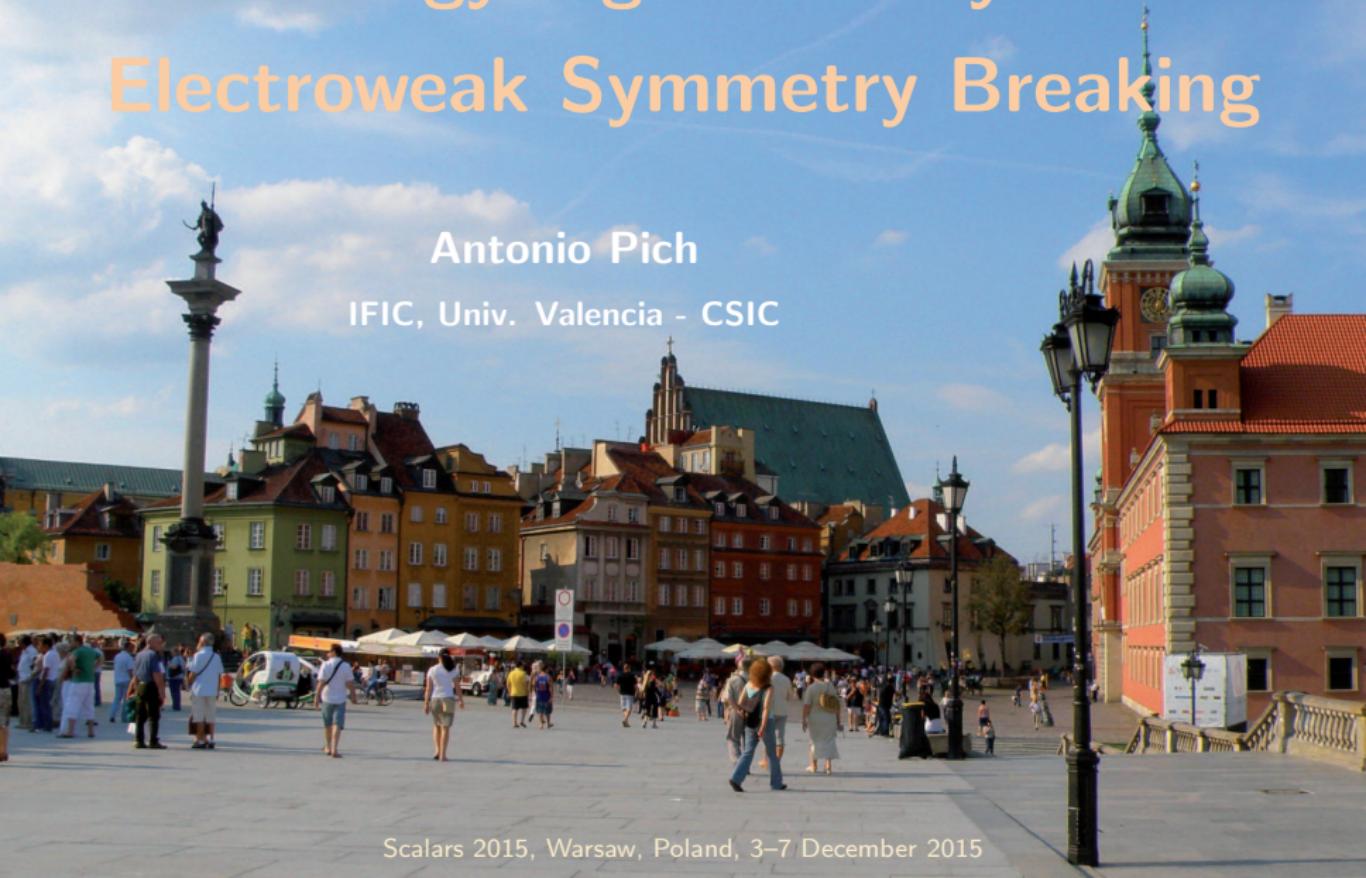


Low-Energy Signals of Dynamical Electroweak Symmetry Breaking

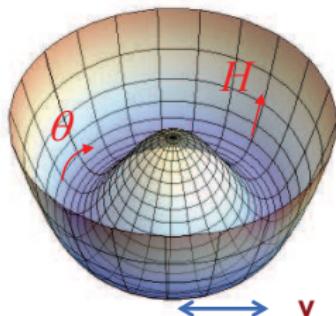
Antonio Pich

IFIC, Univ. Valencia - CSIC



Great success of the Standard Model

BEGHHK (\equiv Higgs) Mechanism

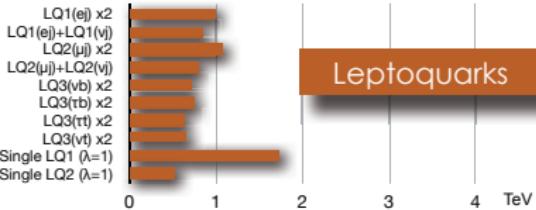


$$SU(2)_L \otimes U(1)_Y \quad v = 246 \text{ GeV}$$

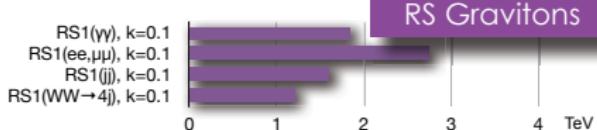
$$M_Z \cos \theta_W = M_W = \frac{1}{2} v g$$



Fundación
Príncipe de Asturias

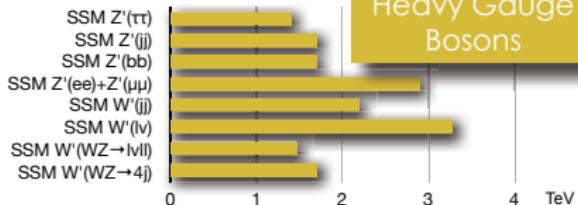


Leptoquarks

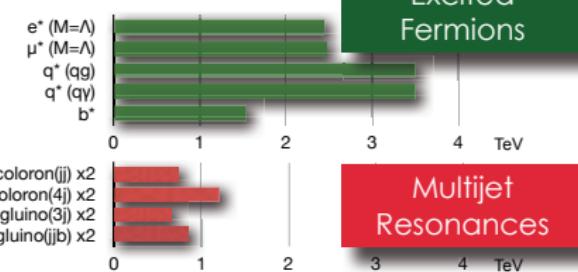


RS Gravitons

CMS Preliminary

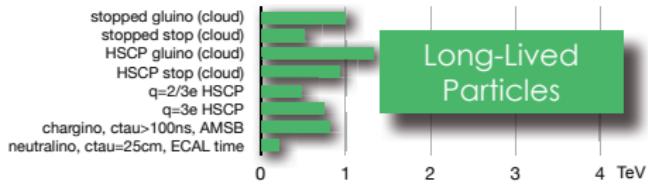


Heavy Gauge Bosons



Excited Fermions

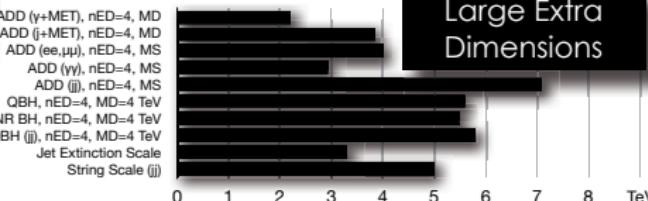
Multijet Resonances



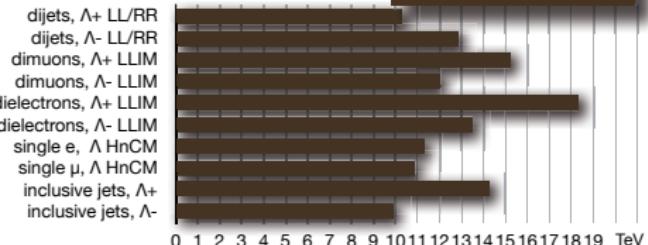
Long-Lived Particles



Dark Matter



Large Extra Dimensions



Compositeness

Energy Scale

Fields

Effective Theory

$\Lambda_{\text{NP}} \sim \text{TeV}$

S_n, P_n, V_n, A_n, F_n
 H, W, Z, γ, g
 τ, μ, e, ν_i
 t, b, c, s, d, u

Underlying Dynamics

----- Energy Gap -----



M_W

H, W, Z, γ, g
 τ, μ, e, ν_i
 t, b, c, s, d, u

Standard Model

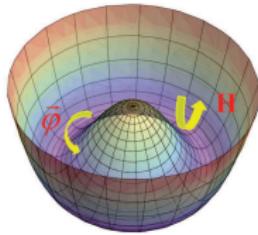
Effective Field Theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(4)} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda_{\text{NP}}^{D-4}} \mathcal{O}_i^{(D)}$$

- Most general Lagrangian with the SM gauge symmetries
- Light ($m \ll \Lambda_{\text{NP}}$) fields only
- The SM Lagrangian corresponds to $D = 4$
- $c_i^{(D)}$ contain information on the underlying dynamics:

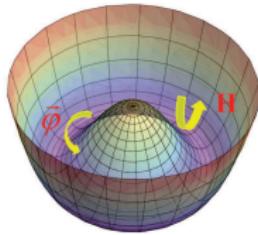
$$\mathcal{L}_{\text{NP}} \doteq g_X (\bar{q}_L \gamma^\mu q_L) X_\mu \quad \rightarrow \quad \frac{g_X^2}{M_X^2} (\bar{q}_L \gamma^\mu q_L) (\bar{q}_L \gamma_\mu q_L)$$

- Options for H(125):
 - SU(2)_L doublet (SM)
 - Scalar singlet
 - Additional light scalars



$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$$

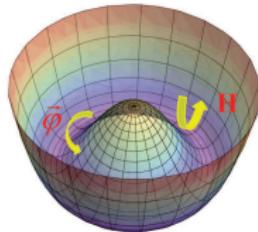
$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix}$$



$$\begin{aligned}\mathcal{L}_\Phi &= (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 \\ &= \frac{1}{2} \text{Tr} [(D^\mu \Sigma)^\dagger D_\mu \Sigma] - \frac{\lambda}{4} \left(\text{Tr} [\Sigma^\dagger \Sigma] - v^2 \right)^2\end{aligned}$$

Custodial Symmetry

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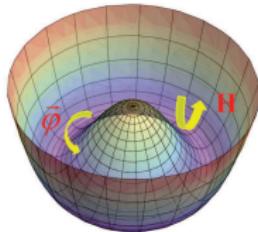
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$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{L+R}$ Symmetry: $\Sigma \rightarrow g_L \Sigma g_R^\dagger$

Custodial Symmetry

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} (\nu + H) U(\vec{\theta})$$

$$U(\vec{\varphi}) \equiv \exp \left\{ i \vec{\sigma} \cdot \frac{\vec{\varphi}}{\nu} \right\}$$



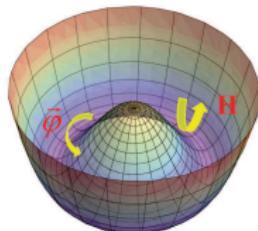
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Same Goldstone Lagrangian as QCD pions:

$$f_\pi \rightarrow \nu \quad , \quad \vec{\pi} \rightarrow \vec{\varphi} \rightarrow W_L^\pm, Z_L$$

Goldstone Electroweak Effective Theory

$$\mathcal{L}_{\text{EW}}^{(2)} = -\frac{1}{2g^2} \langle \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \rangle - \frac{1}{2g'^2} \langle \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \rangle + \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle$$

$$U(\varphi) = \exp \left\{ \frac{i\sqrt{2}}{v} \Phi \right\} \quad , \quad \Phi \equiv \frac{1}{\sqrt{2}} \vec{\sigma} \cdot \vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}} \varphi^0 \end{pmatrix}$$

$$D^\mu U = \partial^\mu U - i \hat{W}^\mu U + i U \hat{B}^\mu \quad , \quad D^\mu U^\dagger = \partial^\mu U^\dagger + i U^\dagger \hat{W}^\mu - i \hat{B}^\mu U^\dagger \quad , \quad \langle A \rangle \equiv \text{Tr}(A)$$

$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i [\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i [\hat{B}^\mu, \hat{B}^\nu]$$

$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{L+R}$ Symmetry: $U(\varphi) \rightarrow g_L U(\varphi) g_R^\dagger$

$$\hat{W}^\mu \rightarrow g_L \hat{W}^\mu g_L^\dagger + i g_L \partial^\mu g_L^\dagger \quad , \quad \hat{B}^\mu \rightarrow g_R \hat{B}^\mu g_R^\dagger + i g_R \partial^\mu g_R^\dagger$$

SM Symmetry Breaking: $\hat{W}^\mu = -\frac{g}{2} \vec{\sigma} \cdot \vec{W}^\mu \quad , \quad \hat{B}^\mu = -\frac{g'}{2} \sigma_3 B^\mu$

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\text{EW}}^{(4)} \Big|_{\text{Bosonic}} = \sum_i \mathcal{F}_i(h/v) \mathcal{O}_i \quad \mathcal{F}_i(h/v) = \sum_{n=0} \mathcal{F}_{i,n} \left(\frac{h}{v}\right)^n$$

Appelquist-Bernard, Longhitano, Buchalla et al, Alonso et al, Pich et al...

$\mathcal{O}(p^4)$ \mathcal{P} -even bosonic operators

(A.P., Rosell, Santos, Sanz-Cillero)

$$\mathcal{O}_1 = \frac{1}{4} \langle f_+^{\mu\nu} f_{\mu\nu}^+ - f_-^{\mu\nu} f_{\mu\nu}^- \rangle$$

$$\mathcal{O}_2 = \frac{1}{2} \langle f_+^{\mu\nu} f_{\mu\nu}^+ + f_-^{\mu\nu} f_{\mu\nu}^- \rangle$$

$$\mathcal{O}_3 = \frac{i}{2} \langle f_+^{\mu\nu} [u_\mu, u_\nu] \rangle$$

$$\mathcal{O}_4 = \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle$$

$$\mathcal{O}_5 = \langle u_\mu u^\mu \rangle^2$$

$$\mathcal{O}_6 = \frac{1}{v^2} (\partial_\mu h)(\partial^\mu h) \langle u_\nu u^\nu \rangle$$

$$\mathcal{O}_7 = \frac{1}{v^2} (\partial_\mu h)(\partial_\nu h) \langle u^\mu u^\nu \rangle$$

$$\mathcal{O}_8 = \frac{1}{v^4} (\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$$

$$\mathcal{O}_9 = \frac{1}{v} (\partial_\mu h) \langle f_-^{\mu\nu} u_\nu \rangle$$

$$U = u^2 = \exp \left\{ \frac{i}{v} \vec{\sigma} \cdot \vec{\varphi} \right\} \quad , \quad u_\mu \equiv i u (D_\mu U)^\dagger u = u_\mu^\dagger \quad , \quad f_\pm^{\mu\nu} = u^\dagger \hat{W}^{\mu\nu} u \pm u \hat{B}^{\mu\nu} u^\dagger$$

Custodial symmetry assumed

EW Resonance Effective Theory

- Towers of heavy states are usually present in strongly-coupled models of EWSB: Technicolour, Walking TC...
- The low-energy constants (**LECs**) of the Goldstone Lagrangian contain information on the heavier states. **The lightest states not included in the Lagrangian dominate**

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- ① Build $\mathcal{L}_{\text{eff}}(\varphi_i, R_k)$ with the lightest R_k coupled to the φ_i
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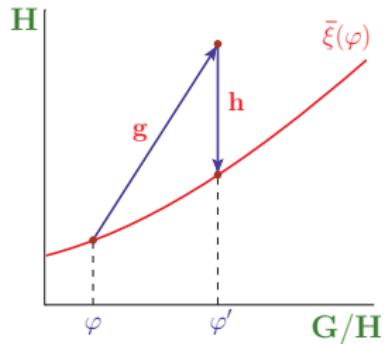
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- ③ Match the two effective Lagrangians \rightarrow LECs

This program works in QCD: $R\chi T$ (Ecker–Gasser–Leutwyler–Pich–de Rafael)

Good dynamical understanding at large N_c

Coset Space Coordinates:

$$G \equiv SU(2)_L \otimes SU(2)_R \rightarrow H \equiv SU(2)_V$$



$$\bar{\xi}(\varphi) \equiv (\xi_L(\varphi), \xi_R(\varphi)) \in G$$

$$\xi_L(\varphi) \xrightarrow{G} g_L \xi_L(\varphi) h^\dagger(\varphi, g)$$

$$\xi_R(\varphi) \xrightarrow{G} g_R \xi_R(\varphi) h^\dagger(\varphi, g)$$

$$\mathbf{U}(\varphi) \equiv \xi_L(\varphi) \xi_R^\dagger(\varphi) \xrightarrow{G} g_L \mathbf{U}(\varphi) g_R^\dagger$$

Canonical choice: $\xi_L(\varphi) = \xi_R(\varphi)^\dagger \equiv \mathbf{u}(\varphi) \xrightarrow{G} g_L \mathbf{u}(\varphi) h^\dagger(\varphi, g) = h(\varphi, g) \mathbf{u}(\varphi) g_R^\dagger$

$$\mathbf{U}(\varphi) = \mathbf{u}(\varphi)^2 = \exp\left\{\frac{i}{v}\vec{\sigma}\vec{\varphi}\right\}$$

SU(2)_V triplets: $\mathbf{X} \equiv \frac{1}{2} \sigma^a \mathbf{X}^a \xrightarrow{G} \mathbf{h}(\varphi, g) \mathbf{X} \mathbf{h}(\varphi, g)^\dagger$

$$\nabla_\mu \mathbf{X} = \partial_\mu \mathbf{X} + [\Gamma_\mu, \mathbf{X}] \quad , \quad \Gamma_\mu = \frac{1}{2} \left\{ u^\dagger (\partial_\mu - i \hat{W}_\mu) u + u (\partial_\mu - i \hat{B}_\mu) u^\dagger \right\}$$

$$u_\mu \equiv i u D_\mu U^\dagger u = u_\mu^\dagger \quad , \quad f_\pm^{\mu\nu} = u^\dagger \hat{W}^{\mu\nu} u \pm u \hat{B}^{\mu\nu} u^\dagger$$

LO Resonance EW Lagrangian:

Pich–Rosell–Santos–Sanz-Cillero

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{EWET}} + \sum_R \mathcal{L}_R + \sum_{R,R'} \mathcal{L}_{RR'} + \dots$$

Heavy Triplets: $\mathbf{V}(1^{--})$, $\mathbf{A}(1^{++})$, $\mathbf{P}(1^{++})$; **Heavy Singlet:** $\mathbf{S}_1(0^{++})$

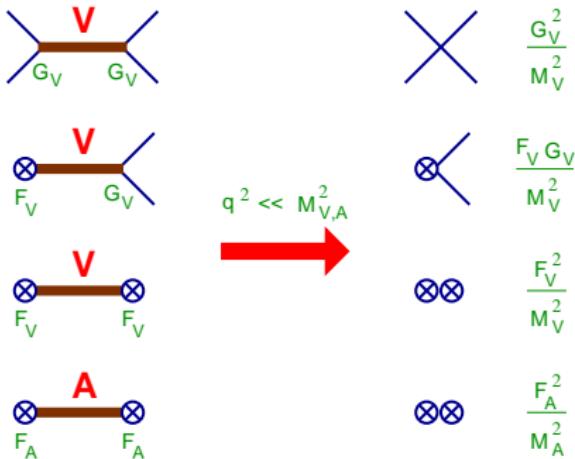
$$\begin{aligned} \sum_R \mathcal{L}_R &= \frac{\nu}{2} \kappa_w h \langle u^\mu u_\mu \rangle + \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu u^\nu] \rangle \\ &+ \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle + \sqrt{2} \lambda_1^{hA} \partial_\mu h \langle A^{\mu\nu} u_\nu \rangle \\ &+ \frac{d_P}{\nu} \partial_\mu h \langle P u^\mu \rangle + \frac{c_d}{\sqrt{2}} S_1 \langle u^\mu u_\mu \rangle + \lambda_{hS_1} \nu h^2 S_1 \end{aligned}$$

$$U = u^2 = \exp \left\{ \frac{i}{\nu} \vec{\sigma} \vec{\varphi} \right\} \quad , \quad u_\mu \equiv i u (D_\mu U)^\dagger u = u_\mu^\dagger \quad , \quad f_\pm^{\mu\nu} = u^\dagger \hat{W}^{\mu\nu} u \pm u \hat{B}^{\mu\nu} u^\dagger$$

Antisymmetric $V_{\mu\nu}$ and $A_{\mu\nu}$ fields (better UV properties):

$$\mathcal{L}_{\text{Kin}} = -\frac{1}{2} \sum_{R=V,A} \langle \nabla^\lambda R_{\lambda\mu} \nabla_\nu R^{\nu\mu} - \frac{1}{2} M_R^2 R_{\mu\nu} R^{\mu\nu} \rangle$$

Resonance Exchange



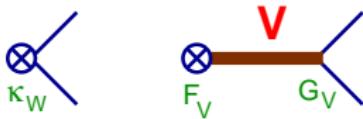
Pich, Rosell, Santos, Sanz-Cillero

$$\begin{aligned}
 \mathcal{F}_1 &= \frac{F_A^2}{4M_A^2} - \frac{F_V^2}{4M_V^2} & , & \quad \mathcal{F}_2 = -\frac{F_A^2}{8M_A^2} - \frac{F_V^2}{8M_V^2} & , & \quad \mathcal{F}_3 = -\frac{F_V G_V}{2M_V^2} \\
 \mathcal{F}_4 &= \frac{G_V^2}{4M_V^2} & , & \quad \mathcal{F}_5 = \frac{c_d^2}{4M_{S_1}^2} - \frac{G_V^2}{4M_V^2} & , & \quad \mathcal{F}_6 = -\frac{(\lambda_1^{hA})^2 v^2}{M_A^2} \\
 \mathcal{F}_7 &= \frac{d_P^2}{2M_P^2} + \frac{(\lambda_1^{hA})^2 v^2}{M_A^2} & , & \quad \mathcal{F}_8 = 0 & , & \quad \mathcal{F}_9 = -\frac{F_A \lambda_1^{hA} v}{M_A^2}
 \end{aligned}$$

Short-Distance Constraints

- Vector Form Factor:

$$\langle \varphi(p_1) \varphi(p_2) | J_V^\mu | 0 \rangle = (p_1 - p_2)^\mu \mathcal{F}_{\varphi\varphi}^V(s)$$



$$\mathcal{F}_{\varphi\varphi}^V(s) = 1 + \frac{F_V G_V}{v^2} \frac{s}{M_V^2 - s}$$

$$\lim_{s \rightarrow \infty} \mathcal{F}_{\varphi\varphi}^V(s) = 0$$

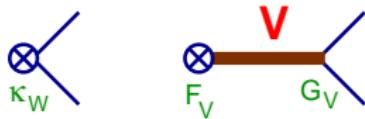


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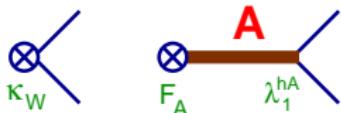
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$$F_V G_V = v^2$$

- Axial Form Factor:

$$\langle h(p_1) \varphi(p_2) | J_A^\mu | 0 \rangle = (p_1 - p_2)^\mu \mathcal{F}_{h\varphi}^A(s)$$



$$\mathcal{F}_{h\varphi}^A(s) = \kappa_W \left(1 + \frac{F_A \lambda_1^{hA}}{\kappa_W v} \frac{s}{M_A^2 - s} \right)$$

$$\lim_{s \rightarrow \infty} \mathcal{F}_{h\varphi}^A(s) = 0$$



$$F_A \lambda_1^{hA} = \kappa_W v$$

Weinberg Sum Rules

Chiral Symmetry:

$$\Pi_{LR}^{\mu\nu}(q) \equiv \int d^4x e^{iqx} \langle 0 | T(J_L^\mu(x) J_R^\nu(0)^\dagger) | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{LR}(q^2) = 0$$

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$$\frac{1}{\pi} \int_0^\infty ds [\text{Im} \Pi_{VV}(s) - \text{Im} \Pi_{AA}(s)] = v^2 \quad (1^{\text{st}} \text{ WSR})$$

$$\frac{1}{\pi} \int_0^\infty ds s [\text{Im} \Pi_{VV}(s) - \text{Im} \Pi_{AA}(s)] = 0 \quad (2^{\text{nd}} \text{ WSR})$$

- WSRs @ LO:

$$\Pi_{LR}(s) = \frac{v^2}{s} + \frac{F_V^2}{M_V^2 - s} - \frac{F_A^2}{M_A^2 - s}$$



- 1st WSR: $F_V^2 - F_A^2 = v^2$
- 2nd WSR: $F_V^2 M_V^2 - F_A^2 M_A^2 = 0$

→ $F_V^2 = v^2 \frac{M_A^2}{M_A^2 - M_V^2}, \quad F_A^2 = v^2 \frac{M_V^2}{M_A^2 - M_V^2}, \quad M_A > M_V$

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- WSRs @ NLO:

$$\kappa_W \equiv \frac{g_{hWW}}{g_{hWW}^{\text{SM}}} = \frac{M_V^2}{M_A^2}$$

Pich–Rosell–Sanz–Cillero

- WSRs @ LO:

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- 1st WSR: $F_V^2 - F_A^2 = v^2$
- 2nd WSR: $F_V^2 M_V^2 - F_A^2 M_A^2 = 0$

→ $F_V^2 = v^2 \frac{M_A^2}{M_A^2 - M_V^2}, \quad F_A^2 = v^2 \frac{M_V^2}{M_A^2 - M_V^2}, \quad M_A > M_V$

- WSRs @ NLO:

$$\kappa_W \equiv \frac{g_{hWW}}{g_{hWW}^{\text{SM}}} = \frac{M_V^2}{M_A^2}$$

Pich–Rosell–Sanz–Cillero

1st WSR likely valid also in gauge theories with non-trivial UV fixed points

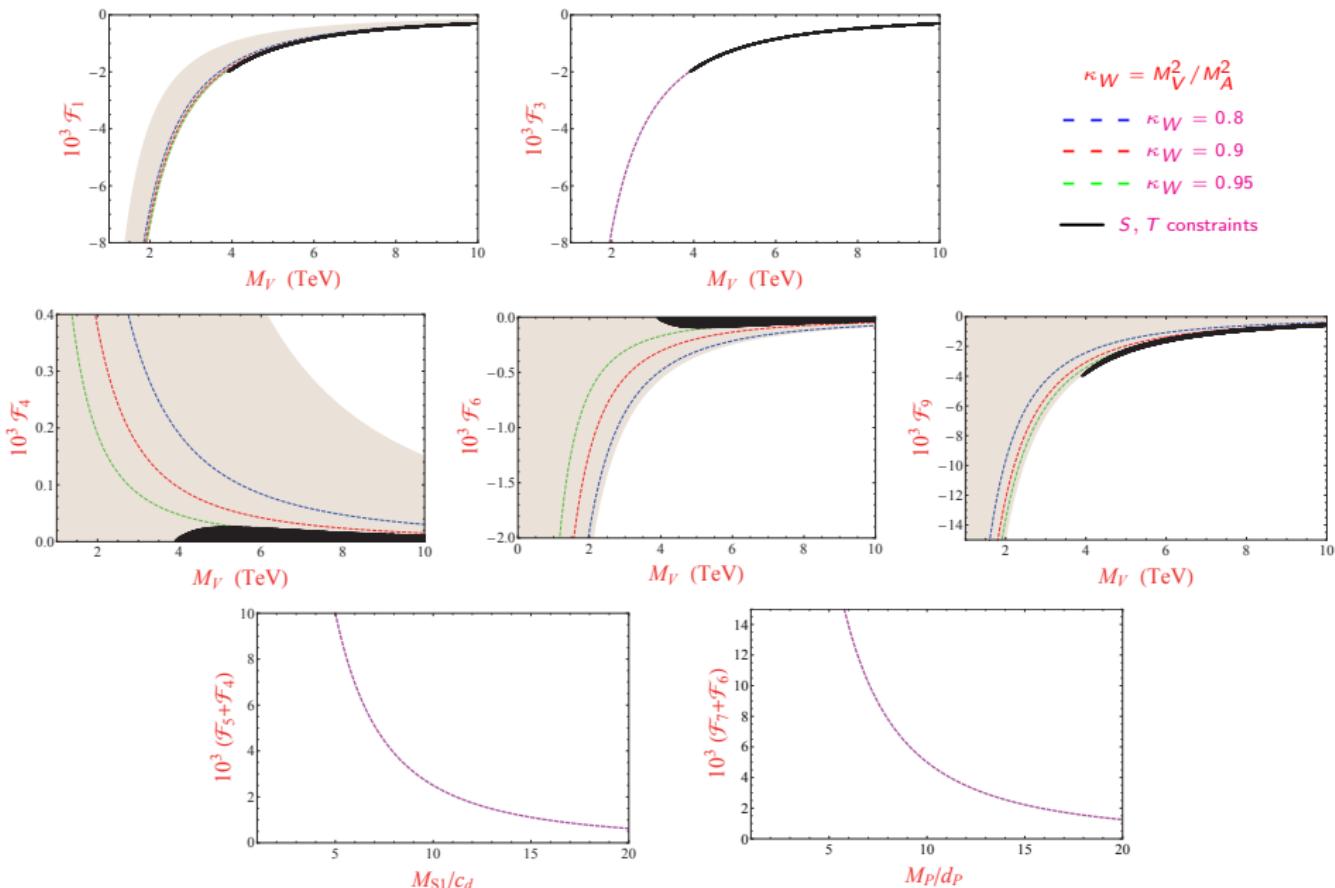
2nd WSR questionable (not valid) in walking (conformal) TC scenarios

Appelquist–Sannino, Orgogozo–Rychkov

Short-distance constraints bring sharper predictions

Pich, Rosell, Santos, Sanz-Cillero

$$\begin{aligned}\mathcal{F}_1 &= \frac{F_A^2}{4M_A^2} - \frac{F_V^2}{4M_V^2} = -\frac{v^2}{4} \left(\frac{1}{M_V^2} + \frac{1}{M_A^2} \right) \\ \mathcal{F}_2 &= -\frac{F_A^2}{8M_A^2} - \frac{F_V^2}{8M_V^2} = -\frac{v^2(M_V^4 + M_A^4)}{8M_V^2 M_A^2 (M_A^2 - M_V^2)} \\ \mathcal{F}_3 &= -\frac{F_V G_V}{2M_V^2} = -\frac{v^2}{2M_V^2} \\ \mathcal{F}_4 &= \frac{G_V^2}{4M_V^2} = \frac{(M_A^2 - M_V^2)v^2}{4M_V^2 M_A^2} \\ \mathcal{F}_5 &= \frac{c_d^2}{4M_{S_1}^2} - \frac{G_V^2}{4M_V^2} = \frac{c_d^2}{4M_{S_1}^2} - \frac{(M_A^2 - M_V^2)v^2}{4M_V^2 M_A^2} \\ \mathcal{F}_6 &= -\frac{(\lambda_1^{hA})^2 v^2}{M_A^2} = -\frac{M_V^2 (M_A^2 - M_V^2)v^2}{M_A^6} \\ \mathcal{F}_7 &= \frac{d_P^2}{2M_P^2} + \frac{(\lambda_1^{hA})^2 v^2}{M_A^2} = \frac{d_P^2}{2M_P^2} + \frac{M_V^2 (M_A^2 - M_V^2)v^2}{M_A^6} \\ \mathcal{F}_8 &= 0 \\ \mathcal{F}_9 &= -\frac{F_A \lambda_1^{hA} v}{M_A^2} = -\frac{M_V^2 v^2}{M_A^4}\end{aligned}$$

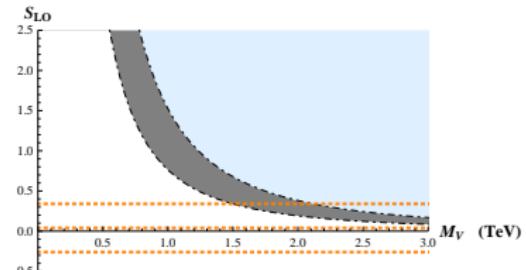
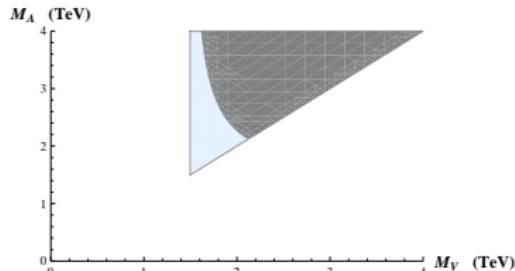


Gauge Boson Self-Energies @ LO



$$S_{\text{LO}} = 4\pi \left(\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right) , \quad T_{\text{LO}} = 0$$

Sensitive to vector and axial states $\rightarrow M_A > M_V > 1.5 \text{ TeV}$



3σ bounds

AP–Rosell–Sanz–Cillero

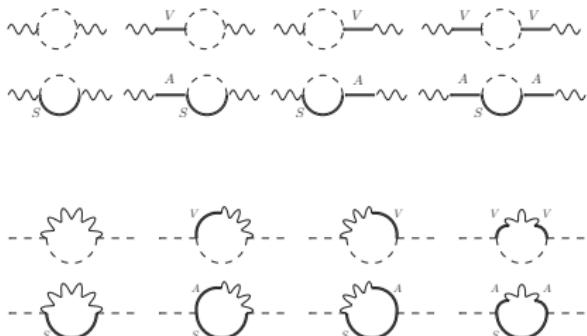
- **1st+2nd WSR:** $S_{\text{LO}} = \frac{4\pi v^2}{M_V^2} \left(1 + \frac{M_V^2}{M_A^2} \right)$

- **1st WSR ($M_A > M_V$):** $S_{\text{LO}} = 4\pi \left\{ \frac{v^2}{M_V^2} + F_A^2 \left(\frac{1}{M_V^2} - \frac{1}{M_A^2} \right) \right\} > \frac{4\pi v^2}{M_V^2} > \frac{4\pi v^2}{M_A^2}$

Gauge Boson Self-Energies @ NLO

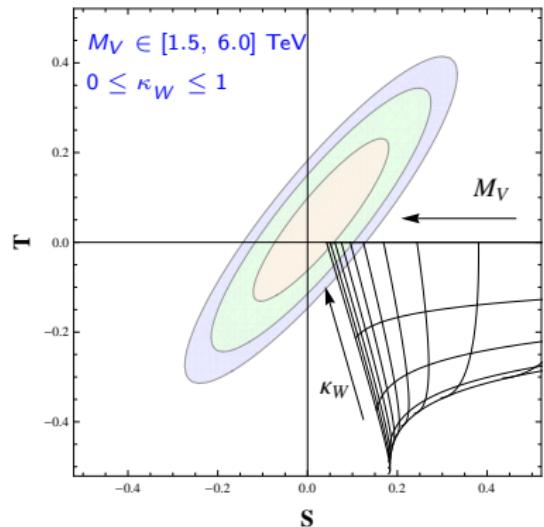
Sensitive to the light scalar $h(125)$

AP, Rosell, Sanz-Cillero

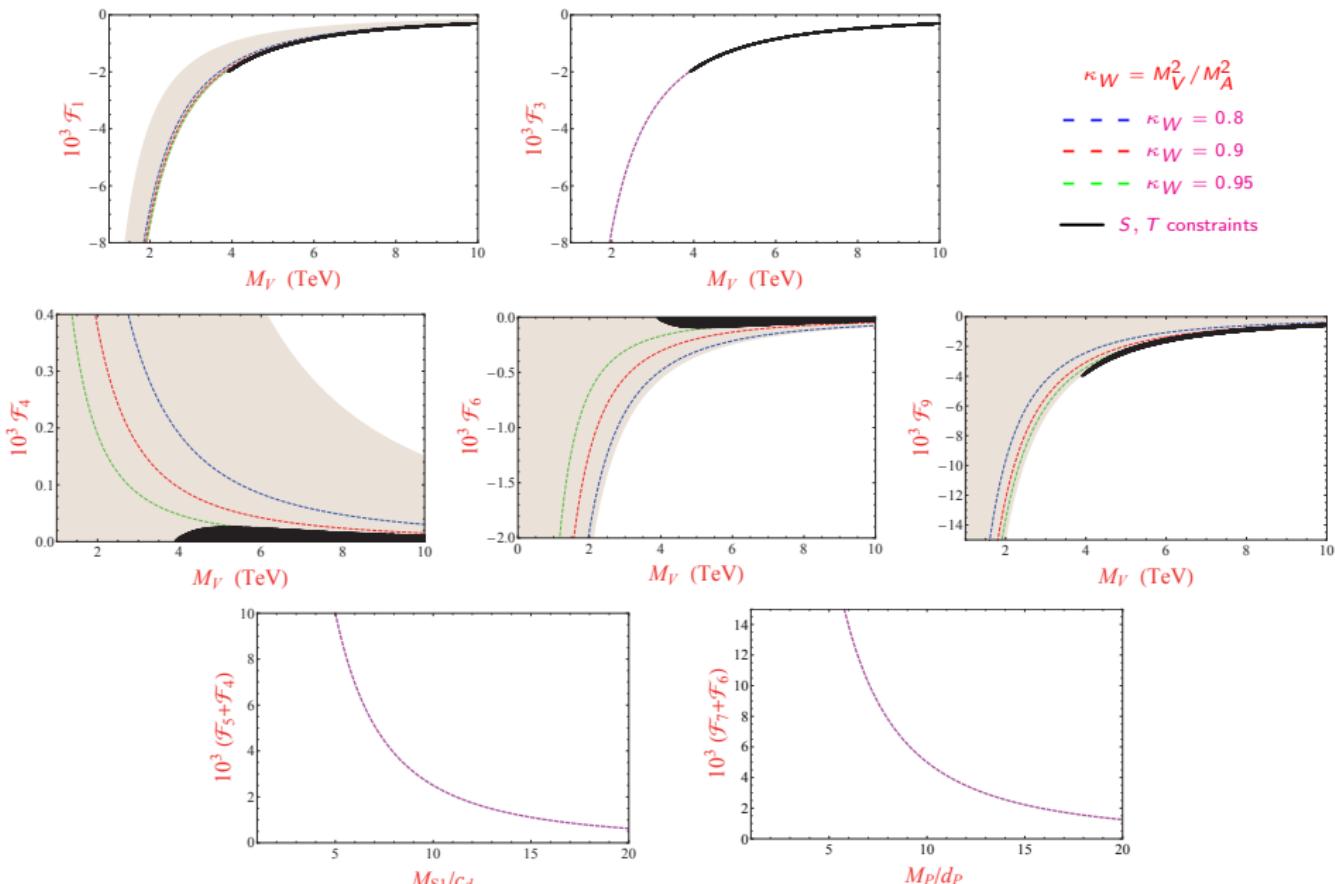


$$\kappa_W \equiv \frac{g_{SWW}}{g_{HWW}^{\text{SM}}} = \frac{M_V^2}{M_A^2} \in [0.94, 1]$$

$M_A \approx M_V > 4 \text{ TeV}$ (95% CL)



1st + 2nd WSRs



OUTLOOK

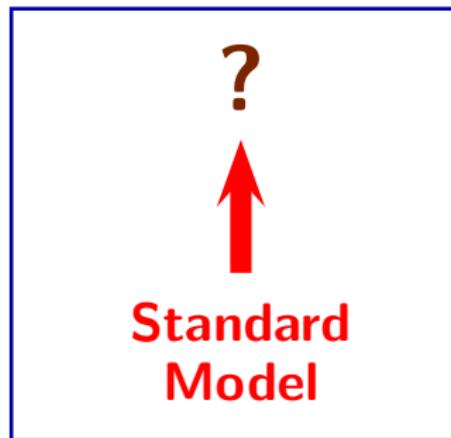
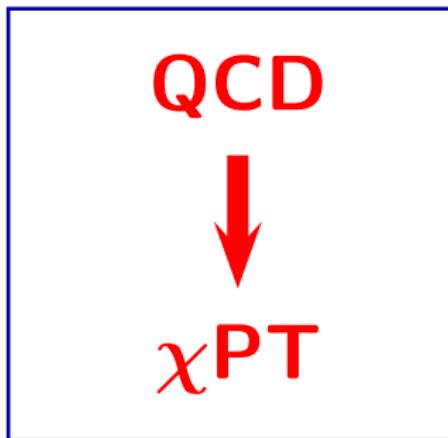
- Effective Field Theory: powerful low-energy tool
- Mass Gap: $E, m_{\text{light}} \ll \Lambda_{\text{NP}}$
- Assumption: relevant symmetries (breakings) & light fields
- Most general $\mathcal{L}_{\text{eff}}(\phi_{\text{light}})$ allowed by symmetry
- Short-distance dynamics encoded in LECs
- LECs constrained phenomenologically
- Goal: get hints on the underlying fundamental dynamics



New Physics

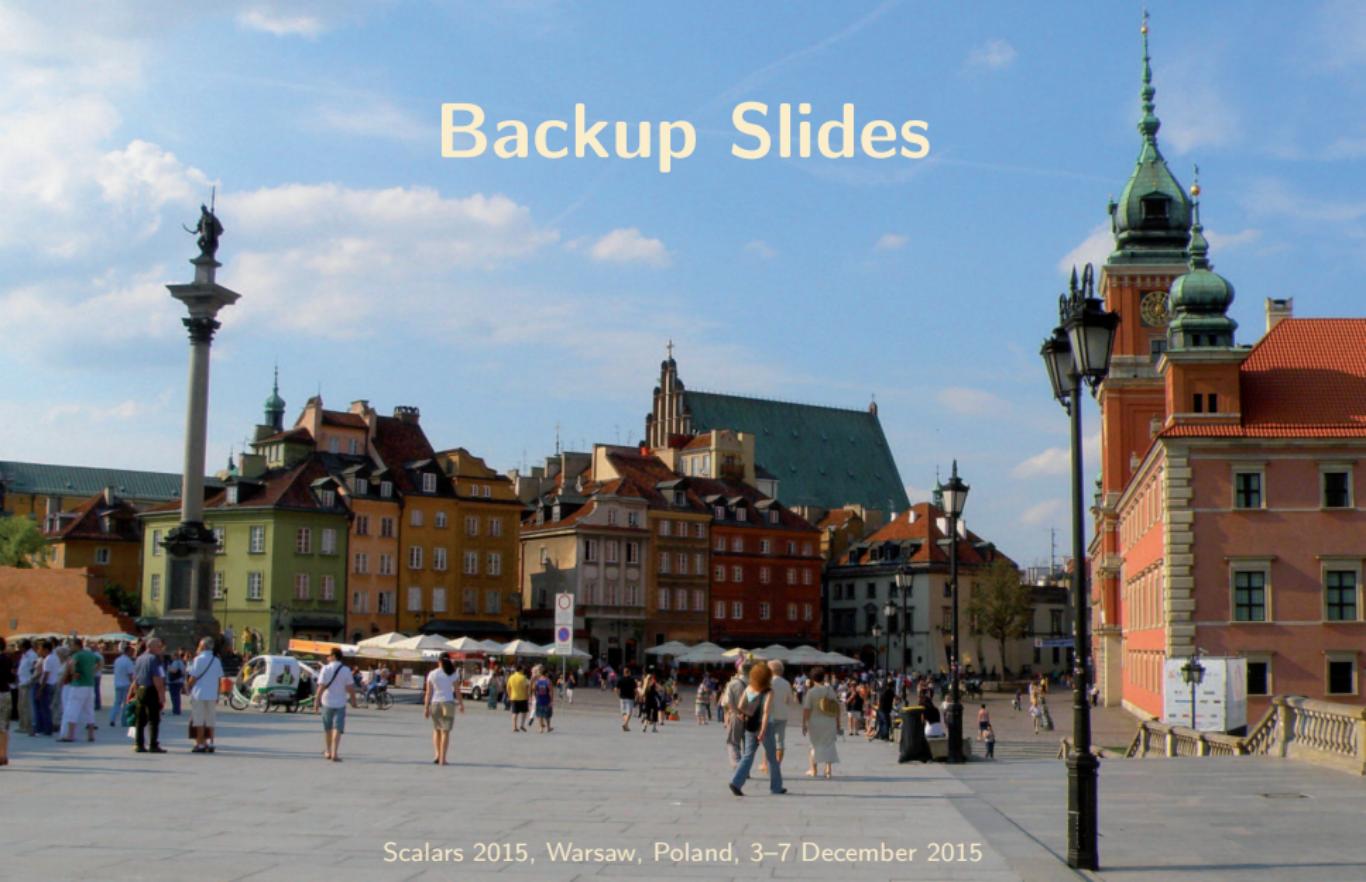
Learning from QCD experience. EW problem more difficult

Fundamental Underlying Theory unknown



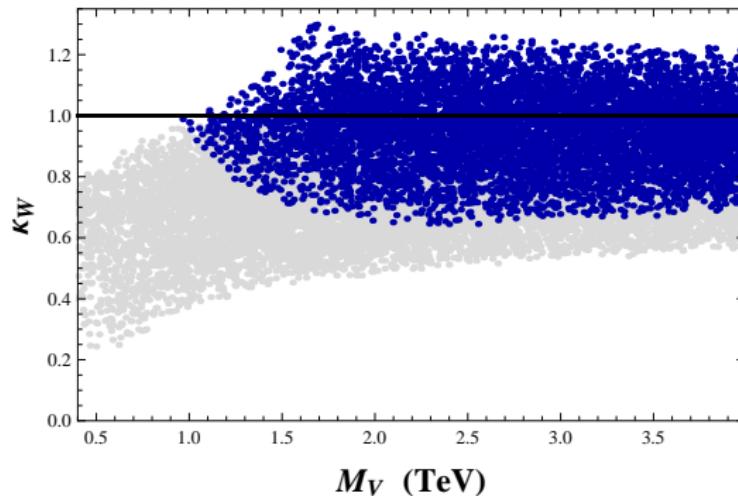
Additional dynamical input (fresh ideas!) needed

Backup Slides



Gauge Boson Self-Energies @ NLO

Weaker assumptions: 1st WSR only , $M_A > M_V > 0.4 \text{ TeV}$

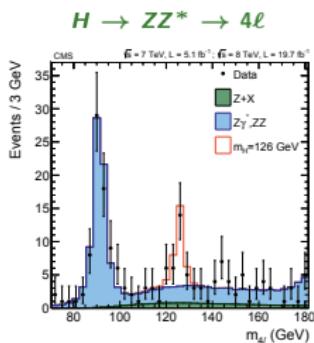
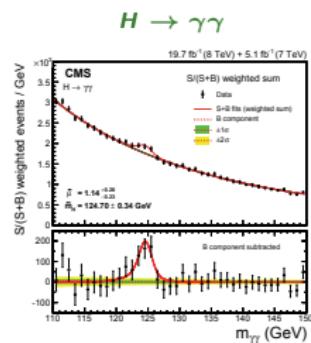
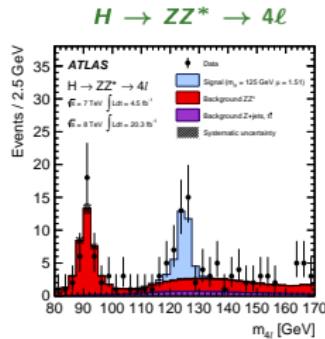
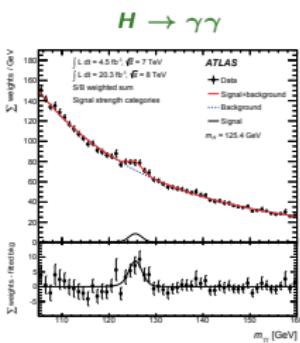
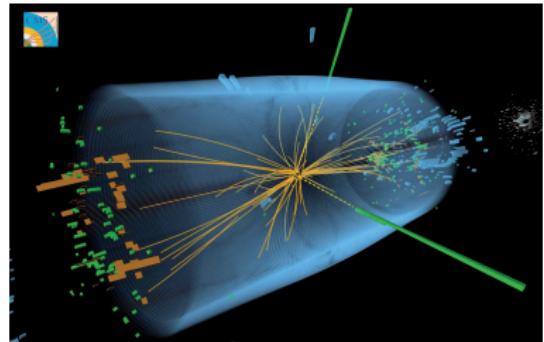
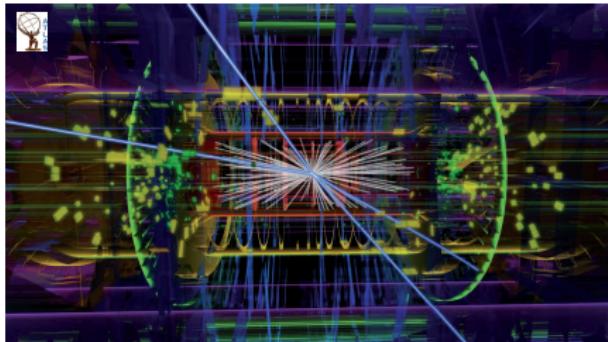


AP, Rosell, Sanz-Cillero

- $0.2 < \frac{M_V}{M_A} < 1$
- $0.02 < \frac{M_V}{M_A} < 0.2$

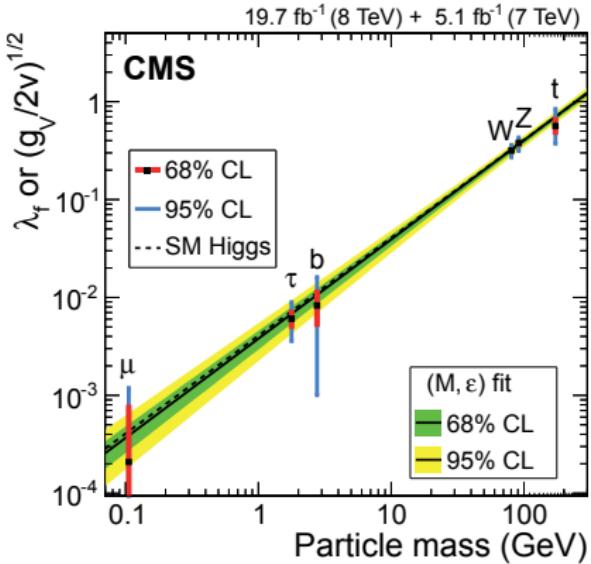
$\kappa_W \equiv g_{sWW}/g_{HWW}^{\text{SM}}$ very different from one
requires large (unnatural) mass splittings

A New Higgs-Like Boson



$$M_H = (125.09 \pm 0.21 \pm 0.11) \text{ GeV}$$

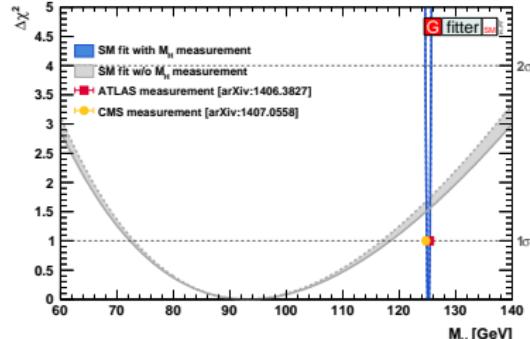
It is a Higgs Boson



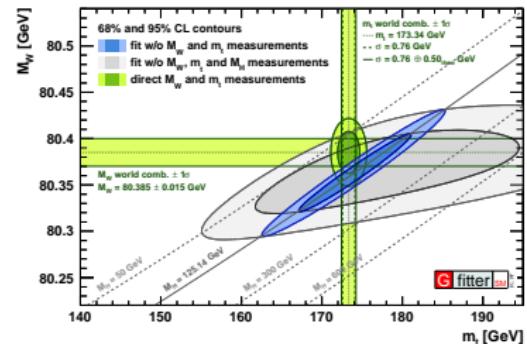
$$\lambda_f = \left(\frac{m_f}{M} \right)^{1+\epsilon} , \quad \left(\frac{g_V}{2v} \right)^{1/2} = \left(\frac{M_V}{M} \right)^{1+\epsilon} \quad (95\% \text{ CL})$$

$$\epsilon \in [-0.054, 0.100] , \quad M \in [217, 279] \text{ GeV}$$

SM: $\epsilon = 0$, $M = v = 246 \text{ GeV}$



SM Fit



Higgs Mechanism:

Gauge invariance

Massless W^\pm, Z (spin 1)

3×2 polarizations = 6

Higgs Mechanism: 3 additional degrees of freedom $\varphi_i(x)$

Gauge invariance

Massless W^\pm, Z (spin 1)

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+

3 Goldstones $\varphi_i(x)$

SSB
↓

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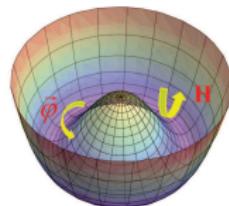
SSB
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Massive W^\pm, Z

3×3 polarizations = 9

Spontaneous Symmetry Breaking

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$



$$\mu^2 < 0$$

$$\Phi(x) = \exp \left\{ i \vec{\sigma} \cdot \frac{\vec{\varphi}(x)}{v} \right\} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

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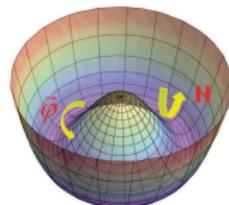
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$$D_\mu \Phi = (\partial_\mu + \frac{i}{2} g \vec{\sigma} \cdot \vec{W}_\mu + \frac{i}{2} g' B_\mu) \Phi \quad ; \quad v^2 = -\mu^2/\lambda$$

$$(D_\mu \Phi)^\dagger D^\mu \Phi \rightarrow M_W^2 W_\mu^\dagger W^\mu + \frac{M_Z^2}{2} Z_\mu Z^\mu$$

$$M_W = M_Z \cos \theta_W = \frac{1}{2} g v$$

EFFECTIVE LAGRANGIAN:

$$\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$$

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$$\text{Parity} \rightarrow \text{even dimension} ; \quad U U^\dagger = 1 \rightarrow 2n \geq 2$$

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**Derivative
Coupling**

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Derivative
Coupling

Goldstones become free at zero momenta

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4} \text{Tr} \left(D_\mu U^\dagger D^\mu U \right) \xrightarrow{U=1} \mathcal{L}_2 = M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$
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$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i [\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i [\hat{B}^\mu, \hat{B}^\nu]$$

$$\hat{W}^\mu = -\frac{g}{2} \vec{\sigma} \cdot \vec{W}^\mu \quad , \quad \hat{B}^\mu = -\frac{g'}{2} \sigma_3 B^\mu \quad (\text{explicit symmetry breaking})$$

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- EW Goldstones are responsible for $M_{W,Z}$ (not the Higgs!)
- QCD pions also generate small W, Z masses: $\delta_\pi M_W = \frac{1}{2} g f_\pi$

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\text{EW}}^{(4)} \Big|_{\text{CP-even}} = \sum_{i=0}^{14} a_i \mathcal{O}_i \quad (\text{Appelquist, Longhitano})$$

$$\mathcal{O}_0 = v^2 \langle T_L V_\mu \rangle^2$$

$$\mathcal{O}_1 = \langle U \hat{B}_{\mu\nu} U^\dagger \hat{W}^{\mu\nu} \rangle$$

$$\mathcal{O}_3 = i \langle \hat{W}_{\mu\nu} [V^\mu, V^\nu] \rangle$$

$$\mathcal{O}_5 = \langle V_\mu V^\mu \rangle^2$$

$$\mathcal{O}_2 = i \langle U \hat{B}_{\mu\nu} U^\dagger [V^\mu, V^\nu] \rangle$$

$$\mathcal{O}_4 = \langle V_\mu V_\nu \rangle \langle V^\mu V^\nu \rangle$$

$$\mathcal{O}_{11} = \langle (D_\mu V^\mu)^2 \rangle$$

$$V_\mu \equiv D_\mu U U^\dagger \quad , \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - i [\hat{W}_\mu, V_\nu] \quad , \quad (V_\mu, D_\mu V_\nu, T_L) \rightarrow g_L (V_\mu, D_\mu V_\nu, T_L) g_L^\dagger$$

Symmetry breaking: $T_L \equiv U \frac{\sigma_3}{2} U^\dagger$, $\hat{B}_{\mu\nu} \equiv -g' \frac{\sigma_3}{2} B_{\mu\nu}$

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$$\mathcal{O}_7 = 4 \langle V_\mu V^\mu \rangle \langle T_L V_\nu \rangle^2$$

$$\mathcal{O}_9 = -2 \langle T_L \hat{W}_{\mu\nu} \rangle \langle T_L [V^\mu, V^\nu] \rangle$$

$$\mathcal{O}_{11} = \langle (D_\mu V^\mu)^2 \rangle$$

$$\mathcal{O}_{13} = 2 \langle T_L D_\mu V_\nu \rangle^2$$

$$\mathcal{O}_2 = i \langle U \hat{B}_{\mu\nu} U^\dagger [V^\mu, V^\nu] \rangle$$

$$\mathcal{O}_4 = \langle V_\mu V_\nu \rangle \langle V^\mu V^\nu \rangle$$

$$\mathcal{O}_6 = 4 \langle V_\mu V_\nu \rangle \langle T_L V^\mu \rangle \langle T_L V^\nu \rangle$$

$$\mathcal{O}_8 = \langle T_L \hat{W}_{\mu\nu} \rangle^2$$

$$\mathcal{O}_{10} = 16 \{ \langle T_L V_\mu \rangle \langle T_L V_\nu \rangle \}^2$$

$$\mathcal{O}_{12} = 4 \langle T_L D_\mu D_\nu V^\nu \rangle \langle T_L V^\mu \rangle$$

$$\mathcal{O}_{14} = -2i \varepsilon^{\mu\nu\rho\sigma} \langle \hat{W}_{\mu\nu} V_\rho \rangle \langle T_L V_\sigma \rangle$$

$$V_\mu \equiv D_\mu U U^\dagger \quad , \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - i [\hat{W}_\mu, V_\nu] \quad , \quad (V_\mu, D_\mu V_\nu, T_L) \rightarrow g_L (V_\mu, D_\mu V_\nu, T_L) g_L^\dagger$$

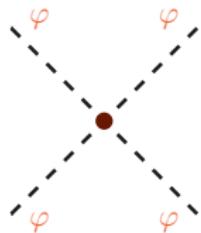
Symmetry breaking: $T_L \equiv U \frac{\sigma_3}{2} U^\dagger \sim \mathcal{O}(p)$, $\hat{B}_{\mu\nu} \equiv -g' \frac{\sigma_3}{2} B_{\mu\nu} \sim \mathcal{O}(p^2)$

Goldstone interactions are determined by the underlying symmetry

$$\begin{aligned} \frac{v^2}{4} \langle \partial_\mu U^\dagger \partial^\mu U \rangle &= \partial_\mu \varphi^- \partial^\mu \varphi^+ + \frac{1}{2} \partial_\mu \varphi^0 \partial^\mu \varphi^0 \\ &+ \frac{1}{6v^2} \left\{ \left(\varphi^+ \overleftrightarrow{\partial}_\mu \varphi^- \right) \left(\varphi^+ \overleftrightarrow{\partial}^\mu \varphi^- \right) + 2 \left(\varphi^0 \overleftrightarrow{\partial}_\mu \varphi^+ \right) \left(\varphi^- \overleftrightarrow{\partial}^\mu \varphi^0 \right) \right\} \\ &+ \mathcal{O}\left(\varphi^6/v^4\right) \end{aligned}$$

Goldstone interactions are determined by the underlying symmetry

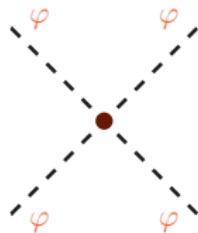
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$$T(\varphi^+ \varphi^- \rightarrow \varphi^+ \varphi^-) = \frac{s+t}{v^2}$$

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$$T(\varphi^+ \varphi^- \rightarrow \varphi^+ \varphi^-) = \frac{s+t}{v^2}$$

Non-Linear Lagrangian: $2\varphi \rightarrow 2\varphi, 4\varphi \dots$ related

$$\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d:$$

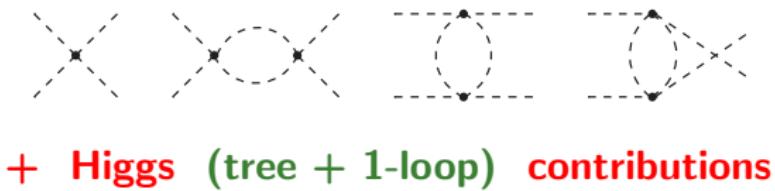


$$A(\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d) = A(s, t, u) \delta_{ab} \delta_{cd} + A(t, s, u) \delta_{ac} \delta_{bd} + A(u, t, s) \delta_{ad} \delta_{bc}$$

$$\begin{aligned}
A(s, t, u) &= \frac{s}{v^2} + \frac{4}{v^2} [a_4^r(\mu)(t^2 + u^2) + 2a_5^r(\mu)s^2] \\
&+ \frac{1}{16\pi^2 v^2} \left\{ \frac{5}{9}s^2 + \frac{13}{18}(t^2 + u^2) + \frac{1}{12}(s^2 - 3t^2 - u^2) \log\left(\frac{-t}{\mu^2}\right) \right. \\
&\quad \left. + \frac{1}{12}(s^2 - t^2 - 3u^2) \log\left(\frac{-u}{\mu^2}\right) - \frac{1}{2}s^2 \log\left(\frac{-s}{\mu^2}\right) \right\}
\end{aligned}$$

$$a_i = a_i^r(\mu) + \frac{\gamma_i}{16\pi^2} \left[\frac{2\mu^{D-4}}{4-D} + \log(4\pi) - \gamma_E \right] \quad , \quad \gamma_4 = -\frac{1}{12} \quad , \quad \gamma_5 = -\frac{1}{24}$$

$$\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d$$



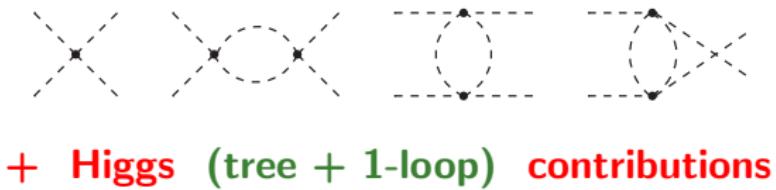
$$\mathcal{L} = \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle \left[1 + 2 \color{red}{a} \frac{H}{v} + \color{red}{b} \frac{H^2}{v^2} \right]$$

Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

$$\begin{aligned}
 A(s, t, u) = & \frac{s}{v^2} (1 - a^2) + \frac{4}{v^2} \left[a_4^r(\mu) (t^2 + u^2) + 2 a_5^r(\mu) s^2 \right] \\
 & + \frac{1}{16\pi^2 v^2} \left\{ \frac{1}{9} (14 a^4 - 10 a^2 - 18 a^2 b + 9 b^2 + 5) s^2 + \frac{13}{18} (1 - a^2)^2 (t^2 + u^2) \right. \\
 & \quad - \frac{1}{2} (2 a^4 - 2 a^2 - 2 a^2 b + b^2 + 1) s^2 \log \left(\frac{-s}{\mu^2} \right) \\
 & \quad \left. + \frac{1}{12} (1 - a^2)^2 \left[(s^2 - 3t^2 - u^2) \log \left(\frac{-t}{\mu^2} \right) + (s^2 - t^2 - 3u^2) \log \left(\frac{-u}{\mu^2} \right) \right] \right\}
 \end{aligned}$$

$$\gamma_4 = -\frac{1}{12} (1 - a^2)^2 \quad , \quad \gamma_5 = -\frac{1}{48} (2 + 5 a^4 - 4 a^2 - 6 a^2 b + 3 b^2)$$

$$\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d$$



$$\mathcal{L} = \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle \left[1 + 2 \textcolor{red}{a} \frac{H}{v} + \textcolor{red}{b} \frac{H^2}{v^2} \right]$$

Espliu–Mescia–Yencho, Delgado–Dobado–Llanes–Estrada

$$\begin{aligned} A(s, t, u) &= \frac{s}{v^2} (1 - a^2) + \frac{4}{v^2} \left[a_4^r(\mu) (t^2 + u^2) + 2 a_5^r(\mu) s^2 \right] \\ &+ \frac{1}{16\pi^2 v^2} \left\{ \frac{1}{9} (14 a^4 - 10 a^2 - 18 a^2 b + 9 b^2 + 5) s^2 + \frac{13}{18} (1 - a^2)^2 (t^2 + u^2) \right. \\ &\quad - \frac{1}{2} (2 a^4 - 2 a^2 - 2 a^2 b + b^2 + 1) s^2 \log \left(\frac{-s}{\mu^2} \right) \\ &\quad \left. + \frac{1}{12} (1 - a^2)^2 \left[(s^2 - 3t^2 - u^2) \log \left(\frac{-t}{\mu^2} \right) + (s^2 - t^2 - 3u^2) \log \left(\frac{-u}{\mu^2} \right) \right] \right\} \end{aligned}$$

$$\gamma_4 = -\frac{1}{12} (1 - a^2)^2 \quad , \quad \gamma_5 = -\frac{1}{48} (2 + 5 a^4 - 4 a^2 - 6 a^2 b + 3 b^2)$$

SM: $a = b = 1$, $a_4 = a_5 = 0$



$$A(s, t, u) \sim \mathcal{O}(M_H^2/v^2)$$