Generating hierarchies on the fermion masses in a 3HDM limit of the pSHUT model

António P. Morais[1] Roman Pasechnik[2]

[1]Center for Research and Development in Mathematics and Applications (CIDMA)
Aveiro University, Aveiro, Portugal

[2]Department of Theoretical Physics
Lund university, Lund, Sweden

October 12, 2017

Outline

Outline

The high scale Grand Unified Principles

1) Phys. Rev. D95 (2017) 075031, JECM, **APM**, AO, RP, MS, JW
2) arXiv:1711.05199[hep-ph], JECM, **APM**, AO, RP, JW

- Supersymmetric trinification model extended with a family $\mathrm{SU}(3)_\mathrm{F}$ symmetry:

$$[\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R} \times \mathrm{SU}(3)_\mathrm{C}] \ltimes \mathbb{Z}_3 \times \{\mathrm{SU}(3)_\mathrm{F}\},$$

- $W = \lambda_U \varepsilon_{ijk} \left(Q^i_\mathrm{L}\right)^x{}_l \left(Q^j_\mathrm{R}\right)^r{}_x \left(L^k\right)^l{}_r + W_{\cancel{L},\cancel{R},\cancel{F}}\left(\Delta^{\text{heavy}}_{\text{adjoint}}\right)$

$$\left(L^i\right)^l{}_r = \begin{pmatrix} H^0_\mathrm{u} & H^-_\mathrm{d} & e_\mathrm{L} \\ H^+_\mathrm{u} & H^0_\mathrm{d} & \nu_\mathrm{L} \\ e_\mathrm{R} & \nu_\mathrm{R} & \phi \end{pmatrix}^i, \qquad \begin{aligned} \left(Q^i_\mathrm{L}\right)^x{}_l &= \begin{pmatrix} u^x_\mathrm{L} & d^x_\mathrm{L} & D^x_\mathrm{L} \end{pmatrix}^i, \\ \left(Q^i_\mathrm{R}\right)^r{}_x &= \begin{pmatrix} u^c_{\mathrm{R}x} & d^c_{\mathrm{R}x} & D^c_{\mathrm{R}x} \end{pmatrix}^{\top\, i}. \end{aligned}$$

- Higgs and leptons unified in L
  - **S**upersymmetric **H**iggs **U**nified **T**rinification (SHUT)
- **No μ-problem** as Higgs sits in fundamental sector
- **No tree-level lepton masses $\rightarrow$ naturally light**

Breaking of the T-GUT symmetry... and beyond

- Soft SUSY breaking above TeV-scale: $\mathrm{TeV} < \mathrm{M_{soft}} \ll \mathrm{M_{GUT}}$

$$[\mathrm{SU(3)_C \times SU(3)_L \times SU(3)_R}] \times \{\mathrm{SU(3)_F}\}$$

$$\Big\downarrow \; \left\langle \widetilde{\Delta}^8_{\mathrm{L,R,F}} \right\rangle \sim M_{\mathrm{GUT}}$$

$$[\mathrm{SU(3)_C \times SU(2)_L \times U(1)_L \times SU(2)_R \times U(1)_R}] \times \{\mathrm{SU(2)_F \times U(1)_F}\}$$

$$\Big\downarrow \; M_{\mathrm{soft}} \sim \left\langle \widetilde{\phi}^3 \right\rangle \gtrsim \left\langle \widetilde{\phi}^2 \right\rangle \gtrsim \langle \widetilde{\nu}^1_{\mathrm{R}} \rangle$$

$$[\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}] \times \{\mathrm{U(1)_T}\}$$

$$T_{\mathrm{Y}} = -2T^3_{\mathrm{R}} - \frac{2}{\sqrt{3}} \left(T^8_{\mathrm{L}} + T^8_{\mathrm{R}} \right), \qquad T_{\mathrm{T}} = 4\left(T^3_{\mathrm{F}} - \frac{3}{2} T^3_{\mathrm{R}} \right) - \frac{2}{\sqrt{3}} \left(T^8_{\mathrm{L}} - T^8_{\mathrm{R}} - 2T^8_{\mathrm{F}} \right)$$

Outline

The low scale theory

- Build a low scale *phenomenological* model *inspired* by the SHUT framework: **pSHUT**
- A new scale set by soft-interactions: Choose $M_{\rm soft} \simeq \left\langle \widetilde{\phi}^3 \right\rangle \sim 10^{3-4}\ \mathrm{TeV}$
- Consider that the VEVs $\left\langle \widetilde{\phi}^3 \right\rangle$, $\left\langle \widetilde{\phi}^2 \right\rangle$ and $\langle \widetilde{\nu}_R^1 \rangle$ can occur up to two orders of magnitude separation

$$\left\langle \widetilde{\phi}^3 \right\rangle \equiv p\,, \qquad \underbrace{\left\langle \widetilde{\phi}^2 \right\rangle \equiv f = \xi_1 p}_{\mathrm{SU(2)_F-breaking}}\,, \qquad \underbrace{\langle \widetilde{\nu}_R^1 \rangle \equiv \omega = \xi_2 p}_{\mathrm{SU(2)_R-breaking}}\,, \qquad 10^{-2} \leqslant \xi_1,\, \xi_2 \leqslant 1\,.$$

The scalar sector

- 9 $\mathrm{SU}(2)_\mathrm{L}$ doublets at our disposal from $\widetilde{L}$ tri-triplet
- Study the case of a 3HDM low-scale limit with $H_\mathrm{u}^{1,2}$ and H_d^{3}.

Boson	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(2)_\mathrm{L}$	$\mathrm{U}(1)_\mathrm{Y}$	$\{\mathrm{U}(1)_\mathrm{T}\}$
$H_\mathrm{u}^{1\,l}$, $H_\mathrm{d}^{3\,*\,l}$	$\mathbf{1}$	$\mathbf{2}^l$	1	5
$H_\mathrm{u}^{2\,l}$	$\mathbf{1}$	$\mathbf{2}^l$	1	1

$$
\begin{aligned}
V\left(H^1, H^2, H^3\right) =& m_i^2 \left|H^i\right|^2 + \left(m_{13}^2 H^{1\,\dagger} H^3 + \mathrm{h.c.}\right) + \tfrac{1}{2}\lambda_{ij} \left|H^i\right|^2 \left|H^j\right|^2 + \tfrac{1}{2}\lambda'_{ij} \left|H^{i\,\dagger} H^j\right|^2 \\
&+ \left[\delta_{13} \left(H^{1\,\dagger} H^3\right)^2 + \delta_{13i} \left(H^{1\,\dagger} H^3\right) \left|H^i\right| + \mathrm{h.c}\right] \\
&+ \kappa_{13} \left(H^{1\,\dagger} H^3\right) \left(H^{3\,\dagger} H^1\right) .
\end{aligned}
$$

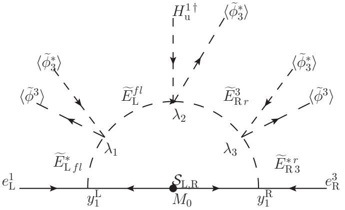
- The choice of the Higgs sector has an impact in the fermion masses:
  - If e.g. $H_\mathrm{u}^{1,2,3}$ bottom quark mass would be unacceptably light.

Outline

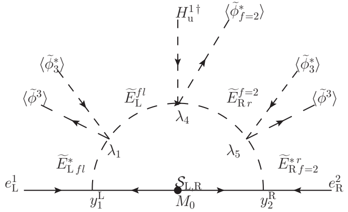
Charged lepton masses

- Due to $\mathrm{SU}(3)_\mathrm{F}$ there are no tree-level lepton masses $\left(W \supset \varepsilon_{ijk} \boldsymbol{L}^i \boldsymbol{Q}_\mathrm{L}^j \boldsymbol{Q}_\mathrm{R}^k\right)$
- **ALL Yukawa couplings and bilinears generated at quantum level**

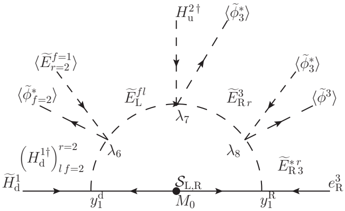
$$\begin{aligned}
\mathcal{L}_\mathrm{C} = {} & \left(H_\mathrm{u}^{1\dagger} + H_\mathrm{d}^3\right) \sum_{i=2}^{3} \left(Y^\mathrm{e}_{11i} E_\mathrm{L}^1 e_\mathrm{R}^i + Y^\mathrm{e}_{1i1} E_\mathrm{L}^i e_\mathrm{R}^1\right) + \sum_{j=2}^{3} \sum_{k=2}^{3} Y^\mathrm{e}_{ijk} H_\mathrm{u}^{2\dagger} E_\mathrm{L}^j e_\mathrm{R}^k \\
& + Y^\mathrm{eh}_{111} \left(H_\mathrm{u}^{1\dagger} + H_\mathrm{d}^3\right) \widetilde{H}_\mathrm{d}^1 e_\mathrm{R}^1 + \sum_{j=2}^{3} \sum_{k=2}^{3} H_\mathrm{u}^{2\dagger} \left(Y^\mathrm{eh}_{i1j} \widetilde{H}_\mathrm{d}^1 e_\mathrm{R}^j + Y^\mathrm{eh}_{ik1} \widetilde{H}_\mathrm{d}^k e_\mathrm{R}^1\right) \\
& + \sum_{i=2}^{3} \sum_{j=2}^{3} \sum_{k=2}^{3} M^\mathrm{eh}_{ij} e_\mathrm{L}^i \widetilde{H}_\mathrm{d}^j + \sum_{i=2}^{3} M^\mathrm{h}_{1i} \widetilde{H}_\mathrm{d}^1 \widetilde{H}_\mathrm{u}^i + \sum_{i=2}^{3} M^\mathrm{h}_{i1} \widetilde{H}_\mathrm{d}^i \widetilde{H}_\mathrm{u}^1
\end{aligned}$$



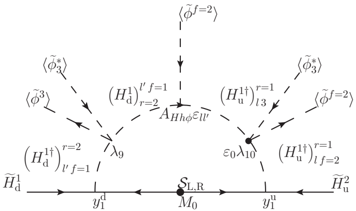
$Y^{e}_{113} \propto \varepsilon_0\lambda_1\lambda_2\lambda_3 y^{L}_1 y^{R}_1 \equiv \delta^{e}_{113}\varepsilon_0$



$Y^{e}_{112} \propto \xi_1\varepsilon_0\lambda_1\lambda_4\lambda_5 y^{L}_1 y^{R}_2 \equiv \delta^{e}_{112}\xi_1\varepsilon_0$



$Y^{eh}_{213} \propto \xi_1\xi_2\varepsilon_0\lambda_6\lambda_7\lambda_8 y^{d}_1 y^{R}_1 \equiv \delta^{eh}_{213}\xi_1\xi_2\varepsilon_0$



$M^{h}_{12} \propto \xi_1^2\varepsilon_0^2\lambda_9\lambda_{10} y^{d}_1 y^{u}_1 \kappa p \equiv \delta^{h}_{12}\xi_1\varepsilon_0^2\kappa p$

Sources of suppression:

$$\varepsilon_0 = \frac{1}{16\pi^2} \qquad \kappa = \frac{A_{Hh\phi}}{p} \qquad \xi_1 = \frac{f}{p} \qquad \xi_2 = \frac{\omega}{p} \qquad \delta^{x}_{ijk},\ \delta^{x}_{ij} = \lambda_m \cdots \lambda_q y_n y_s$$

Numerical estimate

- Quantum effects also present in δ^{x}_{ijk} and δ^{x}_{ij} due to RG-flow

Numerical estimate

- Quantum effects also present in $\delta^{\mathrm{x}}_{ijk}$ and δ^{x}_{ij} due to RG-flow
- Once EW symmetry is broken, consider $v = \sqrt{v_1^2 + v_2^2 + v_3^2} = 246$ GeV

$$\mathcal{L}_C = \begin{pmatrix} e^1_{\mathrm{L}} & e^2_{\mathrm{L}} & e^3_{\mathrm{L}} & \widetilde{H}^{1\,-}_{\mathrm{d}} & \widetilde{H}^{2\,-}_{\mathrm{d}} & \widetilde{H}^{3\,-}_{\mathrm{d}} \end{pmatrix} \mathcal{M}^C \begin{pmatrix} e^1_{\mathrm{R}} & e^2_{\mathrm{R}} & e^3_{\mathrm{R}} & \widetilde{H}^{1\,+}_{\mathrm{u}} & \widetilde{H}^{2\,+}_{\mathrm{u}} & \widetilde{H}^{3\,+}_{\mathrm{u}} \end{pmatrix}^{\top} + h.c..$$

$$\mathcal{M}^C = \varepsilon_0 \left(\begin{array}{ccc|ccc} 0 & a^{\mathrm{e}}_{12} v & a^{\mathrm{e}}_{13} v & 0 & 0 & 0 \\ a^{\mathrm{e}}_{21} v & b^{\mathrm{e}}_{22} v & b^{\mathrm{e}}_{23} v & 0 & c^{\mathrm{eh}}_{25} p & c^{\mathrm{eh}}_{26} p \\ a^{\mathrm{e}}_{31} v & b^{\mathrm{e}}_{32} v & \varepsilon_0 b^{\mathrm{e}}_{33} v & 0 & c^{\mathrm{eh}}_{35} p & c^{\mathrm{eh}}_{36} p \\ \hline a^{\mathrm{eh}}_{41} v & b^{\mathrm{eh}}_{42} v & b^{\mathrm{eh}}_{43} v & 0 & c^{\mathrm{h}}_{45} p & c^{\mathrm{h}}_{46} p \\ b^{\mathrm{eh}}_{51} v & 0 & 0 & c^{\mathrm{h}}_{54} p & 0 & 0 \\ b^{\mathrm{eh}}_{61} v & 0 & 0 & c^{\mathrm{h}}_{64} p & 0 & 0 \end{array}\right),$$

$$a^{\mathrm{e}}_{12} = k_1 \delta^{\mathrm{e}}_{121} \xi_1 + k_3 \delta^{\mathrm{e}}_{123} \xi_1 \kappa_3 \qquad b^{\mathrm{e}}_{22} = k_2 \delta^{\mathrm{e}}_{222} \xi_1 \qquad c^{\mathrm{h}}_{54} = \kappa_2 \delta^{\mathrm{h}}_{54} \xi_1^2$$

$$k_i = v_i / v \qquad \sum_i k_i^2 = 1$$

Two example parameter points

Physical lepton	Point 1	Point 2
m_e (keV)	510.7	511.4
m_μ (MeV)	105.2	105.8
m_τ (MeV)	1778	1779
m_E (TeV)	0.3331	0.4258
m_M (TeV)	4.249	0.4258
$m_{\mathcal{T}}$ (TeV)	4.254	1.534

$p \simeq 1.06 \times 10^3$ TeV $\quad \xi_1 \simeq 0.924 \quad \xi_2 \simeq 0.0926 \quad \kappa_2 \simeq 1.67 \quad \kappa_3 \simeq 1.1$

$k_1 \simeq 0.428 \quad k_2 \simeq 0.849 \quad k_d \simeq 0.310$

- δ^{x}_{ijk} and δ^{x}_{ij} preferably from $\mathcal{O}(10^{-6})$ to $\mathcal{O}(1)$
- Fitted values put constraints on the high scale theory parameters

Quark masses and CKM mixing

- A subset of quark Yulawa couplings are generated at tree-level $\left(W \supset \varepsilon_{ijk} \boldsymbol{L}^i \boldsymbol{Q}_{\mathrm{L}}^j \boldsymbol{Q}_{\mathrm{R}}^k\right)$
- Up and down quark sectors with tree-level contributions

Quark masses and CKM mixing

- A subset of quark Yulawa couplings are generated at tree-level $\left(W \supset \varepsilon_{ijk}\boldsymbol{L}^i\boldsymbol{Q}^j_{\mathrm{L}}\boldsymbol{Q}^k_{\mathrm{R}}\right)$
- Up and down quark sectors with tree-level contributions

$$\begin{aligned}\mathcal{L}_{\mathrm{q}} = & \sum_{i=2}^{3} Y^u_{1\,i} Q^1_{\mathrm{L}} H^2_{\mathrm{u}} u^i_{\mathrm{R}} + Y^d_{1\,2} Q^1_{\mathrm{L}} H^{2\,\dagger}_{\mathrm{u}} d^2_{\mathrm{R}} + \sum_{i=2}^{3} Y^u_{i\,1} Q^i_{\mathrm{L}} H^2_{\mathrm{u}} u^1_{\mathrm{R}} + \sum_{i=2}^{3} Y^d_{i\,3} Q^i_{\mathrm{L}} H^{2\,\dagger}_{\mathrm{u}} d^3_{\mathrm{R}} \\ & + Y^d_{1\,3} Q^1_{\mathrm{L}} H^{1\,\dagger}_{\mathrm{u}} d^3_{\mathrm{R}} + \sum_{i,j=2}^{3} Y^u_{ij} Q^i_{\mathrm{L}} H^1_{\mathrm{u}} u^j_{\mathrm{R}} + \sum_{i=2}^{3} Y^d_{i\,1} Q^i_{\mathrm{L}} H^{1\,\dagger}_{\mathrm{u}} d^1_{R} + Y^d_{1\,4} Q^1_{\mathrm{L}} H^{1\,\dagger}_{\mathrm{u}} \mathcal{D}_{\mathrm{R}} \\ & + \sum_{i=2}^{3} Y^d_{i\,4} Q^i_{\mathrm{L}} H^{2\,\dagger}_{\mathrm{u}} \mathcal{D}_{\mathrm{R}} + m_D \mathcal{D}_{\mathrm{L}} \mathcal{D}_{\mathrm{R}} + \left(H^2_{\mathrm{u}} \to H^{3\,\dagger}_{\mathrm{d}}\right) + h.c.\end{aligned}$$

CKM mixing – Tree-level structure

$$d_R^3 = a_1 d_R^3 + a_2 d_R^2 + a_3 D_R^1 \qquad \mathcal{D}_R = b_1 d_R^3 + b_2 d_R^2 + b_3 D_R^1$$

take $a_2 \sim 1 \Rightarrow d_L^1 d_R^3 H_d^3 \simeq d_L^1 d_R^2 H_d^3$ tree-level

$$\mathcal{M}^u = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 0 & k_2 Y_{13}^u \\ 0 & 0 & k_1 Y_{23}^u \\ k_1 Y_{31}^u & k_2 Y_{32}^u & 0 \end{pmatrix} \qquad \mathcal{M}^d = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 0 & Y_{13}^d k_d & 0 \\ Y_{21}^d k_d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} m_D / v \end{pmatrix}$$

Mass eigenstates: $\tilde{u}_{L,R}^i = (u_{L,R},\, c_{L,R},\, t_{L,R}) \quad \tilde{d}_{L,R}^i = (d_{L,R},\, s_{L,R},\, b_{L,R}\,, D_{L,R})$

Bi-unitary transformations: $u_{L,R}^i = \left(V_{L,R}^u\right)^{ij} \tilde{u}_{L,R}^j\,,\; d_{L,R}^I = \left(V_{L,R}^d\right)^{IJ} \tilde{d}_{L,R}^J$

Tree-level CKM mixing:

$$V'_{CKM} = V_L^{d\dagger} E V_L^u = \left(\begin{array}{ccc} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \\ \hline 0 & 0 & 0 \end{array}\right) \qquad \tan\theta_C = \frac{k_1 Y_{13}^u}{k_2 Y_{13}^u} \qquad E = \left(\mathbb{1}_{3\times 3}\ 0\right)^\top$$

- Cabibbo form readily at tree-level

Quantum effects

$$d_R^3 = 0.71 d_R^3 + 0.06 d_R^2 + 0.7 D_R^1 \qquad \mathcal{D}_R = -0.11 d_R^3 + 0.99 d_R^2 + 0.04 D_R^1$$

$$\mathcal{M}^u = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & \beta_{12}^{\mathrm{u}} & k_2 Y_{13}^u \\ \beta_{12}^{\mathrm{u}} & \alpha_{22}^{\mathrm{u}} & k_1 Y_{23}^u \\ k_1 Y_{31}^u & k_2 Y_{32}^u & \alpha_{33}^{\mathrm{u}} \end{pmatrix} \qquad \mathcal{M}^d = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & \beta_{12}^{\mathrm{d}} & 0.06 Y_{13}^d k_d & 0.99 Y_{13}^d k_d \\ k_d Y_{21}^d & 0 & \beta_{23}^{\mathrm{d}} & \beta_{24}^{\mathrm{d}} \\ \alpha_{31}^{\mathrm{d}} & 0 & \beta_{33}^{\mathrm{d}} & \beta_{34}^{\mathrm{d}} \\ 0 & 0 & 0 & \sqrt{2} m_D / v \end{pmatrix}$$

Spectrum and CKM mixing: (Compatible with previous charged lepton points)

$$\boldsymbol{m_t = 173.2\ \mathrm{GeV} \qquad m_c = 1338\ \mathrm{MeV} \qquad m_u = 3.552\ \mathrm{MeV}}$$

$$\boldsymbol{m_D = 2.369\ \mathrm{TeV} \qquad m_b = 4022\ \mathrm{MeV} \qquad m_s = 105.4\ \mathrm{MeV} \qquad m_d = 2.595\ \mathrm{MeV}}$$

$$V'_{\mathrm{CKM}} \simeq \left(\begin{array}{ccc} 0.972436 & 0.232173 & 0.00438568 \\ 0.232023 & 0.972665 & 0.0200507 \\ 0.0092939 & 0.0216599 & 0.999756 \\ \hline 0.0015349 & 0.00786742 & 0.000153942 \end{array}\right)$$

- Tree-level entries on the last row have a suppression factor $\propto 1/m_D$
- VLQ mass depends on the $\mathrm{SU}(2)_{\mathrm{R}}$-breaking scale ($\xi_2 = 0.0326$)

Neutrino masses

Using the same strategy for 15 neutral states...

$$\Psi_N = \left(\phi^1 \; \phi^2 \; \phi^3 \; \nu_R^1 \; \nu_R^2 \; \nu_R^3 \; \nu_L^1 \; \nu_L^2 \; \nu_L^3 \; \widetilde{H}_d^{1\,0} \; \widetilde{H}_d^{2\,0} \; \widetilde{H}_d^{3\,0} \; \widetilde{H}_u^{1\,0} \; \widetilde{H}_u^{2\,0} \; \widetilde{H}_u^{3\,0} \right)$$

Mass matrix

$$\mathcal{L}_N = \Psi_N \mathcal{M}^N \Psi_N^\top$$

$$\mathcal{M}^N = \varepsilon_0 \times$$

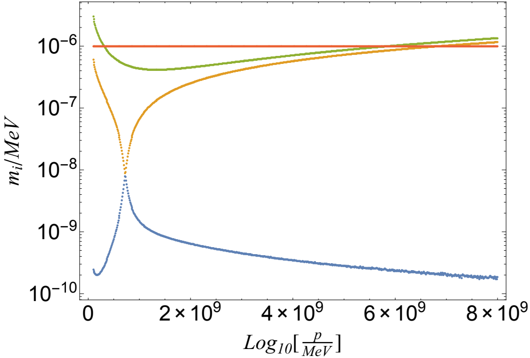
$$\left(\begin{array}{ccccccccccccccc}
0 & 0 & 0 & 0 & p\delta_{12}^{\phi R}\xi_1\xi_2 & p\delta_{13}^{\phi R}\xi_2 & 0 & 0 & 0 & 0 & vk_3\delta_{312}^{\phi d}\kappa & vk_2\delta_{213}^{\phi d}\kappa & 0 & vk_1\delta_{112}^{\phi u}\xi_1 & vk_1\delta_{113}^{\phi u}\varepsilon_0 \\
0 & p\delta_{21}^{\phi} & p\delta_{23}^{\phi}\xi_1 & p\delta_{21}^{\phi R}\xi_1\xi_2 & 0 & 0 & 0 & vk_2\delta_{222}^{\phi L}\xi_2\varepsilon_0 & vk_2\delta_{223}^{\phi L}\xi_1\xi_2\varepsilon_0 & vk_3\delta_{321}^{\phi d}\kappa & vk_3\delta_{322}^{\phi d}\varepsilon_0\kappa\xi_1 & vk_1\delta_{123}^{\phi d}\kappa & vk_1\delta_{121}^{\phi u}\xi_1 & vk_2\delta_{222}^{\phi u}\xi_1 & vk_3\delta_{323}^{\phi u}\xi_1 \\
0 & p\delta_{23}^{*\phi}\xi_1 & p\delta_{33}^{\phi} & p\delta_{31}^{\phi R}\xi_2 & 0 & 0 & 0 & vk_3\delta_{332}^{\phi L}\xi_2\varepsilon_0 & vk_3\delta_{333}^{\phi L}\xi_1\xi_2\varepsilon_0 & vk_3\delta_{331}^{\phi d}\kappa\xi_1 & vk_1\delta_{132}^{\phi d}\varepsilon_0\kappa\xi_1^2 & vk_1\delta_{133}^{\phi d}\kappa\xi_1 & vk_1\delta_{131}^{\phi u} & vk_2\delta_{232}^{\phi u} & vk_3\delta_{333}^{\phi u} \\
0 & p\delta_{21}^{*\phi R}\xi_1\xi_2 & p\delta_{33}^{*\phi} & p\delta_{31}^{\phi R}\xi_2 & 0 & 0 & 0 & vk_3\delta_{312}^{RL}\kappa & vk_3\delta_{313}^{RL}\kappa & vk_3\delta_{311}^{Rd}\kappa\xi_1\xi_2 & vk_1\delta_{112}^{Rd}\varepsilon_0\kappa\xi_1^2\xi_2 & vk_1\delta_{113}^{Rd}\kappa\xi_1\xi_2 & vk_1\delta_{111}^{Ru}\xi_2 & vk_2\delta_{212}^{Ru}\xi_1^2\xi_2 & vk_3\delta_{313}^{Ru} \\
p\delta_{12}^{*\phi\nu}\xi_1\xi_2 & 0 & 0 & 0 & 0 & 0 & vk_3\delta_{121}^{RL}\kappa & vk_1\delta_{122}^{RL}\kappa\xi_1 & vk_1\delta_{123}^{RL}\kappa & vk_1\delta_{121}^{Rd}\kappa\xi_2 & 0 & 0 & vk_2\delta_{221}^{Ru}\kappa\xi_1^2\xi_2 & 0 & 0 \\
vk_1\delta_{112}^{*\phi u}\xi_1 & 0 & 0 & 0 & 0 & 0 & vk_2\delta_{231}^{RL}\kappa & vk_1\delta_{132}^{RL}\kappa\xi_1 & vk_1\delta_{133}^{RL}\kappa\xi_1 & vk_1\delta_{131}^{Rd}\kappa\xi_1\xi_2 & 0 & 0 & vk_2\delta_{231}^{Ru}\kappa\xi_1\xi_2 & 0 & 0 \\
0 & 0 & 0 & 0 & vk_3\delta_{121}^{*RL}\kappa & vk_2\delta_{231}^{*RL}\kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & vk_2\delta_{222}^{*\phi L}\xi_2\varepsilon_0 & vk_3\delta_{332}^{*\phi L}\xi_2\varepsilon_0 & vk_3\delta_{312}^{*RL}\kappa & vk_1\delta_{122}^{*RL}\kappa\xi_1 & vk_1\delta_{*132}^{RL}\kappa\xi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p\delta_{22}^{Lu}\kappa\xi_1\xi_2 & p\delta_{23}^{Lu}\kappa\xi_2 \\
0 & vk_2\delta_{223}^{*\phi L}\xi_1\xi_2\varepsilon_0 & vk_3\delta_{333}^{*\phi L}\xi_1\xi_2\varepsilon_0 & vk_3\delta_{313}^{*RL}\kappa & vk_1\delta_{123}^{*RL}\kappa & vk_1\delta_{133}^{*RL}\kappa\xi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p\delta_{32}^{Lu}\kappa\xi_2 & p\delta_{33}^{Lu}\kappa\xi_1\xi_2 \\
0 & vk_3\delta_{321}^{*\phi d}\kappa & vk_3\delta_{331}^{*\phi d}\kappa\xi_1 & vk_3\delta_{311}^{*Rd}\kappa\xi_1\xi_2 & vk_1\delta_{121}^{*Rd}\kappa\xi_2 & vk_1\delta_{131}^{*Rd}\kappa\xi_1\xi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p\delta_{12}^{du}\varepsilon_0\kappa\xi_1^2 & p\delta_{13}^{du}\kappa\xi_1 \\
vk_3\delta_{312}^{*\phi d}\kappa & vk_3\delta_{322}^{*\phi d}\varepsilon_0\kappa\xi_1 & vk_1\delta_{132}^{\phi d}\varepsilon_0\kappa\xi_1^2 & vk_1\delta_{112}^{*Rd}\varepsilon_0\kappa\xi_1^2\xi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p\delta_{21}^{du}\varepsilon_0\kappa\xi_1^2 & 0 & 0 \\
vk_2\delta_{213}^{*\phi d}\kappa & vk_1\delta_{123}^{*\phi d}\kappa & vk_1\delta_{133}^{*\phi d}\kappa\xi_1 & vk_1\delta_{113}^{*Rd}\kappa\xi_1\xi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p\delta_{31}^{du}\kappa\xi_1 & 0 & 0 \\
0 & vk_1\delta_{121}^{*\phi u}\xi_1 & vk_1\delta_{131}^{\phi u} & vk_1\delta_{111}^{*Ru}\xi_2 & vk_2\delta_{221}^{*Ru}\kappa\xi_1^2\xi_2 & vk_2\delta_{231}^{*Ru}\kappa\xi_1\xi_2 & 0 & 0 & 0 & 0 & p\delta_{21}^{*du}\varepsilon_0\kappa\xi_1^2 & p\delta_{31}^{*du}\kappa\xi_1 & 0 & 0 & 0 \\
vk_1\delta_{112}^{*\phi u}\xi_1 & vk_2\delta_{222}^{*\phi u}\xi_1 & vk_2\delta_{232}^{*\phi u} & vk_2\delta_{212}^{*Ru}\xi_1^2\xi_2 & 0 & 0 & 0 & p\delta_{22}^{*Lu}\kappa\xi_1\xi_2 & p\delta_{32}^{*Lu}\kappa\xi_2 & p\delta_{12}^{*du}\varepsilon_0\kappa\xi_1^2 & 0 & 0 & 0 & 0 & 0 \\
vk_1\delta_{113}^{*\phi u}\varepsilon_0 & vk_3\delta_{323}^{*\phi u}\xi_1 & vk_3\delta_{333}^{*\phi u} & vk_3\delta_{313}^{*Ru} & 0 & 0 & 0 & p\delta_{23}^{*Lu}\kappa\xi_2 & p\delta_{33}^{*Lu}\kappa\xi_1\xi_2 & p\delta_{13}^{*du}\kappa\xi_1 & 0 & 0 & 0 & 0 & 0
\end{array}\right)$$

$p \simeq 1.06 \times 10^3$ TeV $\quad \xi_1 \simeq 0.924 \quad \xi_2 \simeq 0.0926 \quad \kappa_2 \simeq 1.67 \quad \kappa_3 \simeq 1.1$

$m_{N_{1,2}} \simeq 11.80$ TeV $\quad m_{N_3} \simeq 4.654$ GeV $\quad m_{N_{4,5}} \simeq 840.0$ GeV $\quad \simeq m_{N_6} \simeq 683.9$ GeV

$m_{N_{7,8}} \simeq 303.7$ GeV $\quad m_{N_9} \simeq 151.3$ GeV $\quad m_{N_{10,11}} \simeq 137.9$ MeV $\quad m_{N_{12}} \simeq 109.1$ MeV

$\mathbf{m_3 \simeq 0.4260}$ **eV** $\quad \mathbf{m_2 \simeq 0.07885}$ **eV** $\quad \mathbf{m_1 \simeq 0.001302}$ **eV** GREAT!



- All neutrinos have Majorana component which is small for $N_{1,2,4,5,7,8,10,11}$

Outline

Conclusions

- Developed a **phenomenological** model *inspired* buy a T-GUT framework
- 3HDM low scale limit reproduces the observed fermion masses in the SM
  - **interplay between $\mathbf{SU(3)_F}$, quantum effects and ratios between breaking scales (ξ_1, ξ_2).**
- Extra exotic fermions $\longrightarrow$ collider signatures? (Future studies)
  - 1 generation of VLQ
  - Heavy electrons
  - $\mathcal{O}(1-10\mathrm{TeV})$ and $\mathcal{O}(100\ \mathrm{MeV})$ neutrinos
  - Sterile neutrinos? dark matter? (Future studies)
- Cabibbo mixing emerges at tree-level
- CKM structure from quantum effects

Outline

Quantum numbers for leptons

Lepton	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(2)_\mathrm{L}$	$\mathrm{U}(1)_\mathrm{Y}$	$\{\mathrm{U}(1)_\mathrm{T}\}$
E_L^1	$\mathbf{1}$	$\mathbf{2}$	-1	3
$E_\mathrm{L}^{2,3}$	$\mathbf{1}$	$\mathbf{2}$	-1	-1
$\widetilde{H}_\mathrm{u}^1$	$\mathbf{1}$	$\mathbf{2}$	1	5
$\widetilde{H}_\mathrm{u}^{2,3}$	$\mathbf{1}$	$\mathbf{2}$	1	1
$\widetilde{H}_\mathrm{d}^1$	$\mathbf{1}$	$\mathbf{2}$	-1	-1
$\widetilde{H}_\mathrm{d}^{2,3}$	$\mathbf{1}$	$\mathbf{2}$	-1	-5
e_R^1	$\mathbf{1}$	$\mathbf{1}$	2	6
$e_\mathrm{R}^{2,3}$	$\mathbf{1}$	$\mathbf{1}$	2	2
ν_R^1	$\mathbf{1}$	$\mathbf{1}$	0	0
$\nu_\mathrm{R}^{2,3}$	$\mathbf{1}$	$\mathbf{1}$	0	-4
ϕ^1	$\mathbf{1}$	$\mathbf{1}$	0	4
$\phi^{2,3}$	$\mathbf{1}$	$\mathbf{1}$	0	0

Quantum numbers for quarks

Quark	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(2)_\mathrm{L}$	$\mathrm{U}(1)_\mathrm{Y}$	$\{\mathrm{U}(1)_\mathrm{T}\}$
Q_L^1	$\mathbf{3}$	$\mathbf{2}$	$1/3$	3
$Q_\mathrm{L}^{2,3}$	$\mathbf{3}$	$\mathbf{2}$	$1/3$	-1
u_R^1	$\mathbf{3}$	$\mathbf{1}$	$-4/3$	0
$u_\mathrm{R}^{2,3}$	$\mathbf{3}$	$\mathbf{1}$	$-4/3$	-4
d_R^1	$\mathbf{3}$	$\mathbf{1}$	$2/3$	6
d_R^2	$\mathbf{3}$	$\mathbf{1}$	$2/3$	-2
d_R^3	$\mathbf{3}$	$\mathbf{1}$	$2/3$	2
$\mathcal{D}_\mathrm{L}$	$\mathbf{3}$	$\mathbf{1}$	$-2/3$	-2
$\mathcal{D}_\mathrm{R}$	$\mathbf{3}$	$\mathbf{1}$	$2/3$	2

$$\left(D_\mathrm{R}^2,\, D_\mathrm{R}^3\right) \mapsto \left(d_\mathrm{R}^2,\, \mathcal{D}_\mathrm{R}^1\right) \qquad \left(D_\mathrm{R}^1,\, d_\mathrm{R}^2,\, d_\mathrm{R}^3\right) \mapsto \left(d_\mathrm{R}^3,\, \mathcal{D}_\mathrm{R}^2,\, \mathcal{D}_\mathrm{R}^3\right)$$

Selecting the matter content

- We have shown recently, for a non-SUSY T-GUT, that it is possible to generate mass hierarchy upon running
  10.1007/JHEP09(2016)129 JECM, **APM**, RP, JW
- Our approach:
  - > All squarks get p-scale masses (hypothesis)
  - > Gauginos $\widetilde{g}$, $\mathcal{T}_{\mathrm{L,R}}$ and $\mathcal{S}_{\mathrm{L,R}}$ are p-scale heavy (exact)
  - > Part of $\widetilde{L}$ scalars get p-scale masses (hypothesis)
  - > **Higgs sector of pSHUT** $\rightarrow$ The part of $\widetilde{L}$ lighter than p-scale (hypothesis)

$$\left(\tilde{L}^i\right)^l{}_r = \left(\begin{array}{c|c|c} H_u^0 & H_d^- & \tilde{e}_L \\ H_u^+ & H_d^0 & \tilde{\nu}_L \\ \hline \tilde{e}_R & \tilde{\nu}_R & \phi \end{array}\right)^i$$

- 9 $\mathrm{SU}(2)_\mathrm{L}$ doublets: $\boldsymbol{H}_\mathrm{u}^{1,2,3}$, $\boldsymbol{H}_\mathrm{d}^{1,2,3}$ and $\widetilde{\boldsymbol{E}}_\mathrm{L}^{1,2,3}$
- 2 real singlets $(\widetilde{\phi}^2, \widetilde{\nu}_\mathrm{R}^1) \mapsto (\widetilde{S}_1, \widetilde{S}_2)$
- 2 complex singlets: $(\widetilde{\phi}^1, \widetilde{\nu}_\mathrm{R}^2, \widetilde{\nu}_\mathrm{R}^3) \mapsto (G, \widetilde{N}_\mathrm{R}^2, \widetilde{N}_\mathrm{R}^3)$
- 2 charged singlets: $(\widetilde{e}_\mathrm{R}^2, \widetilde{e}_\mathrm{R}^3) \mapsto (\widetilde{R}_2, \widetilde{R}_3)$

$$(\tilde{L}^i)^l{}_r = \left(\begin{array}{cc|c} H_u^0 & H_d^- & \tilde{e}_L \\ H_u^+ & H_d^0 & \tilde{\nu}_L \\ \hline \tilde{e}_R & \tilde{\nu}_R & \phi \end{array}\right)^i$$

- 9 $\mathrm{SU}(2)_\mathrm{L}$ doublets: $\boldsymbol{H}_u^{1,2,3}$, $\boldsymbol{H}_d^{1,2,3}$ and $\widetilde{\boldsymbol{E}}_L^{1,2,3}$
- 2 real singlets $(\widetilde{\phi}^2, \widetilde{\nu}_R^1) \mapsto (\widetilde{S}_1, \widetilde{S}_2)$
- 2 complex singlets: $(\widetilde{\phi}^1, \widetilde{\nu}_R^2, \widetilde{\nu}_R^3) \mapsto (G, \widetilde{N}_R^2, \widetilde{N}_R^3)$
- 2 charged singlets: $(\widetilde{e}_R^2, \widetilde{e}_R^3) \mapsto (\widetilde{R}_2, \widetilde{R}_3)$

1. Choose three light doublets based on their $\mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)_\mathrm{T}$ charges

$$(1,\ 1) : H_u^{2,3},\ H_d^{*1},\ \widetilde{E}_L^{*2,3}, \qquad (1,\ 5) : H_u^1,\ H_d^{*2,3}, \qquad (1,\ -3) : \widetilde{E}_L^{*1}.$$

2. Add all massless fermions (Exact)
3. Add 1 generation of TeV scale vector-like quarks (VLQ) (Exact)
   - After p-VEV two generations get heavy via $m_D D_L^f D_R^{f'} \varepsilon_{ff'}$
   - f and ω VEVs induce mixing yielding and one TeV(ish) scale VLQ (Exact)

$$(D_R^2,\ D_R^3) \mapsto (d_R^2,\ \mathcal{D}_R^2) \qquad (D_R^1,\ d_R^2,\ d_R^3) \mapsto (d_R^3,\ \mathcal{D}_R^1,\ \mathcal{D}_R^3)$$

$$(\tilde{L}^i)^l{}_r = \left(\begin{array}{c|c|c} H_u^0 & H_d^- & \tilde{e}_L \\ H_u^+ & H_d^0 & \tilde{\nu}_L \\ \hline \tilde{e}_R & \tilde{\nu}_R & \phi \end{array}\right)^i$$

- 9 $\mathrm{SU}(2)_\mathrm{L}$ doublets: $\boldsymbol{H}_\mathrm{u}^{1,2,3}$, $\boldsymbol{H}_\mathrm{d}^{1,2,3}$ and $\widetilde{\boldsymbol{E}}_\mathrm{L}^{1,2,3}$
- 2 real singlets $(\widetilde{\phi}^2, \widetilde{\nu}_\mathrm{R}^1) \mapsto (\widetilde{S}_1, \widetilde{S}_2)$
- 2 complex singlets: $(\widetilde{\phi}^1, \widetilde{\nu}_\mathrm{R}^2, \widetilde{\nu}_\mathrm{R}^3) \mapsto (G, \widetilde{N}_\mathrm{R}^2, \widetilde{N}_\mathrm{R}^3)$
- 2 charged singlets: $(\widetilde{e}_\mathrm{R}^2, \widetilde{e}_\mathrm{R}^3) \mapsto (\widetilde{R}_2, \widetilde{R}_3)$

1. Choose three light doublets based on their $\mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)_\mathrm{T}$ charges

$$(1,\ 1) : H_\mathrm{u}^{2,3},\ H_\mathrm{d}^{*1},\ \widetilde{E}_\mathrm{L}^{*2,3}, \qquad (1,\ 5) : H_\mathrm{u}^1,\ H_\mathrm{d}^{*2,3}, \qquad (1,\ -3) : \widetilde{E}_\mathrm{L}^{*1}.$$

2. Add all massless fermions (Exact)
3. Add 1 generation of TeV scale vector-like quarks (VLQ) (Exact)
   - After p-VEV two generations get heavy via $m_D D_\mathrm{L}^f D_\mathrm{R}^{f'} \varepsilon_{ff'}$
   - f and ω VEVs induce mixing yielding and one TeV(ish) scale VLQ (Exact)

$$(D_\mathrm{R}^2,\ D_\mathrm{R}^3) \mapsto (d_\mathrm{R}^2,\ \mathcal{D}_\mathrm{R}^2) \qquad (D_\mathrm{R}^1,\ d_\mathrm{R}^2,\ d_\mathrm{R}^3) \mapsto (d_\mathrm{R}^3,\ \mathcal{D}_\mathrm{R}^1,\ \mathcal{D}_\mathrm{R}^3)$$

We have the pSHUT model!