# Probing Inflationary PBHs for the LIGO GW events

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Based on 1611.06130 (to be published in PRD)

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# Introduction

# Introduction

## ■ What is **"Primordial" BH**?

- BH formed **before** any astrophysical objects exist (even in RD era).
- Need **large density perturbation** for the formation.

$$\rho \sim M R_s^{-3} \sim 10^{17}\mathrm{g/cm^3}\left(\frac{M}{M_\odot}\right)^{-2} \quad \text{where} \quad R_s \sim 3\mathrm{km}\left(\frac{M}{M_\odot}\right)$$

$(1\mathrm{g} \simeq 5.6 \times 10^{23}\mathrm{GeV})$ ✤ Comparable to that of radiation at T ~ GeV.

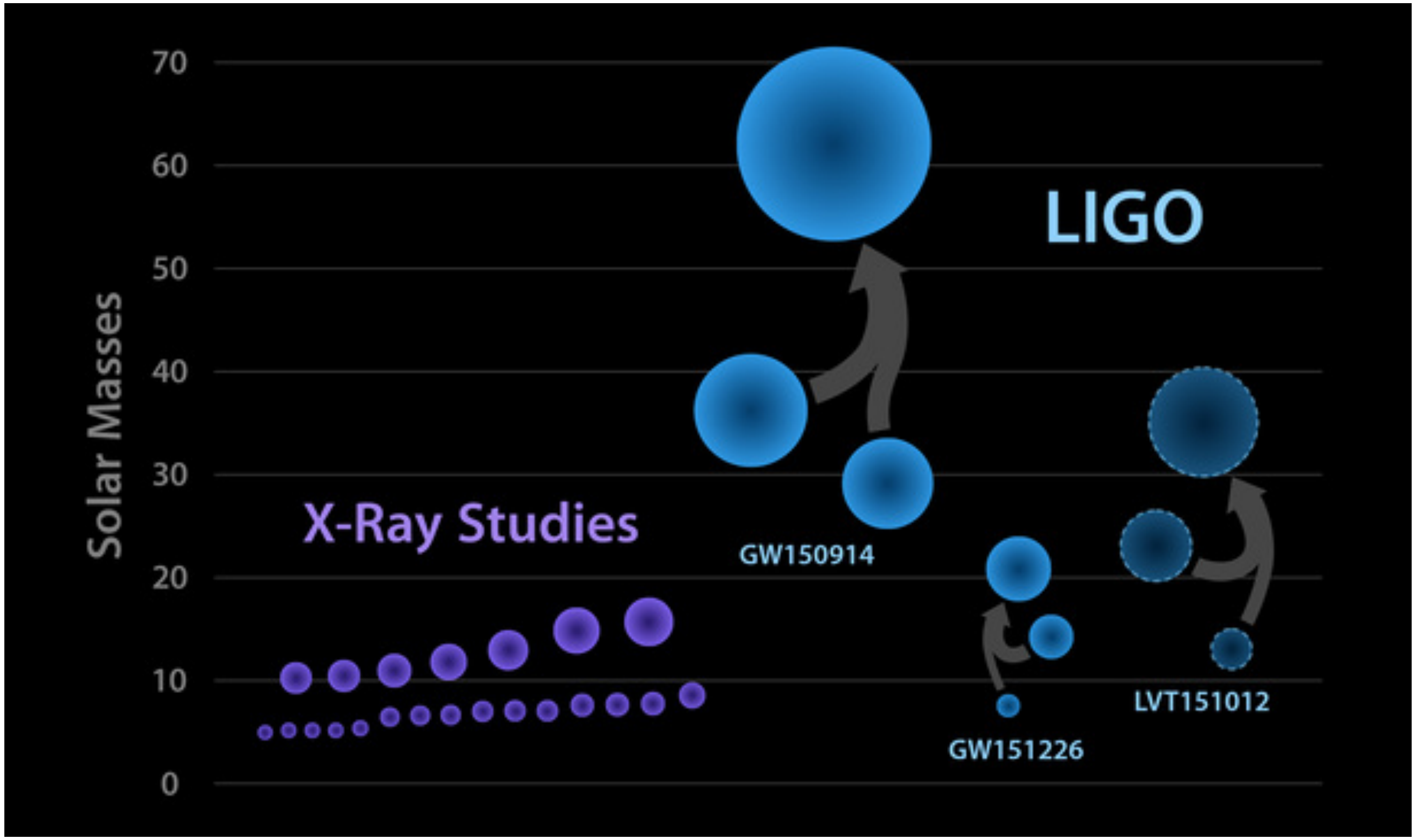
## ■ Is there any motivation to study them?

- **Non-particle candidate of DM**
  - Behave as cold matter, interact gravitationally, and quasi-stable for $M > 10^{15}$g.
- **Candidate of gravitational wave events observed by LIGO**
  - Merger of PBHs with O(10) solarmasses may be responsible for the observed events.
- **Constrain other DM models**
  - e.g., constraining **WIMP** via Ultra Compact Mini Halo, (**axion** via superradiance, …)

# Introduction

- **Merging of Binary BH observed by LIGO**

[Ref. https://www.ligo.caltech.edu]

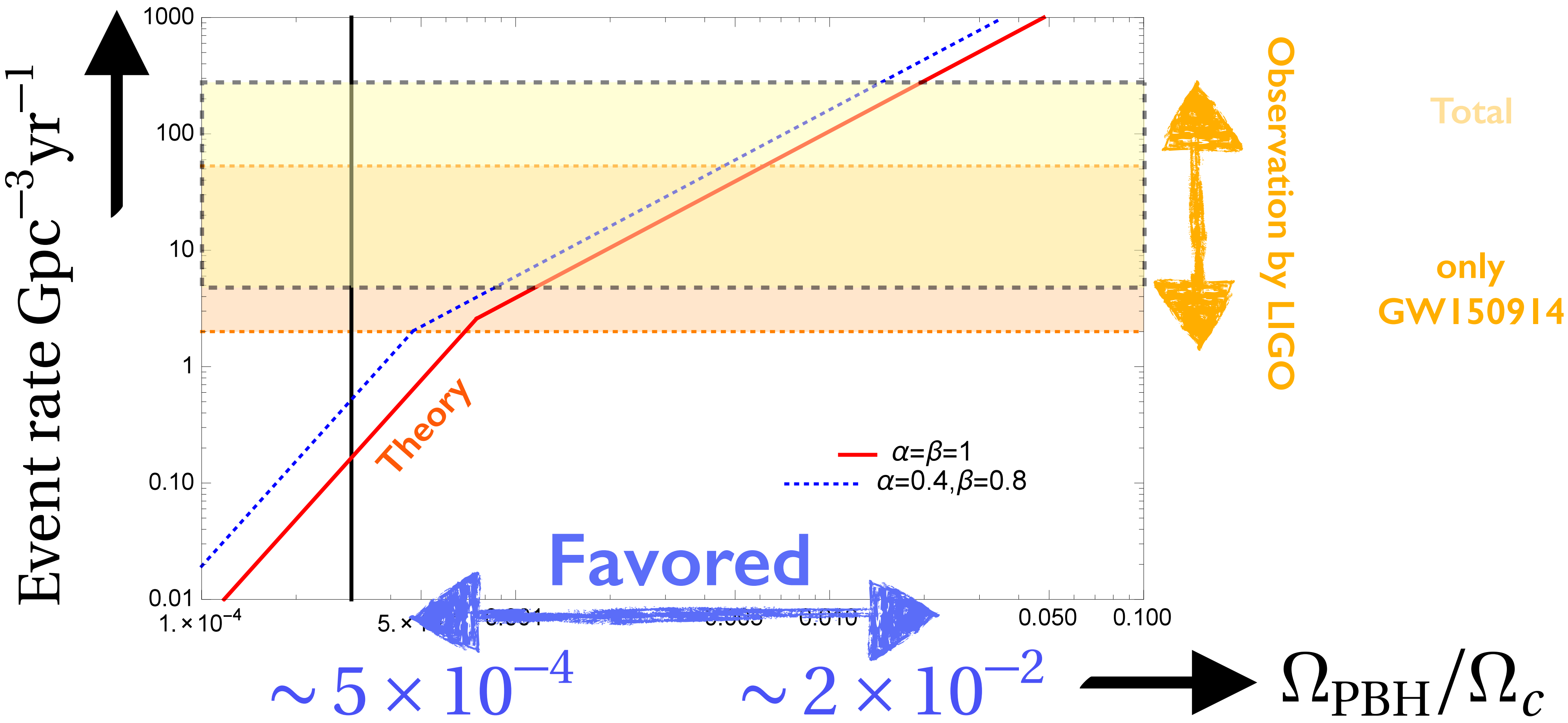



- Masses of BHs: **GW150914** $(36^{+5}_{-4}\,M_\odot, 29^{+4}_{-4}\,M_\odot)$, **GW151226** $(14.2^{+8.3}_{-3.7}\,M_\odot, 7.5^{+2.3}_{-2.3}\,M_\odot)$
- Estimated event rate: **GW150914** $2-53\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$, **Total** $9-240\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$

# Introduction

## ■ Indicated PBH abundance: $f = \Omega_{PBH}/\Omega_c \sim O(10^{-4} - 10^{-2})$

- Merger rate as a function of PBH fraction estimated in Sasaki et al. 1603.08338



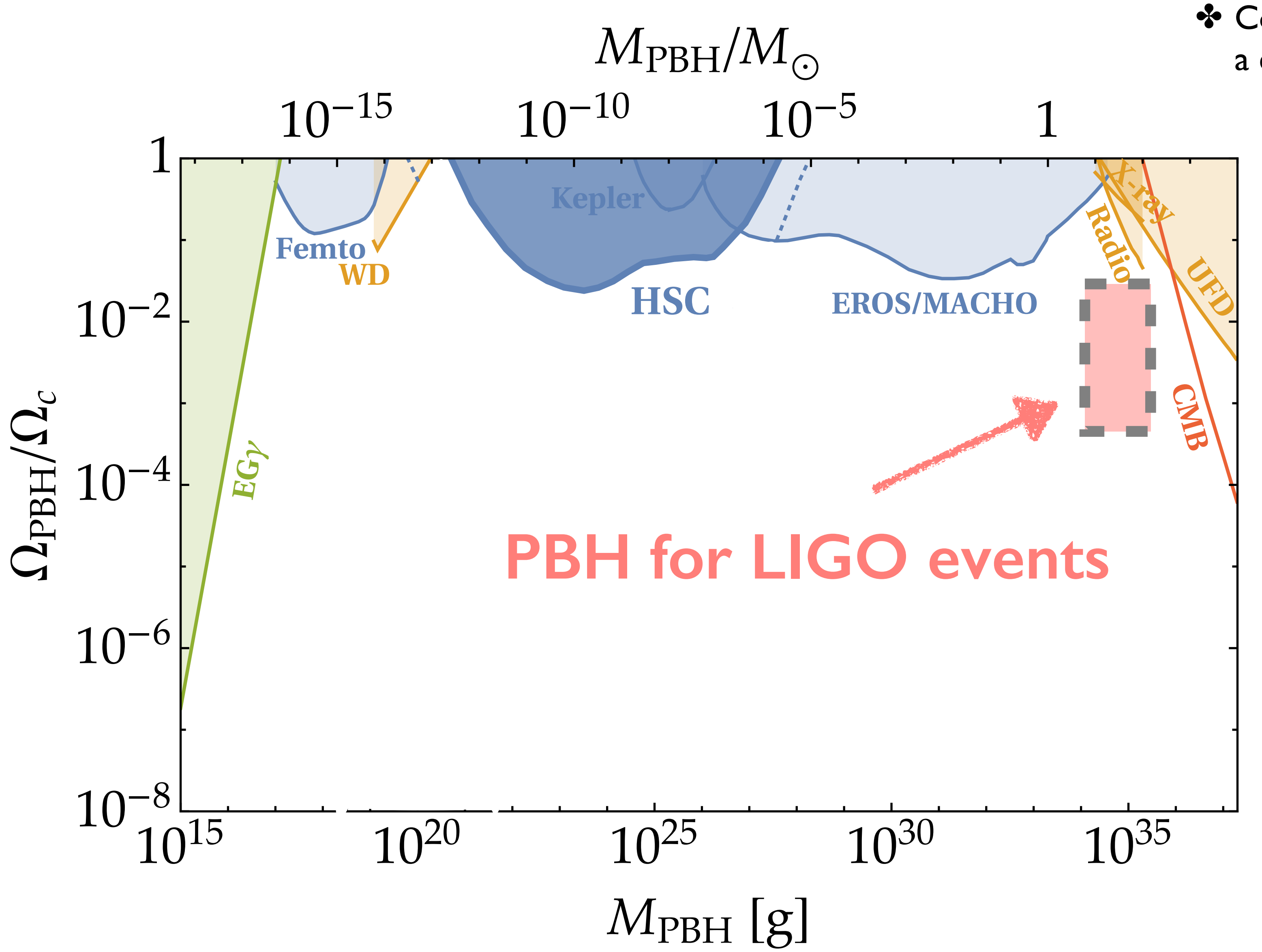

- **Possible corrections?**
  - PBHs should have a continuous mass function, not a delta function.
  - Surrounding DM halo (UCMH): Minihalo of DM is accompanied for f < 1. [Yu.N. Eroshenko 1604.04932]
  - Surrounding Baryons: efficient angular momentum transfer of binary by baryon disk(?) [Hayasaki et al. 0909.1738]

# Introduction

## ■ Current observational constraints

- **PBHs for all DM**: marginal, maybe excluded/probed soon.
- PBHs for LIGO events: seems to be viable.

✤ Constraints from Neutron Star capture are evaded for a conservative value of DM inside the globular clusters.
[See e.g. Kusenko+, 1310.8642; Carr+,1607.06077]




**Hawking radiation**
EGγ: 0912.5297

**Gravitational lensing**
Femto: 1204.2056
**HSC: 1701.02151**
Kepler: PhysRevLett.111.181302
EROS/MACHO: 0607207

**Dynamical**
WD: 1505.04444
UFD: 1605.03665

**Accretion**
CMB: 1612.05644 (1612.06811, 1612.07264)
Radio/Xray: 1612.00457

# Outline

- **Introduction**
- **Formation of PBHs (by cosmic inflation)**
- **Induced gravitational waves and pulsar timing array**

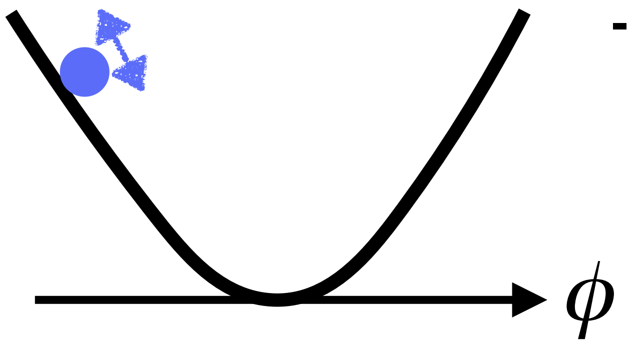
# Formation of PBHs

# Formation of PBHs

## ■ Mechanisms to generate large density

- **Collapse of localized configuration**
  - **Collision of Bubbles**: Bubble formation rate should be tuned to obtain a prolonged duration of PT.
  - **Cosmic Strings**: string may get into a configuration within Schwarzschild-radius with a small probability.
  - **Collapse of Domain-Wall network (?)**: ongoing with TTY and MK.

- **Collapse of "large" primordial density perturbations**
  - Inflation as an origin of primordial density perturbations




- **Vacuum fluctuation**

$$\delta\phi \sim \frac{H}{2\pi}$$

- **Time fluctuation**

$$\delta t \sim \frac{\delta\phi}{\dot{\phi}}$$

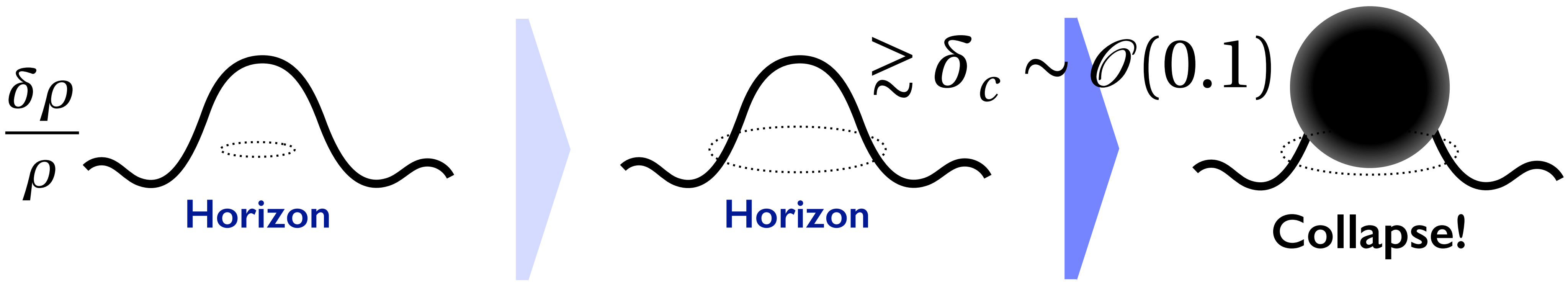
- **Density perturbation**

$$\delta\rho \sim \rho H \delta t$$

# Formation of PBHs

## ■ PBH formation by large primordial density perturbations

- Need large perturbations for **Gravity force > Pressure**




- **PBH mass** v.s. **scale ($k^{-1}$) of large perturbation:**

$$M = \gamma\rho\frac{4\pi H^{-3}}{3} \simeq M_\odot\left(\frac{\gamma}{0.2}\right)\left(\frac{g_*}{3.36}\right)^{-\frac{1}{6}}\left(\frac{k/(2\pi)}{3\times10^{-9}\,\mathrm{Hz}}\right)^{-2}$$

- **Abundance is determined by** $\mathcal{P}_\zeta$ (assump. Gaussian distribution)

  - Probability of the PBH formation: $\beta(M) = \int_{\delta_c} \mathrm{d}\delta\, \frac{\mathrm{e}^{-\frac{\delta^2}{2\sigma^2(M)}}}{\sqrt{2\pi\sigma^2(M)}} \sim \sigma(M)\,\mathrm{e}^{-\frac{\delta_c^2}{2\sigma^2(M)}}$

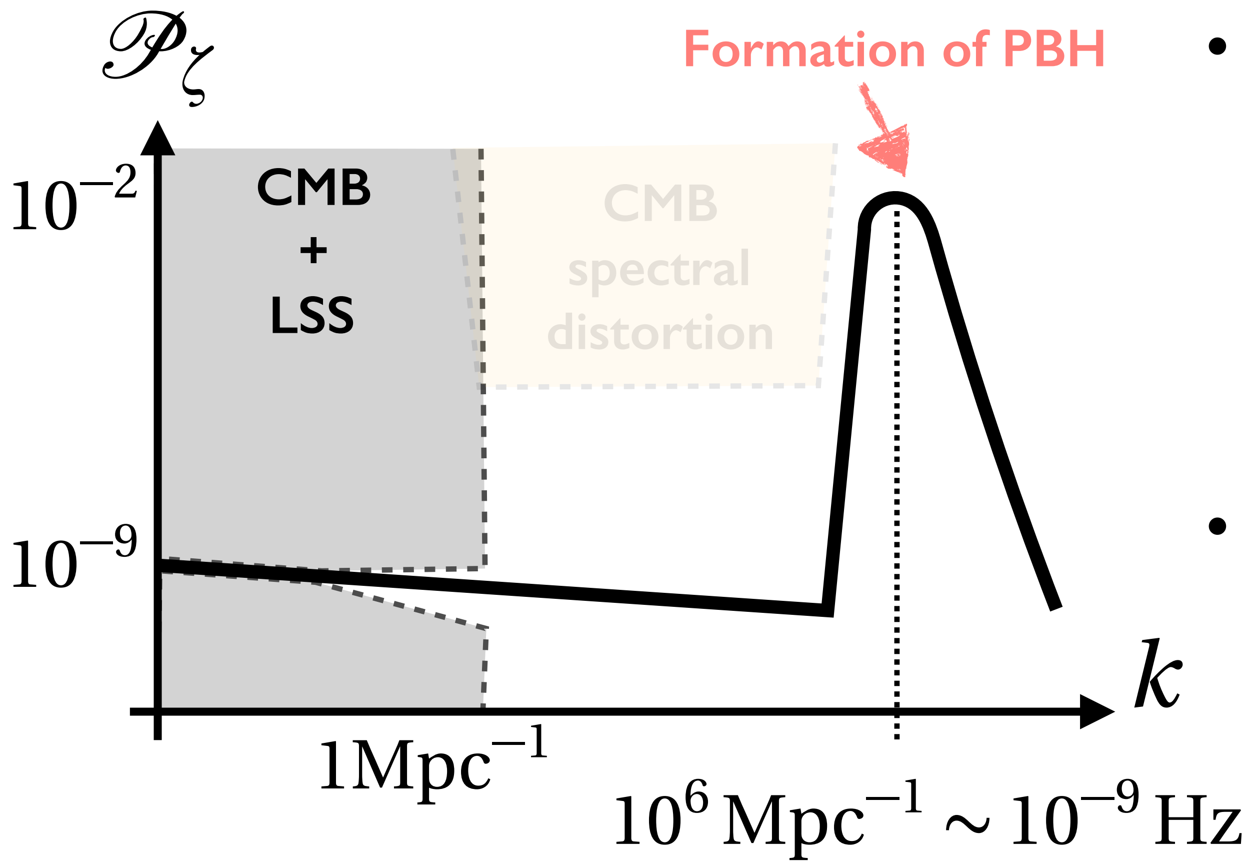
    ◉ **Standard deviation ~ Curvature perturbation:** $\sigma^2(M) \sim \mathcal{P}_\zeta(k)$

**For LIGO events, large** $\mathcal{P}_\zeta(k) \sim 10^{-2}$ **at** $f \equiv k/2\pi \sim 10^{-9}\,\mathrm{Hz}$

# Formation of PBHs

## ■ Inflation as the origin of PBHs

- How to enhance curvature perturbation at small scales ?




- **Fluctuation of slow-rolling inflaton**
  - **Double inflation**: pre-inflation for large scales and second-inflation for small scales.
  - **Running mass inflation**: flatten the potential at small scales.
  - …
- **Fluctuation of other fields than inflaton**
  - **Parametric resonance**: gauge preheating,…
  - **Curvaton**: Axion curvaton,…
  - …

- Crucial constraints on curvature perturbations at this scale.

# Formation of PBHs

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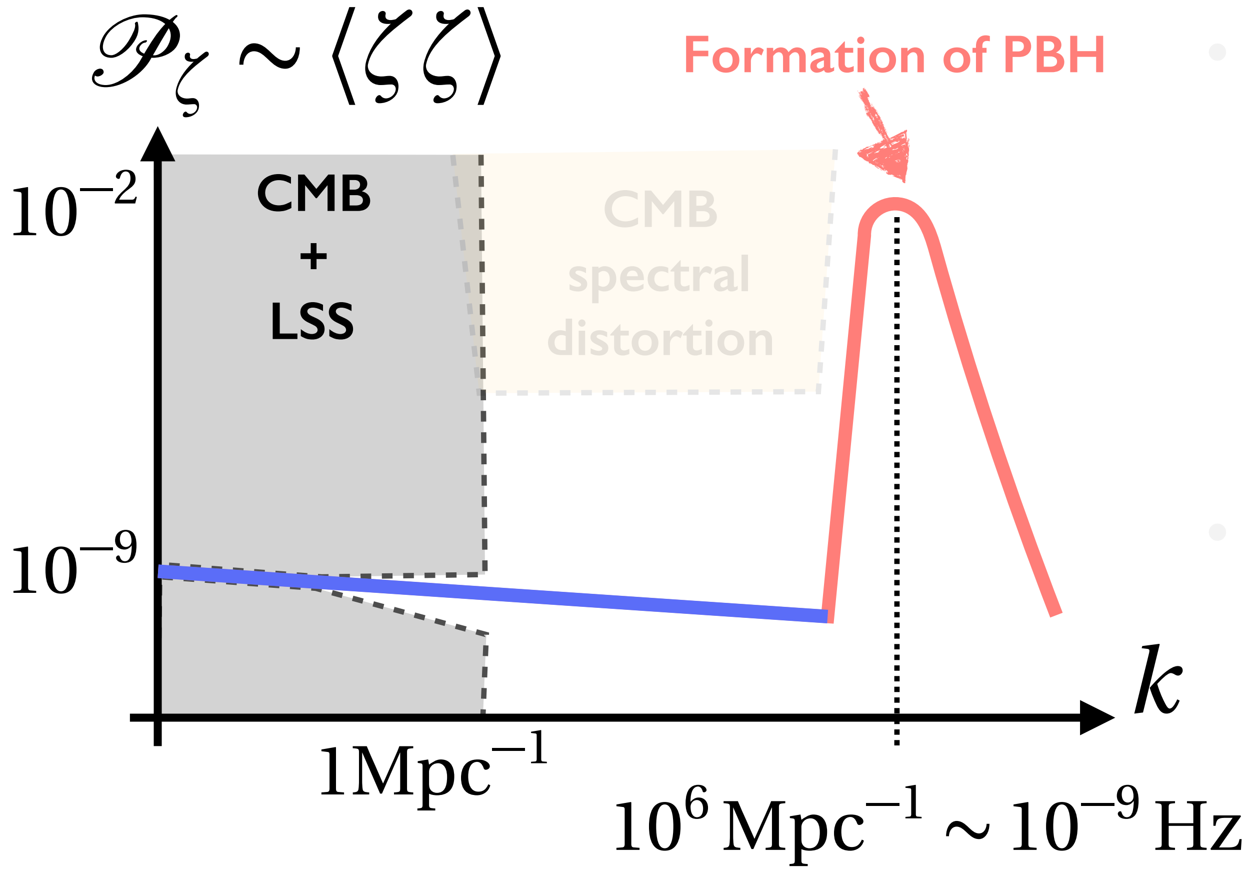
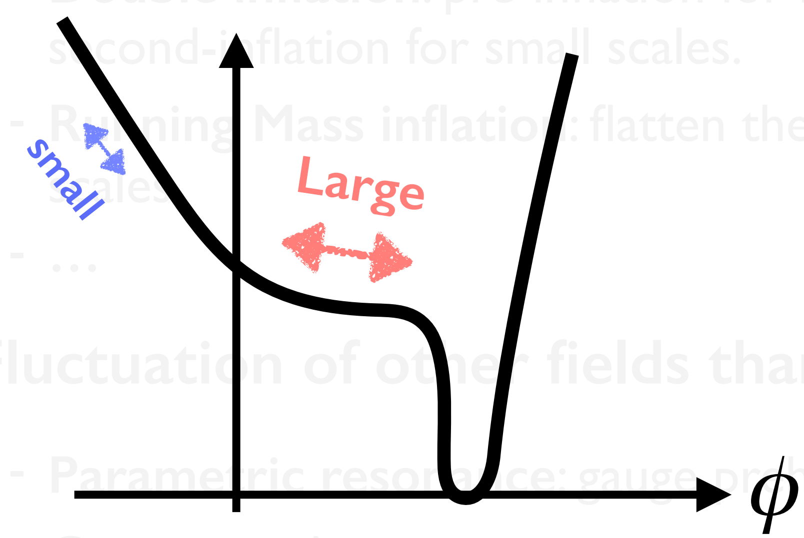
- How to enhance curvature perturbation at small scales ? → **Flatten your potential!**




$$\zeta \sim \frac{\delta\rho}{\rho} \sim H\delta t \sim H\frac{\delta\phi}{\dot{\phi}} \sim \left|\frac{V^{3/2}}{V' M_{\rm pl}^3}\right|$$

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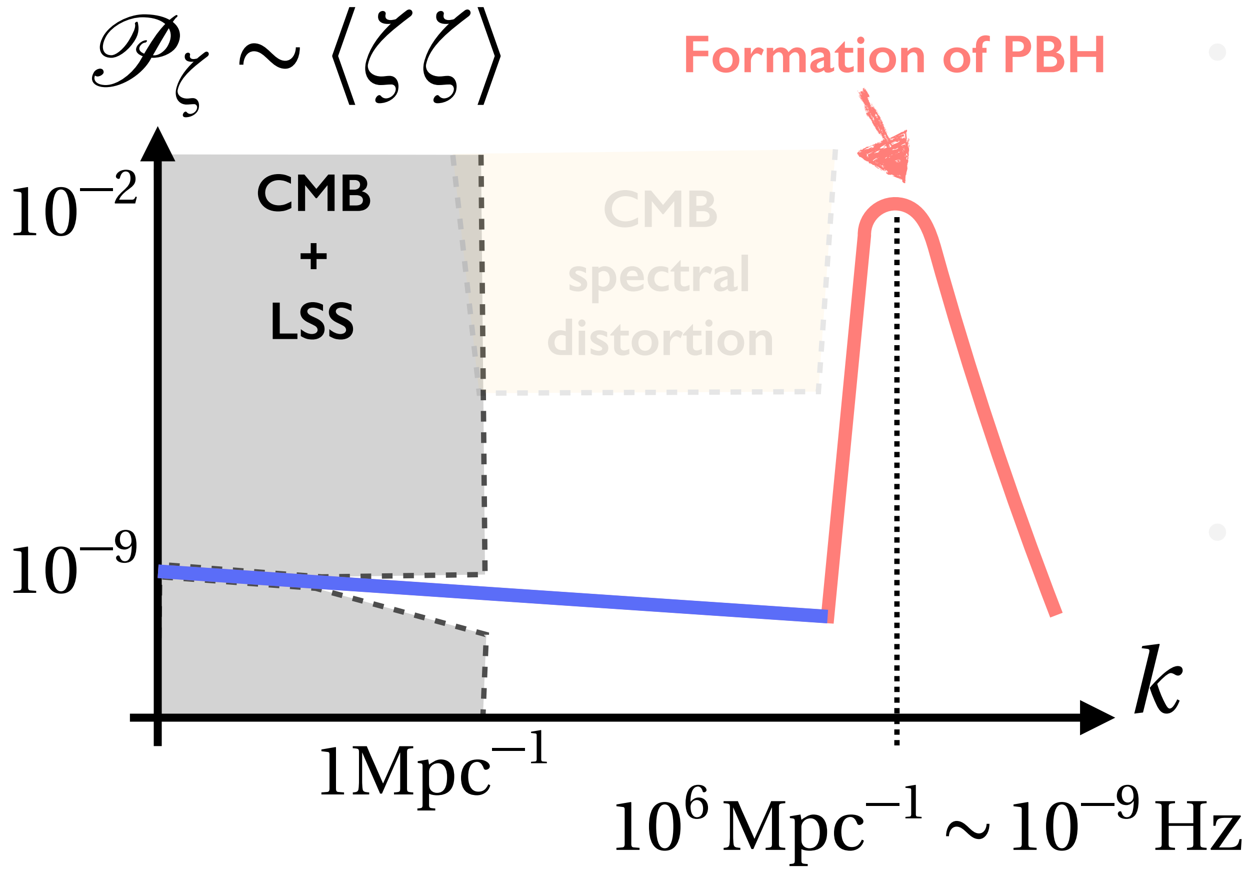
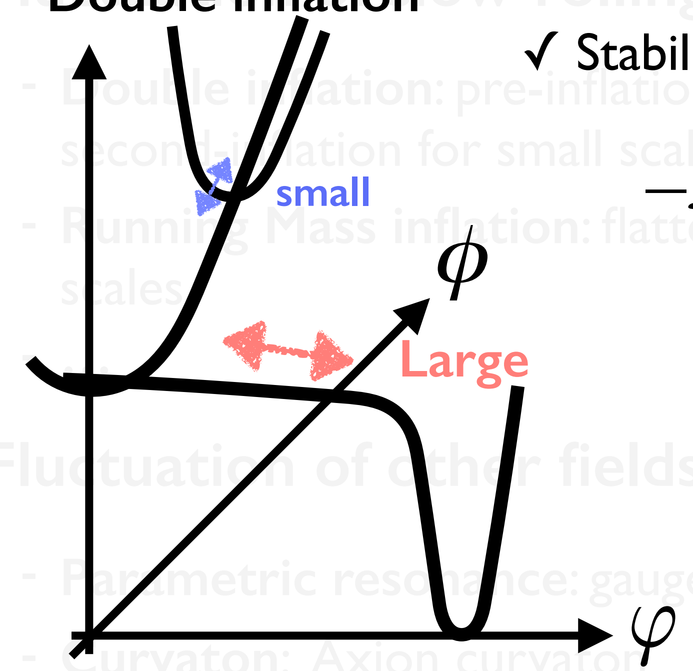



- Double inflation




✓ Stabilize during chaotic inflation

$$-\mathscr{L}_{\rm int} \propto \frac{V_{\rm ch}(\phi)}{M_{\rm pl}^2}\varphi^2$$

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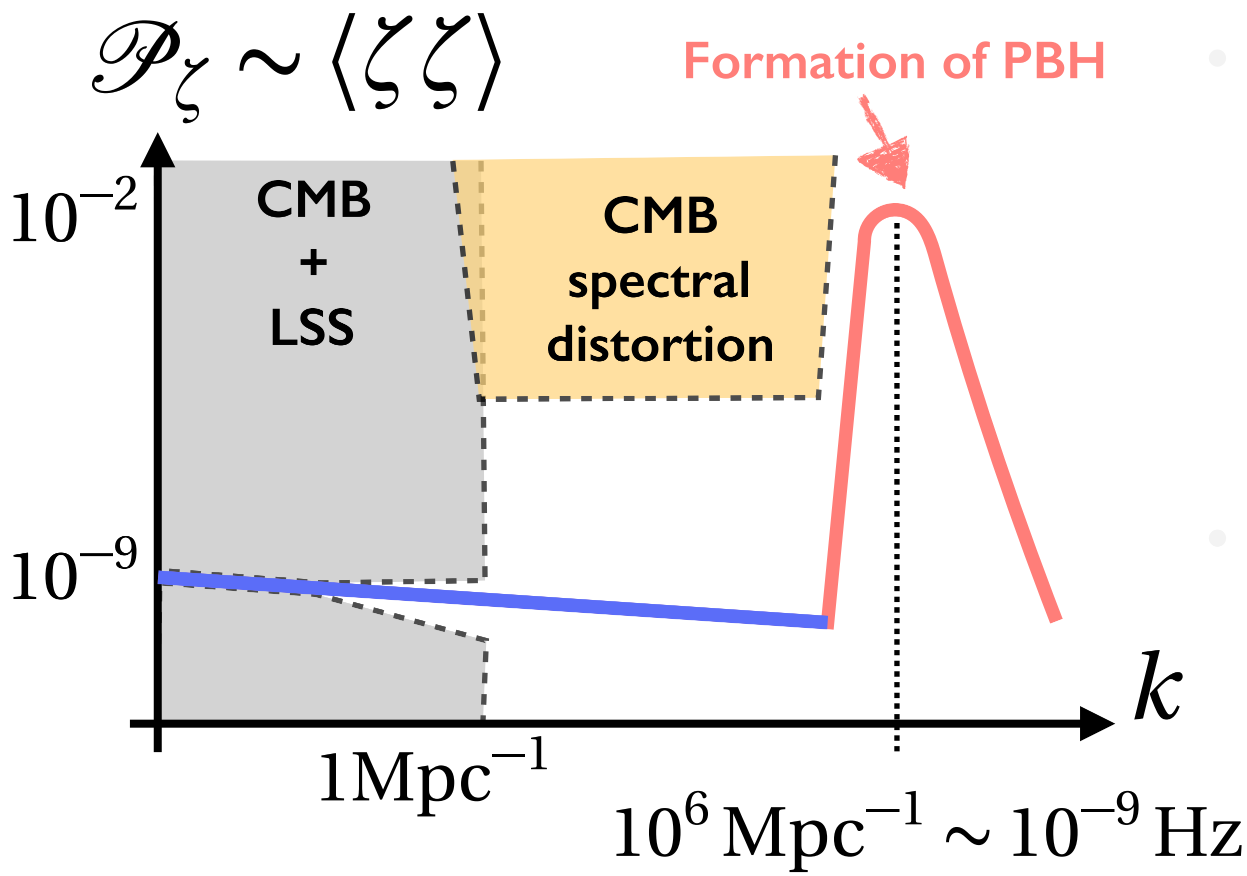


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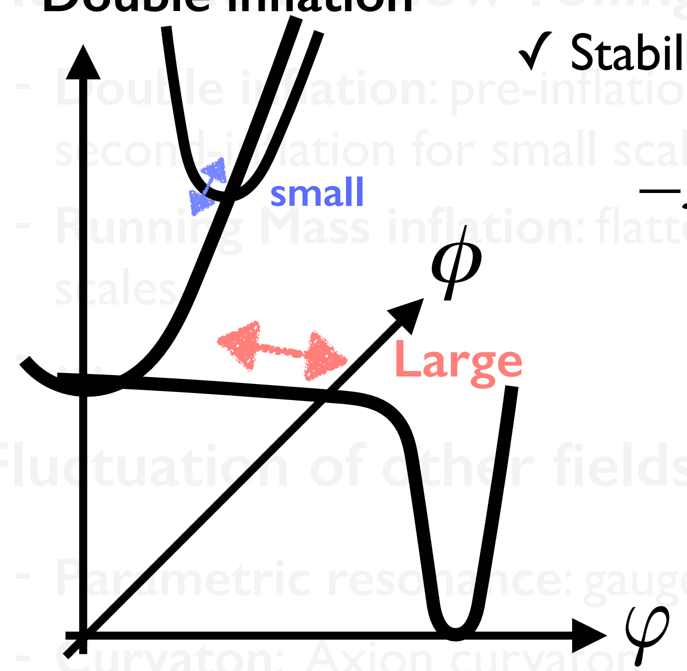


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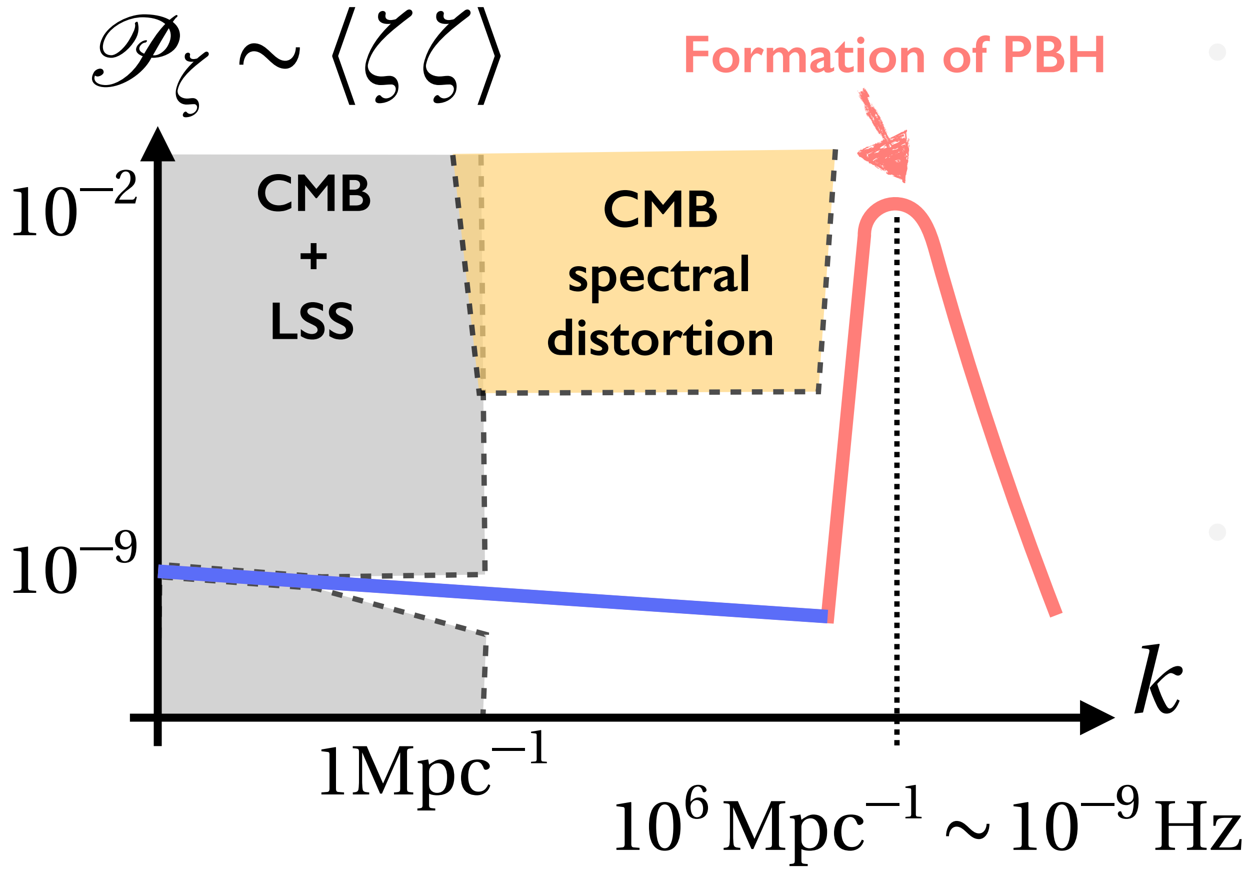


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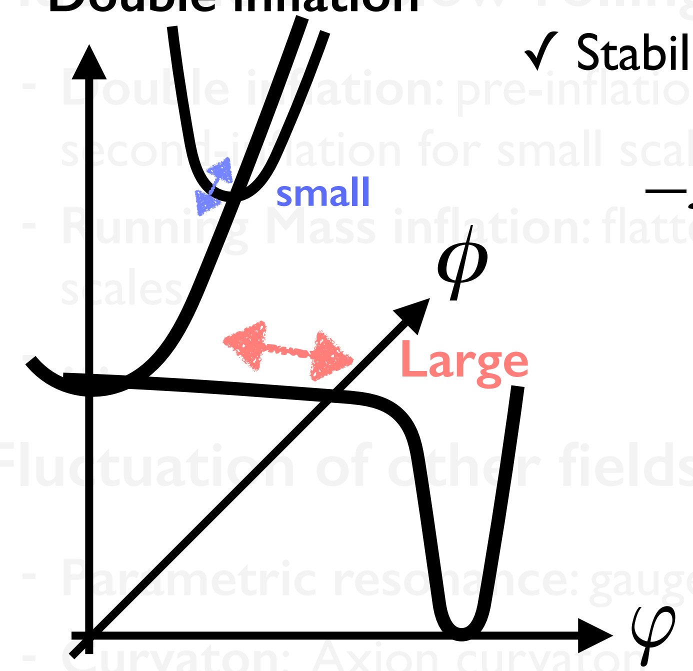


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- Crucial constraints on curvature perturbations at this scale.
  - **CMB spectral distortion at small scales:** mu-distortion,…
  - **Induced GWs via second order effects: constrained by pulsar timing array (PTA).**

# Induced GWs
# and
# Pulsar Timing Array

# Induced GWs

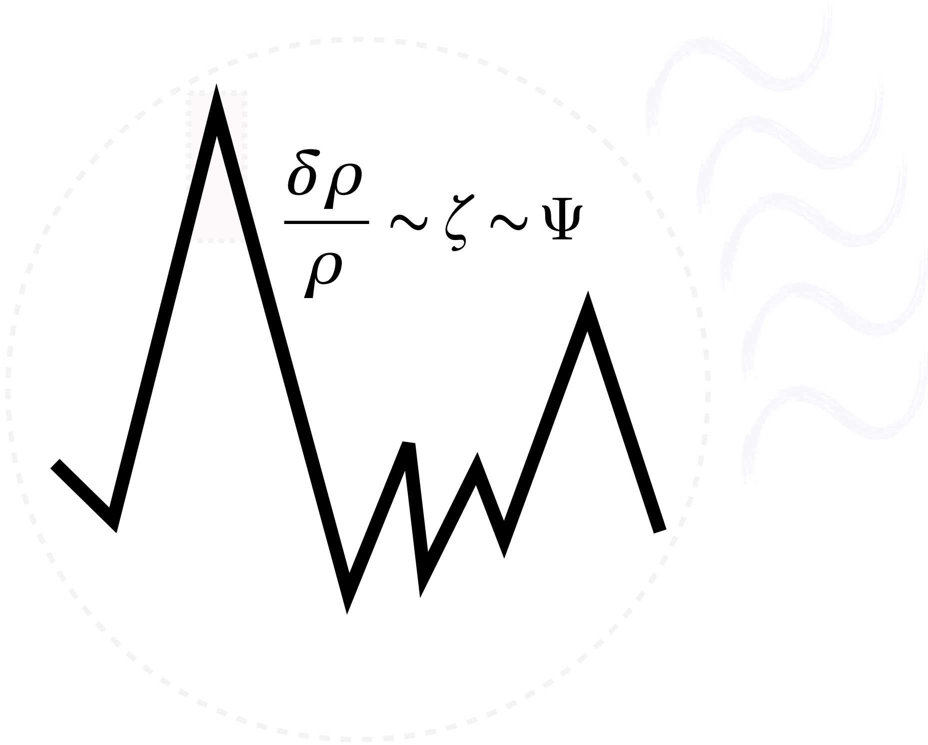
## ■ Large scalar perturbations act as source terms of GWs

- Equation of motion for tensor perturbation [Saito, Yokoyama; Bugaev, Klimai]

$$h''_{ij} + 2\mathcal{H} h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij;kl} S_{kl}$$

projection to transverse-traceless part

**Source term:** $S_{ij} \equiv 4\Psi\partial_i\partial_j\Psi + 2\partial_i\Psi\partial_j\Psi - \frac{4}{3(1+w)}\partial_i\left(\frac{\Psi'}{\mathcal{H}} + \Psi\right)\partial_j\left(\frac{\Psi'}{\mathcal{H}} + \Psi\right)$ where $\Psi \sim \zeta$




Production of GW by second order effects

$$h_{ij} \propto \Psi^2 \sim \zeta^2$$

$$\Omega_{\rm GW}(k)h^2 \sim 10^{-9}\left(\frac{\mathcal{P}_\zeta(k)}{10^{-2}}\right)^2$$

where $\Omega_{\rm GW,tot} = \int d\log k\,\Omega_{\rm GW}(k)$

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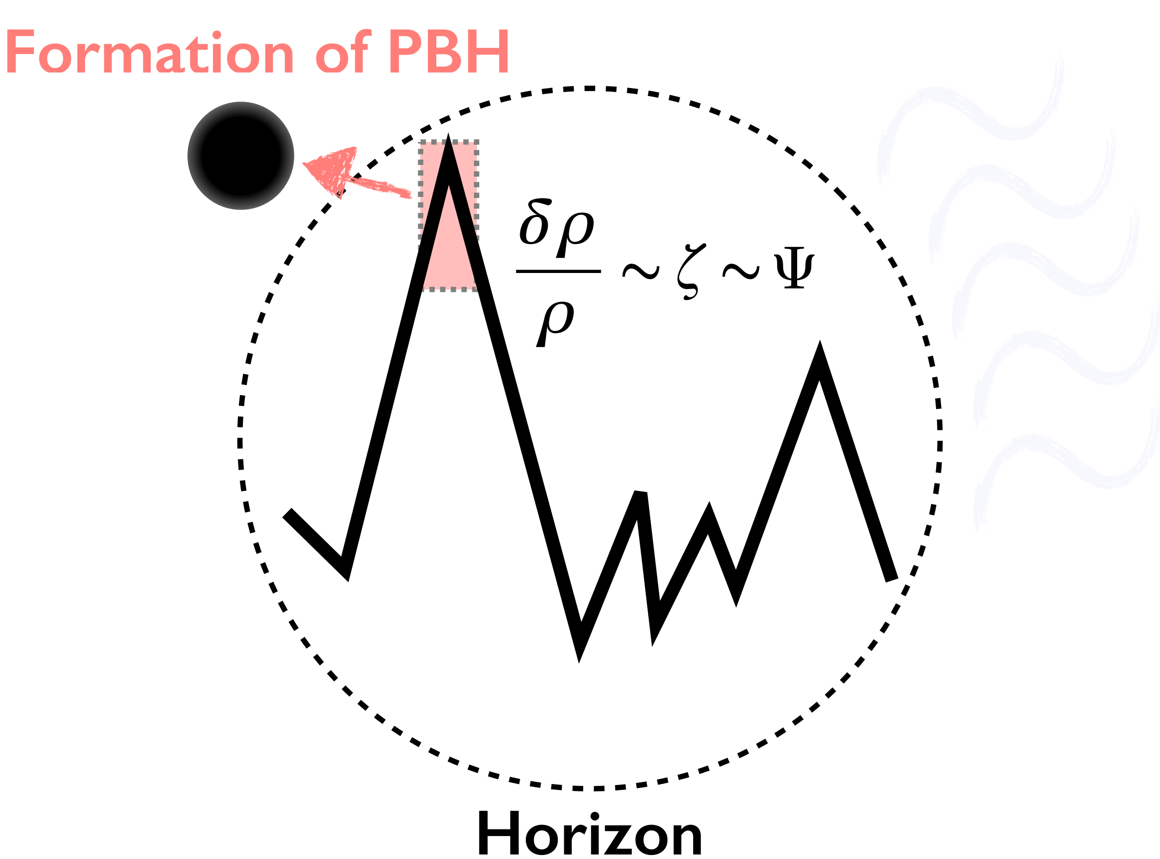


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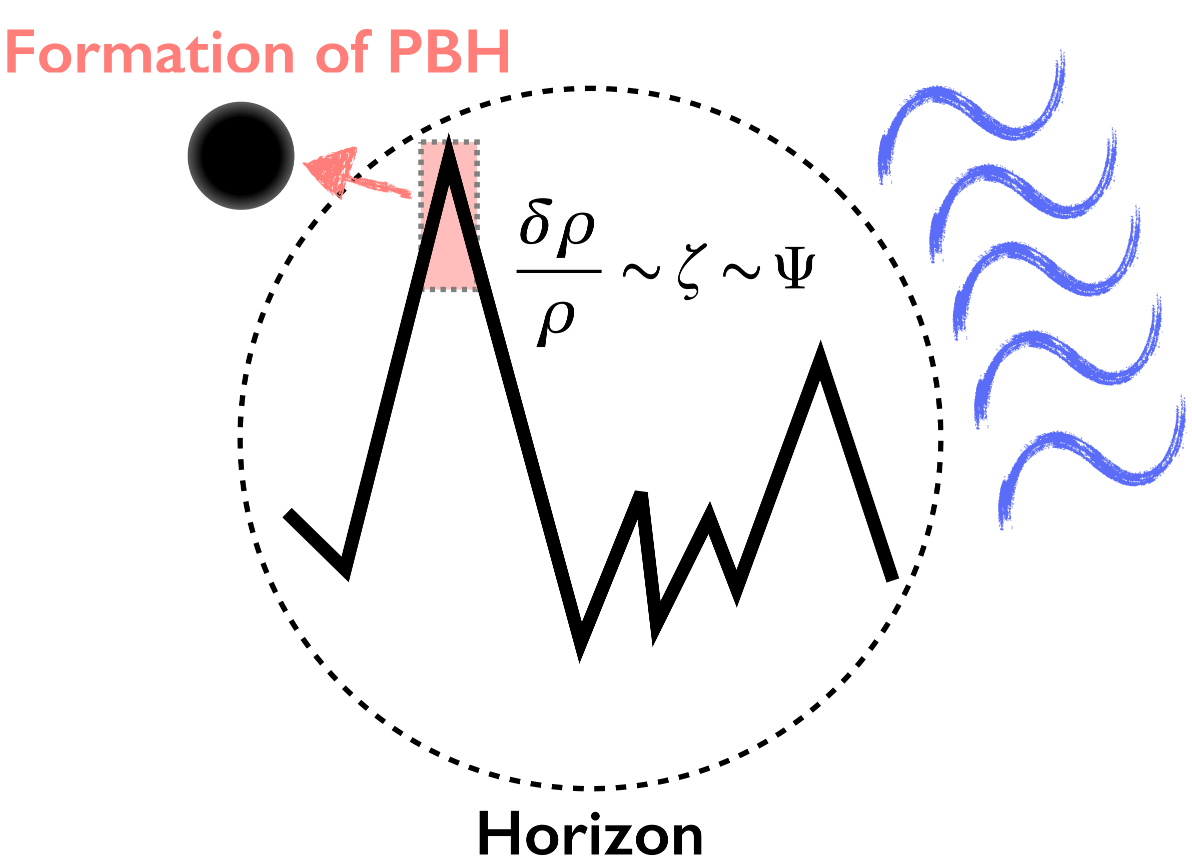


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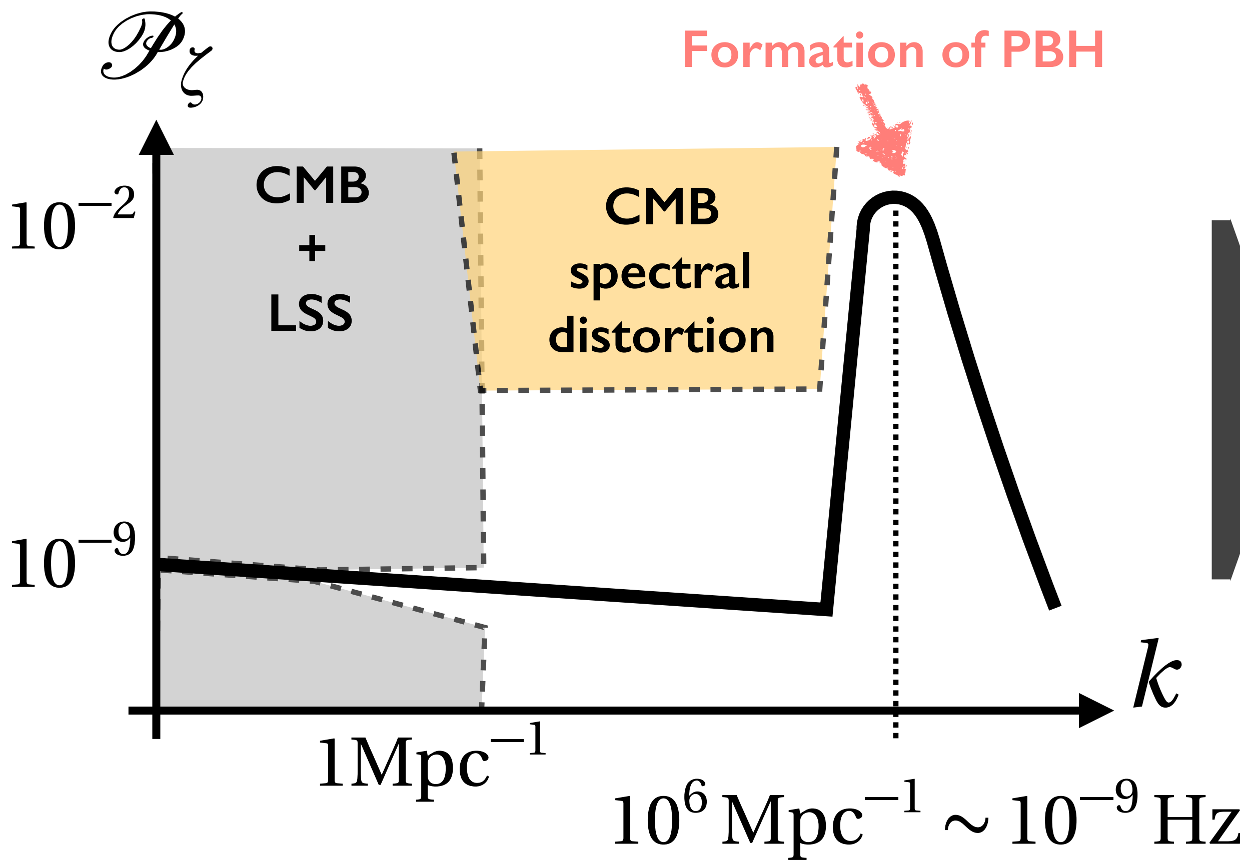
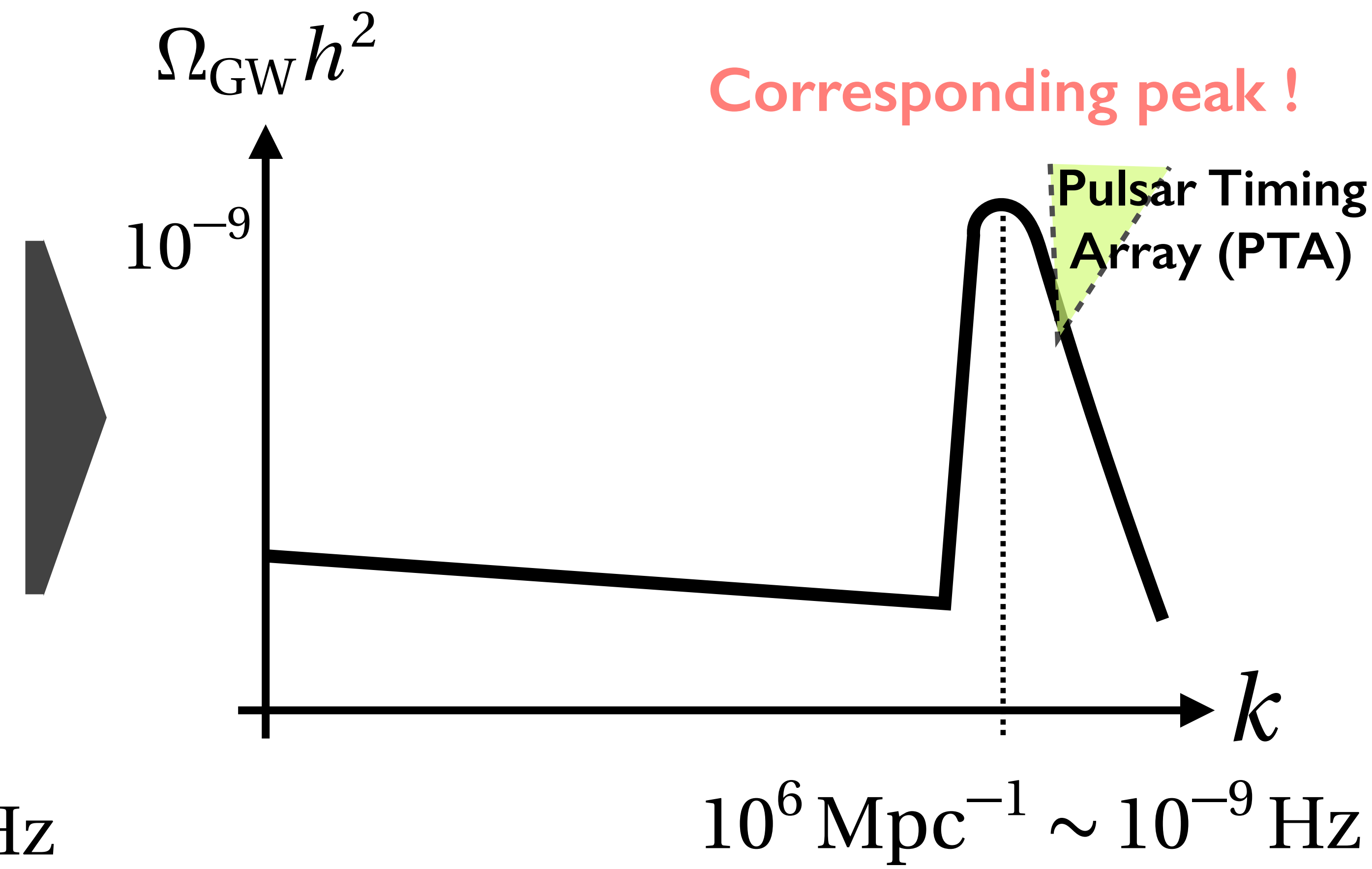
# Constraints on induced GWs

## ■ Large scalar perturbations act as source terms of GWs

- **GW-spectrum has a peak at the same k!**

[K.Inomata, M.Kawasaki, **KM**, Y.Tada, T.T.Yanagida; 1611.06130]
See also Orlofsky, Pierce, Wells; 1612.05279, Nakama, Silk, Kamionkowsky; 1612.06264

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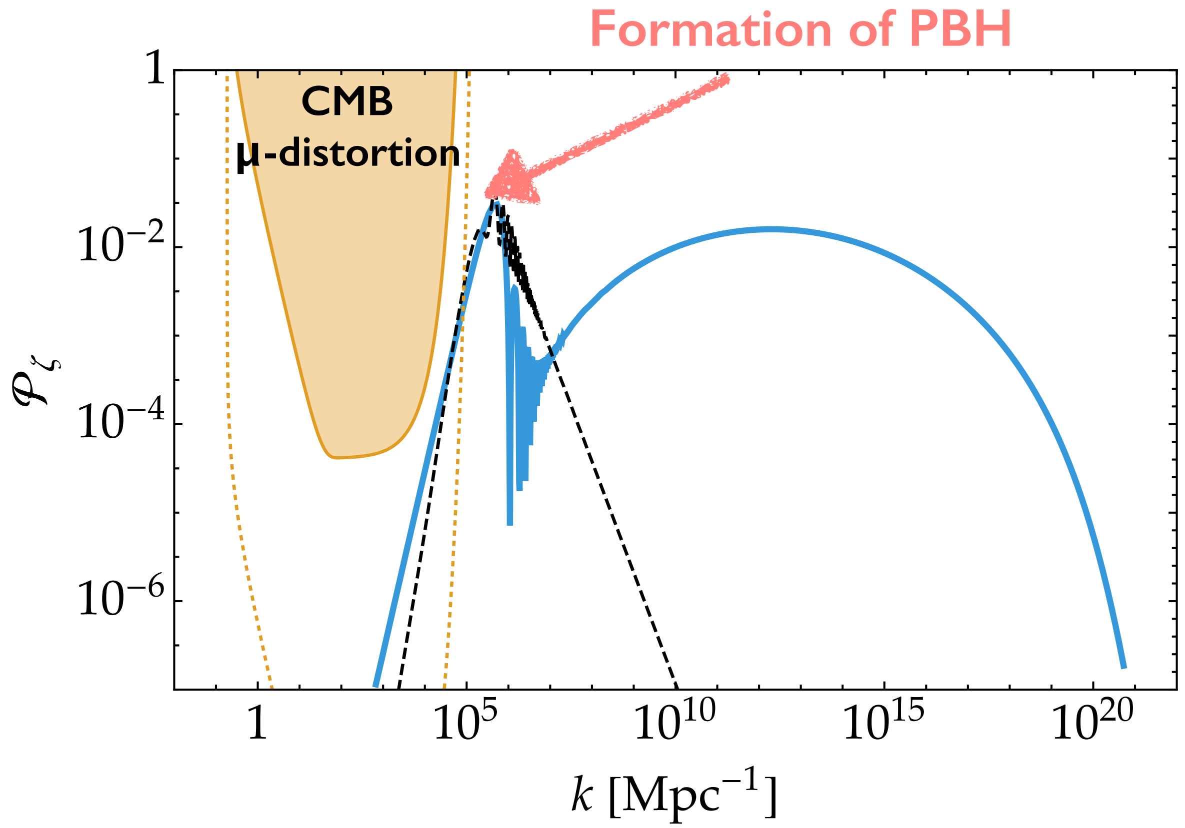
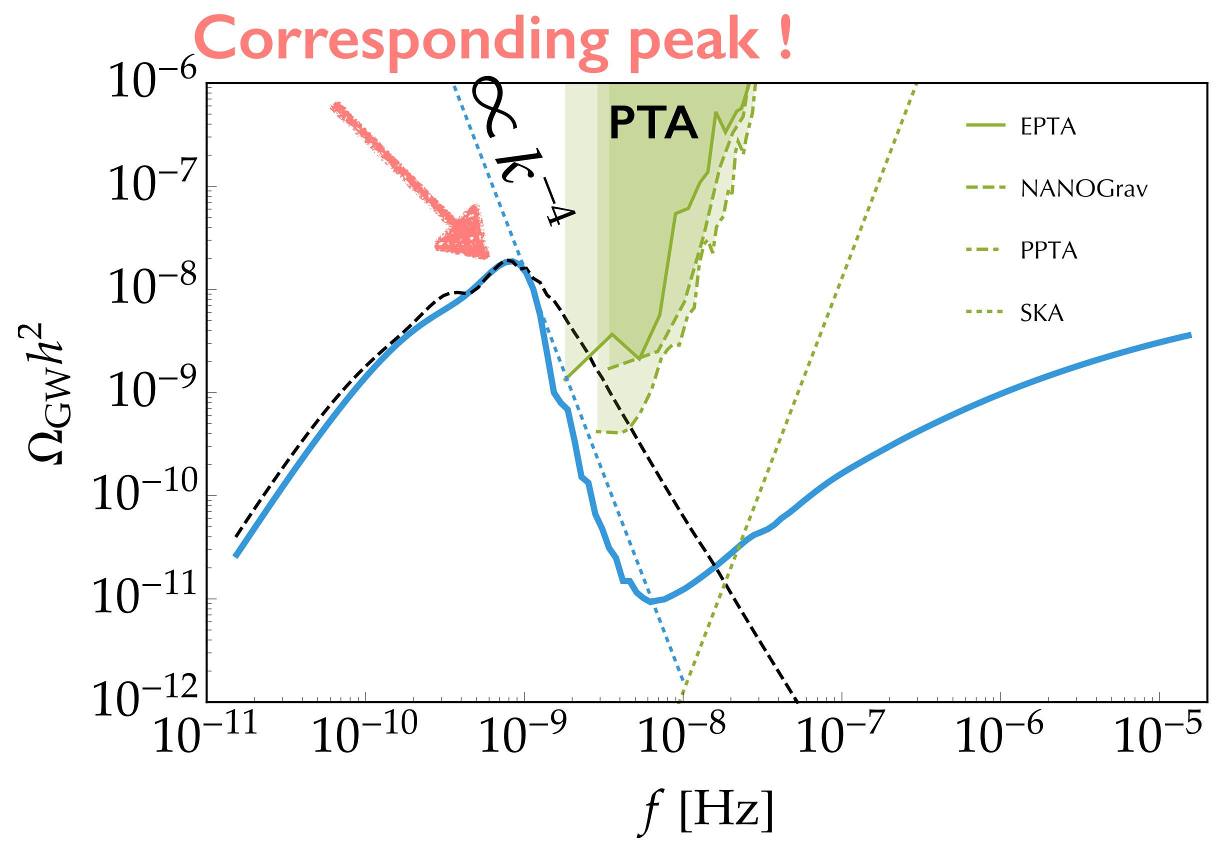
- An example: **Double Inflation**




- **Spectrum should be so steep.**

$$\mathcal{P}_\zeta \propto \begin{cases} k^x & \cdots\ k < k_{\rm peak} \\ k^{-y} & \cdots\ k > k_{\rm peak} \end{cases}$$

- **Current constraints**

$$x \gtrsim 1.5,\ \ y \gtrsim 2$$

**(cf.) Slow Roll condition**

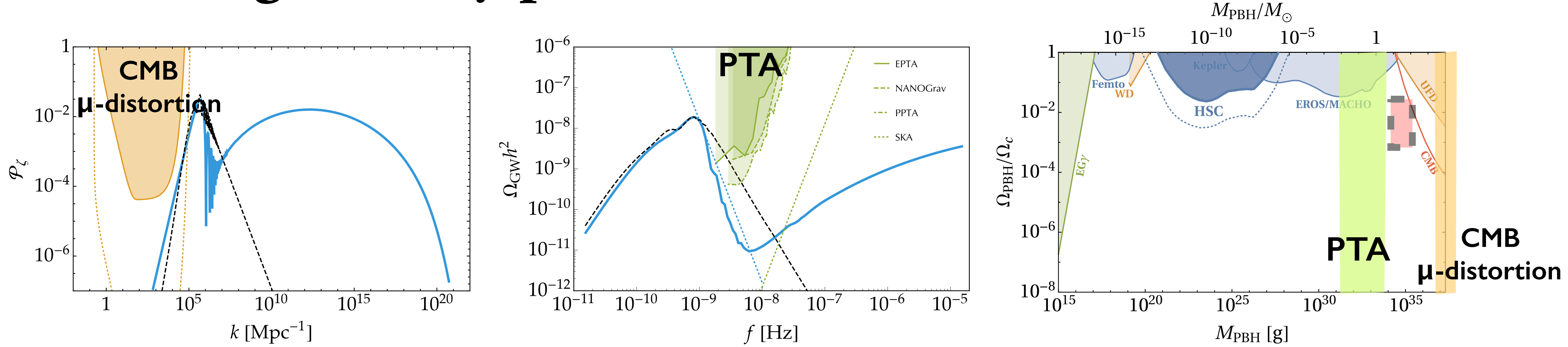
$$y = -(n_s - 1)$$
$$\simeq -2\eta < 2$$

❖ ε≪|η| @ single small field inflation

# Conclusions and Discussion

# Summary

- **Inflation** can produce PBHs whose mergers account for the LIGO GW events.
- But, need **large density perturbations @ f ~ $10^{-9}$ Hz.**
- **Small scale CMB distortions** and **induced GWs** can probe such **large density perturbations.** ✤ Assume almost Gaussian




[K.Inomata, M.Kawasaki, **KM**, Y.Tada, T.T.Yanagida; 1611.06130]

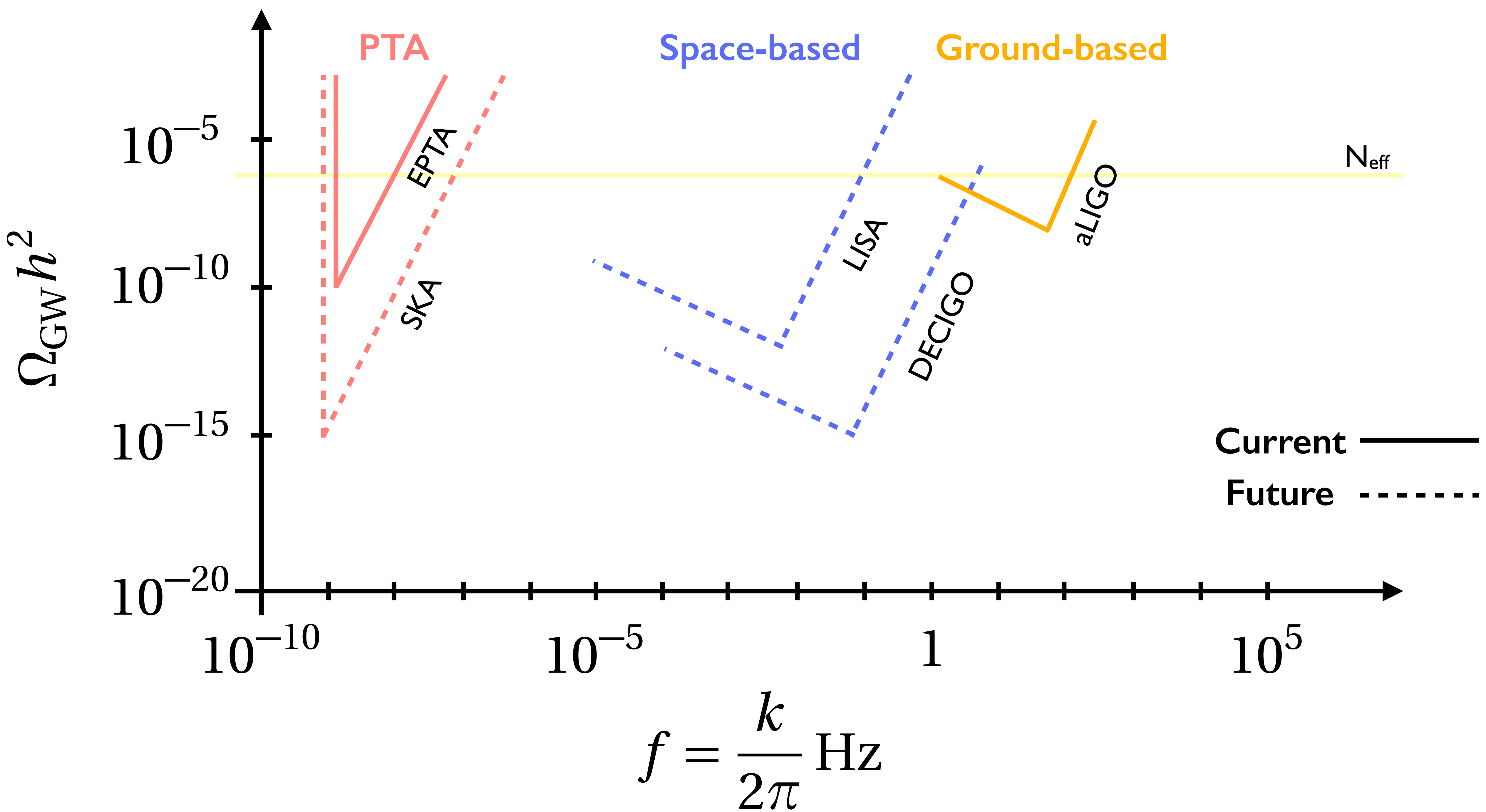
- Enhanced **Non-Gaussianity** at small scales could generate PBHs with a smaller $P_\zeta$, and thus may evade these constraints.

[Nakama, Silk, Kamionkowsky; 1612.06264]

# Backup

# Constraints on induced GWs

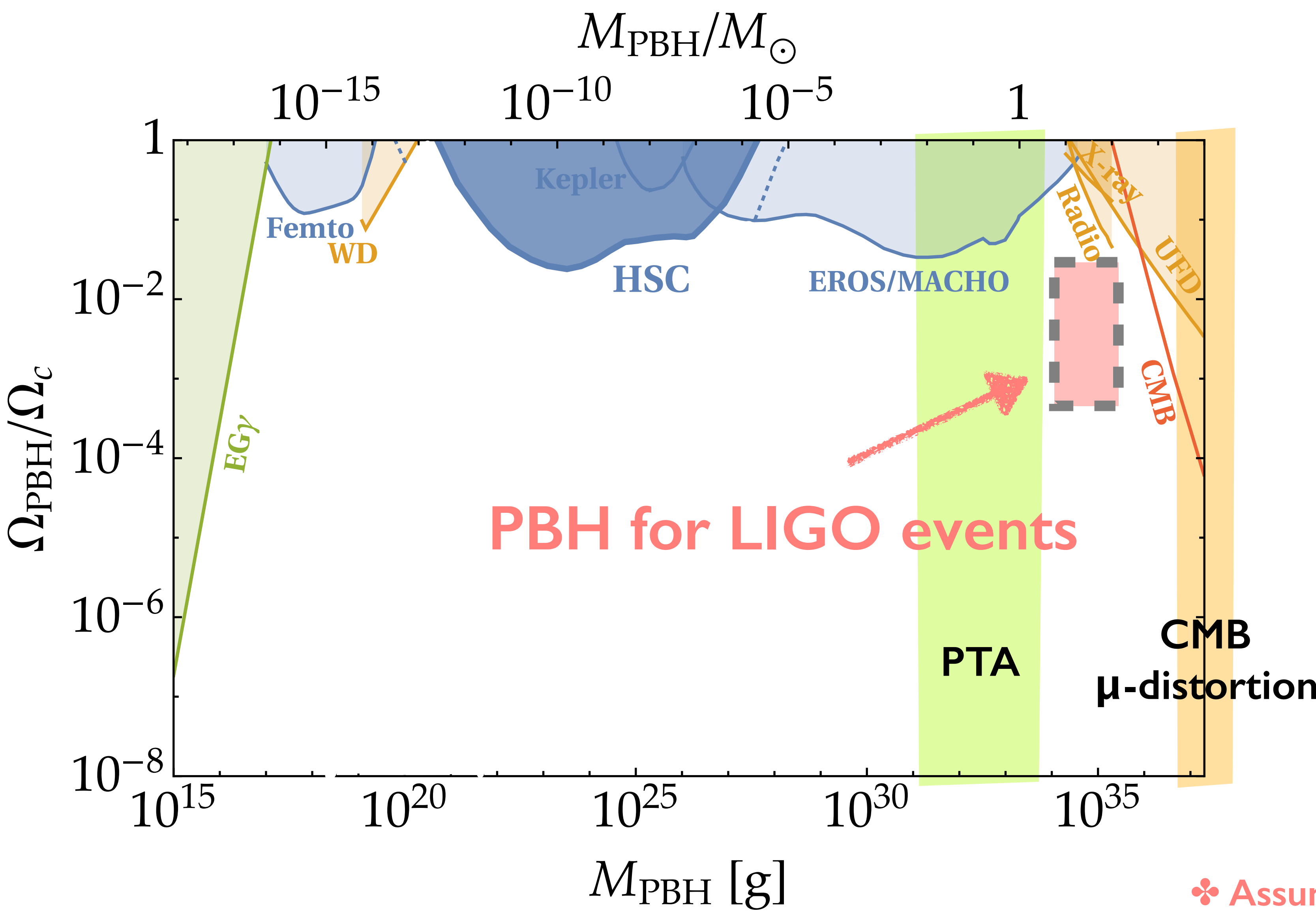
■ **Current and Future constraints summary of GWs**



# Constraints on PBH abundance

■ **Constraints on *inflationary* PBHs**

- If one specifies the production mechanism, there may be another way to probe PBHs.
- **Inflationary PBHs → Small scale distortions of CMB** and **Induced GWs**




✤ Assume almost Gaussian