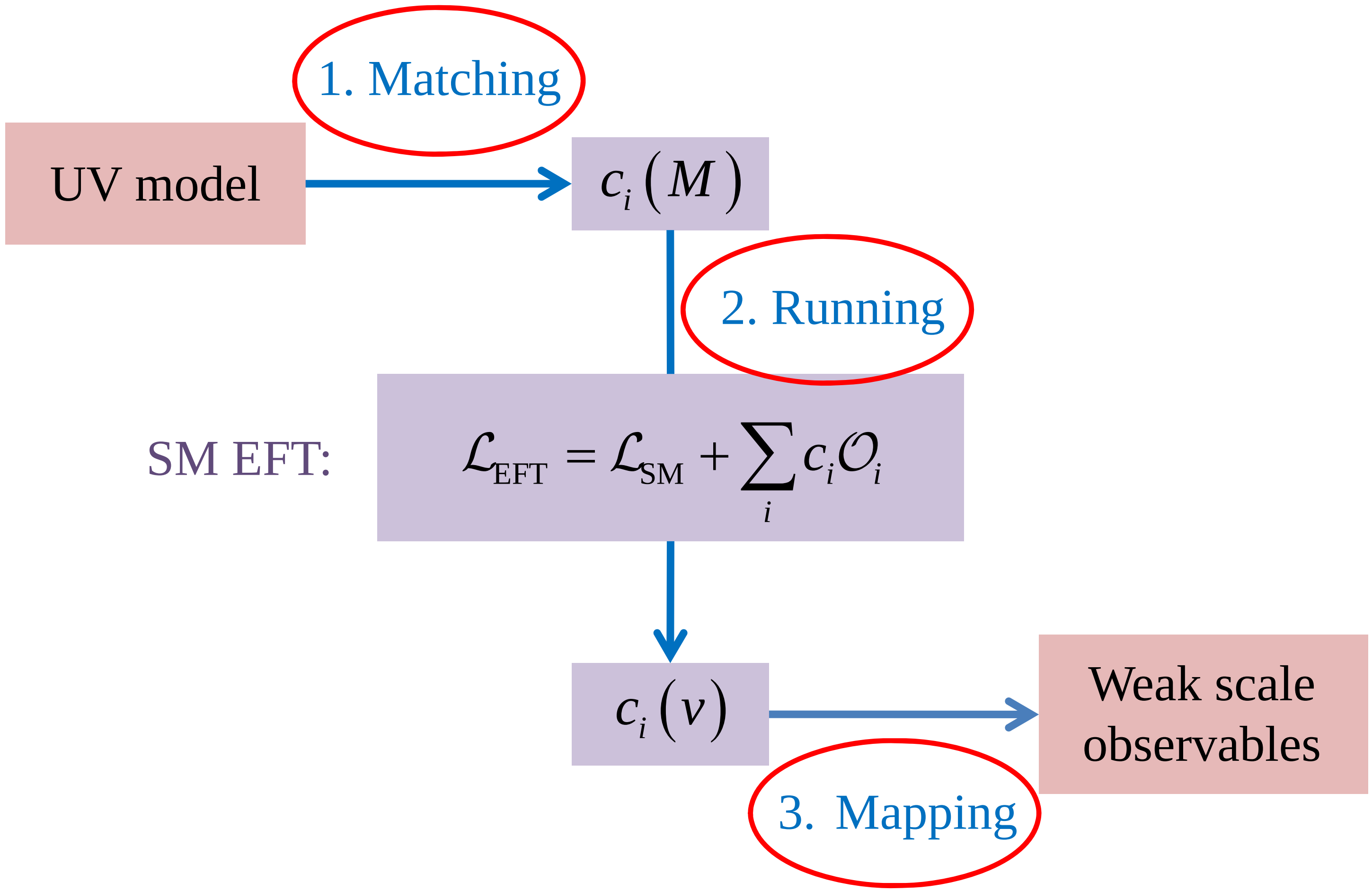
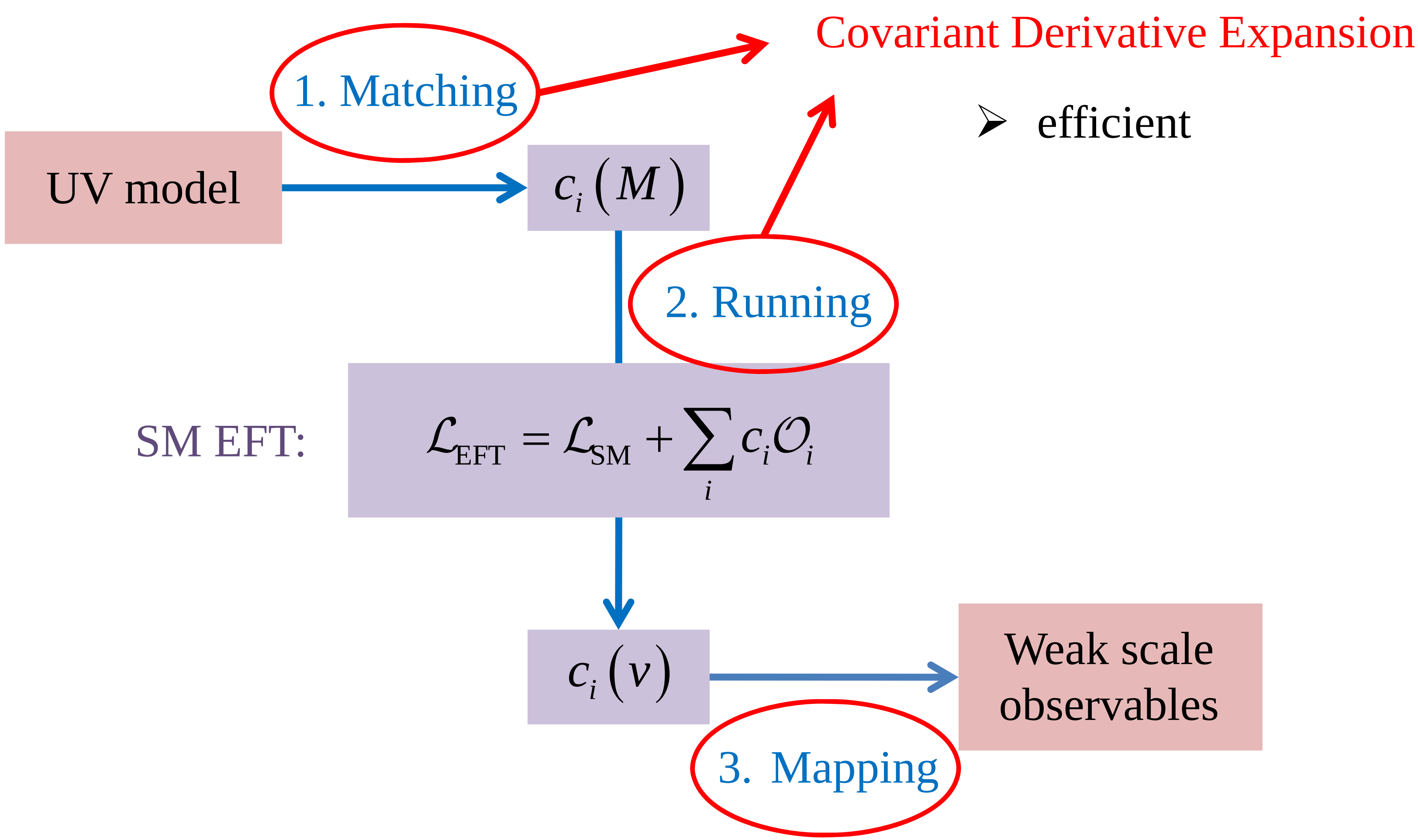
3 steps in using SM EFT

SM EFT: $$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

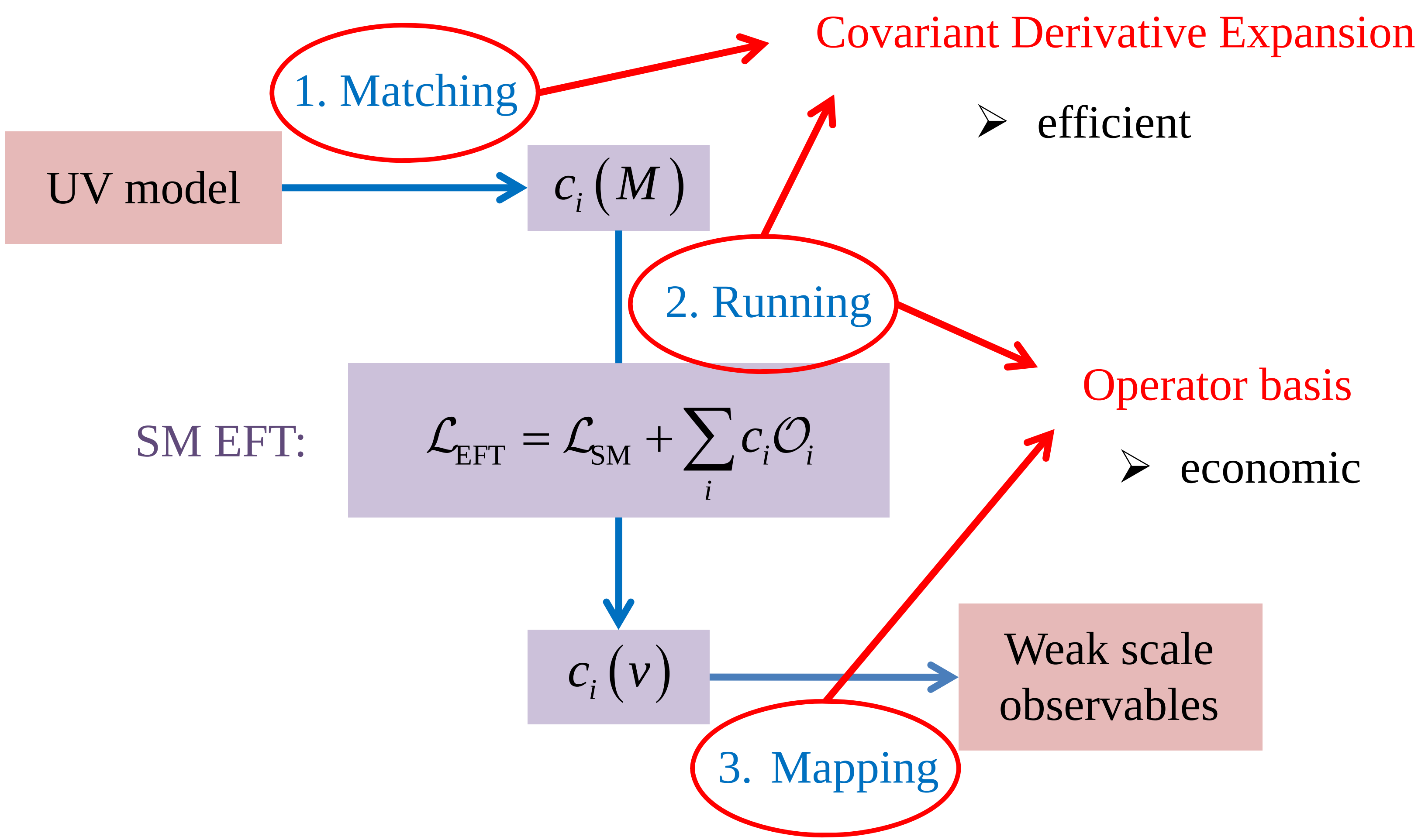
3 steps in using SM EFT



3 steps in using SM EFT



3 steps in using SM EFT



Functional method

$$S[\phi] = \int d^4 x \mathcal{L}(\phi)$$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \frac{\int D\phi e^{iS[\phi]} \phi(x_1) \cdots \phi(x_n)}{\int D\phi e^{iS[\phi]}}$$

Functional method

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$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \frac{\int D\phi e^{iS[\phi]} \phi(x_1) \cdots \phi(x_n)}{\int D\phi e^{iS[\phi]}}$$

1PI effective action: $\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{1PI}} = i \dfrac{\delta^n \Gamma[\phi]}{\delta\phi(x_1) \cdots \delta\phi(x_n)}$

Covariant derivative expansion

Functional method

$$S[\phi] = \int d^4x \mathcal{L}(\phi)$$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \frac{\int D\phi e^{iS[\phi]} \phi(x_1) \cdots \phi(x_n)}{\int D\phi e^{iS[\phi]}}$$

1PI effective action: $$\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{1PI}} = i \frac{\delta^n \Gamma[\phi]}{\delta\phi(x_1) \cdots \delta\phi(x_n)}$$

Up to 1-loop level: $$\Gamma[\phi] = S[\phi] + \frac{i}{2} \log\det\left(-\frac{\delta^2 S[\phi]}{\delta\phi^2} \right)$$

Functional method in matching

$$\mathcal{L}_{\text{EFT}}(\phi) = \mathcal{L}_{\phi}(\phi) + \sum_i c_i(\mu)\mathcal{O}_i(\phi)$$

$$\mathcal{L}_{\text{UV}}(\phi,\Phi) = \mathcal{L}_{\phi}(\phi) + \mathcal{L}_{\Phi}(\phi,\Phi)$$

$$\Gamma_{\text{L,EFT}}[\phi](\mu = M) = \Gamma_{\text{L,UV}}[\phi](\mu = M)$$

Functional method in matching

$$\mathcal{L}_{\text{EFT}}(\phi) = \mathcal{L}_{\phi}(\phi) + \sum_i c_i(\mu)\mathcal{O}_i(\phi)$$

$$\mathcal{L}_{\text{UV}}(\phi, \Phi) = \mathcal{L}_{\phi}(\phi) + \mathcal{L}_{\Phi}(\phi, \Phi)$$

$$\Gamma_{\text{L,EFT}}[\phi](\mu = M) = \Gamma_{\text{L,UV}}[\phi](\mu = M)$$

- Tree level:

$$\sum_i c_i^{(0)}(M)\mathcal{O}_i = \mathcal{L}_{\Phi}(\phi, \Phi_c[\phi]) \qquad \left.\frac{\delta S_{\text{UV}}[\phi, \Phi]}{\delta \Phi}\right|_{\Phi_c[\phi]} = 0$$

Functional method in matching

$$\mathcal{L}_{\mathrm{EFT}}(\phi) = \mathcal{L}_{\phi}(\phi) + \sum_i c_i(\mu)\mathcal{O}_i(\phi) \qquad \mathcal{L}_{\mathrm{UV}}(\phi,\Phi) = \mathcal{L}_{\phi}(\phi) + \mathcal{L}_{\Phi}(\phi,\Phi)$$

$$\Gamma_{\mathrm{L,EFT}}[\phi](\mu = M) = \Gamma_{\mathrm{L,UV}}[\phi](\mu = M)$$

- Tree level:

$$\sum_i c_i^{(0)}(M)\mathcal{O}_i = \mathcal{L}_{\Phi}(\phi,\Phi_c[\phi]) \qquad \left.\frac{\delta S_{\mathrm{UV}}[\phi,\Phi]}{\delta\Phi}\right|_{\Phi_c[\phi]} = 0$$

- 1-loop level:

$$\int d^4x \sum_i c_i^{(1)}(M)\mathcal{O}_i = \frac{i}{2}\log\det\left(-\left.\frac{\delta^2 S_{\mathrm{UV}}[\phi,\Phi]}{\delta(\phi,\Phi)^2}\right|_{\Phi=\Phi_c[\phi]}\right) - \frac{i}{2}\log\det\left(-\frac{\delta^2 S_{\mathrm{EFT}}^{(0)}[\phi]}{\delta\phi^2}\right)$$

Covariant derivative expansion

Functional method in running

$$\mathcal{L}(\phi) \supset \mathcal{O}_K(\phi) + \lambda(\mu)\mathcal{O}_\lambda(\phi) \quad \Rightarrow \quad \beta_\lambda \equiv \mu \frac{d}{d\mu}\lambda(\mu) = ?$$

Functional method in running

$$\mathcal{L}(\phi) \supset \mathcal{O}_K(\phi) + \lambda(\mu)\mathcal{O}_\lambda(\phi) \quad \Rightarrow \quad \beta_\lambda \equiv \mu \frac{d}{d\mu}\lambda(\mu) = ?$$

$$\begin{aligned} \Gamma[\phi] &\supset \int d^4x \left[a_K(\mu)\mathcal{O}_K(\phi) + a_\lambda(\mu)\mathcal{O}_\lambda(\phi) \right] \\ &\to \int d^4x \left[\mathcal{O}_K(\phi) + a'_\lambda(\mu)\mathcal{O}_\lambda(\phi) \right] \end{aligned}$$

Covariant derivative expansion

Functional method in running

$$\mathcal{L}(\phi) \supset \mathcal{O}_K(\phi) + \lambda(\mu)\mathcal{O}_\lambda(\phi) \quad \Rightarrow \quad \beta_\lambda \equiv \mu \frac{d}{d\mu}\lambda(\mu) = ?$$

$$\Gamma[\phi] \supset \int d^4x \left[a_K(\mu)\mathcal{O}_K(\phi) + a_\lambda(\mu)\mathcal{O}_\lambda(\phi) \right]$$
$$\to \int d^4x \left[\mathcal{O}_K(\phi) + a'_\lambda(\mu)\mathcal{O}_\lambda(\phi) \right]$$

Renormalization group equation:

$$\mu \frac{d}{d\mu} a'_\lambda(\mu) = 0$$

Covariant derivative expansion

Advantages compared to Feynman diagram method

- Do not need to enumerate Feynman diagrams, nor remember Feynman rules

  --- everything comes out of expansion correctly

- Do not need a prior knowledge of the EFT operators

  --- Wilson coefficients are directly read off from the result

- Can deal with different correlators simultaneously

  --- A framework that gauge covariance can be transparent

Covariant derivative expansion

Evaluating functional traces

$$\mathcal{L} \supset \frac{1}{2}\Phi\left[-D^2 - M^2 - U(x)\right]\Phi + \cdots$$

elliptic operator

$$\Gamma^{(1)}[\Phi] = \frac{i}{2}\log\det\left(-\frac{\delta^2 S[\Phi]}{\delta\Phi^2}\right) = \frac{i}{2}\mathrm{Tr}\log\left[D^2 + M^2 + U(x)\right]$$

$$= \frac{i}{2}\mathrm{Tr}\log\left(D^2 + M^2\right) - \frac{i}{2}\mathrm{Tr}\log\left[\frac{1}{-D^2 - M^2}U(x) + \frac{1}{2}\frac{1}{-D^2 - M^2}U(x)\frac{1}{-D^2 - M^2}U(x) + \cdots\right]$$

$$\mathrm{Tr}\left[\frac{1}{-D^2 - M^2}U(x)\right]$$

Partial derivative expansion

$$D_\mu = \partial_\mu - igA_\mu(x)$$

$$\frac{1}{-D^2 - M^2}U = \frac{1}{-\partial^2 - M^2}U - \frac{1}{-\partial^2 - M^2}\left(igA^\mu\partial_\mu + ig\partial_\mu A^\mu + g^2A^2\right)\frac{1}{-\partial^2 - M^2}U + \cdots$$

$$\mathrm{Tr}[\ \cdots\ ] = \int d^4x \langle x|\ \cdots\ |x\rangle$$

$$1 = \int d^4x |x\rangle\langle x| \quad , \quad 1 = \int \frac{d^4p}{(2\pi)^4}|p\rangle\langle p|$$

Gauge covariance not manifest

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}U(x)\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\langle p|x\rangle\langle x\left|\frac{1}{(iD)^2-M^2}U(x)\right|p\rangle$$

$$\langle x|\,f(\hat{x},\hat{p})=f(x,i\partial_x)\langle x|$$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}U(x)\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\langle p|x\rangle\langle x\left|\frac{1}{(iD)^2-M^2}U(x)\right|p\rangle$$

$$\langle x|\,f(\hat{x},\hat{p})=f(x,i\partial_x)\langle x|$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}e^{ipx}\frac{1}{(iD)^2-M^2}U(x)e^{-ipx}\cdot 1(x,p)$$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}U(x)\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\langle p|x\rangle\underline{\left\langle x\left|\frac{1}{(iD)^2-M^2}U(x)\right|}p\rangle$$

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$$e^{ipx}iD_\mu e^{-ipx}=iD_\mu+p_\mu$$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}U(x)\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\langle p|x\rangle\underline{\left\langle x\left|\frac{1}{(iD)^2-M^2}U(x)\right|}p\right\rangle$$

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$$=\int d^4x\frac{d^4p}{(2\pi)^4}\frac{1}{(iD+p)^2-M^2}U(x)$$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}U(x)\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\langle p|x\rangle\left\langle x\left|\frac{1}{(iD)^2-M^2}U(x)\right|p\right\rangle$$

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Expand in terms of D

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}U(x)\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\langle p|x\rangle\underline{\left\langle x\left|\frac{1}{(iD)^2-M^2}U(x)\right|p\right\rangle}$$

$$\langle x|f(\hat{x},\hat{p})=f(x,i\partial_x)\langle x|$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}\underline{e^{ipx}\frac{1}{(iD)^2-M^2}U(x)e^{-ipx}}\cdot 1(x,p)$$

$$e^{ipx}iD_\mu e^{-ipx}=iD_\mu+p_\mu$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}\underline{\frac{1}{(iD+p)^2-M^2}}U(x)$$

Expand in terms of D

$$\mathrm{Tr}\left[\frac{(-D^2)^{k_1}}{-D^2-M_1^2}U_1(x)\frac{(-D^2)^{k_2}}{-D^2-M_2^2}U_2(x)\cdots\frac{(-D^2)^{k_n}}{-D^2-M_n^2}U_n(x)\right]$$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}U(x)\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\langle p|x\rangle\underline{\left\langle x\left|\frac{1}{(iD)^2-M^2}U(x)\right|p\right\rangle}$$

$$\langle x|f(\hat{x},\hat{p})=f(x,i\partial_x)\langle x|$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}\underline{e^{ipx}\frac{1}{(iD)^2-M^2}U(x)e^{-ipx}}\cdot 1(x,p)$$

$$e^{ipx}iD_\mu e^{-ipx}=iD_\mu+p_\mu$$

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Expand in terms of D

$$\mathrm{Tr}\left[\frac{(-D^2)^{k_1}}{-D^2-M_1^2}U_1(x)\frac{(-D^2)^{k_2}}{-D^2-M_2^2}U_2(x)\cdots\frac{(-D^2)^{k_n}}{-D^2-M_n^2}U_n(x)\right]$$

- Exception: $U_n(x)=\mathrm{const}$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2 - M^2}\right] = \int d^4x \frac{d^4p}{(2\pi)^4} \frac{1}{(iD+p)^2 - M^2} \cdot 1(x,p)$$

$$= \int d^4x \frac{d^4p}{(2\pi)^4} f_1(\partial_p) \frac{1}{(iD-p)^2 - M^2} f_2(\partial_p) \cdot 1(x,p) \qquad f_1(0) = f_2(0) = 1$$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\frac{1}{(iD+p)^2-M^2}\cdot 1(x,p)$$

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$$f_1(0)=f_2(0)=1$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}e^{iD\partial_p}\frac{1}{(iD-p)^2-M^2}e^{-iD\partial_p}\cdot 1(x,p)$$

$$e^{iD\partial_p}\left(iD_\mu\right)e^{-iD\partial_p}=iD_\mu+\left[iD_\nu\partial_p^\nu,iD_\mu\right]+\cdots$$

$$e^{iD\partial_p}\left(p_\mu\right)e^{-iD\partial_p}=p_\mu+iD_\mu+\cdots$$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\frac{1}{(iD+p)^2-M^2}\cdot 1(x,p)$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}f_1(\partial_p)\frac{1}{(iD-p)^2-M^2}f_2(\partial_p)\cdot 1(x,p) \qquad f_1(0)=f_2(0)=1$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}e^{iD\partial_p}\frac{1}{(iD-p)^2-M^2}e^{-iD\partial_p}\cdot 1(x,p)$$

$$e^{iD\partial_p}(iD_\mu)e^{-iD\partial_p}=iD_\mu+\left[iD_\nu\partial_p^\nu,iD_\mu\right]+\cdots$$

$$e^{iD\partial_p}(p_\mu)e^{-iD\partial_p}=p_\mu+iD_\mu+\cdots$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}\frac{1}{\left(-p_\mu+G'_{\mu\nu}\partial_p^\nu\right)^2-M^2}\cdot 1(x,p)$$

$$G'_{\mu\nu}\equiv\sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}i^n\left(D_{\alpha_1}D_{\alpha_2}\cdots D_{\alpha_n}\left[D_\mu,D_\nu\right]\right)\partial_p^{\alpha_1}\partial_p^{\alpha_2}\cdots\partial_p^{\alpha_n}$$

Covariant derivative expansion

$$\mathrm{Tr}\left[\frac{1}{-D^2-M^2}\right]=\int d^4x\frac{d^4p}{(2\pi)^4}\frac{1}{(iD+p)^2-M^2}\cdot 1(x,p)$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}f_1(\partial_p)\frac{1}{(iD-p)^2-M^2}f_2(\partial_p)\cdot 1(x,p)$$

$$f_1(0)=f_2(0)=1$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}e^{iD\partial_p}\frac{1}{(iD-p)^2-M^2}e^{-iD\partial_p}\cdot 1(x,p)$$

$$e^{iD\partial_p}(iD_\mu)e^{-iD\partial_p}=iD_\mu+\left[iD_\nu\partial_p^\nu,iD_\mu\right]+\cdots$$

$$e^{iD\partial_p}(p_\mu)e^{-iD\partial_p}=p_\mu+iD_\mu+\cdots$$

$$=\int d^4x\frac{d^4p}{(2\pi)^4}\underline{\frac{1}{\left(-p_\mu+G'_{\mu\nu}\partial_p^\nu\right)^2-M^2}}\cdot 1(x,p)$$

Expand in terms of $G'_{\mu\nu}$

$$G'_{\mu\nu}\equiv\sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}i^n\left(D_{\alpha_1}D_{\alpha_2}\cdots D_{\alpha_n}\left[D_\mu,D_\nu\right]\right)\partial_p^{\alpha_1}\partial_p^{\alpha_2}\cdots\partial_p^{\alpha_n}$$

Covariant derivative expansion

$$\mathrm{Tr}\log\left(D^2+M^2+U\right)=\int d^4x\frac{-i}{(4\pi)^2}\mathrm{tr}\left\{M^4\left[-\frac{1}{2}\left(\ln\frac{M^2}{\mu^2}-\frac{3}{2}\right)\right]\right.$$

$$+M^2\left[-\left(\ln\frac{M^2}{\mu^2}-1\right)U\right]$$

$$G_{\mu\nu}\equiv\left[D_\mu,D_\nu\right]$$

$$+M^0\left[-\frac{1}{2}\ln\frac{M^2}{\mu^2}U^2-\frac{1}{12}\left(\ln\frac{M^2}{\mu^2}-1\right)G_{\mu\nu}^2\right]$$

$$+\frac{1}{M^2}\left[-\frac{1}{6}U^3+\frac{1}{12}(DU)^2-\frac{1}{12}UG_{\mu\nu}^2+\frac{1}{60}(D^\mu G_{\mu\nu})^2-\frac{1}{90}G_\mu^\nu G_\nu^\rho G_\rho^\mu\right]$$

$$+\frac{1}{M^4}\left[\frac{1}{24}U^4-\frac{1}{12}U(DU)^2+\frac{1}{120}(D^2U)^2+\frac{1}{60}(D^\mu U)(D^\nu U)G_{\mu\nu}\right.$$

$$\left.+\frac{1}{40}U^2G_{\mu\nu}^2+\frac{1}{60}(UG_{\mu\nu})^2\right]$$

$$+\frac{1}{M^6}\left[-\frac{1}{60}U^5+\frac{1}{20}U^2(DU)^2+\frac{1}{30}(UDU)^2\right]$$

$$\left.+\frac{1}{M^8}\left[\frac{1}{120}U^6\right]\right\}$$

Universal Result for elliptic operator

Operator basis

$$\mathcal{L}_{\text{EFT}}(\phi) = \mathcal{L}_{\text{SM}}(\phi) + \sum_i c_i \mathcal{O}_i(\phi)$$

- Redundancies:

  --- Group identities

  --- Integration by Part (IBP) $\quad \mathcal{O}_1 = \mathcal{O}_2 + \partial \cdot \mathcal{O}$

  --- Equations of Motion (EOM) $\quad \mathcal{O}_1 = \mathcal{O}_2 + \mathcal{O}\dfrac{\delta \mathcal{L}_{\text{SM}}}{\delta \phi}$

- operator basis: $\mathcal{K} \supset \mathcal{O}_i$ a complete set of independent operators

  --- Economic for running and mapping

Operator basis

$$\text{EOM} \Leftrightarrow \text{Field redefinition}$$

Scherer and Fearing,
arXiv: hep-ph/9408298

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots + \mathcal{L}^{(k)} + \underline{\mathcal{L}^{(k+1)} + \cdots + \mathcal{L}^{(n)}} \quad , \quad \mathcal{L}^{(k)} \propto \frac{1}{\Lambda^k}$$

$$\supset \frac{1}{\Lambda^k} \mathcal{O} \frac{\delta \mathcal{L}^{(0)}}{\delta \phi}$$

Operator basis

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$$\supset \frac{1}{\Lambda^k} \mathcal{O} \frac{\delta \mathcal{L}^{(0)}}{\delta \phi}$$

$$\frac{1}{\Lambda^k} \mathcal{O} \left[-\frac{\delta \mathcal{L}^{(1)}}{\delta \phi} - \frac{\delta \mathcal{L}^{(2)}}{\delta \phi} - \cdots \right]$$

EOM $\Leftrightarrow$ Field redefinition

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$$\mathcal{L}_{\text{EFT}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots + \mathcal{L}^{(k)} + \underline{\mathcal{L}^{(k+1)} + \cdots + \mathcal{L}^{(n)}} \quad , \quad \mathcal{L}^{(k)} \propto \frac{1}{\Lambda^k}$$

$$\supset \frac{1}{\Lambda^k}\mathcal{O}\frac{\delta\mathcal{L}^{(0)}}{\delta\phi}$$

$$\frac{1}{\Lambda^k}\mathcal{O}\left[-\frac{\delta\mathcal{L}^{(1)}}{\delta\phi} - \frac{\delta\mathcal{L}^{(2)}}{\delta\phi} - \cdots\right]$$

$$\phi \to \phi + \Delta\phi \quad \Rightarrow \quad \Delta\mathcal{L}_{\text{EFT}} = \frac{\delta\mathcal{L}^{(0)}}{\delta\phi}\Delta\phi + \frac{1}{2}\frac{\delta^2\mathcal{L}^{(0)}}{\delta\phi^2}(\Delta\phi)^2 + \cdots$$
$$+ \frac{\delta\mathcal{L}^{(1)}}{\delta\phi}\Delta\phi + \frac{1}{2}\frac{\delta^2\mathcal{L}^{(1)}}{\delta\phi^2}(\Delta\phi)^2 + \cdots$$
$$+ \frac{\delta\mathcal{L}^{(2)}}{\delta\phi}\Delta\phi + \cdots$$

EOM $\Leftrightarrow$ Field redefinition

Scherer and Fearing, arXiv: hep-ph/9408298

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots + \mathcal{L}^{(k)} + \underline{\mathcal{L}^{(k+1)} + \cdots + \mathcal{L}^{(n)}} \quad , \quad \mathcal{L}^{(k)} \propto \frac{1}{\Lambda^k}$$

$$\supset \frac{1}{\Lambda^k}\mathcal{O}\frac{\delta\mathcal{L}^{(0)}}{\delta\phi}$$

$$\frac{1}{\Lambda^k}\mathcal{O}\left[-\frac{\delta\mathcal{L}^{(1)}}{\delta\phi} - \frac{\delta\mathcal{L}^{(2)}}{\delta\phi} - \cdots\right]$$

$$\phi \to \phi + \Delta\phi \quad \Rightarrow \quad \Delta\mathcal{L}_{\text{EFT}} = \frac{\delta\mathcal{L}^{(0)}}{\delta\phi}\Delta\phi + \frac{1}{2}\frac{\delta^2\mathcal{L}^{(0)}}{\delta\phi^2}(\Delta\phi)^2 + \cdots$$

$$+\frac{\delta\mathcal{L}^{(1)}}{\delta\phi}\Delta\phi + \frac{1}{2}\frac{\delta^2\mathcal{L}^{(1)}}{\delta\phi^2}(\Delta\phi)^2 + \cdots$$

$$+\frac{\delta\mathcal{L}^{(2)}}{\delta\phi}\Delta\phi + \cdots$$

$$\Delta\phi = -\frac{1}{\Lambda^k}\mathcal{O}$$

Operator basis

$$\text{EOM} \Leftrightarrow \text{Field redefinition}$$

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$$\supset \frac{1}{\Lambda^k} \mathcal{O} \frac{\delta \mathcal{L}^{(0)}}{\delta \phi}$$

$$\frac{1}{\Lambda^k} \mathcal{O} \left[-\frac{\delta \mathcal{L}^{(1)}}{\delta \phi} - \frac{\delta \mathcal{L}^{(2)}}{\delta \phi} - \cdots \right]$$

$$\phi \to \phi + \Delta\phi \quad \Rightarrow \quad \Delta\mathcal{L}_{\text{EFT}} = \frac{\delta \mathcal{L}^{(0)}}{\delta \phi} \Delta\phi + \frac{1}{2} \frac{\delta^2 \mathcal{L}^{(0)}}{\delta \phi^2} (\Delta\phi)^2 + \cdots$$
$$+ \frac{\delta \mathcal{L}^{(1)}}{\delta \phi} \Delta\phi + \frac{1}{2} \frac{\delta^2 \mathcal{L}^{(1)}}{\delta \phi^2} (\Delta\phi)^2 + \cdots$$
$$+ \frac{\delta \mathcal{L}^{(2)}}{\delta \phi} \Delta\phi + \cdots$$

$$\Delta\phi = -\frac{1}{\Lambda^k} \mathcal{O}$$

$$\text{EOM} \;\Leftrightarrow\; \text{Field redefinition}$$

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$$\mathcal{L}_{\text{EFT}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots + \mathcal{L}^{(k)} + \underline{\mathcal{L}^{(k+1)} + \cdots + \mathcal{L}^{(n)}} \quad , \quad \mathcal{L}^{(k)} \propto \frac{1}{\Lambda^k}$$

$$\supset \frac{1}{\Lambda^k}\mathcal{O}\frac{\delta\mathcal{L}^{(0)}}{\delta\phi}$$

Different from using
Higher order EOM

$$\frac{1}{\Lambda^k}\mathcal{O}\left[-\frac{\delta\mathcal{L}^{(1)}}{\delta\phi}-\frac{\delta\mathcal{L}^{(2)}}{\delta\phi}-\cdots\right]$$

$$\phi \to \phi + \Delta\phi$$

$$\Delta\phi = -\frac{1}{\Lambda^k}\mathcal{O}$$

$$\begin{aligned}\Delta\mathcal{L}_{\text{EFT}} = &\frac{\delta\mathcal{L}^{(0)}}{\delta\phi}\Delta\phi + \frac{1}{2}\frac{\delta^2\mathcal{L}^{(0)}}{\delta\phi^2}(\Delta\phi)^2 + \cdots \\ &+ \frac{\delta\mathcal{L}^{(1)}}{\delta\phi}\Delta\phi + \frac{1}{2}\frac{\delta^2\mathcal{L}^{(1)}}{\delta\phi^2}(\Delta\phi)^2 + \cdots \\ &+ \frac{\delta\mathcal{L}^{(2)}}{\delta\phi}\Delta\phi + \cdots\end{aligned}$$

Operator basis

It is not easy to enumerate $\mathcal{K}$ by hand…

dim 6

B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, arXiv: 1008.4884

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu \tau^I q_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu \tau^I l_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$
$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating			
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$		
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$		
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$		

Table 3: Four-fermion operators.

SM EFT counting history

- dim 6, $n_g = 1$ — 1986 — Buchmuller and Wyler
  Nucl. Phys. B 268 (1986) 621

  2010 — Grzadkowski, Iskrzynski, Misiak, and Rosiek
  arXiv: 1008.4884

- dim 6, general n_g — 2013 — Alonso, Jenkins, Manohar, and Trott
  arXiv: 1312.2014

- $\begin{cases} \text{dim 7, general } n_g \\ \text{dim 8, } n_g = 1 \end{cases}$ — 2014 - 15 — L. Lehman and A. Martin
  arXiv: 1410.4193, 1503.07537, 1510.00372

our goal: making it systematic

Operator basis

Example: a real scalar field ϕ

EOM: $\partial^2\phi = 0$

traceless symmetric

EOM removed: $\mathcal{J} = \mathbb{R}\left[\phi,\quad \partial_{\mu_1}\phi,\quad \partial_{\{\mu_1}\partial_{\mu_2\}}\phi,\quad \partial_{\{\mu_1}\partial_{\mu_2}\partial_{\mu_3\}}\phi,\quad \cdots\right] = \mathbb{R}\left[R_\phi\right]$

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$\mathcal{J}$ is generated by $R_\phi = \begin{pmatrix} \phi \\ \partial_{\mu_1}\phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{pmatrix}$ which forms a representation of the conformal group $SO(d+2,\mathbb{C})$

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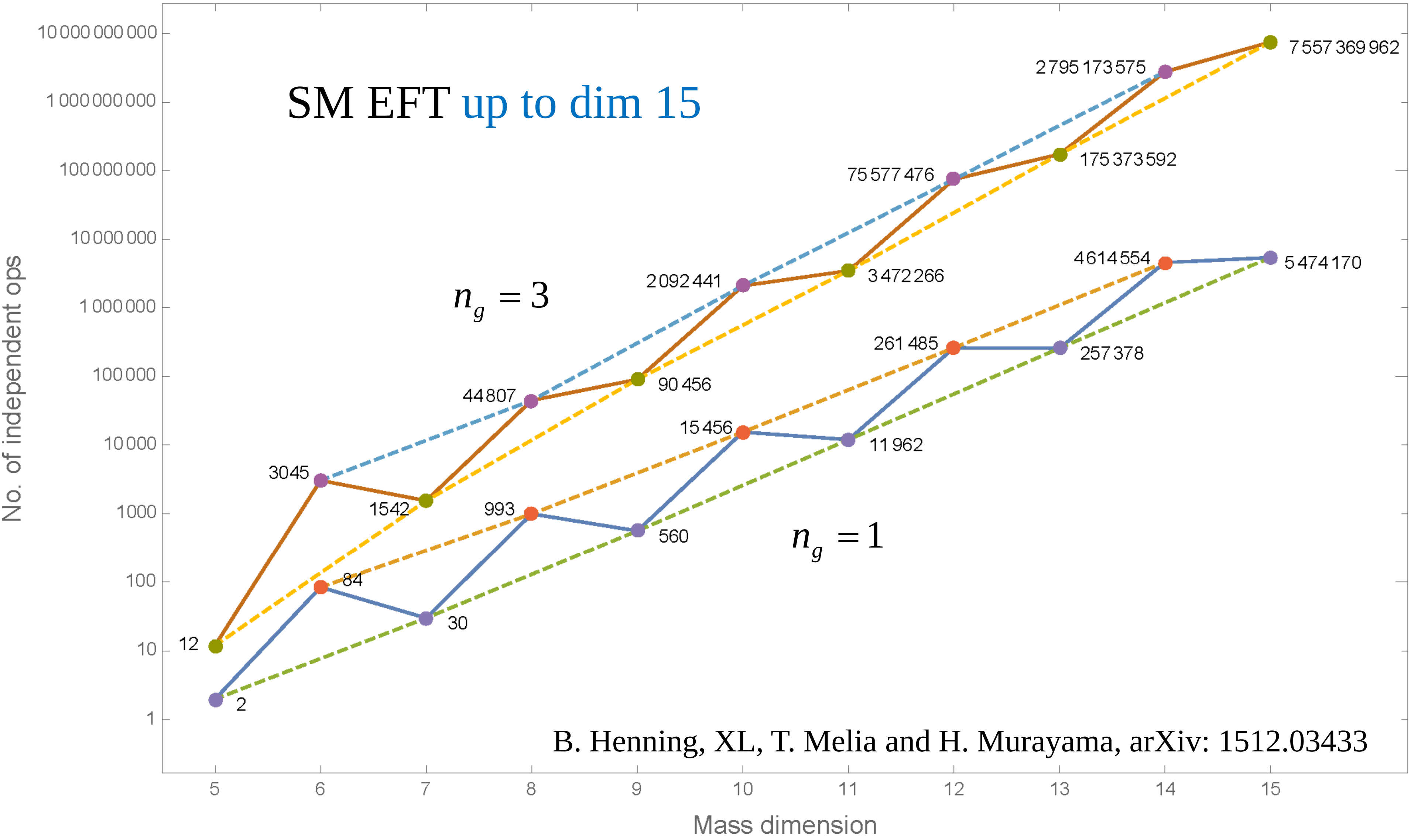
$\mathcal{J}$ is generated by $R_\phi = \begin{pmatrix} \phi \\ \partial_{\mu_1}\phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{pmatrix}$ which forms a representation of the conformal group $SO(d+2,\mathbb{C})$

operator basis: $\mathcal{K} = \left[\mathcal{J} / \partial\mathcal{J}\right]^{SO(d)}$ $\rightarrow$ scalar conformal primaries in $\mathcal{J}$

Operator basis



Towards constructing operator basis

$$\phi^n \partial^k = \phi_1 \cdots \phi_n \partial_{\mu_1} \cdots \partial_{\mu_k} = \int dp_1 \cdots dp_n \exp\left(i \sum_{i=1}^{n} p_i^\mu x_\mu \right) \tilde{\phi}_1(p_1) \cdots \tilde{\phi}_n(p_n) F_n^k\left(p_1^\mu, \cdots, p_n^\mu\right)$$

$$\mathcal{O}_n^k = \phi^n \partial^k \quad \leftrightarrow \quad F_n^k\left(p_1^\mu, \cdots, p_n^\mu\right) \qquad \begin{cases} \text{EOM}: & p_i^2 = 0 \\ \text{IBP}: & \sum_{i=1}^{n} p_i^\mu = 0 \end{cases}$$

$$\mathcal{K} = \oplus_n \left\{ \left[\mathbb{R}\left[p_1^\mu, \cdots, p_n^\mu \right] \big/ \left\langle p_1^2, \cdots, p_n^2, p_1^\mu + \cdots + p_n^\mu \right\rangle \right]^{SO(d) \times S_n} \right\}$$

Summary

- Generically 3 steps in using an EFT

  --- matching, running, and mapping

- Covariant derivative expansion technique

  --- useful for matching and running

- Studying operator basis: counting and constructing

  --- important for running and mapping

Thank you!