



# Neutrino and dark matter phenomenology from an $A_4$ Discrete Dark Matter Model

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Based on [Phys. Rev. D 94, 055007] and [in prep.]

in collaboration with E. Peinado

20th Planck Conference

Warsaw, May 25<sup>th</sup> 2017





# Motivation

Alternating group ( $A_4$ ): Flavour symmetry group. [E. Ma, *et al.* '01]

Non-abelian, discrete group. It has:

Tree 1-dim. irreps.:  $\mathbf{1}_1$ ,  $\mathbf{1}_2$ ,  $\mathbf{1}_3$ .

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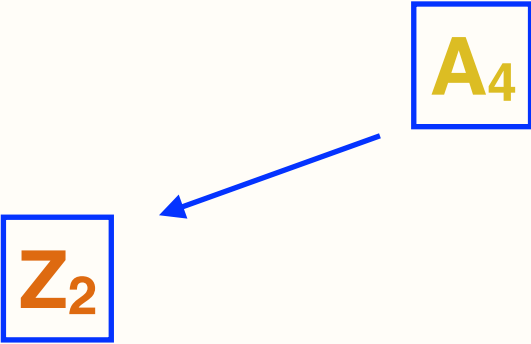
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Generators in the a  
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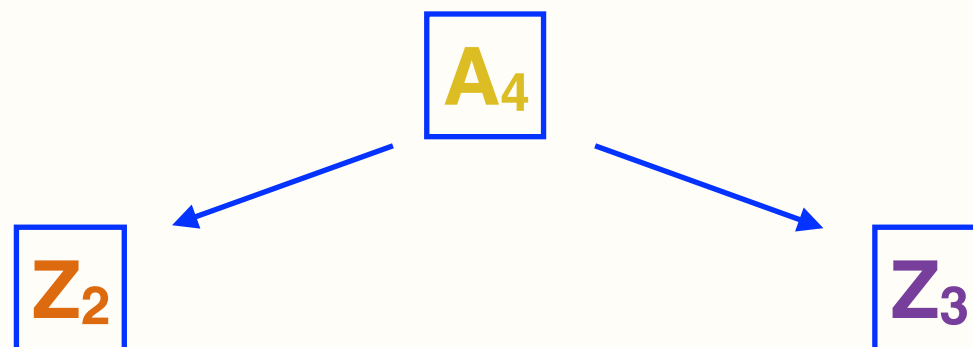
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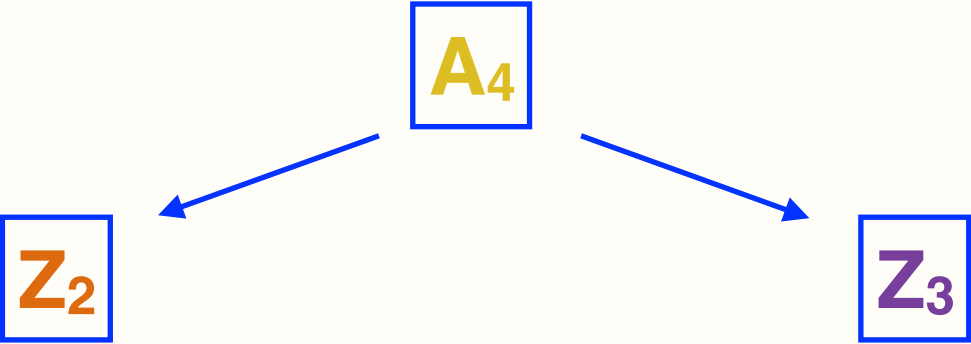
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Discrete Dark Matter Model: [M. Hirsch, *et al.* '10]

Non-Abelian discrete flavour symmetry breaking into one of its subgroups, by means of EWSB, accounts for the neutrino masses and mixing and dark matter stability.

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# The Model

## Particle assignments

	$L_e$	$L_\mu$	$L_\tau$	$l_e^c$	$l_\mu^c$	$l_\tau^c$	$N_T$	$N_4$	$N_5$
SU(2)	2	2	2	1	1	1	1	1	1
$A_4$	1	$1'$	$1''$	1	$1''$	$1'$	3	1	$1'$

	$H$	$\eta$	$\phi$
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SM

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New particles

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$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

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$1''$

Another model,  
analogous pheno.

# The Model

Symmetry breaking

**$A_4$**

# The Model

Flavour symmetry  
breaking (at a scale  
close to the seesaw  
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\*Seesaw  
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**A<sub>4</sub>**



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Induced by the flavon,  
 $\phi$ , VEV alignment:

$$\langle \phi_1 \rangle = v_\phi \neq 0, \quad \langle \phi_{2,3} \rangle = 0.$$

Let **S** (**Z<sub>2</sub>**)  
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**Z<sub>2</sub>**

Effective **Z<sub>2</sub>** potential at the EW scale.  
(Flavon dynamics integrated out)

**EWSB**

Scalar fields, **H** and  $\eta_{1,2,3}$ , VEVs  
alignments:

$$\langle H^0 \rangle = v_h \neq 0, \quad \langle \eta_1^0 \rangle = v_\eta \neq 0, \quad \langle \eta_{2,3}^0 \rangle = 0$$



# The Model

## The Yukawa Lagrangian

Quarks are singlets:  $Q_L q_R H$

$$\begin{aligned}\mathcal{L}_Y^{(A)} = & y_e L_e l_e^c H + y_\mu L_\mu l_\mu^c H + y_\tau L_\tau l_\tau^c H \\ & + y_1^\nu L_e [N_T \eta]_1 + y_2^\nu L_\mu [N_T \eta]_{1''} + y_3^\nu L_\tau [N_T \eta]_{1'} + y_4^\nu L_e N_4 H + y_5^\nu L_\tau N_5 H \\ & + M_1 N_T N_T + M_2 N_4 N_4 + y_1^N [N_T \phi]_3 N_T + y_2^N [N_T \phi]_1 N_4 + y_3^N [N_T \phi]_{1''} N_5 + h.c.\end{aligned}$$

Diagonal mass matrix for charged leptons.

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The  $Z_2$  effective potential (at the EW Scale):

$$\begin{aligned} V = & \mu_1^2 \left( \eta_1^\dagger \eta_1 \right) + \mu_2^2 \left( h^\dagger h \right) + \mu_3^2 \left( h^\dagger \eta_1 + \eta_1^\dagger h \right) + \mu_4^2 \left( \eta_2^\dagger \eta_2 + \eta_3^\dagger \eta_3 \right) + \mu_5^2 \left( \eta_2^\dagger \eta_3 + \eta_3^\dagger \eta_2 \right) \\ & + \lambda_1 \left( (\eta_1^\dagger \eta_1)^2 + (\eta_2^\dagger \eta_2)^2 + (\eta_3^\dagger \eta_3)^2 \right) + \lambda_2 \left( h^\dagger h \right)^2 \\ & + \lambda_3 \left( \eta_1^\dagger \eta_1 \eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1 \eta_3^\dagger \eta_3 + \eta_2^\dagger \eta_2 \eta_3^\dagger \eta_3 \right) \\ & + \lambda_4 \left( h^\dagger h \right) \left( \eta_1^\dagger \eta_1 + \eta_2^\dagger \eta_2 + \eta_3^\dagger \eta_3 \right) \\ & + \lambda_5 \left( \eta_2^\dagger \eta_1 \eta_1^\dagger \eta_2 + \eta_3^\dagger \eta_1 \eta_1^\dagger \eta_3 + \eta_3^\dagger \eta_2 \eta_2^\dagger \eta_3 \right) \\ & + \lambda_6 \left( \eta_1^\dagger h h^\dagger \eta_1 + \eta_2^\dagger h h^\dagger \eta_2 + \eta_3^\dagger h h^\dagger \eta_3 \right) \\ & + \lambda_7 \left( (\eta_3^\dagger \eta_1)^2 + (\eta_1^\dagger \eta_2)^2 + (\eta_2^\dagger \eta_3)^2 \right) \\ & + \lambda_7^* \left( (\eta_2^\dagger \eta_1)^2 + (\eta_3^\dagger \eta_2)^2 + (\eta_1^\dagger \eta_3)^2 \right) \\ & + \lambda_8 \left( (\eta_1^\dagger h)^2 + (\eta_2^\dagger h)^2 + (\eta_3^\dagger h)^2 \right) \\ & + \lambda_8^* \left( (h^\dagger \eta_1)^2 + (h^\dagger \eta_2)^2 + (h^\dagger \eta_3)^2 \right) \\ & + \lambda_9 \left( \eta_2^\dagger \eta_3 \eta_1^\dagger h + \eta_3^\dagger \eta_1 \eta_2^\dagger h + \eta_1^\dagger \eta_2 \eta_3^\dagger h \right) \\ & + \lambda_9^* \left( \eta_3^\dagger \eta_2 h^\dagger \eta_1 + \eta_1^\dagger \eta_3 h^\dagger \eta_2 + \eta_2^\dagger \eta_1 h^\dagger \eta_3 \right) \\ & + \lambda_{10} \left( \eta_3^\dagger \eta_2 \eta_1^\dagger h + \eta_1^\dagger \eta_3 \eta_2^\dagger h + \eta_2^\dagger \eta_1 \eta_3^\dagger h \right) \\ & + \lambda_{10}^* \left( \eta_2^\dagger \eta_3 h^\dagger \eta_1 + \eta_3^\dagger \eta_1 h^\dagger \eta_2 + \eta_1^\dagger \eta_2 h^\dagger \eta_3 \right) \end{aligned}$$

10 couplings and 5  $\mu$ -terms.



# The Model

The  $Z_2$  effective potential (at the EW Scale):

$$\begin{aligned}
 V = & \mu_1^2 \left( \eta_1^\dagger \eta_1 \right) + \mu_2^2 (h^\dagger h) + \mu_3^2 \left( h^\dagger \eta_1 + \eta_1^\dagger h \right) + \mu_4^2 \left( \eta_2^\dagger \eta_2 + \eta_3^\dagger \eta_3 \right) + \mu_5^2 \left( \eta_2^\dagger \eta_3 + \eta_3^\dagger \eta_2 \right) \\
 & + \lambda_1 \left( (\eta_1^\dagger \eta_1)^2 + (\eta_2^\dagger \eta_2)^2 + (\eta_3^\dagger \eta_3)^2 \right) + \lambda_2 (h^\dagger h)^2 \\
 & + \lambda_3 \left( \eta_1^\dagger \eta_1 \eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1 \eta_3^\dagger \eta_3 + \eta_2^\dagger \eta_2 \eta_3^\dagger \eta_3 \right) \\
 & + \lambda_4 (h^\dagger h) \left( \eta_1^\dagger \eta_1 + \eta_2^\dagger \eta_2 + \eta_3^\dagger \eta_3 \right) \\
 & + \lambda_5 \left( \eta_2^\dagger \eta_1 \eta_1^\dagger \eta_2 + \eta_3^\dagger \eta_1 \eta_1^\dagger \eta_3 + \eta_3^\dagger \eta_2 \eta_2^\dagger \eta_3 \right) \\
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 & + \lambda_8^* \left( (h^\dagger \eta_1)^2 + (h^\dagger \eta_2)^2 + (h^\dagger \eta_3)^2 \right) \\
 & + \lambda_9 \left( \eta_2^\dagger \eta_3 \eta_1^\dagger h + \eta_3^\dagger \eta_1 \eta_2^\dagger h + \eta_1^\dagger \eta_2 \eta_3^\dagger h \right) \\
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 \end{aligned}$$

New  $\mu$ - terms appear  
due to  $\phi$  vev

10 couplings and 5  $\mu$ -terms.

# The Model

The  $Z_2$  effective potential (at the EW Scale):

$$\begin{aligned} V = & \mu_1^2 \left( \eta_1^\dagger \eta_1 \right) + \mu_2^2 \left( h^\dagger h \right) + \mu_3^2 \left( h^\dagger \eta_1 + \eta_1^\dagger h \right) + \mu_4^2 \left( \eta_2^\dagger \eta_2 + \eta_3^\dagger \eta_3 \right) + \mu_5^2 \left( \eta_2^\dagger \eta_3 + \eta_3^\dagger \eta_2 \right) \\ & + \lambda_1 \left( (\eta_1^\dagger \eta_1)^2 + (\eta_2^\dagger \eta_2)^2 + (\eta_3^\dagger \eta_3)^2 \right) + \lambda_2 \left( h^\dagger h \right)^2 \\ & + \lambda_3 \left( \eta_1^\dagger \eta_1 \eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1 \eta_3^\dagger \eta_3 + \eta_2^\dagger \eta_2 \eta_3^\dagger \eta_3 \right) \\ & + \lambda_4 \left( h^\dagger h \right) \left( \eta_1^\dagger \eta_1 + \eta_2^\dagger \eta_2 + \eta_3^\dagger \eta_3 \right) \\ & + \lambda_5 \left( \eta_2^\dagger \eta_1 \eta_1^\dagger \eta_2 + \eta_3^\dagger \eta_1 \eta_1^\dagger \eta_3 + \eta_3^\dagger \eta_2 \eta_2^\dagger \eta_3 \right) \\ & + \lambda_6 \left( \eta_1^\dagger h h^\dagger \eta_1 + \eta_2^\dagger h h^\dagger \eta_2 + \eta_3^\dagger h h^\dagger \eta_3 \right) \\ & + \lambda_7 \left( (\eta_3^\dagger \eta_1)^2 + (\eta_1^\dagger \eta_2)^2 + (\eta_2^\dagger \eta_3)^2 \right) \\ & + \lambda_7^* \left( (\eta_2^\dagger \eta_1)^2 + (\eta_3^\dagger \eta_2)^2 + (\eta_1^\dagger \eta_3)^2 \right) \\ & + \lambda_8 \left( (\eta_1^\dagger h)^2 + (\eta_2^\dagger h)^2 + (\eta_3^\dagger h)^2 \right) \\ & + \lambda_8^* \left( (h^\dagger \eta_1)^2 + (h^\dagger \eta_2)^2 + (h^\dagger \eta_3)^2 \right) \\ & + \lambda_9 \left( \eta_2^\dagger \eta_3 \eta_1^\dagger h + \eta_3^\dagger \eta_1 \eta_2^\dagger h + \eta_1^\dagger \eta_2 \eta_3^\dagger h \right) \\ & + \lambda_9^* \left( \eta_3^\dagger \eta_2 h^\dagger \eta_1 + \eta_1^\dagger \eta_3 h^\dagger \eta_2 + \eta_2^\dagger \eta_1 h^\dagger \eta_3 \right) \\ & + \lambda_{10} \left( \eta_3^\dagger \eta_2 \eta_1^\dagger h + \eta_1^\dagger \eta_3 \eta_2^\dagger h + \eta_2^\dagger \eta_1 \eta_3^\dagger h \right) \\ & + \lambda_{10}^* \left( \eta_2^\dagger \eta_3 h^\dagger \eta_1 + \eta_3^\dagger \eta_1 h^\dagger \eta_2 + \eta_1^\dagger \eta_2 h^\dagger \eta_3 \right) \end{aligned}$$

Mixing between:  
 $h, \eta_1$  ( $Z_2$  even)  
 $\eta_2, \eta_3$  ( $Z_2$  odd)

# The Model

## Scalar mass spectrum

$$M_h^2 = \frac{2v_h^3\lambda_2 - \mu_3^2 v_\eta}{v_h} \cos(\alpha)^2 + \frac{2v_\eta^3\lambda_1 - \mu_3^2 v_h}{v_\eta} \sin(\alpha)^2 + (\mu_3^2 + v_h v_\eta L) \sin(2\alpha),$$

$$M_{\eta_1}^2 = \frac{2v_h^3\lambda_2 - \mu_3^2 v_\eta}{v_h} \cos(\alpha)^2 + \frac{2v_\eta^3\lambda_1 - \mu_3^2 v_h}{v_\eta} \sin(\alpha)^2 - (\mu_3^2 + v_h v_\eta L) \sin(2\alpha),$$

$$M_{A_1}^2 = -\frac{(v_h \cos(\alpha) + v_\eta \sin(\alpha))^2 (\mu_3^2 + 2v_h v_\eta \lambda_8)}{v_h v_\eta},$$

$$M_{\eta_1^+}^2 = -\frac{(v_h \cos(\alpha) + v_\eta \sin(\alpha))^2 (\mu_3^2 + 2v_h v_\eta O)}{v_h v_\eta},$$

$$M_{\eta_2}^2 = \frac{1}{2} (2\mu_4^2 + v_\eta^2 K + v_h^2 L + (2\mu_5^2 + 3v_\eta v_h M) \sin(2\alpha)) ,$$

$$M_{\eta_3}^2 = \frac{1}{2} (2\mu_4^2 + v_\eta^2 K + v_h^2 L - (2\mu_5^2 + 3v_\eta v_h M) \sin(2\alpha)) ,$$

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$$K = \lambda_3 + \lambda_5 + 2\lambda_7, \quad K' = \lambda_3 + \lambda_5 - 2\lambda_7,$$

$$L = \lambda_4 + \lambda_6 + 2\lambda_8, \quad L' = \lambda_4 + \lambda_6 - 2\lambda_8,$$

$$M = \lambda_9 + \lambda_{10}, \quad O = \lambda_6 + 2\lambda_8.$$

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$Z_2$  even  
(2HDM)

$$M_{\eta_2}^2 = \frac{1}{2} (2\mu_4^2 + v_\eta^2 K + v_h^2 L + (2\mu_5^2 + 3v_\eta v_h M) \sin(2\alpha)) ,$$

$$M_{\eta_3}^2 = \frac{1}{2} (2\mu_4^2 + v_\eta^2 K + v_h^2 L - (2\mu_5^2 + 3v_\eta v_h M) \sin(2\alpha)) ,$$

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**Z<sub>2</sub> odd  
(2-IDM)**

$$\begin{aligned} K &= \lambda_3 + \lambda_5 + 2\lambda_7, & K' &= \lambda_3 + \lambda_5 - 2\lambda_7, \\ L &= \lambda_4 + \lambda_6 + 2\lambda_8, & L' &= \lambda_4 + \lambda_6 - 2\lambda_8, \\ M &= \lambda_9 + \lambda_{10}, & O &= \lambda_6 + 2\lambda_8. \end{aligned}$$

$$\begin{aligned} M_{\eta_2}^2 &= \frac{1}{2} (2\mu_4^2 + v_\eta^2 K + v_h^2 L + (2\mu_5^2 + 3v_\eta v_h M) \sin(2\alpha)), \\ M_{\eta_3}^2 &= \frac{1}{2} (2\mu_4^2 + v_\eta^2 K + v_h^2 L - (2\mu_5^2 + 3v_\eta v_h M) \sin(2\alpha)), \\ M_{A_2}^2 &= \frac{1}{2} (2\mu_4^2 + v_\eta^2 K' + v_h^2 L' + (2\mu_5^2 + v_h v_\eta M) \sin(2\alpha)), \\ M_{A_3}^2 &= \frac{1}{2} (2\mu_4^2 + v_\eta^2 K' + v_h^2 L' - (2\mu_5^2 + v_h v_\eta M) \sin(2\alpha)), \\ M_{\eta_2^+}^2 &= \frac{1}{2} (2\mu_4^2 + \lambda_3 v_\eta^2 + \lambda_4 v_h^2 + (2\mu_5^2 + 2v_\eta v_h M) \sin(2\alpha)), \\ M_{\eta_3^+}^2 &= \frac{1}{2} (2\mu_4^2 + \lambda_3 v_\eta^2 + \lambda_4 v_h^2 - (2\mu_5^2 + 2v_\eta v_h M) \sin(2\alpha)). \end{aligned}$$

# Neutrino Phenomenology

Light LH Majorana neutrinos get masses via Type-I Seesaw:

$$m_\nu = -m_{\text{D}_{3 \times 5}} M_{\text{R}_{5 \times 5}}^{-1} m_{\text{D}_{3 \times 5}}^{\text{T}}$$

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Two-zero textures  
[P. H. Frampton, *et al.* '02],  
[P. O. Ludl, *et al.* '11,  
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These textures:  
Consistent with experimental data.  
Allows both mass orderings (NO, IO).

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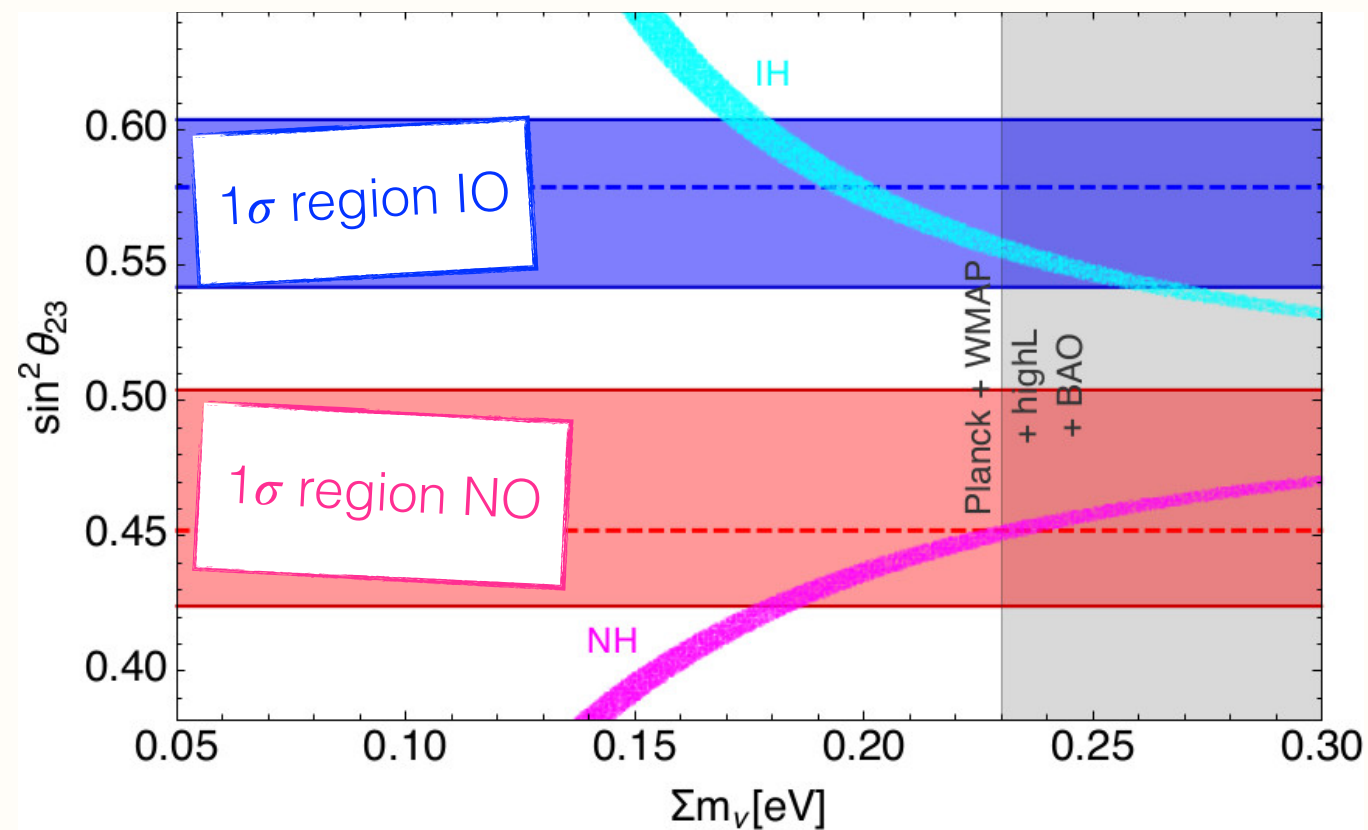
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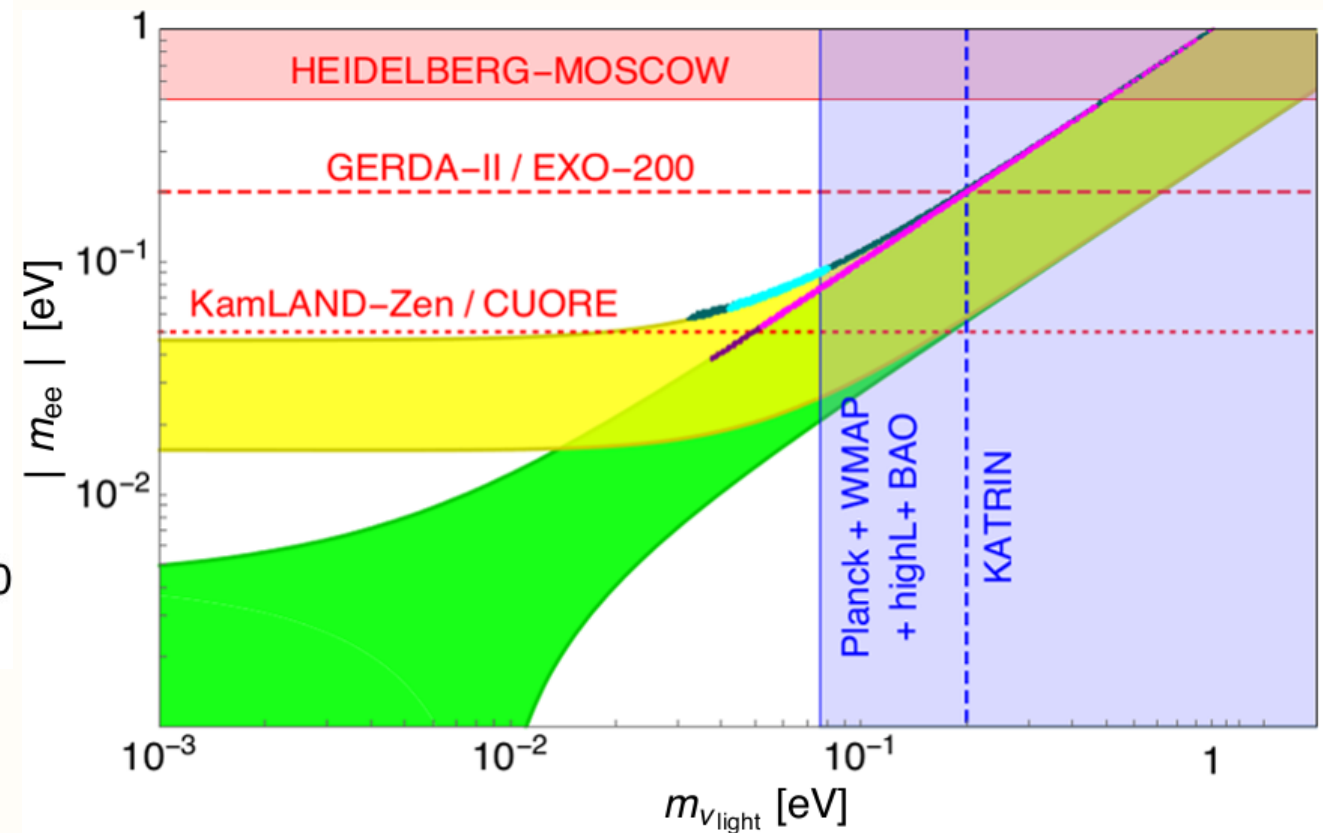
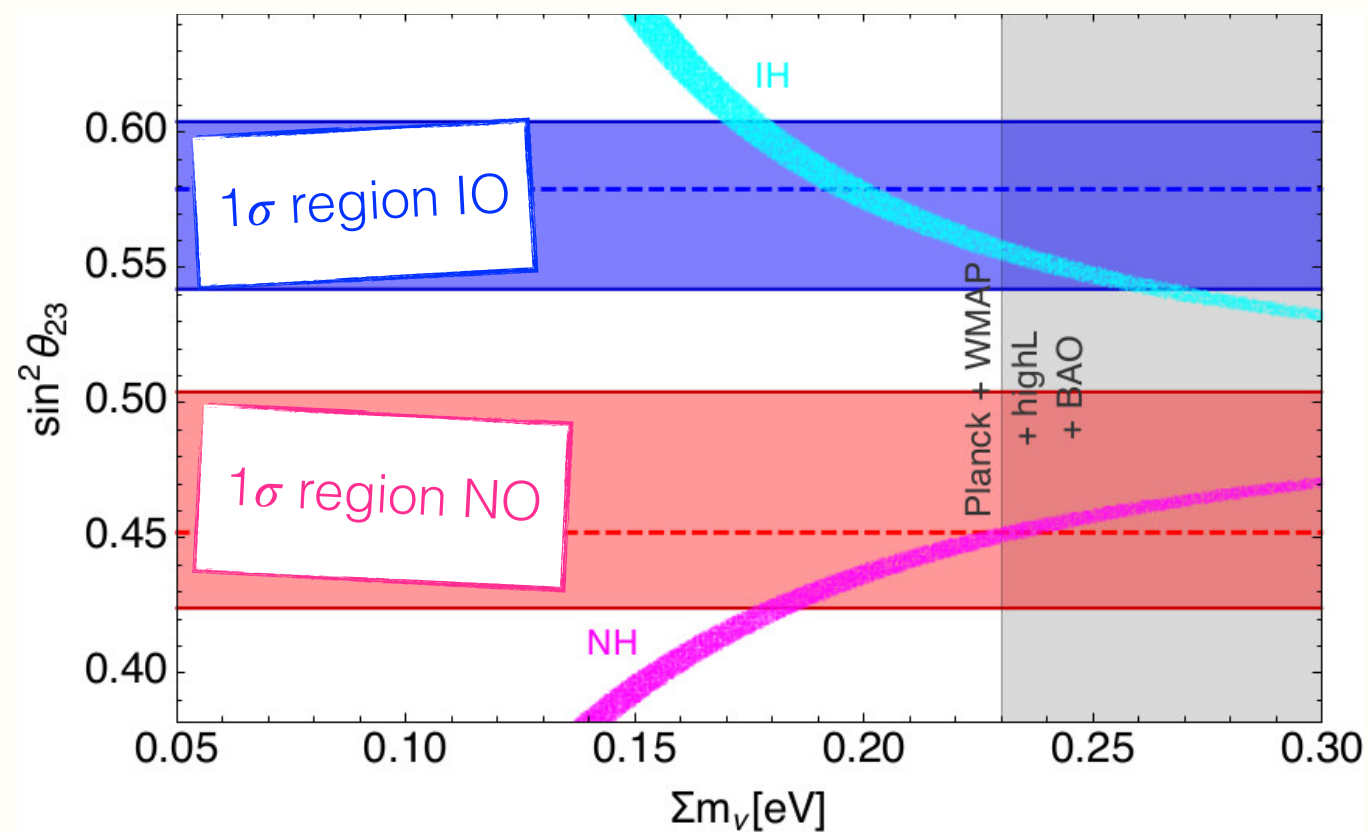
Correlation: Atmospheric mixing angle and sum of light neutrino masses



Input data: [Gonzalez-Garcia, *et al.* '15]

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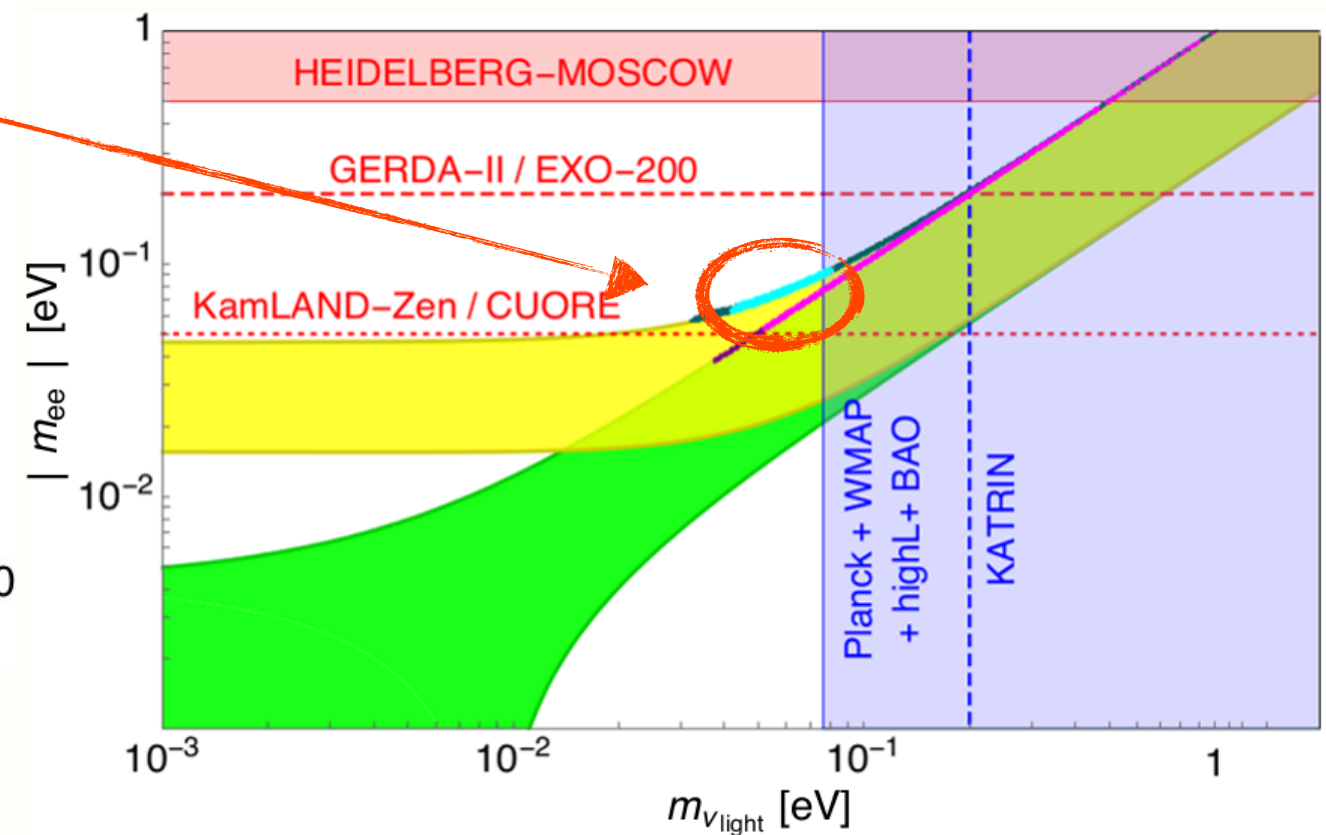
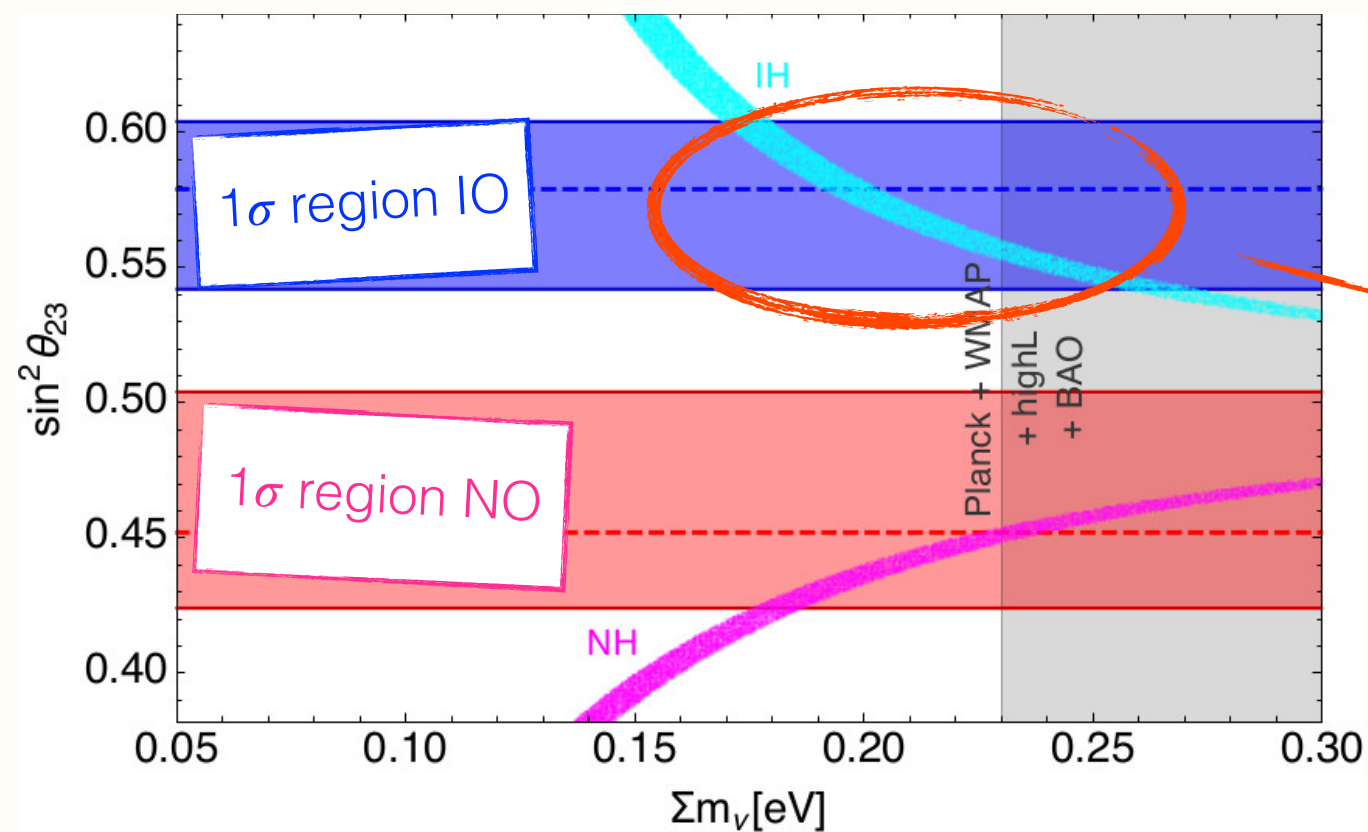
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Numerical scan using MicrOMEGAs [G. Belanger, *et al.* '08] over a 13 dim. CP conserving potential (10 real couplings and 3  $\mu$ -terms) constraining the parameter region.

Constrains

Vacuum stability:

Potential perturbativity:

Experimental bounds to masses: [A. Pierce et al. '07, E. Lundstrom et al '03]

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$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_1 + \lambda_2 + 3(\lambda_4 + \lambda_6 + \min\{-2\lambda_8, 0\}) + 3(\lambda_3 + \lambda_5 + \min\{-2|\lambda_7|, 0\}) - 6(|\lambda_9| + |\lambda_{10}|) > 0.$$

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$$M_{\eta_{2,3}^\pm} > 70\text{GeV}, \quad M_{A_{2,3}} > 110\text{GeV}.$$

# DM Phenomenology

Low mass region ( $M_{\eta_2} < M_W$ )<sup>[M. S. Boucenna *et al.* '11]</sup>:

$$h, \eta_1, (A_1, Z) \\ \eta_2 \eta_2 (A_2) \longrightarrow f f,$$

Intermediate & high mass region ( $M_W \lesssim M_{\eta_2} \lesssim 500 \text{ GeV}$ ):

$$h, \eta_1, A_2, \eta_2^+, Z, * \\ \eta_2 \eta_2 (A_2) \longrightarrow W^+ W^-, Z Z.$$

Indirect Detection (gamma ray)  
Fermilat<sup>[M. Ackermann *et al.* '15]</sup>, Hess<sup>[Hess collab, '11]</sup>.

Measured relic  
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in progress

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Fermilab [M. Ackermann *et al.* '15], Hess [Hess collab,

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Measured relic  
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# Conclusions

- Interesting Neutrino and DM phenomenology arises from an  $A_4$  flavour symmetric model, broken by a flavon  $\phi$  into a  $Z_2$ .
- Three RH neutrinos ( $Z_2$  even) are responsible for giving mass to the light neutrino (via type I seesaw).
- Light neutrino mass and mixing in agreement with the experimental data and both neutrino mass orderings (NO, IO).
- Correlation between  $\theta_{23}$  and the sum of the lightest neutrino masses is obtained.
- Lower bound for neutrinoless double beta decay effective mass,  $|m_{ee}|$ .
- DM (relic density) constrains parameter region. But more statistic is needed.

Dziękuję.  
*/dʑɛŋ'ku.jɛ/*

(Thank you.)

# Backup

Flavour Symmetry Breaking of  $A_4$  into  $Z_2$  by choosing the vev alignments:

$$\langle H^0 \rangle = v_h \neq 0, \quad \langle \eta_1^0 \rangle = v_\eta \neq 0, \quad \langle \eta_{2,3}^0 \rangle = 0, \quad \langle \phi_1 \rangle = v_\phi \neq 0, \quad \langle \phi_{2,3} \rangle = 0.$$

The Majorana neutrino mass matrix:

$$M_R = \begin{pmatrix} M_1 & 0 & 0 & y_2^N v_\phi & y_3^N v_\phi \\ 0 & M_1 & y_1^N v_\phi & 0 & 0 \\ 0 & y_1^N v_\phi & M_1 & 0 & 0 \\ y_2^N v_\phi & 0 & 0 & M_2 & 0 \\ y_3^N v_\phi & 0 & 0 & 0 & 0 \end{pmatrix}.$$

# Backup

The Dirac Neutrino mass matrix:

Model A

$$m_D^{(A)} = \begin{pmatrix} y_1^\nu v_\eta & 0 & 0 & y_4^\nu v_h & 0 \\ y_2^\nu v_\eta & 0 & 0 & 0 & 0 \\ y_3^\nu v_\eta & 0 & 0 & 0 & y_5^\nu v_h \end{pmatrix}$$

Model B

$$m_D^{(B)} = \begin{pmatrix} y_1^\nu v_\eta & 0 & 0 & y_4^\nu v_h & 0 \\ y_2^\nu v_\eta & 0 & 0 & 0 & y_5^\nu v_h \\ y_3^\nu v_\eta & 0 & 0 & 0 & 0 \end{pmatrix}$$

# Backup

- Light neutrinos get Majorana masses (type I see-saw),  $m_\nu = -m_{D_{3 \times 5}} M_{R_{5 \times 5}}^{-1} m_{D_{3 \times 5}}^T$

Model A

$$m_\nu^{(A)} \equiv \begin{pmatrix} a & 0 & b \\ 0 & 0 & c \\ b & c & d \end{pmatrix}$$

where

$$a = \frac{(y_4^\nu v_h)^2}{M_2}, \quad b = \frac{y_1^\nu y_5^\nu v_\eta v_h}{y_3^N v_\phi} - \frac{y_2^N y_4^\nu y_5^\nu v_h^2}{y_3^N M_2},$$

$$c = \frac{y_2^\nu y_5^\nu v_\eta v_h}{y_3^N v_\phi}, \quad d = \frac{(y_2^N y_5^\nu v_h)^2}{(y_3^N)^2 M_2} - \frac{(y_5^\nu v_h)^2 M_1}{(y_3^N v_\phi)^2} + 2 \frac{y_3^\nu y_5^\nu v_\eta v_h}{y_3^N v_\phi}.$$



# Backup

- Model B

$$m_{\nu}^{(\text{B})} \equiv \begin{pmatrix} a & b & 0 \\ b & d & c \\ 0 & c & 0 \end{pmatrix}$$

where

$$a = \frac{(y_4^{\nu} v_h)^2}{M_2}, \quad b = \frac{y_1^{\nu} y_5^{\nu} v_{\eta} v_h}{y_3^N v_{\phi}} - \frac{y_2^N y_4^{\nu} y_5^{\nu} v_h^2}{y_3^N M_2},$$

$$c = \frac{y_3^{\nu} y_5^{\nu} v_{\eta} v_h}{y_3^N v_{\phi}}, \quad d = \frac{(y_2^N y_5^{\nu} v_h)^2}{(y_3^N)^2 M_2} - \frac{(y_5^{\nu} v_h)^2 M_1}{(y_3^N v_{\phi})^2} + 2 \frac{y_2^{\nu} y_5^{\nu} v_{\eta} v_h}{y_3^N v_{\phi}}.$$

- These mass matrices has the two zero textures:  $B_3$  for model A and  $B_4$  for model B.

# Backup

Model B (Two zero texture  $B_4$ )

