From extra particles to the Standard Model Effective Field Theory

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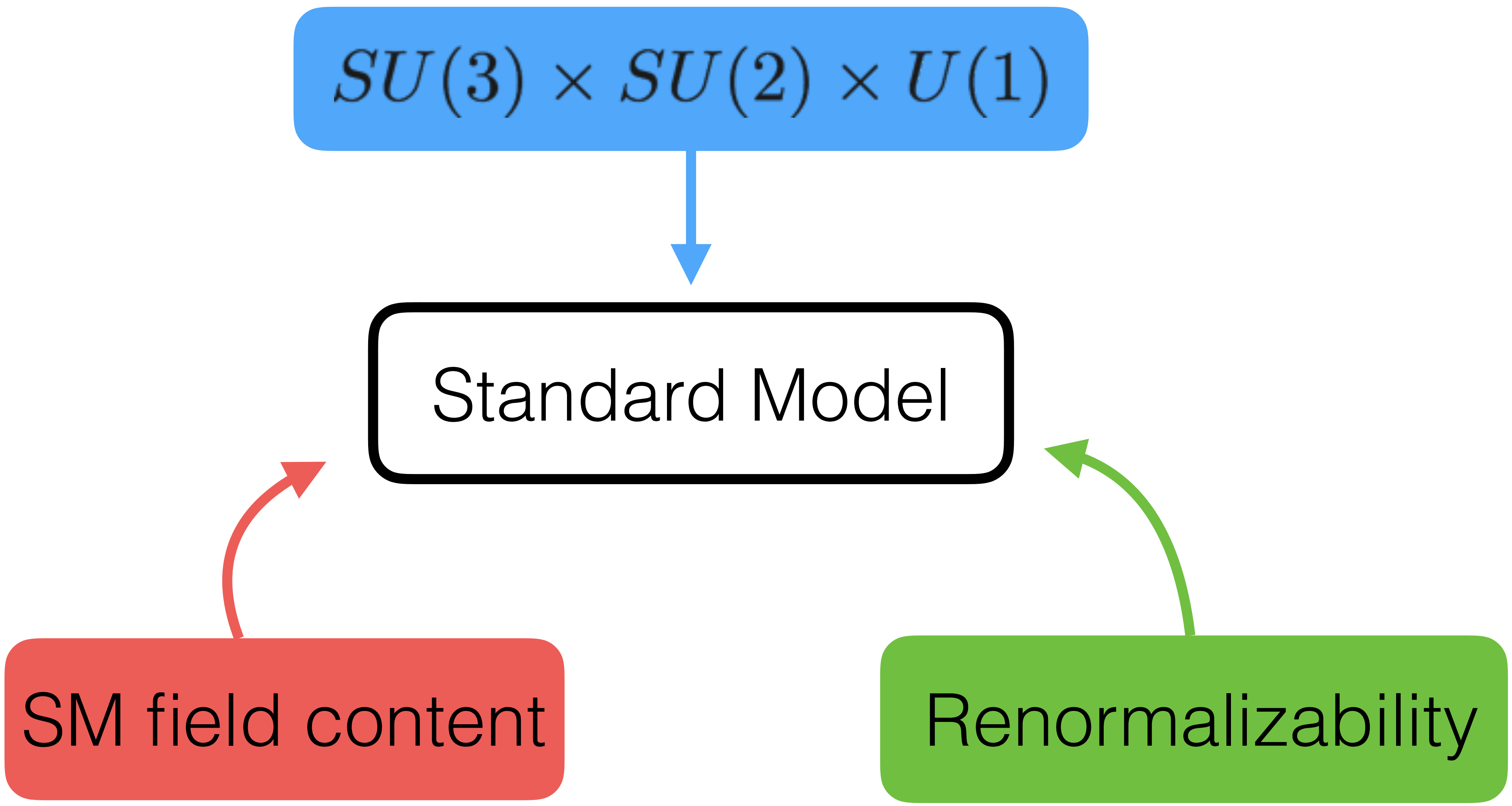
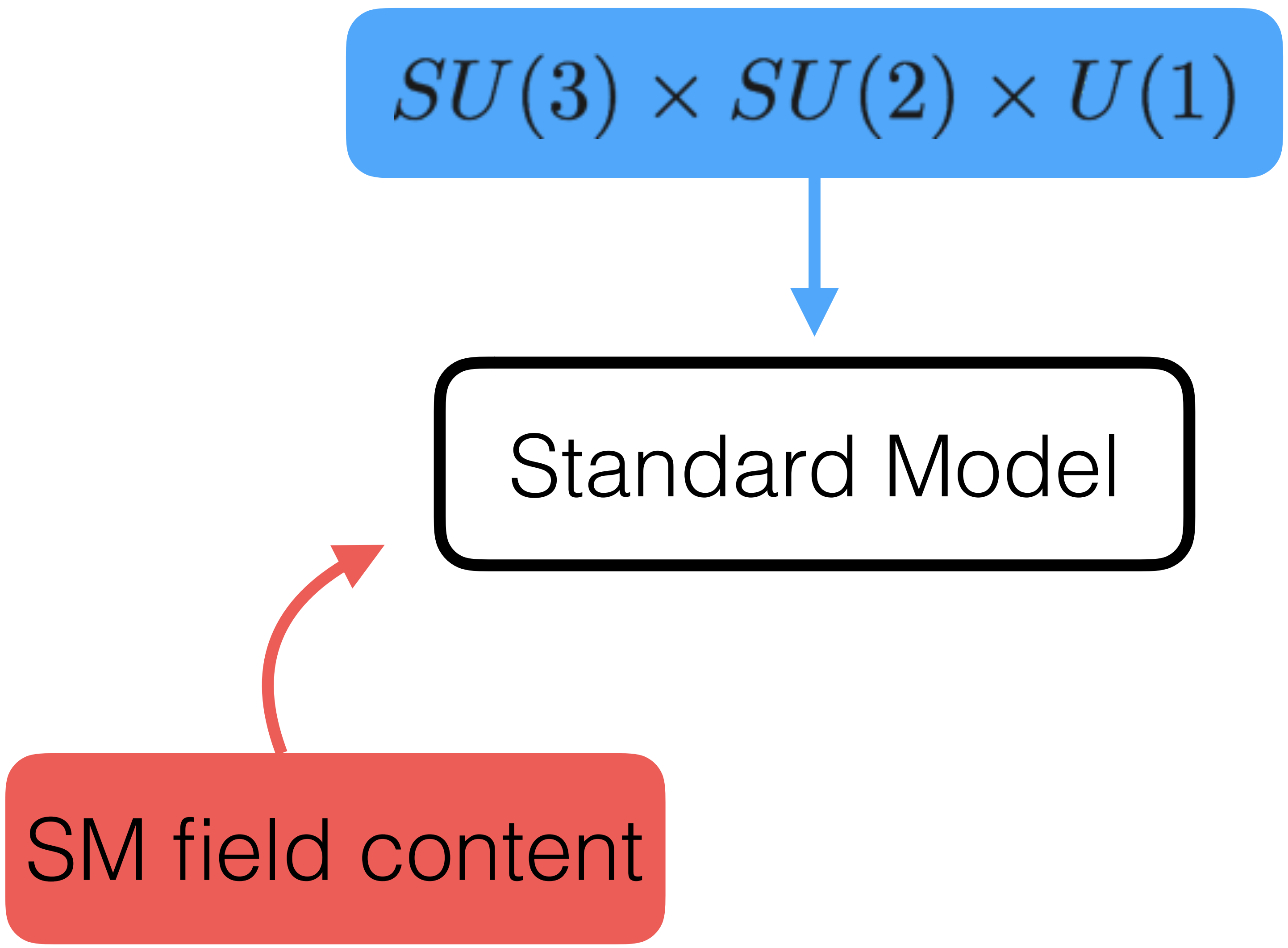
Outline

- Introduction: general extensions of the SM
- Tree level integration: general picture and examples of integration for general extensions
- Mixed contributions to the SM EFT
- Conclusions

General extensions of the Standard Model



Standard Model Effective Field Theory



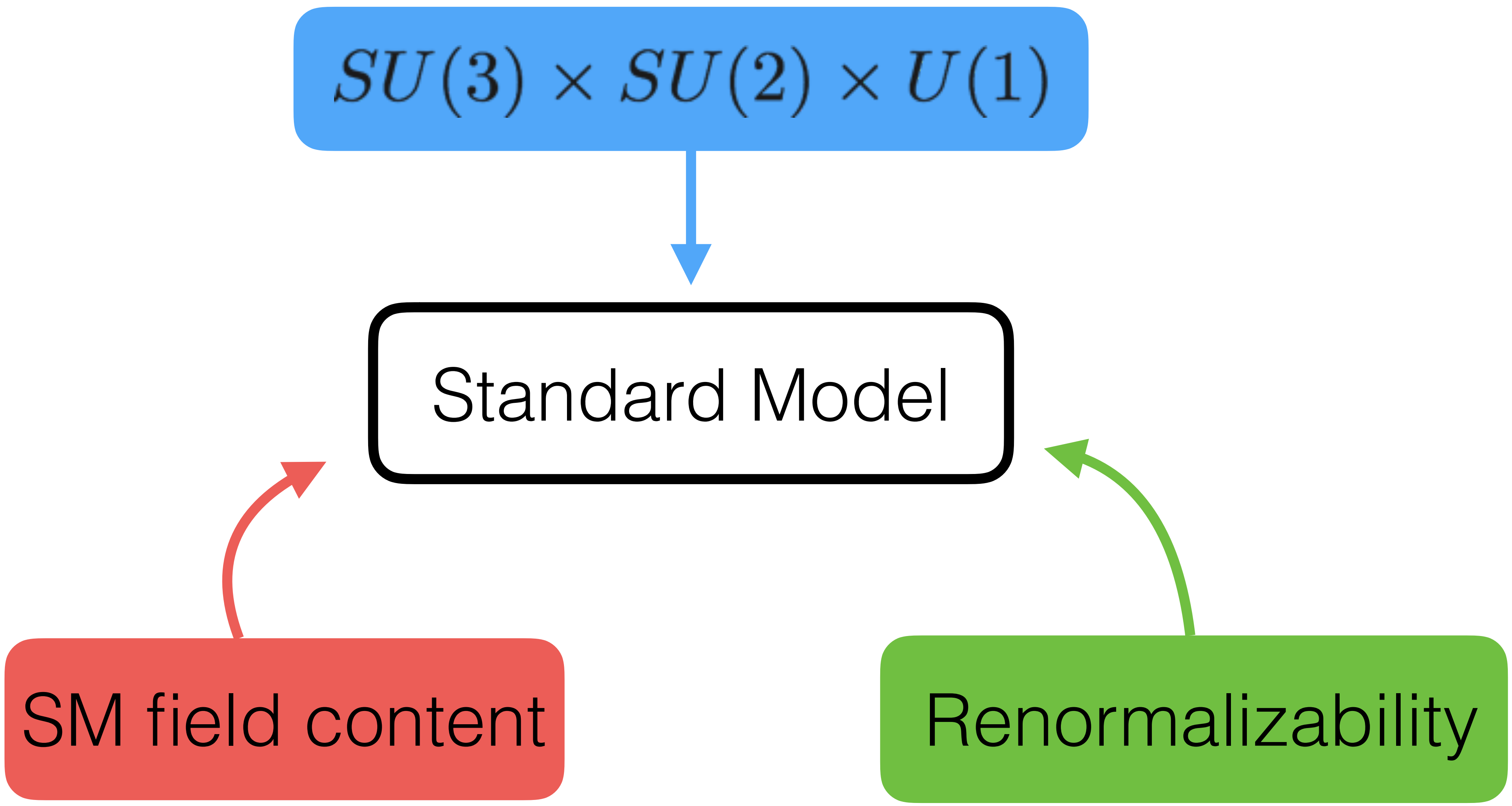
Standard Model Effective Field Theory

$$\mathcal{L}_{eff} = \mathcal{L}_{renorm} + \sum_i \frac{\alpha_i}{\Lambda^{\dim(\mathcal{O}_i)-4}} \mathcal{O}_i$$

The effect of an operator in amplitudes goes roughly as $\sim \left(\frac{E}{\Lambda}\right)^{\dim(\mathcal{O}_i)-4}$

$\Longrightarrow$ **We can cut the expansion at a fixed operator dimension**

General extensions of the SM with new particles



General extensions of the SM with new particles

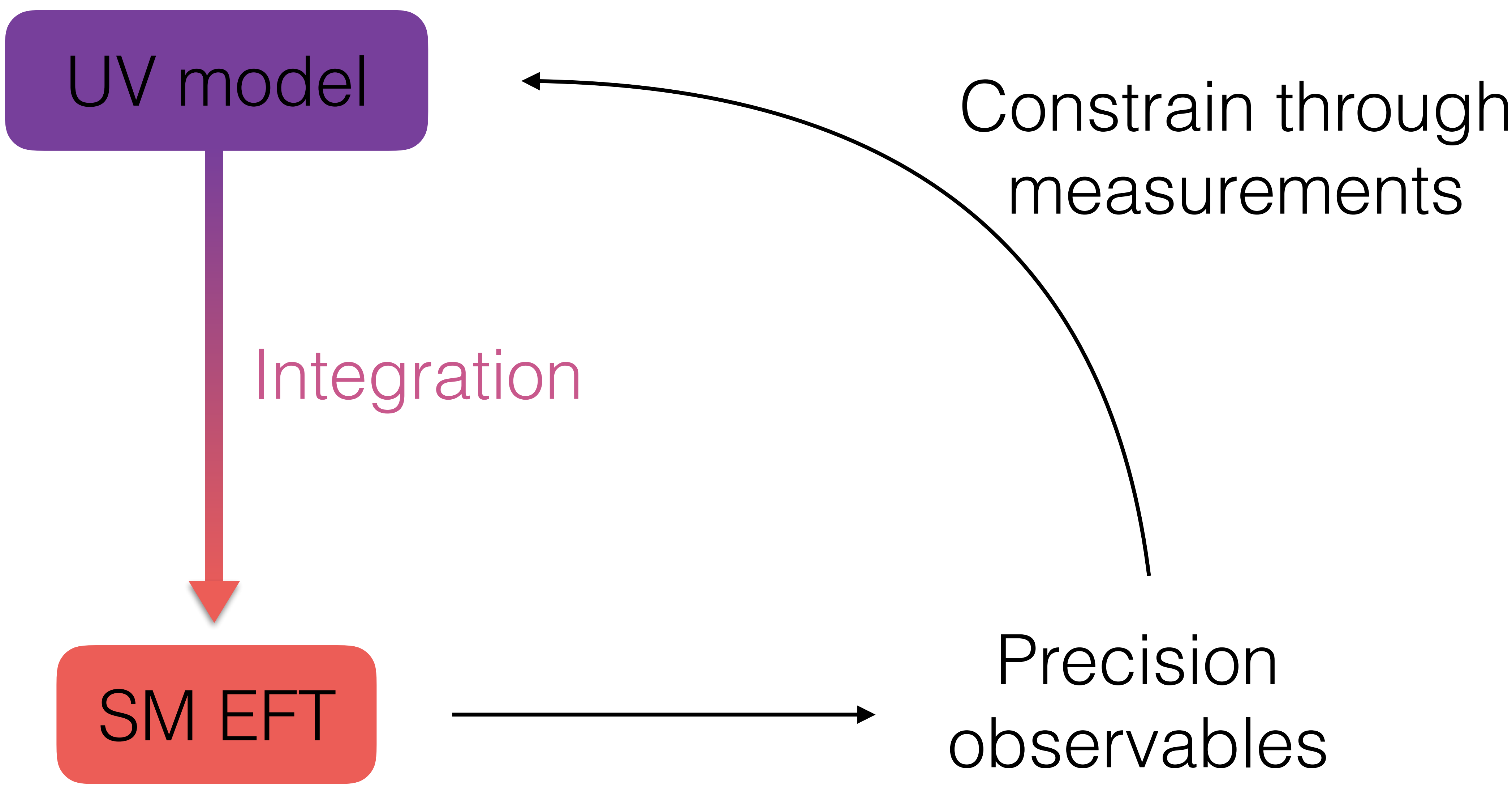
$SU(3) \times SU(2) \times U(1)$

Standard Model

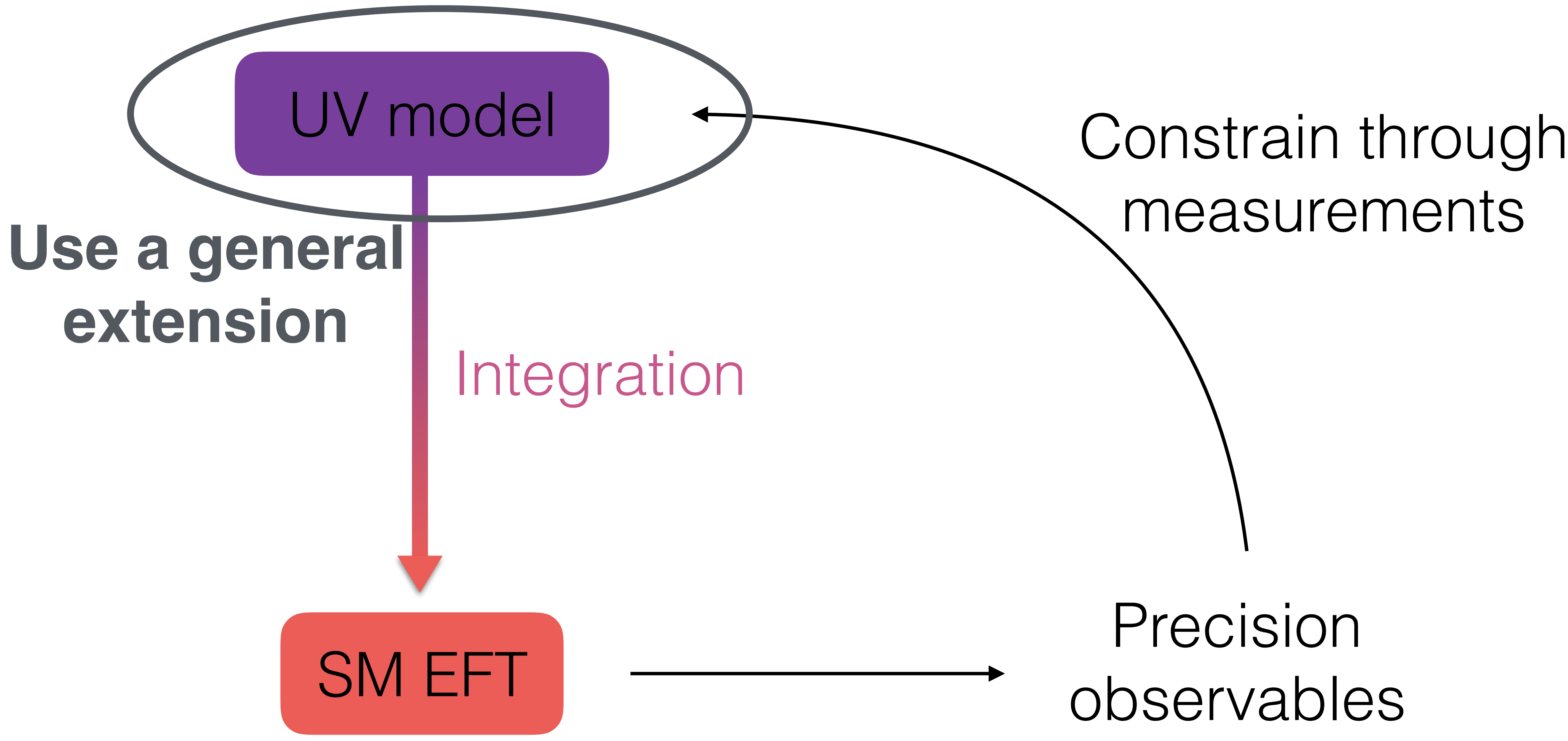
Renormalizability

See the talk "*Parametrizing BSM physics*" by M. Pérez-Victoria

Integrating general extensions with new particles

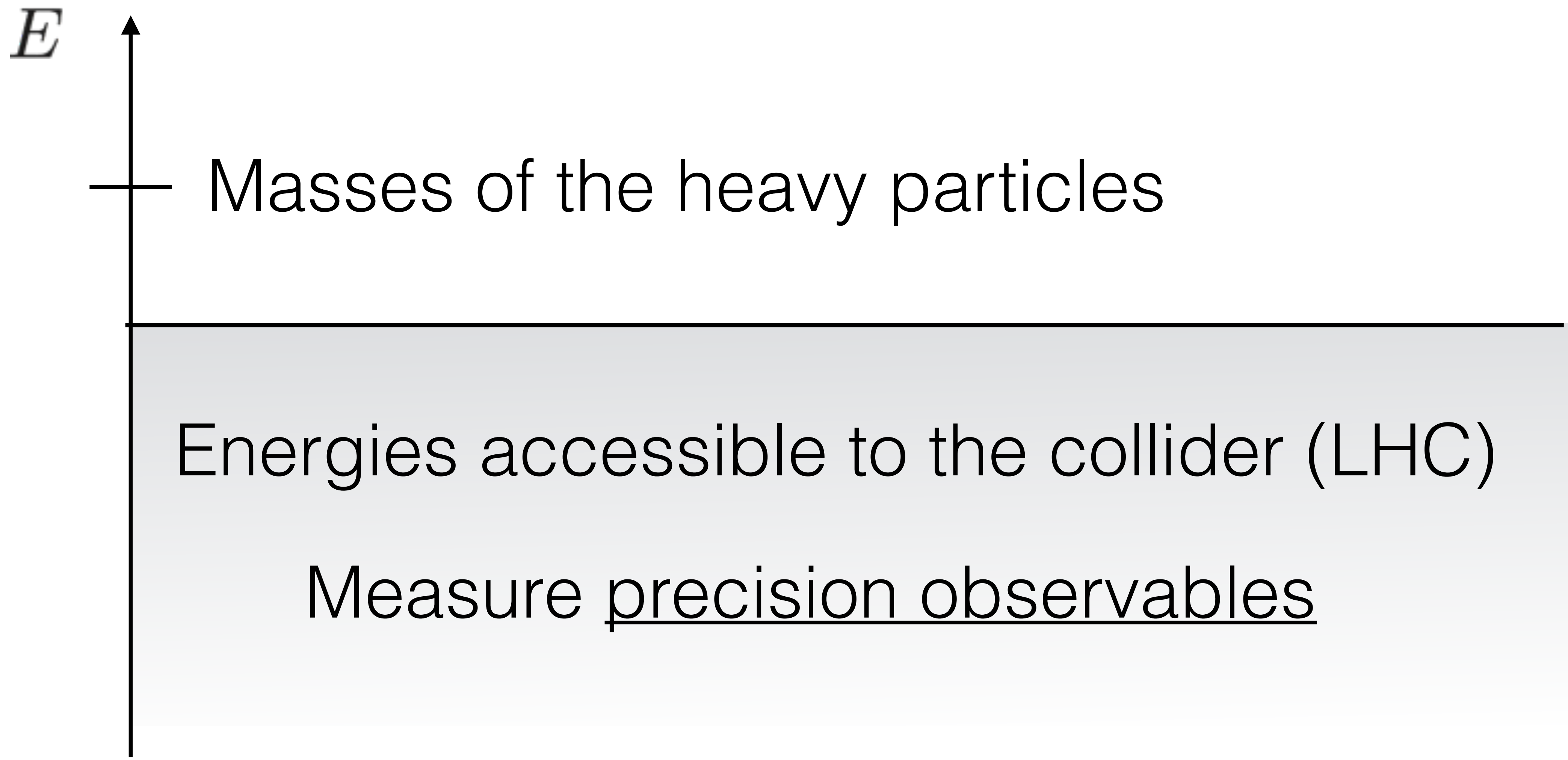


Integrating general extensions with new particles



Integrating general extensions with new particles

Indirect study of new particles



Tree level integration

UV theory

$$S[\phi, \Phi] = S_{quad}[\Phi] + S_{int}[\phi, \Phi]$$

$$S_{quad}[\Phi] = -\int d^4x \Phi^\dagger Q \Phi$$

EOM

$$Q\Phi_c = \frac{\delta S_{int}}{\delta \Phi^\dagger}$$

Classical solution

$$\Phi_c = Q^{-1} \frac{\delta S_{int}}{\delta \Phi^\dagger}$$

Tree level integration

$$\underbrace{S_{eff}[\phi]}_{\text{We only keep terms with dimension 6 or less}} = S[\phi, \Phi_c]$$

Tree level integration

Expand Q^{-1} in powers of the covariant derivative:

$$\Phi_c \sim F_0 + \left(\frac{D}{M}\right)^2 F_1 + \left(\frac{D}{M}\right)^4 F_2 + \cdots$$

$$\longrightarrow$$

Increasing dimension

If some heavy fields appear in the expansion, substitute iteratively until all terms under some dimension depend only on the light fields

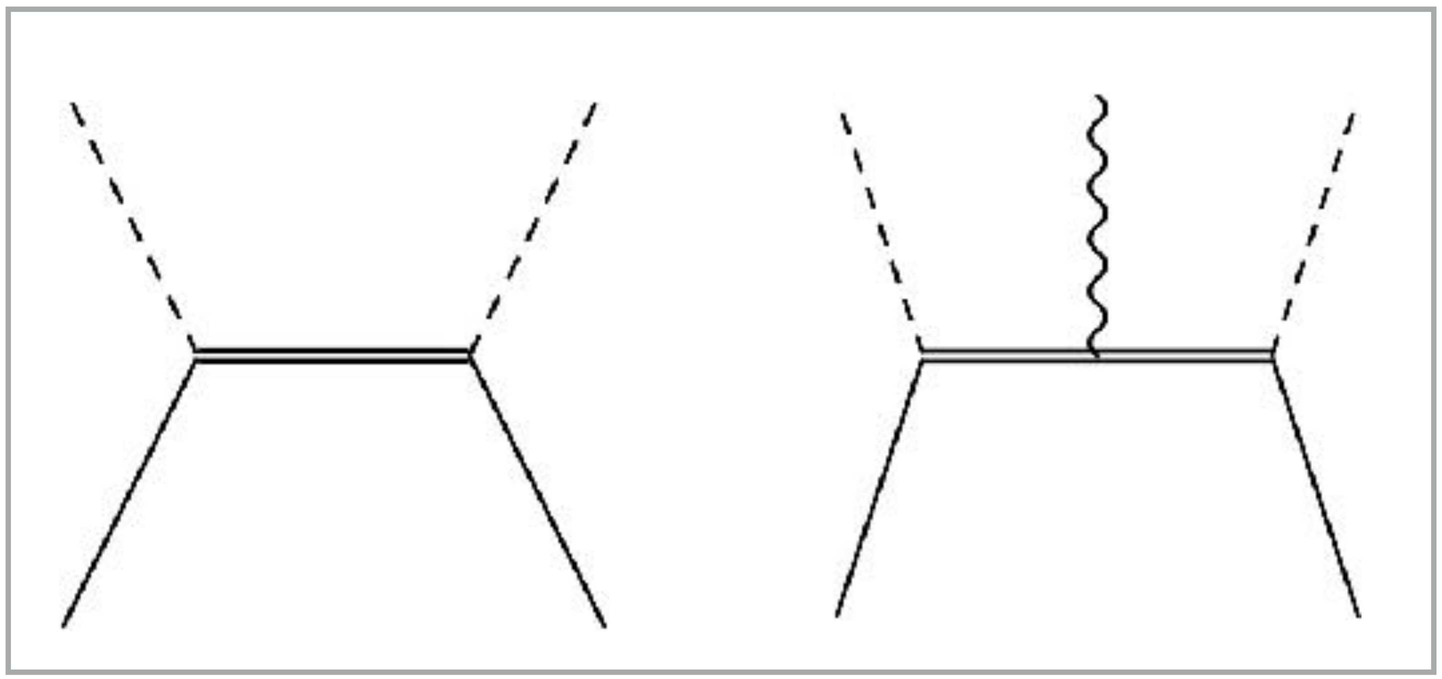
Basis of dim. 6 operators

- Identities for tensor products (such as Fierz)
- Integration by parts
- EOMs for light fields
- …

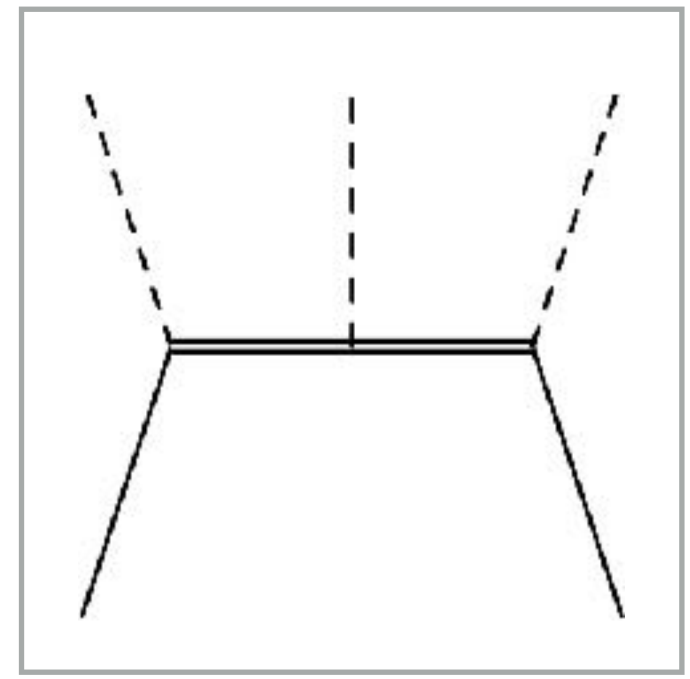
$\Downarrow$

Complete set of independent operators (**basis**)

Heavy quarks



$$\sim (\phi^\dagger i D_\mu \phi)(\bar{q}\gamma^\mu q)$$

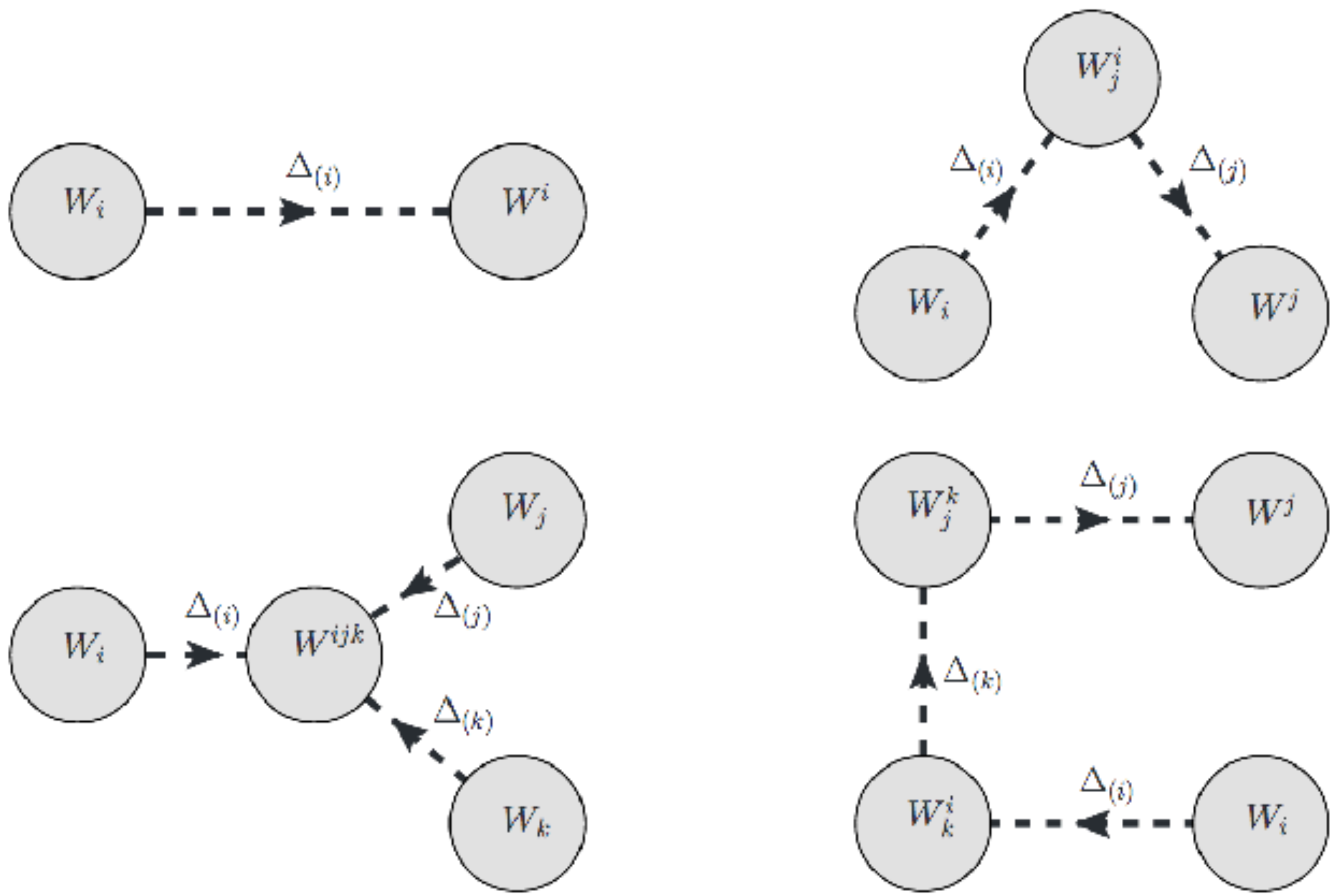


$$\sim (\phi^\dagger \phi)(\bar{q}_1 \phi q_2)$$

Single heavy quark and two heavy quarks contributions

F. del Águila, M. Pérez-Victoria, J. Santiago, [hep-ph/0007316]

Heavy scalars



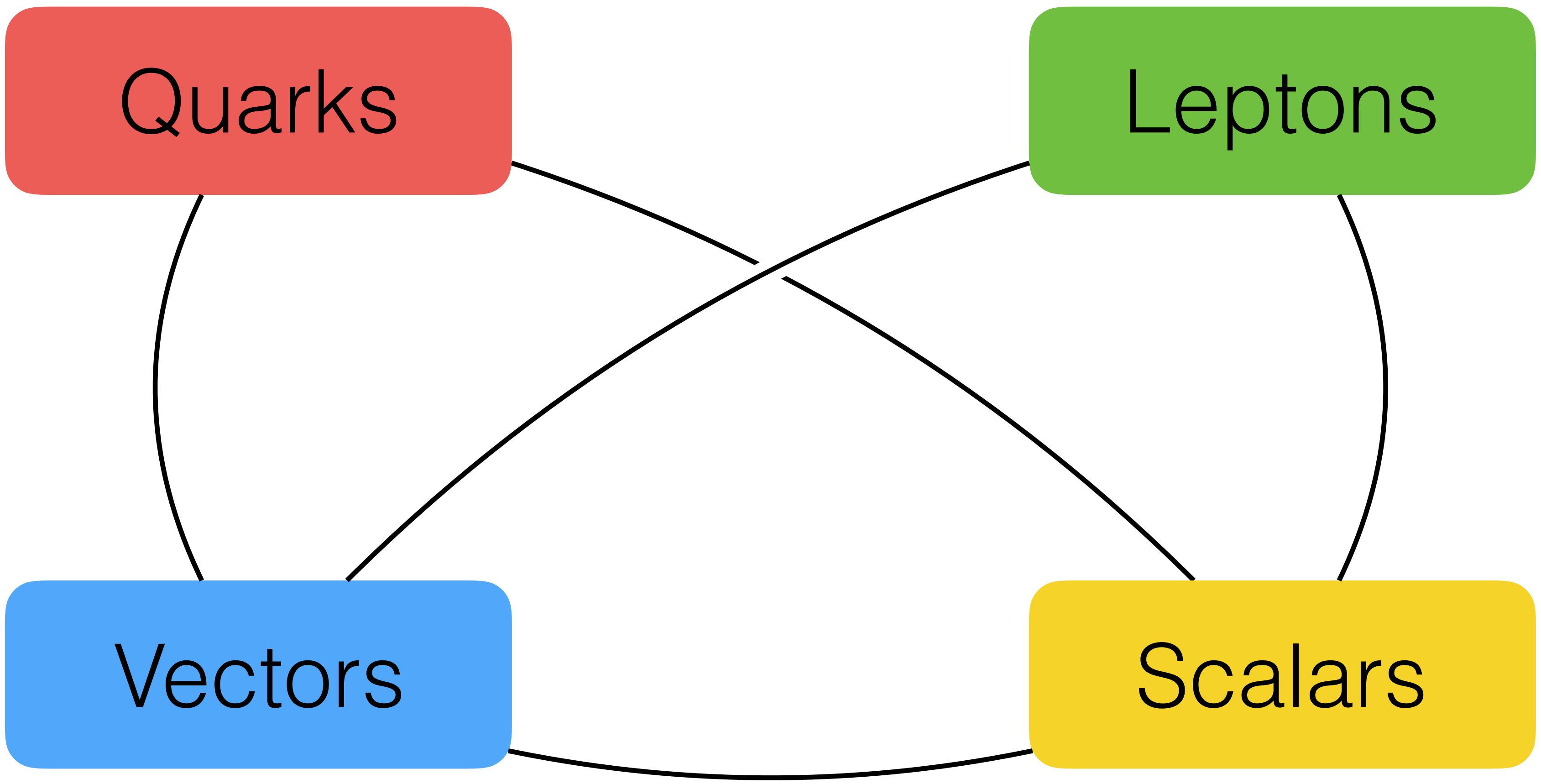
J. de Blas, M. Chala, M. Pérez-Victoria, [1412.8480]

Heavy leptons and vectors

The contributions from each kind of particle have been computed

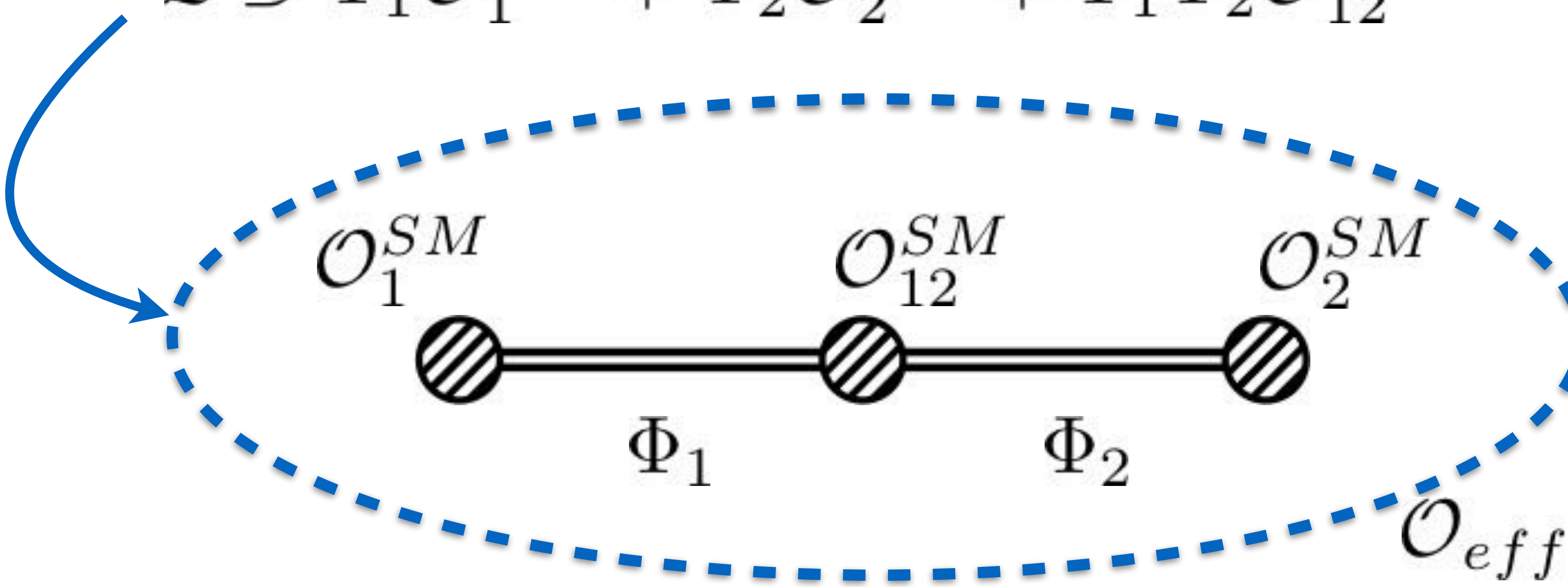
F. del Águila, J. de Blas, M. Pérez-Victoria, [0803.4008, 1005.3998]

Mixed contributions



The importance of dimension 3 linear couplings

$$\mathcal{L} \supset \Phi_1 \mathcal{O}_1^{SM} + \Phi_2 \mathcal{O}_2^{SM} + \Phi_1 \Phi_2 \mathcal{O}_{12}^{SM}$$



$$6 \geq \dim(\mathcal{O}_{eff}) = \dim(\mathcal{O}_1^{SM}) + \dim(\mathcal{O}_2^{SM}) + \dim(\mathcal{O}_{12}^{SM})$$

$\Longrightarrow$ **A linear coupling of dim. 3 is necessary**

Finding the dimension 3 linear couplings

<u>For fermions:</u>

- Each one has dimension 3/2
- They need to couple to at least another fermion

$\boxed{\text{No dimension 3 couplings}}$

Finding the dimension 3 linear couplings

For vectors:

- Each one has dimension 1
- Their Lorentz index can couple to:
  1. A gamma matrix. Brings 2 fermions
  2. A covariant derivative. Adds 1 to the dim.
- We can only add a SM field with dim. 1: Higgs

$$V_\mu D^\mu \phi \implies V_\mu \sim 2_{1/2}$$

Finding the dimension 3 linear couplings

For scalars:

- Each one has dimension 1
- Should couple to SM dim. 2 scalar operator: two Higgs bosons

$$S\phi^\dagger\phi \implies S \sim 1_0$$

$$S^a\phi^\dagger\sigma^a\phi \implies S \sim 3_0$$

$$S^{\dagger a}\tilde{\phi}^\dagger\sigma^a\phi \implies S \sim 3_1$$

Example of mixed contribution

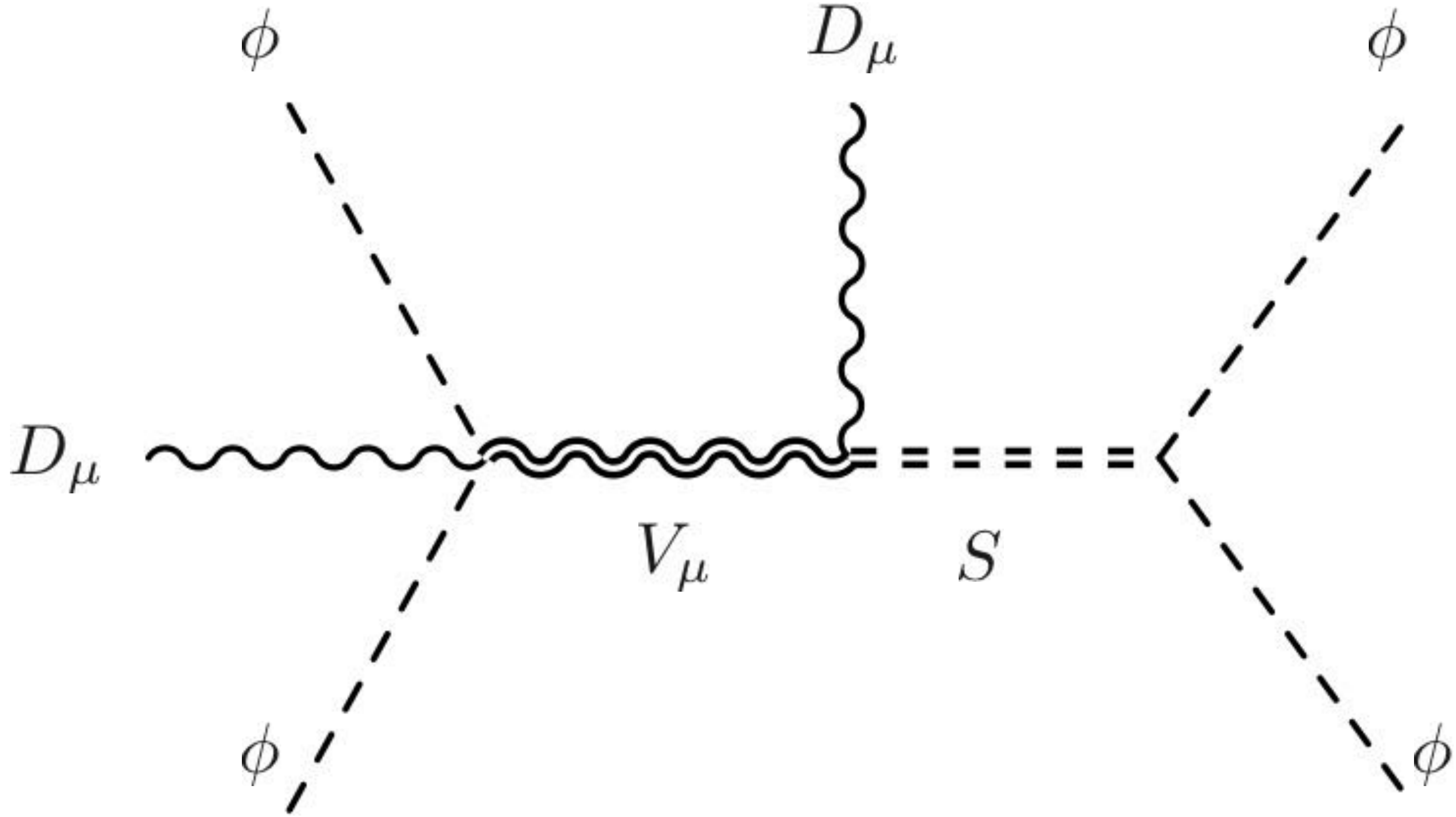
$V_\mu \sim 3_0$

$S \sim 3_0$

$$\mathcal{L} \supset g V_\mu \phi^\dagger i D_\mu \phi + \kappa S^a \phi^\dagger \sigma^a \phi + \gamma V_\mu^a D^\mu S^a$$

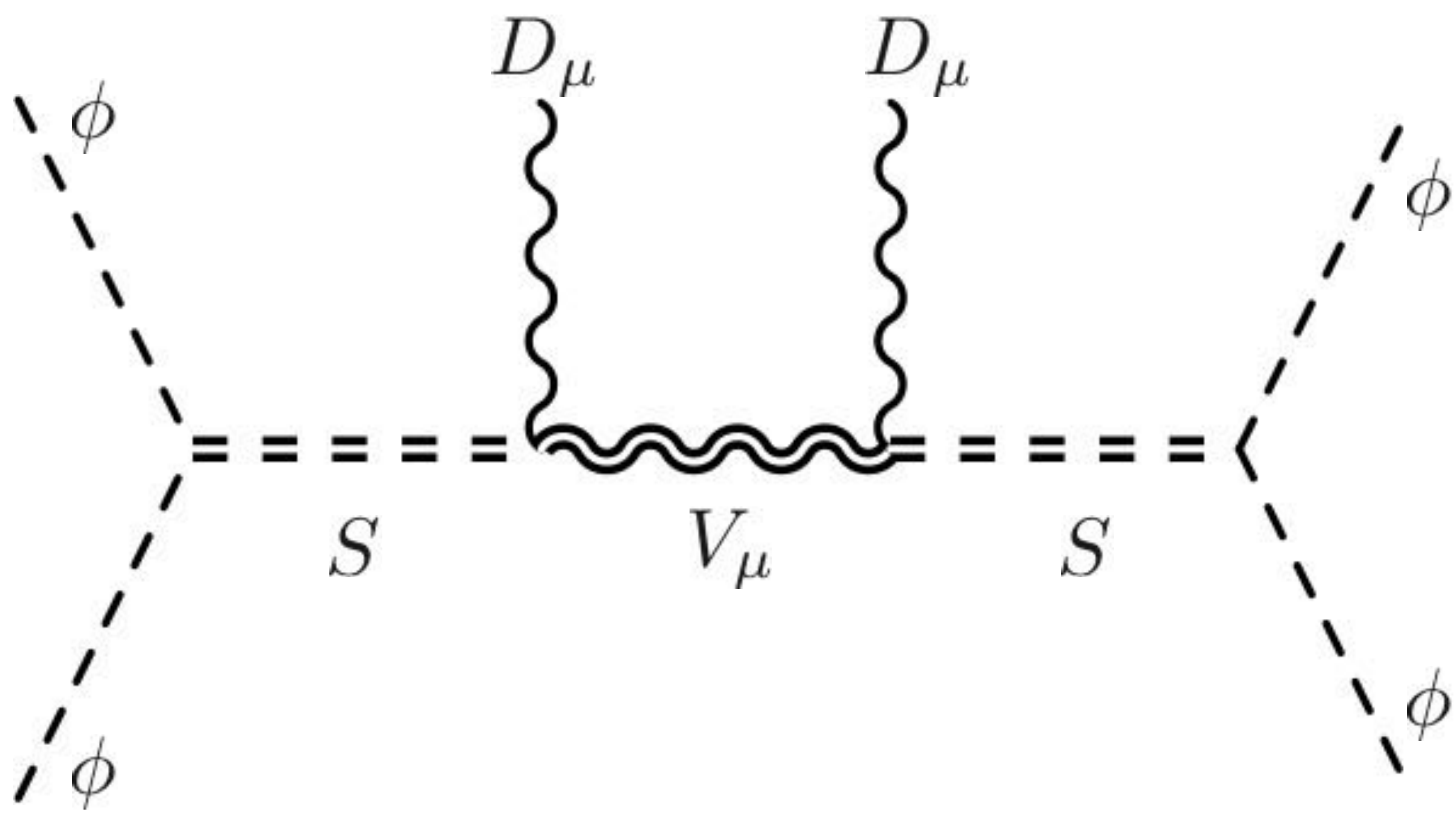
Dimension 3

Example of mixed contribution



$$\sim (\phi^\dagger D_\mu \phi)(D^\mu \phi)^\dagger \phi, \;\; (\phi^\dagger \phi)\Box(\phi^\dagger \phi), \;\; \ldots$$

Example of mixed contribution



$$\sim (\phi^\dagger D_\mu \phi)(D^\mu \phi)^\dagger \phi, \quad (\phi^\dagger \phi)\Box(\phi^\dagger \phi), \quad \ldots$$

Mixed contributions

Establish a **complete** dictionary

Combinations of new particles ⟷ Coefficients of operators of the dim. 6 SM EFT

All tree-level contributions to dim. 6 SM EFT

This work will be published soon

Example coefficient

$$\boxed{\mathcal{O}_{\Box\phi} = (\phi^\dagger\phi)\Box(\phi^\dagger\phi)}$$

$$\begin{aligned}
\frac{\alpha_{\phi\Box}}{\Lambda^2} = {} & \frac{\mathrm{Im}\left(g^{\phi}_{\mathcal{B}_i}\right)\delta^{ij}_{\mathcal{B}}\kappa^{j}_{\mathcal{S}}}{M^2_{\mathcal{B}_i}M^2_{\mathcal{S}_j}} - \frac{\mathrm{Im}\left(g^{\phi}_{\mathcal{W}_i}\right)\delta^{ij}_{\mathcal{W}}\kappa^{j}_{\Xi_0}}{2M^2_{\mathcal{W}_i}M^2_{\Xi_{0j}}} + \frac{\mathrm{Im}\left(g^{\phi}_{\mathcal{W}^1_i}\delta^{ij*}_{\mathcal{W}^1}\kappa^{j*}_{\Xi_1}\right)}{M^2_{\mathcal{W}^1_i}M^2_{\Xi_{1j}}} - \frac{\mathrm{Re}\left(g^{(1)}_{\mathcal{S}_i\mathcal{L}^1_j}\gamma_j\right)\kappa^{i}_{\mathcal{S}}}{M^2_{\mathcal{L}^1_j}M^2_{\mathcal{S}_i}} \\
& + \frac{\mathrm{Re}\left(g^{(1)}_{\Xi_{0i}\mathcal{L}^1_j}\gamma_j\right)\kappa^{i}_{\Xi_0}}{M^2_{\mathcal{L}^1_j}M^2_{\Xi_{0i}}} + \frac{2\mathrm{Re}\left(g^{(1)}_{\Xi_{1i}\mathcal{L}^1_j}\gamma_j\kappa^{i*}_{\Xi_1}\right)}{M^2_{\mathcal{L}^1_j}M^2_{\Xi_{1i}}} + \frac{\mathrm{Re}\left(g^{(2)}_{\mathcal{S}_i\mathcal{L}^1_j}\gamma_j\right)\kappa^{i}_{\mathcal{S}}}{M^2_{\mathcal{L}^1_j}M^2_{\mathcal{S}_i}} - \frac{\mathrm{Re}\left(g^{(2)}_{\Xi_{0i}\mathcal{L}^1_j}\gamma_j\right)\kappa^{i}_{\Xi_0}}{M^2_{\mathcal{L}^1_j}M^2_{\Xi_{0i}}} \\
& - \frac{2\mathrm{Re}\left(g^{(2)}_{\Xi_{1i}\mathcal{L}^1_j}\gamma_j\kappa^{i*}_{\Xi_1}\right)}{M^2_{\mathcal{L}^1_j}M^2_{\Xi_{1i}}} + \frac{\delta^{ij}_{\mathcal{B}}\delta^{ik}_{\mathcal{B}}\kappa^{j}_{\mathcal{S}}\kappa^{k}_{\mathcal{S}}}{2M^2_{\mathcal{B}_i}M^2_{\mathcal{S}_j}M^2_{\mathcal{S}_k}} - \frac{\delta^{ij}_{\mathcal{W}}\delta^{ik}_{\mathcal{W}}\kappa^{j}_{\Xi_0}\kappa^{k}_{\Xi_0}}{2M^2_{\mathcal{W}_i}M^2_{\Xi_{0j}}M^2_{\Xi_{0j}}} - \frac{2\delta^{ij}_{\mathcal{W}^1}\delta^{ik*}_{\mathcal{W}^1}\kappa^{j}_{\Xi_1}\kappa^{k*}_{\Xi_1}}{M^2_{\mathcal{W}^1_i}M^2_{\Xi_{1j}}M^2_{\Xi_{1k}}} \\
& + \frac{\varepsilon^{ijk}_{\mathcal{S}}\kappa^{i}_{\mathcal{S}}\gamma^*_j\gamma_k}{2M^2_{\mathcal{S}_i}M^2_{\mathcal{L}^1_j}M^2_{\mathcal{L}^1_k}} - \frac{\varepsilon^{ijk}_{\Xi_0}\kappa^{i}_{\Xi_0}\gamma^*_j\gamma_k}{2M^2_{\Xi_{0i}}M^2_{\mathcal{L}^1_j}M^2_{\mathcal{L}^1_k}} - 2\frac{\mathrm{Re}\left(\varepsilon^{ijk}_{\Xi_1}\kappa^{i}_{\Xi_1}\gamma^*_j\gamma^*_k\right)}{M^2_{\Xi_{1i}}M^2_{\mathcal{L}^1_j}M^2_{\mathcal{L}^1_k}} \\
& - \frac{\mathrm{Re}\left(\zeta^{ij}_{\mathcal{B}}\gamma^*_i\right)\delta^{jk}_{\mathcal{B}}\kappa^{k}_{\mathcal{S}}}{M^2_{\mathcal{L}^1_i}M^2_{\mathcal{B}_j}M^2_{\mathcal{S}_k}} + \frac{\mathrm{Re}\left(\zeta^{ij}_{\mathcal{W}}\gamma^*_i\right)\delta^{jk}_{\mathcal{W}}\kappa^{k}_{\Xi_0}}{M^2_{\mathcal{L}^1_i}M^2_{\mathcal{W}_j}M^2_{\Xi_{0k}}} + \frac{2\mathrm{Re}\left(\zeta^{ij}_{\mathcal{W}^1}\gamma_i\delta^{jk*}_{\mathcal{W}^1}\kappa^{k*}_{\Xi_1}\right)}{M^2_{\mathcal{L}^1_i}M^2_{\mathcal{W}^1_j}M^2_{\Xi_{1k}}};
\end{aligned}$$

Computer tools

A Python library for these symbolic calculations:

- Tree level integration in any model
- Transformations of the effective lagrangian

This work will be published soon

Sample code: integration

$$\mathcal{L}_{int} = -\kappa \Xi^a \phi^\dagger \sigma^a \phi - \lambda \Xi^a \Xi^a \phi^\dagger \phi$$

```
sigma = TensorBuilder("sigma")
kappa = TensorBuilder("kappa")
lamb = TensorBuilder("lamb")

phi = FieldBuilder("phi", 1, boson)
phic = FieldBuilder("phic", 1, boson)
Xi = FieldBuilder("Xi", 1, boson)

interaction_lag = -OpSum(
    Op(kappa(), Xi(0), phic(1), sigma(0, 1, 2), phi(2)),
    Op(lamb(), Xi(0), Xi(0), phic(1), phi(1)))

heavy_Xi = RealScalar("Xi", 1)
effective_lag = integrate([heavy_Xi], interaction_lag, 6)
```

Sample code: transformation rules

Fierz identities

$$\sigma^a_{ij}\sigma^a_{kl} = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}$$

```
fierz_rule = (
    Op(sigma(0, -1, -2), sigma(0, -3, -4)),
    OpSum(number_op(2) * Op(kdelta(-1, -4), kdelta(-3, -2)),
          -Op(kdelta(-1, -2), kdelta(-3, -4))))
```

Sample code: op. basis

$$\mathcal{O}_{\phi 6} = (\phi^\dagger \phi)^3, \qquad \mathcal{O}_{\phi 4} = (\phi^\dagger \phi)^2,$$

$$\mathcal{O}_{\phi}^{(1)} = \phi^\dagger \phi (D_\mu \phi)^\dagger D^\mu \phi, \qquad \mathcal{O}_{\phi}^{(3)} = (\phi^\dagger D_\mu \phi)(D^\mu \phi)^\dagger \phi,$$

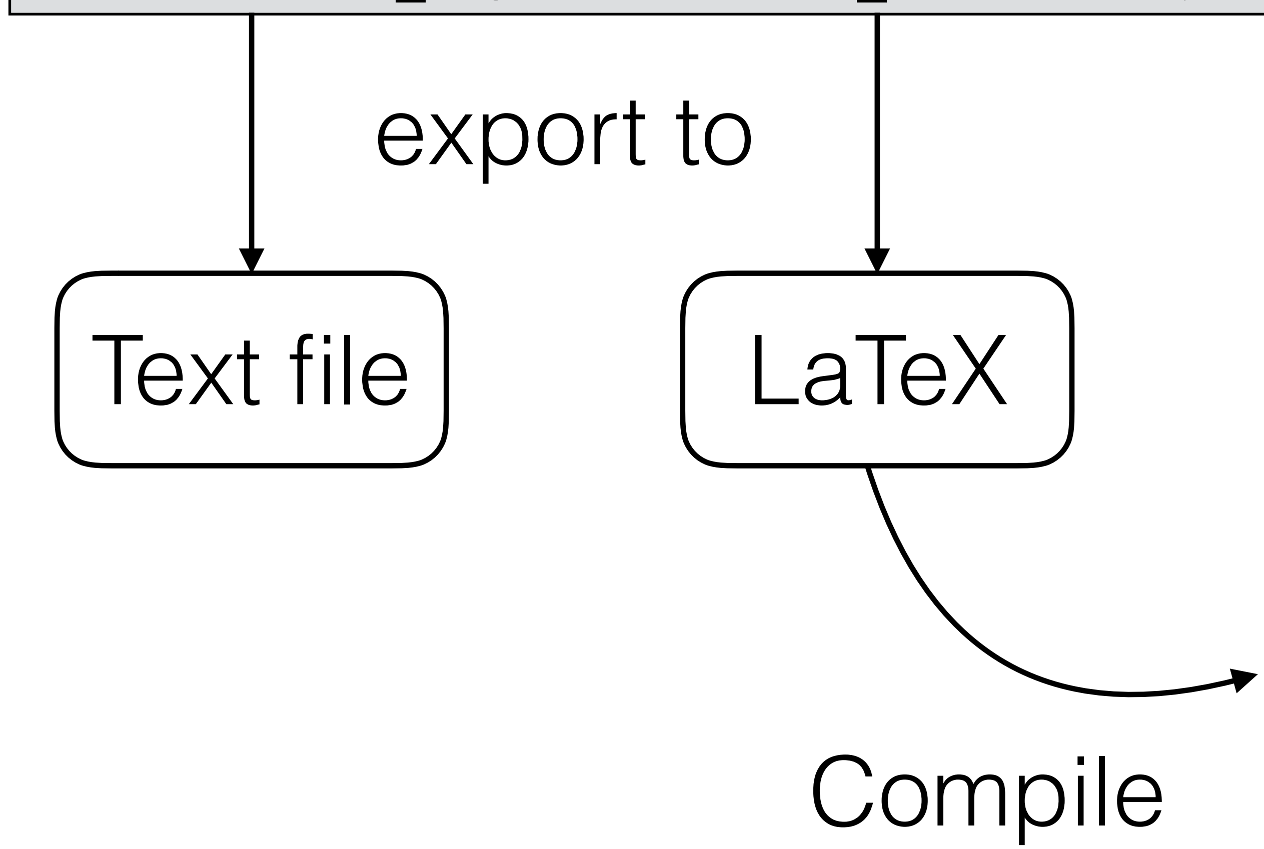
$$\mathcal{O}_{D\phi} = \phi^\dagger (D_\mu \phi) \phi^\dagger D^\mu \phi, \qquad \mathcal{O}_{D\phi}^* = (D_\mu \phi)^\dagger \phi (D^\mu \phi)^\dagger \phi$$

```
Ophi6 = tensor_op("Ophi6")
Ophi4 = tensor_op("Ophi4")
O1phi = tensor_op("O1phi")
...

definition_rules = [
    (Op(phic(0), phi(0), phic(1), phi(1), phic(2), phi(2)),
     OpSum(Ophi6)),
    (Op(phic(0), phi(0), phic(1), phi(1)),
     OpSum(Ophi4)),
    (Op(D(2, phic(0)), D(2, phi(0)), phic(1), phi(1)),
     OpSum(O1phi)),
    ...]
```

Sample results

```
rules = [fierz_rule] + definition_rules
max_iterations = 2
transf_eff_lag = apply_rules(
    effective_lag, rules, max_iterations)
```



$$\frac{\alpha_{\phi}^{(1)}}{\Lambda^2} = +2\,\frac{\kappa\kappa}{M_{\Xi}^4}$$

$$\frac{\alpha_{\phi}^{(3)}}{\Lambda^2} = -\,\frac{\kappa\kappa}{M_{\Xi}^4}$$

$$\frac{\alpha_{D\phi}}{\Lambda^2} = +0.5\,\frac{\kappa\kappa}{M_{\Xi}^4}$$

$$\frac{\alpha_{D\phi}^{*}}{\Lambda^2} = +0.5\,\frac{\kappa\kappa}{M_{\Xi}^4}$$

$$\alpha_{\phi 4} = +0.5\,\frac{\kappa\kappa}{M_{\Xi}^2}$$

$$\frac{\alpha_{\phi 6}}{\Lambda^2} = -\,\frac{\lambda\kappa\kappa}{M_{\Xi}^4}$$

Conclusions

- Complete study of new particles at low energies: integrate general extensions of the Standard Model
- Non-trivial mixed contributions appear
- Results: a dictionary between general new particles and the effective operators of the SM
- The tree-level contribution to dimension 6 of any model is a particular case