New results on CP-violation in multi-Higgs models

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based on: G. C. Branco, I.P.I., arXiv:1511.02764 and a work in progress.

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CPV from the scalar sector

CP-violation from the scalar sector of multi-Higgs-doublet models:

- explicit: the scalar lagrangain does not possess any CP (or gCP) symmetry,
- spontaneous: the lagrangain possesses a set of gCP symmetries, but none of them leaves vevs invariant.

A very brief history:

- T.D.Lee, 1973: 2HDM with spontaneous breaking of CP; but NFC is incompatible with explicit and with spontaneous CPV;
- Weinberg, 1976: 3HDM with NFC and with explicit CPV;
  Branco, 1980: same with spontaneous CPV;
  Branco, Gerard, Grimus, 1984: 3HDM with geometric CPV.
- early review: Branco, Buras, Gerard, 1985;
  Branco, Lavoura, Silva, "CPV", 1999.

Generalized CP symmetries

A reminder: the CP transformation is not uniquely defined *a priori* [e.g. Feinberg, Weinberg, 1959]. In NHDM with N scalar doublets,

$$\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$$

with any $X \in U(N)$ leaves the kinetic term invariant and can play the role of the (general) CP transformation.

- If no gCP symmetry with any X_{ij} exists, the model is explicitly CP-violating;
- If V is invariant under a gCP with *any* X_{ij}, the model is explicitly CP-conserving;
  - If none of the gCP symmetries of V leaves vevs invariant $\to$ spontaneous CPV;
  - If $X_{ij}\langle 0|\phi_j|0\rangle^* = \langle 0|\phi_i|0\rangle$ for some gCP symmetry $\to$ no CPV from the scalar sector.

CP-violation vs. family symmetries

Family symmetries vs. CPV

Imposing a family symmetry on the Higgs doublets has consequences for CPV.

- $\mathbb{Z}_2$ in 2HDM $\to$ neither explicit nor spontaneous CPV;
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ (NFC) in 3HDM $\to$ both explicit or spontaneous CPV possible;
- $\Delta(27)$ 3HDM, Branco, Gerard, Grimus, 1985 $\to$ both explicit or spontaneous CPV possible;
- A_4, S_4 3HDM $\to$ neither explicit nor spontaneous CPV.
- Ivanov, Nishi, 2014: all discrete groups G in 3HDM, in all their vev alignments, follow this "neither/both" pattern.

Conjecture: CPV comes in pairs

Family symmetry group G is compatible with spontaneous CPV if and only if it is compatible with explicit CPV in the (neutral) Higgs sector.

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$\mathbb{Z}_4$ 3HDM

Consider 3HDM with $G = \mathbb{Z}_4$ symmetry generated by $a_4 = \mathrm{diag}(i, -i, 1)$. Then, $V = V_0 + V_1$, where the phase-sensitive part is

$$V_1(\mathbb{Z}_4) = \lambda_1(\phi_1^\dagger\phi_2)^2 + \lambda_2(\phi_1^\dagger\phi_3)(\phi_2^\dagger\phi_3) + h.c.$$

- rephasing freedom $\to$ make $\lambda_{1,2}$ real $\to$ explicitly CP-conserving;
- extremization condition gives $\langle\phi_i^0\rangle = v_i e^{i\xi_i}/\sqrt{2}$, $i = 1, 2, 3$. If all $v_i \neq 0 \to$ phases are rigid, such as

  $$(v_1 e^{i\pi/4}, v_2 e^{-i\pi/4}, v_3),$$

  and there remains a gCP symmetry.
- if $v_1 = 0$, then $(0, v_2 e^{\xi_2}, v_3)$ with arbitrary ξ_2 is OK, but it is a saddle point $\to$ spontaneous CPV is absent.

Proof

We proved the conjecture for any rephasing symmetry group G and any number of doublets, G. C. Branco, I.P.I., arXiv:1511.02764.

An outline of the proof (bears some similarity with the spurion-based technique of Haber, Surujon, 2012):

- V_1 contains k terms built of N doublets. Rephase doublets by α_j. The i-th term picks up phase change $d_{ij}\alpha_j$. The $k \times N$ matrix d_{ij} plays the key role.
- The rephasing symmetry of the model is given by solutions of $d_{ij}\alpha_j = 2\pi n_i$, which are efficiently found with the Smith normal form technique [Ivanov, Keus, Vdovin, 2012].
- At quasiclassical values of fields $\phi_j \to v_j e^{i\xi_j}/\sqrt{2}$, the potential is

$$V_1 = \frac{1}{2}\sum_{i=1}^{k} A_i \cos(d_{ij}\xi_j + \psi_i)\,, \quad A_i = |\lambda_i| \prod_{j=1}^{N} v_j^{|d_{ij}|}\,, \quad \psi_i = \arg\lambda_i\,.$$

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Proof

CP-conserving case: $k = N - 1$, $\mathrm{rank}\, d = N - 1$.

- Enough rephasing freedom to set all $\psi_i = 0 \rightarrow$ explicit CPC.
- $$0 = \frac{\partial V}{\partial \xi_j} = \sum_i A_i s_i d_{ij} \quad \forall j \quad \Rightarrow \quad A_i s_i = 0 \quad \forall i = 1, \ldots, k,$$
  where $s_i \equiv \sin(d_{ij}\xi_j)$.
- if all $v_i \neq 0$, then $s_i = 0$ which implies
  $$d_{ij}\xi_j = -d_{ij}\xi_j \quad \Rightarrow \quad \xi_j = -\xi_j + \alpha_j\,,$$
  where α_j is a symmetry of the model $\rightarrow$ gCP symmetry present.
- if some $v_i = 0$, the proof is more elaborate, but the conclusion is the same: there is no spontaneous CPV.

Proof

CP-violating case: $k = N$, $\operatorname{rank} d = N - 1$.

- Not enough rephasing freedom to set all $\psi_i = 0 \to$ explicit CPV.
- For CP-conserving case, the same system

$$0 = \sum_{i=1}^{N} A_i s_i d_{ij} \quad \forall j = 1, \ldots, N$$

now allows for a non-zero solution: not all $A_i s_i = 0$.

- This solution cannot have any residual rephasing gCP $\to$ spontaneous CPV.

CPV from charged Higgs sector

There is one peculiar situation in which the non-zero solution exists, in the algebraic sense, but cannot be realized via vev alignment.

Consider 4HDM with symmetry group $\mathbb{Z}_4 \times \mathbb{Z}_2$.

$$V_1 = \lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5'(\phi_3^\dagger\phi_4)^2 + \lambda_6(\phi_1^\dagger\phi_3)(\phi_2^\dagger\phi_4) + \lambda_6'(\phi_1^\dagger\phi_4)(\phi_2^\dagger\phi_3) + h.c.$$

invariant under $a_2 = \mathrm{diag}(1,-1,1,-1)$ and $a_4 = \mathrm{diag}(1,1,i,-i)$.

$$d = \begin{pmatrix} -2 & 2 & 0 & 0 \\ 0 & 0 & -2 & 2 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{pmatrix}.$$

The model is explicitly CPV: cannot change the phase between λ_6 and λ_6'.

CPV from charged Higgs sector

However, there is no room for spontaneous CPV in this model!

The non-zero solution of $\sum_{i=1}^{N} A_i s_i d_{ij} = 0$ is $A_i s_i = (0,\, 0,\, 1,\, -1)$, which implies $\lambda_6 = -\lambda_6'$. For generic λ's, this solution cannot be realized via vevs.

One can rewrite the model as

$$\begin{aligned} V_1 &= \lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5'(\phi_3^\dagger\phi_4)^2 + \lambda_6(\phi_1^\dagger\phi_3)(\phi_2^\dagger\phi_4) \\ &\quad + \tilde\lambda_6 \left[(\phi_1^\dagger\phi_3)(\phi_2^\dagger\phi_4) - (\phi_1^\dagger\phi_4)(\phi_2^\dagger\phi_3)\right] + h.c. \end{aligned}$$

with real λ_5, λ_5', λ_6, and complex $\tilde\lambda_6$.

Charged-Higgs-induced CPV

The complex parameter disappears in conditions for vevs; it enters the model only via the charged Higgs sector.

To avoid this exotic situation, we added "neutral" to the conjecture.

Beyond rephasing

I also hope to settle the issue for non-abelian G's (rephasing and permutations).

Example: 3HDM with A_4 and $\Delta(27)$ look similarly,

$$A_4: \qquad \lambda\left[(\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_3)^2 + (\phi_3^\dagger\phi_1)^2\right] + h.c.$$
$$\Delta(27): \qquad \lambda\left[(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_3) + (\phi_2^\dagger\phi_3)(\phi_2^\dagger\phi_1) + (\phi_3^\dagger\phi_1)(\phi_3^\dagger\phi_2)\right] + h.c.$$

but A_4 3HDM is CPC, while $\Delta(27)$ 3HDM is CPV. The difference is in matrices d_{ij}:

$$d(A_4) = \begin{pmatrix} -2 & 2 & 0 \\ 0 & -2 & 2 \\ 2 & 0 & -2 \end{pmatrix}, \quad d(\Delta(54)) = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}.$$

For A_4, $-d = d$ up to permutations, while for $\Delta(27)$, $-d \neq d$.

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Explicit CP-conservation

and

the existence of a real basis

CP conserving Higgs sector $\Leftrightarrow$ existence of a real basis.
Formalized in the most accurate form in Gunion, Haber, 2005:

CP violating or *CP* conserving. The answer to this question is governed by a simple theorem:

Theorem 1.—The Higgs potential is explicitly *CP* conserving if and only if a basis exists in which all Higgs potential parameters are real. Otherwise, *CP* is explicitly violated.

Although Theorem 1 is well known and often stated in the literature, its proof is usually given under the assumption that a convenient basis has been chosen in which the

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The model

Consider 3HDM with the following potential $V = V_0 + V_1$ (notation: $i \equiv \phi_i$):

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2\left[(2^\dagger 2)^2 + (3^\dagger 3)^2\right]$$
$$+\lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4\left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1)\right] + \lambda_4'(2^\dagger 3)(3^\dagger 2)\,,$$

with all parameters real, and

$$V_1 = \lambda_5(3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right] + h.c.$$

with real $\lambda_{5,6}$ and complex $\lambda_{8,9}$. It is invariant under order-4 gCP:

$$J:\ \phi_i \mapsto X_{ij}\phi_j^*\,, \quad X = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}.$$

Its square, $J^2 = \mathrm{diag}(1, -1, -1)$, and $J^4 = \mathbb{I}$.
This model has no other symmetries [Ivanov, Keus, Vdovin, 2012].

The model

There exists no basis change, $\phi_i \mapsto U_{ij}\phi_j$, which could make all coefficients real.

Proof.
Suppose it exists. In the new basis, the potential has the usual CP-symmetry of order 2. But this model does *not* have any order-2 gCP. Contradiction.

NB: In 2HDM, one can impose CP2-symmetry (gCP of order 4) [Ferreira, Haber, Silva, 2009]. But the resulting potential has additional symmetries, including the usual order-2 gCP [Maniatis, von Manteuffel, Nachtmann, 2008]. In this case, there are no additional symmetries, therefore, the real basis does not exist.

Technically, the loophole in the proof of Gunion, Haber, 2005 is the assumption that $\mathcal{T}^2 = \mathbb{I}$, with references to Feinberg, Weinberg, 1959 and Carruthers, 1968. But in those papers, X_{ij} was assumed to be diagonal, which automatically leads to order-2 gCP. In general, X_{ij} can be block-diagonal with 2×2 blocks [Ecker, Grimus, Neufeld, 1987].

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Some phenomenology of the model

The model is similar to the usual Inert Doublet Model (IDM) but with elaborate interaction pattern within the inert sector.

$$V_1 = \underbrace{\lambda_5(3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right]}_{\text{similar to } \lambda_5(\phi_2^\dagger \phi_1)^2} + \underbrace{\lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right]}_{\text{new}} + h.c.$$

- Extending J to the entire lagrangian: $\phi_{2,3}$ decouple from fermions, the J-symmetric minimum is $(v, 0, 0)$, inert scalars protected from decay to SM fields.
- The scalar spectrum is exactly IDM-like: a pair of degenerate $H^\pm$, and two pairs of degenerate neutrals.

Some phenomenology of the model

Possible to diagonalize the mass matrix staying within complex neutral fields with a non-holomorphic map $(\phi_2^0, \phi_3^0) \mapsto (\Phi, \varphi)$:

$$\begin{pmatrix} \Phi \\ \varphi \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2^0 + \phi_3^{0*} \\ \phi_3^0 - \phi_2^{0*} \end{pmatrix} .$$

with $\tan 2\gamma = -\lambda_6/\lambda_5$. Complex fields Φ and φ are eigenstates of mass,

$$M^2,\, m^2 = -m_{22}^2 + \frac{v^2}{2}\left(\lambda_3 + \lambda_4 \pm \sqrt{\lambda_5^2 + \lambda_6^2}\right) ,$$

and are also eigenstates of J with charges $q = +1$:

$$J: \quad \Phi \mapsto i\Phi\,, \quad \varphi \mapsto i\varphi\,.$$

Half-CP-oddness

The real complex fields Φ, φ have weird CP-properties:

$$J: \quad \Phi \mapsto i\Phi\,, \quad \varphi \mapsto i\varphi\,.$$

They are neither CP-even nor CP-odd but are half-CP-odd.

NB: J, which was antiunitary in the ϕ_i doublet space, becomes unitary in (Φ, φ)-space!

Conserved quantum number: not $\mathbb{Z}_2$-parity but the charge q defined modulo 4.

Some phenomenology of the model

The map from (ϕ_2^0, ϕ_3^0) to (Φ, φ) conserves the norm implying

$$|\partial_\mu \phi_2^0|^2 + |\partial_\mu \phi_3^0|^2 = |\partial_\mu \Phi|^2 + |\partial_\mu \varphi|^2\,,$$

while the interaction potential contains only combinations

$$\varphi^*\varphi\,, \quad \varphi^4\,, \quad (\varphi^*)^4\,, \quad \varphi^2(\varphi^*)^2\,, \quad \text{where } \varphi \text{ stands for } \Phi \text{ or } \varphi,$$

all of which conserve q. Transitions $\varphi^* \to \varphi\varphi\varphi$, $\varphi\varphi \to \varphi^*\varphi^*$, or loop-induced $\varphi \leftrightarrow \Phi$ as possible, while $\varphi \to \varphi^*$ are forbidden by q conservation.

Instead of ZHA vertex in CP-conserving 2HDM, with H and A of opposite CP-parities, we have $Z\Phi\varphi$ vertex, with two scalars of the same CP-properties:

$$\text{instead of} \quad (+1)\cdot(-1) = -1 \quad \text{we have} \quad i \cdot i = -1\,.$$

Conclusions

Generation of CP-violation from the scalar sector of multi-Higgs models still has room for surprises.

- We conjectured that the intimate relation between Higgs family symmetry groups and the two forms of CPV, known for decades and so far observed in all cases, is a general phenomenon. We proved this conjecture for rephasing symmetry groups.
- We remarked on a peculiar form of explicit CPV which has no spontaneous CPV counterpart $\to$ deserves further study.
- We found a counterexample to the general claim that the explicit CPV requires existence of a real basis. This counterexample is based on a order-4 gCP in 3HDM without any other symmetry; no such example existed in 2HDM.
- This model resembles the IDM with a more elaborate inert sector and with inert scalars displaying "half-*CP*-oddness".