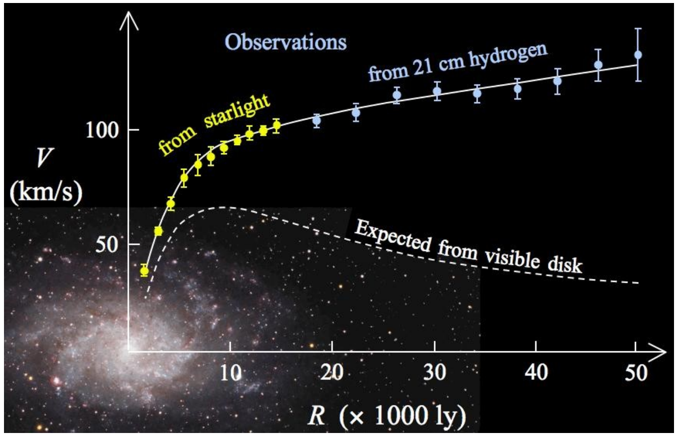
Mateusz Duch
University of Warsaw

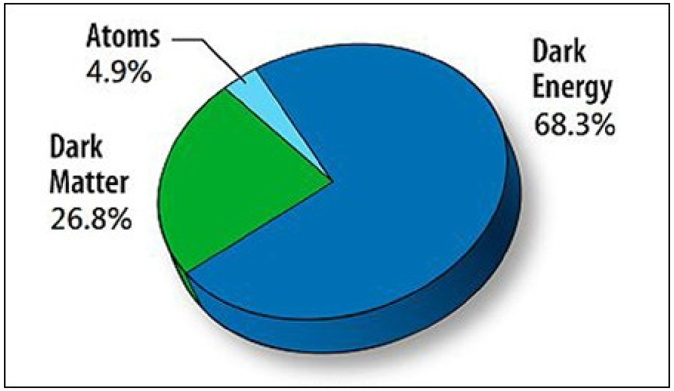
Gauge-invariant approach to dark matter resonant annihilation

Harmonia meeting V
7 December 2018

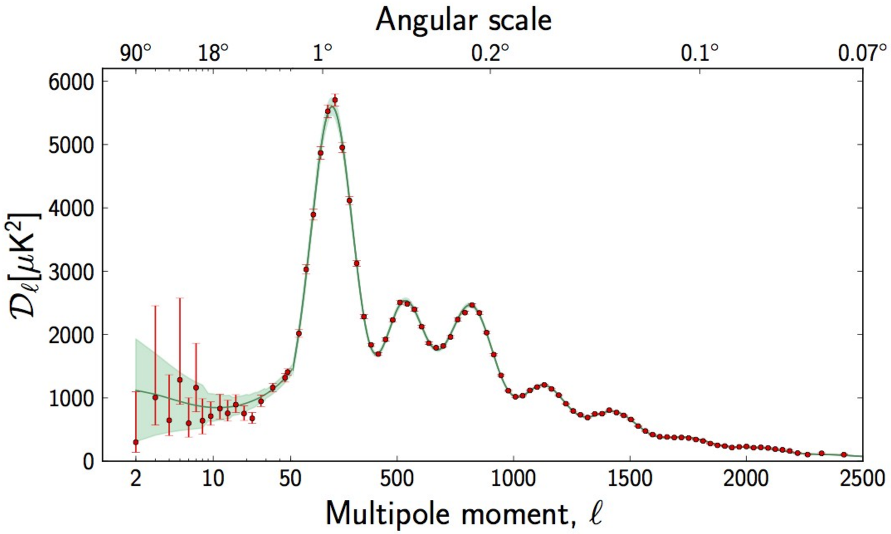
MD, Bohdan Grzadkowski, JHEP 1709 (2017) 159 [1705.10777]
MD, Bohdan Grzadkowski, Apostolos Pilaftsis, Gauge-dependence of the dark matter resonant annihilation, in preparation

Dark matter – motivation





Convincing evidence on various astrophysical and cosmological scales



leading hypothesis → new, unknown particle

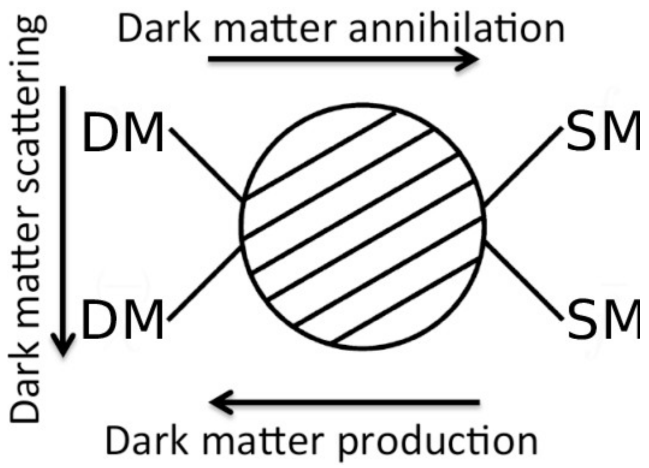
Dark matter – motivation

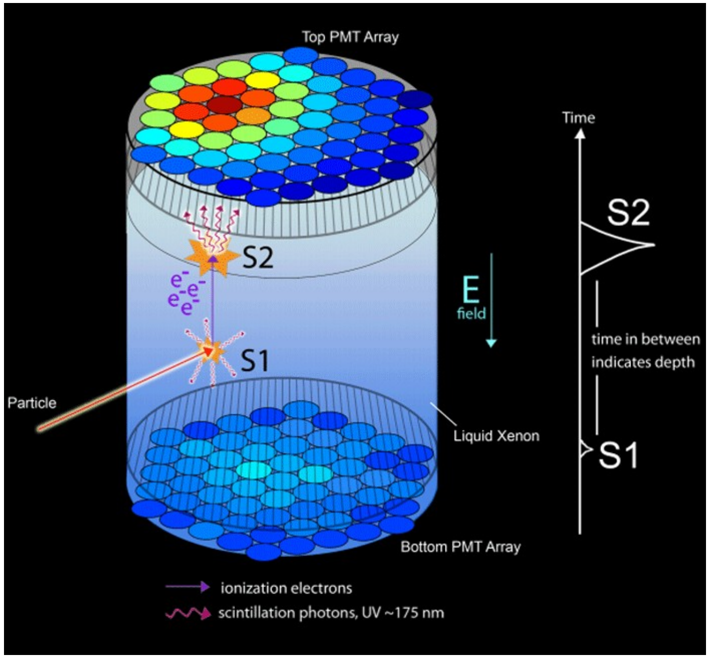
Properties of particle dark matter:

- electrically neutral (non-luminous),
- non-relativistic (cold) (structure formation)
- stable or long-lived
- weakly interacting with ordinary matter

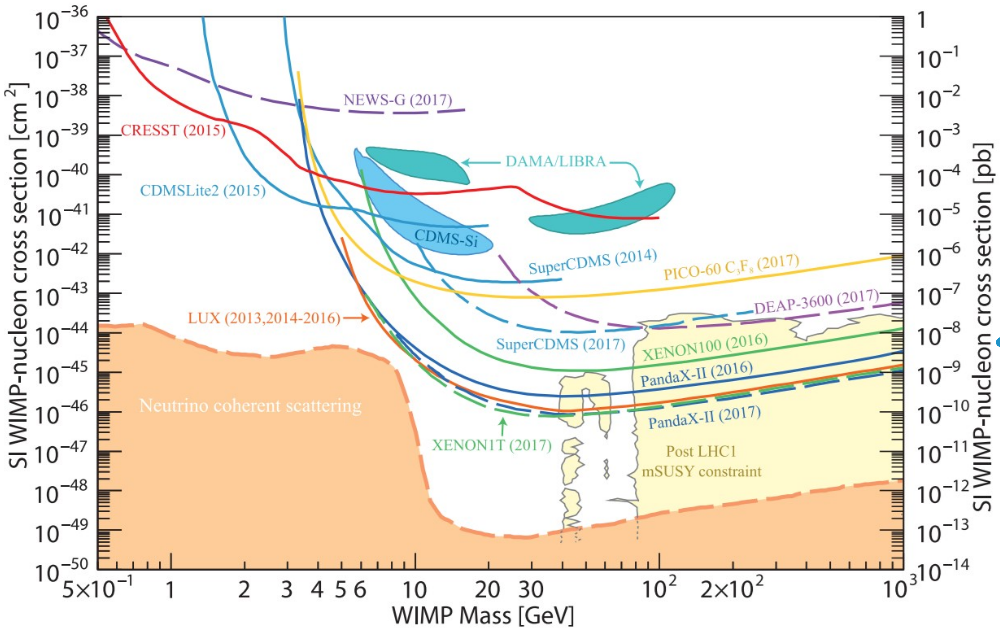
Dark matter interactions:

- annihilation – production in the early universe and indirect detection (FERMI-LAT, MAGIC, H.E.S.S, ...)
- production – collider searches
- scattering on nucleons – direct detection (LUX, PANDA, XENON 1T)

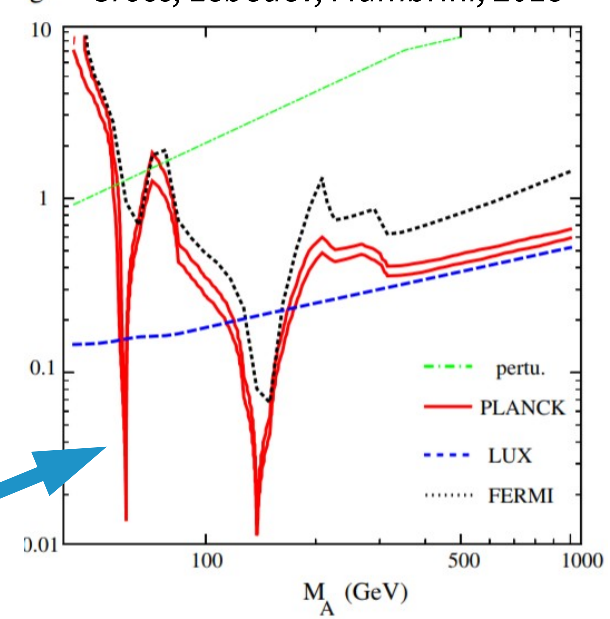




Resonance region

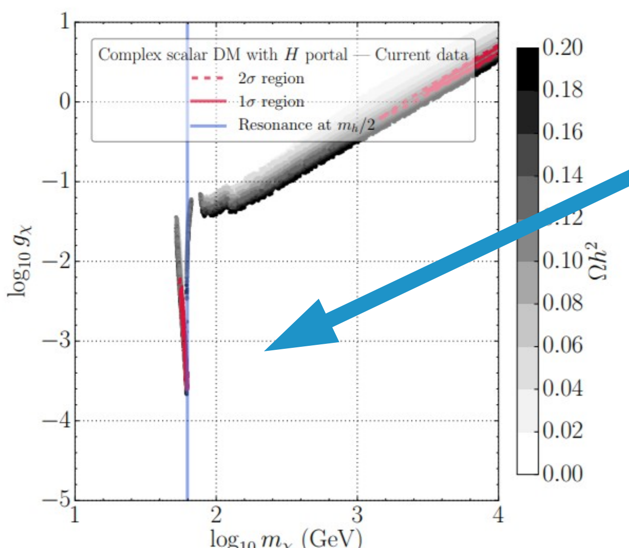


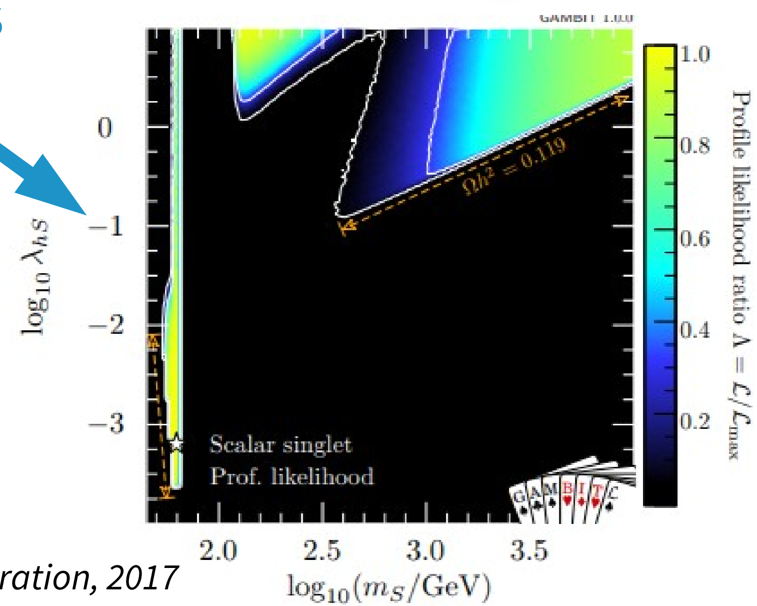
Gross, Lebedev, Mambrini, 2015



STRONG CONSTRAINTS

SMALL COUPLING REGIONS ARE RELEVANT





Ellis, Fowlie, Marzola, Raidal, 2017

GAMBIT Collaboration, 2017

Breit-Wigner resonance

Breit-Wigner resonance $2M_{\mathrm{DM}} \approx M_{\mathrm{R}}$

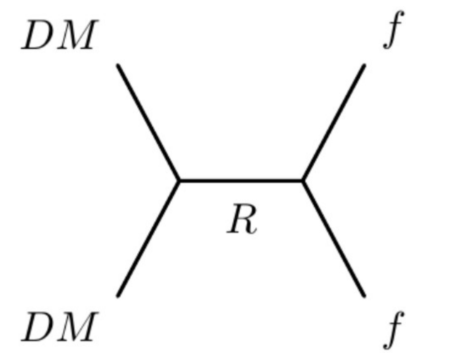
enhanced annihilation → suppressed coupling

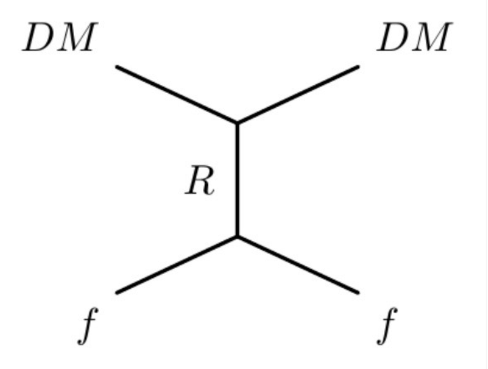
low sensitivity to direct detection

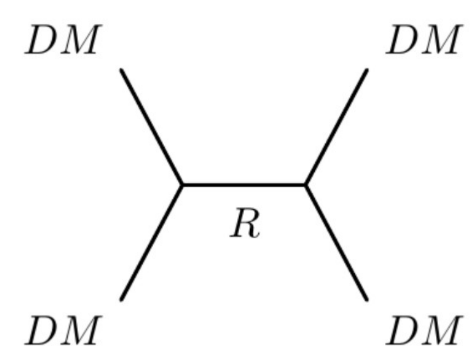
- velocity dependent cross-section → possibility of enhanced indirect detection signals
- kinetic decoupling $T_{\mathrm{DM}} \neq T_{\mathrm{SM}}$
- large self-interaction cross-section constrained by indirect detection

- proper description of annihilation amplitudes

  – gauge-invariance
  – unitarity







Standard freeze-out mechanism

Boltzmann equation for DM phase space density $f_{DM}(\vec{p},t)$ $\quad L[f]=C[f]$

$$g\int L[f_{DM}]\frac{d^3p}{2\pi^3}\to \qquad \frac{dn}{dt}+3Hn=-\langle\sigma v_{\rm rel}\rangle(n^2-n_{\rm EQ}^2) \qquad \leftarrow g\int C[f_{DM}]\frac{d^3p}{2\pi^3}$$

DM yield $\quad Y=n/s, \quad x=M_{\rm DM}/T \quad$ n – number density, s – entropy density

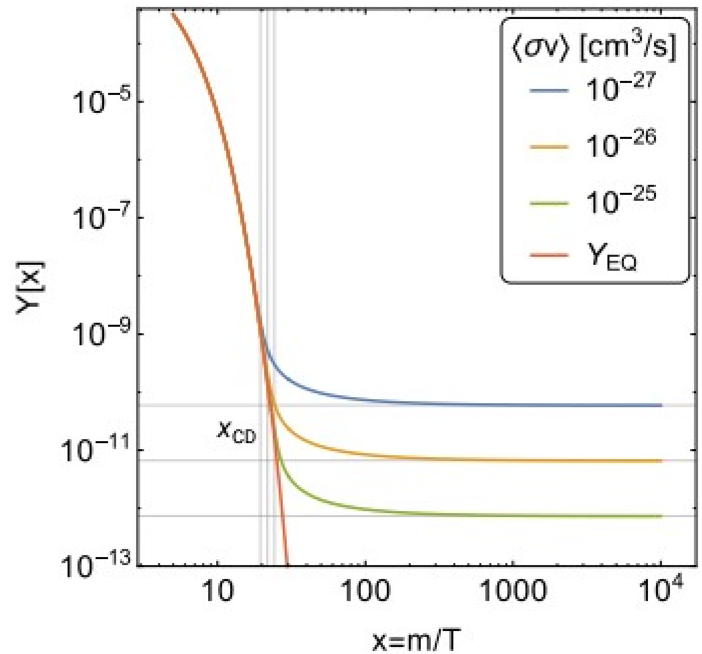
$$\frac{dY}{dx}=-\alpha\frac{\langle\sigma v_{\rm rel}\rangle}{x^2}(Y^2-Y_{\rm EQ}^2), \qquad \alpha=\frac{s(M_{\rm DM})}{H(M_{\rm DM})}$$

DM chemical decoupling

$$n_{EQ}\langle\sigma v_{\rm rel}\rangle\sim H(x)$$

Approximate solution

$$Y_\infty\approx\frac{x_{CD}}{\alpha\langle\sigma v_{\rm rel}\rangle}$$

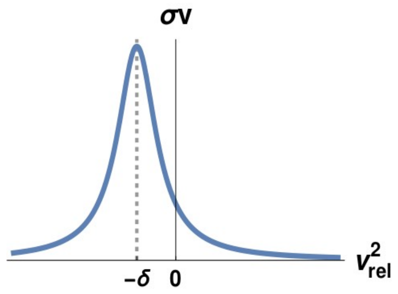


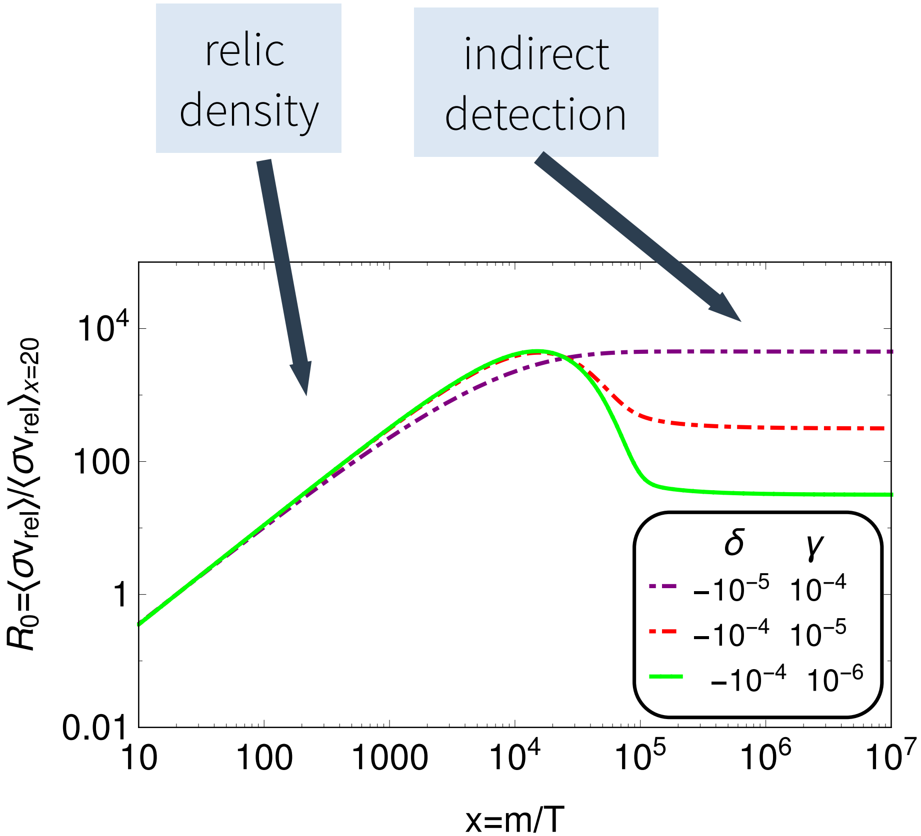
Standard assumption: DM is kinetically coupled to SM during freeze-out,
i.e. it has the same temperature as the SM thermal bath ← not always the case

MD, Bohdan Grządkowski 1705.10777, Binder et al 1706.07433

Breit-Wigner approximation

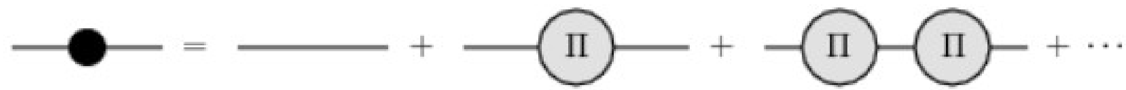
$$\sigma \sim \frac{1}{(s-M_R^2)^2+M_R^2\Gamma_{tot}^2}$$





- thermally averaged cross section grows with falling temperature
- prolonged period of effective annihilation
- strong temperature dependence

Resummed propagator



Dyson resummed propagator

$$\frac{1}{s - m_R^2 + i\,\mathrm{Im}\,\Pi_R(s)}$$

Self-energy

$$\Pi_R(s) = \Pi_{\mathrm{non-DM}}(s) + \Pi_{DM}(s)$$

$$2M_{\mathrm{DM}} \approx M_{\mathrm{R}}$$

other SM or dark sector fields

DM contribution

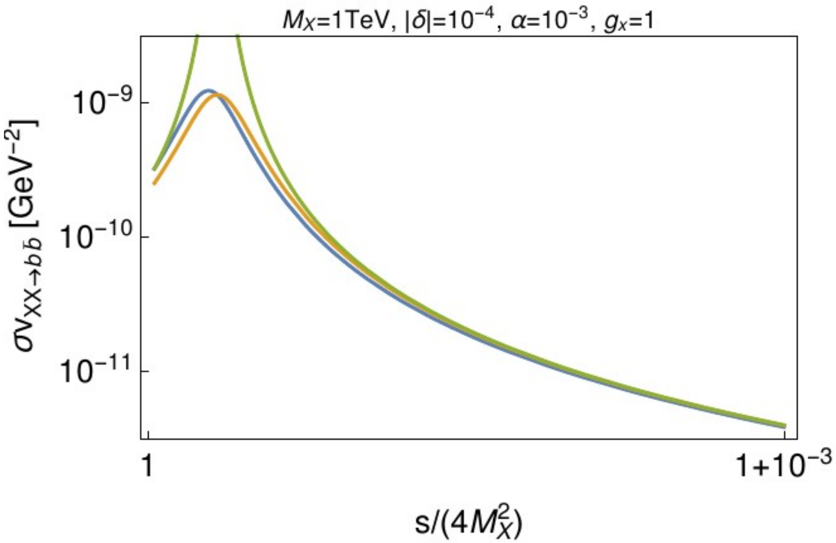
no nearby thresholds ⇨ Breit-Wigner approximation

$$\mathrm{Im}\,\Pi_{\mathrm{non-DM}}(s) \approx \mathrm{Im}\,\Pi_{\mathrm{non-DM}}(M_R^2) = M_{DM}\Gamma_{\mathrm{non-DM}}$$

nearby threshold $s \gtrsim 4M_{DM}^2$

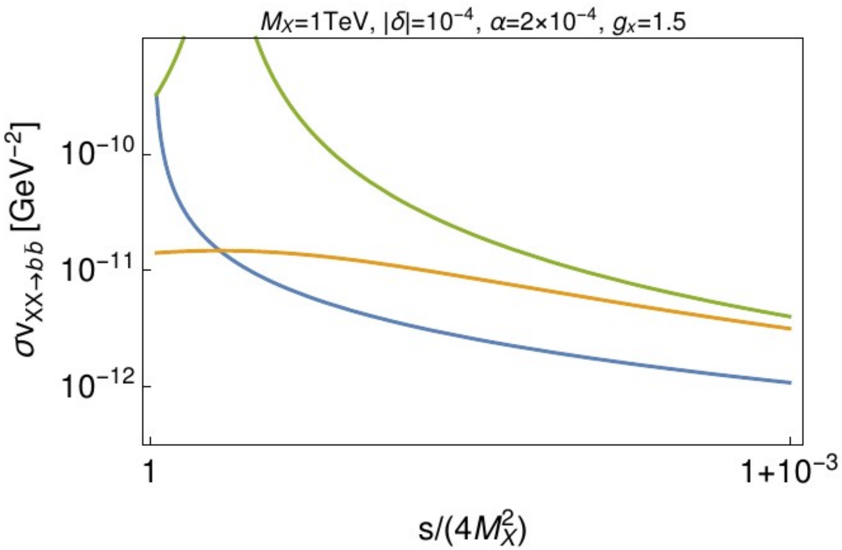
$$\mathrm{Im}\,\Pi_{DM}(s) \sim \sqrt{1 - 4M_{DM}^2/s}$$

Beyond Breit-Wigner approximation



$$\Gamma_{non-DM} \gg \Gamma_{DM}$$

$$\mathrm{Im}\,\Pi_{\mathrm{non-DM}}(s) \approx \mathrm{Im}\,\Pi_{\mathrm{non-DM}}(M_R^2) = M_{DM}\Gamma_{\mathrm{non-DM}}$$



$$\Gamma_{non-DM} \ll \Gamma_{DM}$$

$$\mathrm{Im}\,\Pi_{DM}(s) \sim \sqrt{1 - 4M_{DM}^2/s}$$

Abelian vector dark matter

Additional complex scalar field S

- singlet of $U(1)_Y \times SU(2)_L \times SU(3)_c$, charged under $U(1)_X$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + (D_\mu S)^* D^\mu S + \tilde{V}(H,S)$$

$$V(H,S) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2$$

Vacuum expectation values: $\langle H \rangle = \frac{v_{SM}}{\sqrt{2}}$, $\langle S \rangle = \frac{v_x}{\sqrt{2}}$

Dark $U(1)_X$ vector gauge boson X_μ

- Stability condition - no mixing of $U(1)_X$ with $U(1)_Y$ $\cancel{B_{\mu\nu} V^{\mu\nu}}$

  $\mathcal{Z}_2 : V_\mu \to -V_\mu$, $S \to S^*$, $S = \phi e^{i\sigma}$: $\phi \to \phi$, $\sigma \to -\sigma$
- Higgs mechanism in the hidden sector $M_X = g_x v_x$

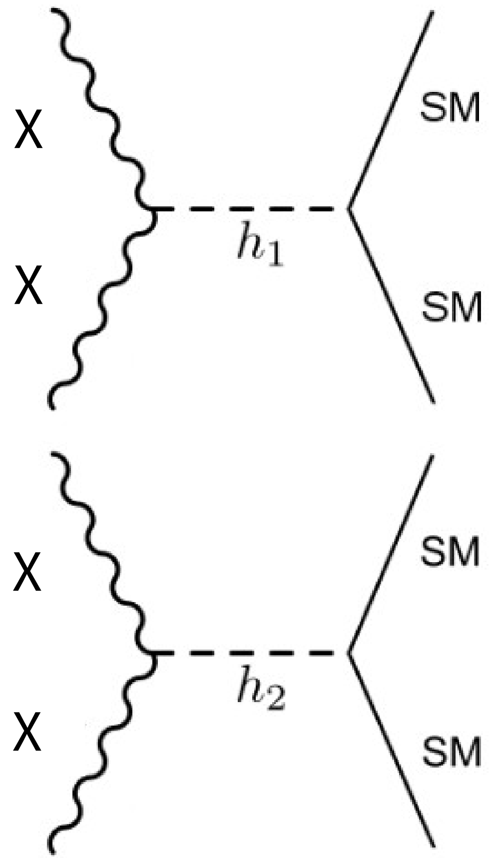
Higgs couplings – mixing angle α, $M_{h_1} = 125$ GeV

$$\mathcal{L} \supset \frac{h_1 c_\alpha + h_2 s_\alpha}{v} \left(2 M_W W_\mu^+ W^{\mu -} + M_Z^2 Z_\mu Z^\mu - m_f \bar{f} f \right) + \frac{h_1 s_\alpha - h_2 c_\alpha}{v_x} M_X^2 X_\mu X^\mu$$

Resonance with a Higgs scalars

$\langle \sigma v_{\rm rel} \rangle \propto \sin\alpha \cos\alpha$

Small α required by relic abundance



Resonance with the SM-like Higgs

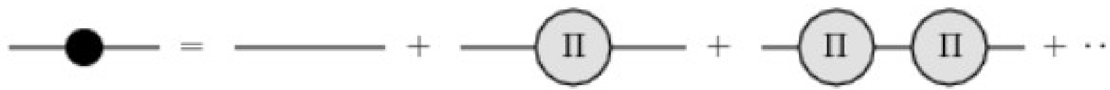
- $M_X \approx 125/2$ GeV
- decay channel $h_1 \to XX$, if open suppressed by $\sin^2\alpha$ and by phase space

$$\sqrt{1 - 4M_X^2/M_{h_1}^2} = \sqrt{\delta} \ll 1 \qquad \Gamma_{h_1 \to XX} \ll \Gamma_{SM}$$

Resonance with the second Higgs

- $M_X \approx M_{h_2}/2$ GeV
- $h_2 \to SMSM$ suppressed by $\sin^2\alpha$, $h_2 \to XX$ can dominate
- $\Gamma_{h_1 \to XX} \sim \sqrt{1 - 4M_X^2/s}$

Resummed propagator – gauge dependence



$$\frac{1}{s - m_R^2 + i\,\mathrm{Im}\,\Pi_R(s)}$$

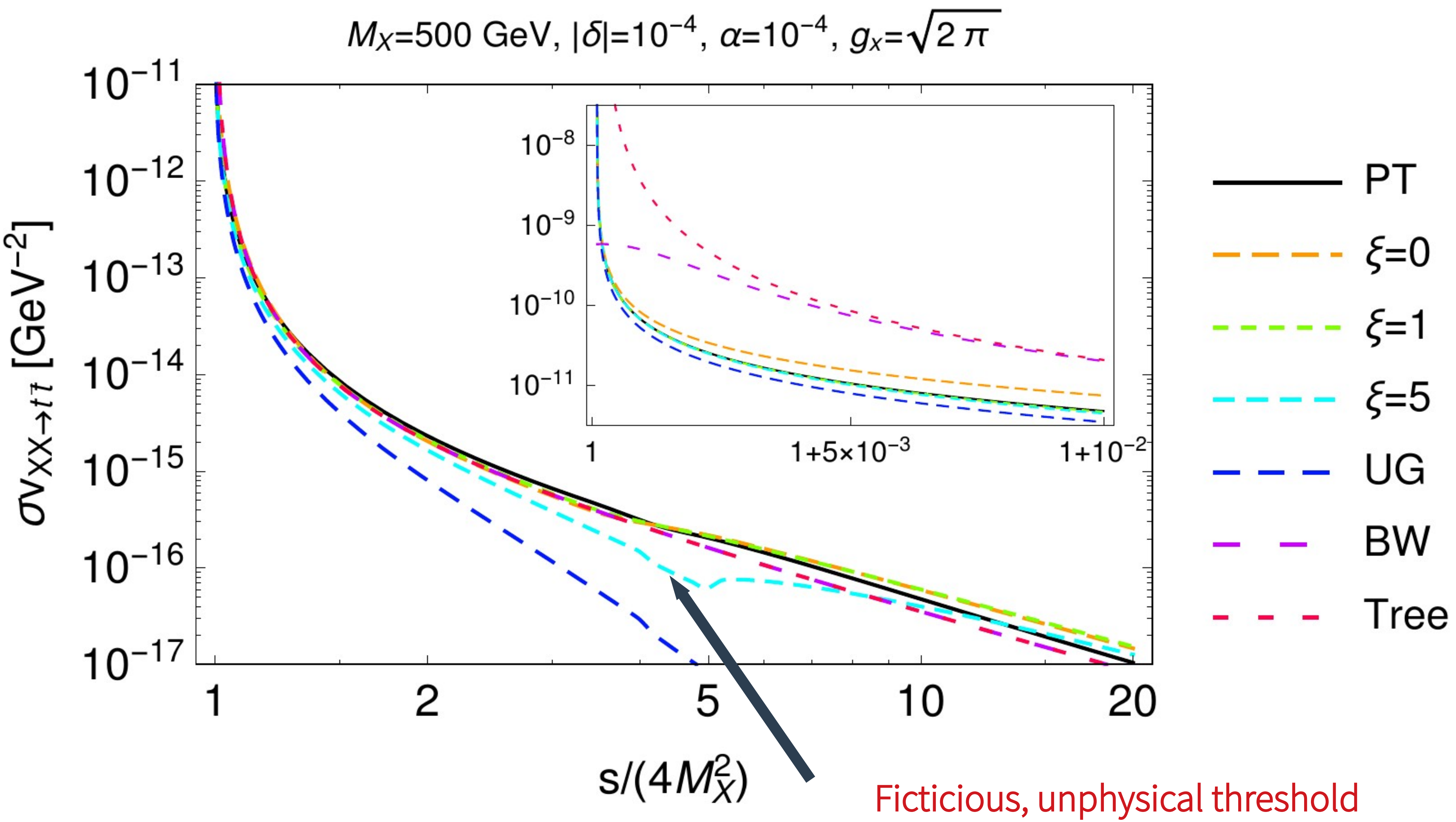
Dark vector boson contribution to the Higgs self-energy in R_ξ gauge

$$\Pi_{ij}^{(XX)}(s) = \frac{g_x^2 R_{2i} R_{2j}}{32\pi^2 M_X^2}\Big[\left(s^2 - 4M_X^2 s + 12 M_X^4\right) B_0(s, M_X^2, M_X^2) \\ - (s^2 - m_i^2 m_j^2) B_0(s, \xi_X M_X^2, \xi_X M_X^2)\Big]$$

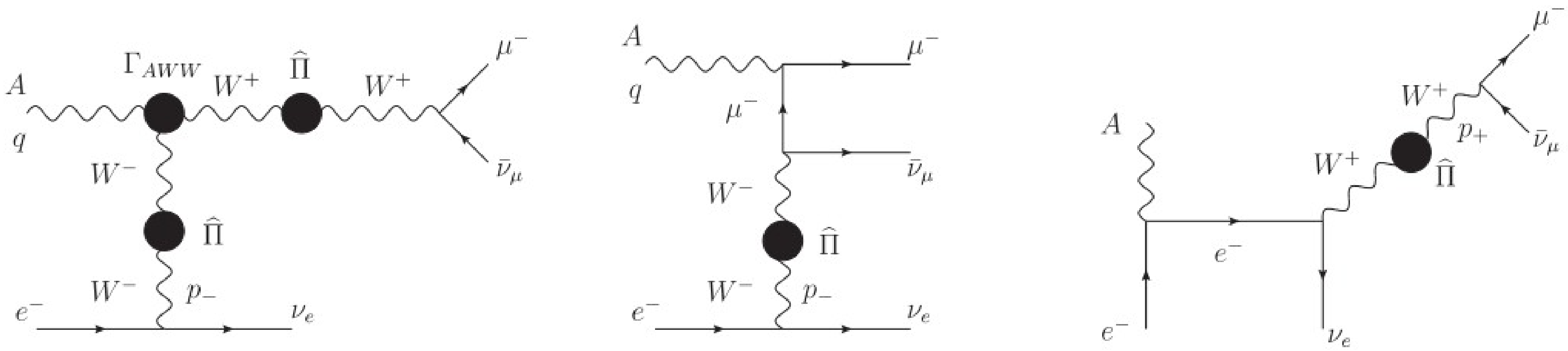
Problems with self-energy

- Explicit dependence on gauge fixing parameter
- Presence of s^2 term – modification of high-energy behaviour
- Problems with unitarity

Cross-section for XX->bb process



External gauge invariance



Gauge invariance imposes a relation for the tree-level amplitude that can be checked using elementary Ward identities:

$$\boxed{q^\alpha T^{(0)}_\alpha = 0}$$

$$k_\nu \gamma^\nu = (\not{k} + \not{p} - m) - (\not{p} - m) \qquad q_\alpha \Gamma^{\alpha\mu\nu}_{AWW} = (p_-^2 - M_W^2) g^{\mu\nu} - (p_+^2 - M_W^2) g^{\mu\nu}$$

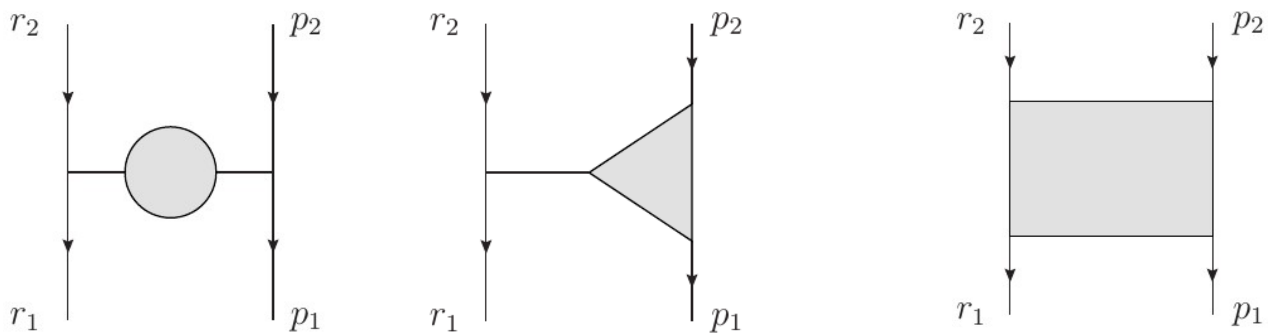
Using the resummed propagator that includes only specific contributions from every order in the perturbative expansion distorts the subtle cancellation that arise order by order in the perturbation theory due to the more complicated Slavnov-Taylor identities, however if we could use

$$\widehat{\Delta}^{\mu\nu}_W(p_\pm) = \frac{-ig^{\mu\nu}}{p_\pm^2 - M_W^2 + \widehat{\Pi}_{WW}(p_\pm^2)} \quad \text{and} \quad q_\alpha \widehat{\Gamma}^{\alpha\mu\nu}_{AWW} = \widehat{\Pi}^{\mu\nu}_{WW}(p_-) - \widehat{\Pi}^{\mu\nu}_{WW}(p_+)$$

we get $q_\alpha \left[\Gamma^{\alpha\mu\nu}_{AWW} + \widehat{\Gamma}^{\alpha\mu\nu}_{AWW}\right] = \widehat{\Delta}^{-1}_W(p_-) g^{\mu\nu} - \widehat{\Delta}^{-1}_W(p_+) g_{\mu\nu}$ and $\boxed{q_\alpha \widehat{T}^\alpha_{\text{res}} = 0}$

Pinch Technique

Reorganization of the sub-amplitudes that have the same kinematical properties



$$T(s,t,m_i) = \widehat{T}_1(s) + \widehat{T}_2(s,m_i) + \widehat{T}_3(s,t,m_i)$$

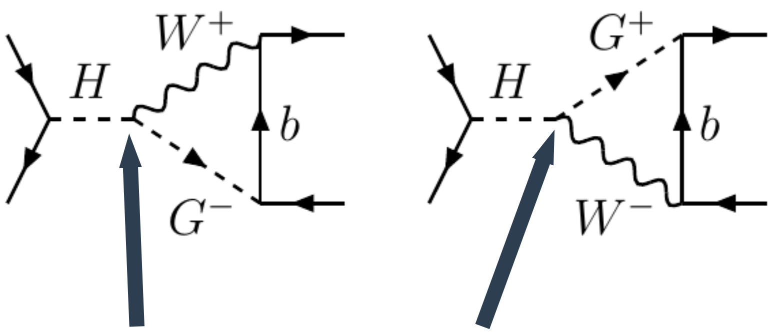
Individually gauge invariant

We have to look for the propagator-like pieces inside vertex and box diagrams

Pinching out loop momenta tt→ H*→ tt

Employing Ward identites $\not{k} P_L = (\not{k} + \not{p} - m_b)P_L - P_R(\not{p} - m_t) + m_b P_L - m_t P_R$

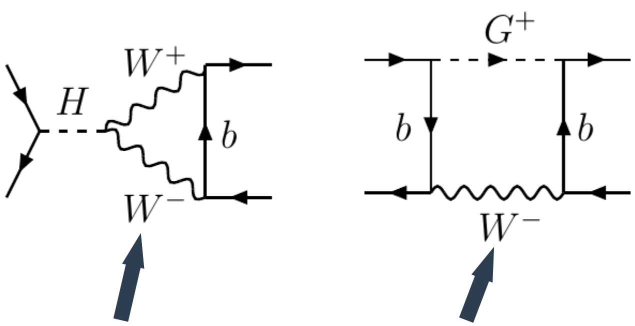
Starting from the Feynman gauge



$$-\frac{\alpha_w}{8\pi} m_t B_0(q^2, M_W^2, M_W^2)\,\bar{v}(p')\,u(p) + \dots$$

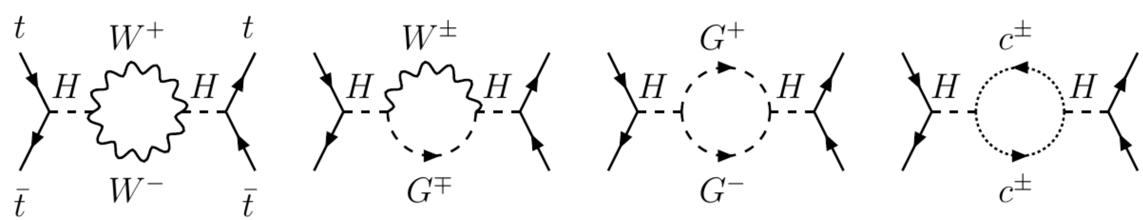
Momentum dependent vertices

In general renormalizable gauge



Momenta in the gauge boson propagator

$$\Delta_{\mu\nu}^{(\xi_w)}(q) = \left[-g_{\mu\nu} + (1-\xi_w)\frac{q_\mu q_\nu}{q^2 - \xi_w M_W^2}\right]\frac{1}{q^2 - M_W^2}$$

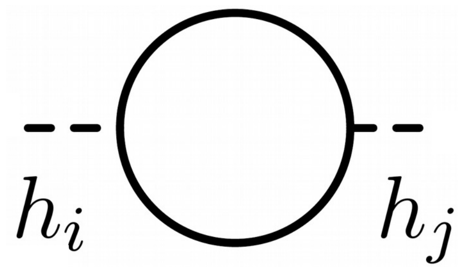


The gauge-fixing parameter dependence cancels because the full amplitude is gauge-independent

Equivalent to calculations in Feynman background field gauge Denner+ 1994, Binosi+ 2002

Model with scalar mixing and vector dark matter

Contributions to Higgs self-energy X, Z, W, f, h



$$\hat{\Pi}^{(XX)}_{ij}(s) = \frac{g_x^2 R_{2i} R_{2j}}{8\pi^2}\left[\frac{(m_i m_j)^2}{4M_X^2} + \frac{m_i^2 + m_j^2}{2} - (2s - 3M_X^2)\right] B_0(s, M_X^2, M_X^2)\,,$$

$$\hat{\Pi}^{(ZZ)}_{ij}(s) = \frac{g^2 R_{1i} R_{1j} M_Z^2}{32\pi^2 M_W^2}\left[\frac{(m_i m_j)^2}{4M_X^2} + \frac{m_i^2 + m_j^2}{2} - (2s - 3M_Z^2)\right] B_0(s, M_Z^2, M_Z^2)\,,$$

$$\hat{\Pi}^{(WW)}_{ij}(s) = \frac{g^2 R_{1i} R_{1j}}{32\pi^2}\left[\frac{(m_i m_j)^2}{4M_X^2} + \frac{m_i^2 + m_j^2}{2} - (2s - 3M_W^2)\right] B_0(s, M_W^2, M_W^2)\,,$$

$$\hat{\Pi}^{(tt)}_{ij}(s) = \frac{3g^2 R_{1i} R_{1j} m_t^2}{32\pi^2 M_W^2}\left(s - 4m_t^2\right) B_0(s, m_t^2, m_t^2)\,,$$

$$\hat{\Pi}^{(h_k h_l)}_{ij}(s) = \frac{-V^h_{ikl} V^h_{jkl}}{32\pi^2} B_0(s, m_{h_k}^2, m_{h_l}^2)\,.$$

no fictitious threshold

no s^2 terms

Resummation of the propagator with scalar mixing

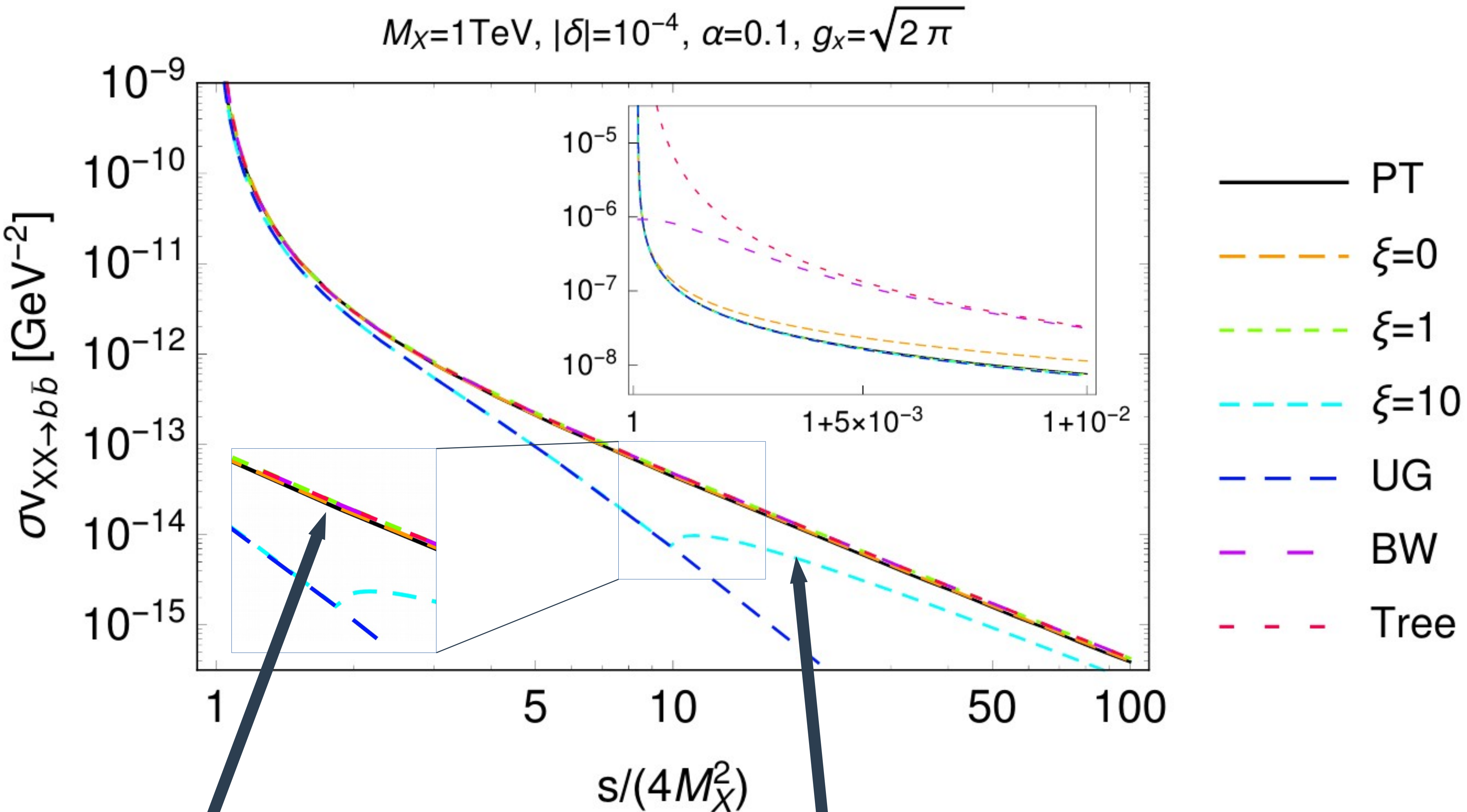
$$i\hat{\Delta} = i\Delta_0 + i\Delta_0\, i\hat{\Pi}\, i\Delta_0 + i\Delta_0\,(i\hat{\Pi}\, i\Delta_0)^2 + \ldots$$

diagonal tree-level propagator

$$\hat{\Delta}(s) = \frac{1}{D(s)}\begin{pmatrix} s - m_2^2 + \hat{\Pi}_{22}(s) & -\hat{\Pi}_{12}(s) \\ -\hat{\Pi}_{21}(s) & s - m_1^2 + \hat{\Pi}_{11}(s) \end{pmatrix}$$

$$D(s) = \left[s - m_1^2 + \hat{\Pi}_{11}(s)\right]\left[(s - m_2^2 + \hat{\Pi}_{22}(s)\right] - \hat{\Pi}_{12}(s)\hat{\Pi}_{21}(s)$$

Cross-section for XX->bb process



Feynman gauge and PT result are similar

Ficticious, unphysical threshold

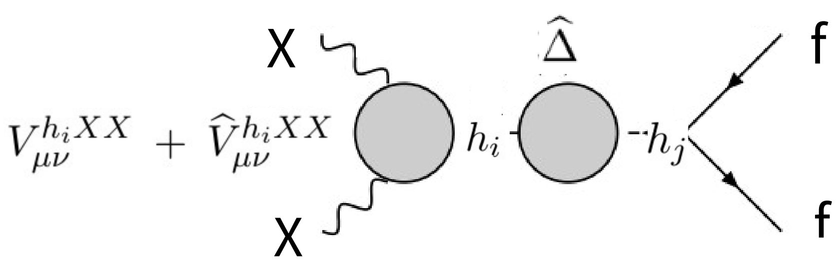
Generalized equivalence theorem (GET)

Tree-level like Ward identites are satisfied by the PT self-energies and vertices

$$
\begin{aligned}
p_2^\nu \widehat{V}^{h_i XX}_{\mu\nu}(q,p_1,p_2) + iM_X \widehat{V}^{h_i X G_X}_{\mu} &= -g_x R_{2i} \widehat{\Pi}^{XG_X}_{\mu}(p_1) \\
p_1^\mu \widehat{V}^{h_i X G_X}_{\mu} + iM_X \widehat{V}^{h_i G_X G_X} &= -g_x \Big[R_{2j} \widehat{\Pi}_{ji}(q^2) + R_{2i} \widehat{\Pi}^{G_X G_X}(p_2) \Big], \\
p_1^\mu p_2^\nu \widehat{V}^{h_i XX}_{\mu\nu} + M_X^2 \widehat{V}^{h_i G_X G_X} &= i g_x M_X \Big[R_{2j} \widehat{\Pi}_{ji}(q^2) + R_{2i} \Big(\widehat{\Pi}^{G_X G_X}(p_1) + \widehat{\Pi}^{G_X G_X}(p_2) \Big) \Big] \\
\widehat{\Pi}^{XG_X}_{\mu}(p) &= -\frac{iM_X p_\mu}{p^2} \widehat{\Pi}^{G_X G_X}(p^2)
\end{aligned}
$$

One can check they lead to the generalized equivalence theorem at tree-level

$$
\begin{aligned}
\mathcal{A}_{X_L(p_1)X_L(p_2)\to \bar{f}f} &= -\mathcal{A}_{G_X(p_1)G_X(p_2)\to \bar{f}f} - i\mathcal{A}_{x^\mu(p_1)G_X(p_2)\to \bar{f}f} \\
&\quad - i\mathcal{A}_{G_X(p_1)x^\nu(p_2)\to \bar{f}f} + \mathcal{A}_{x^\mu(p_1)x^\nu(p_2)\to \bar{f}f} \,.
\end{aligned}
$$



$$x^\mu(p_1) \equiv \epsilon_L^\mu(p_1) - \frac{p_1^\mu}{M_X}$$

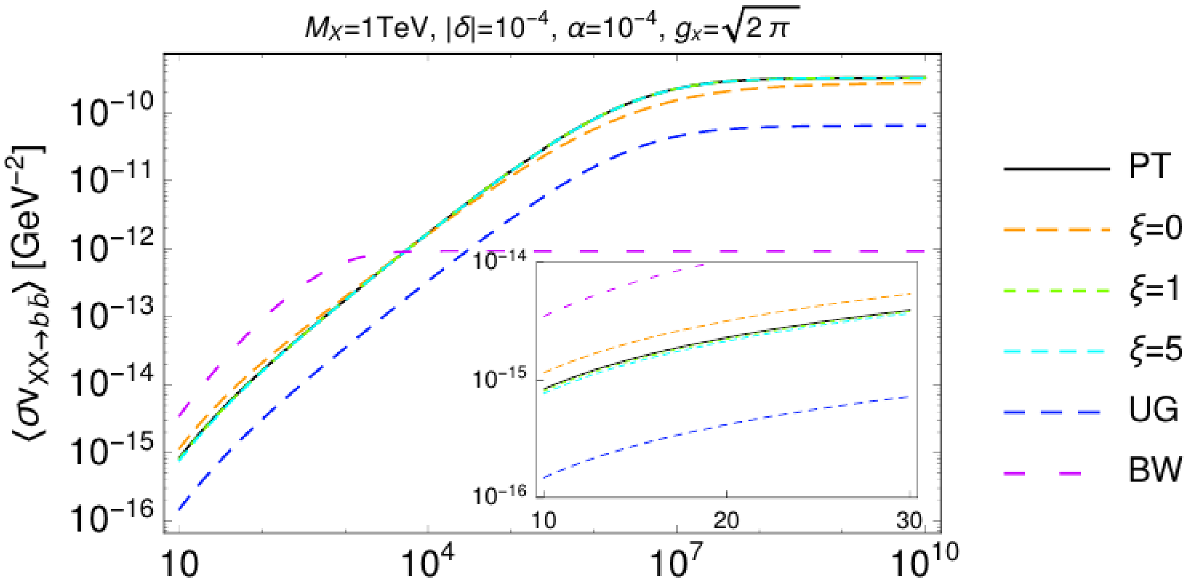
subleading part of longitudinal polarization

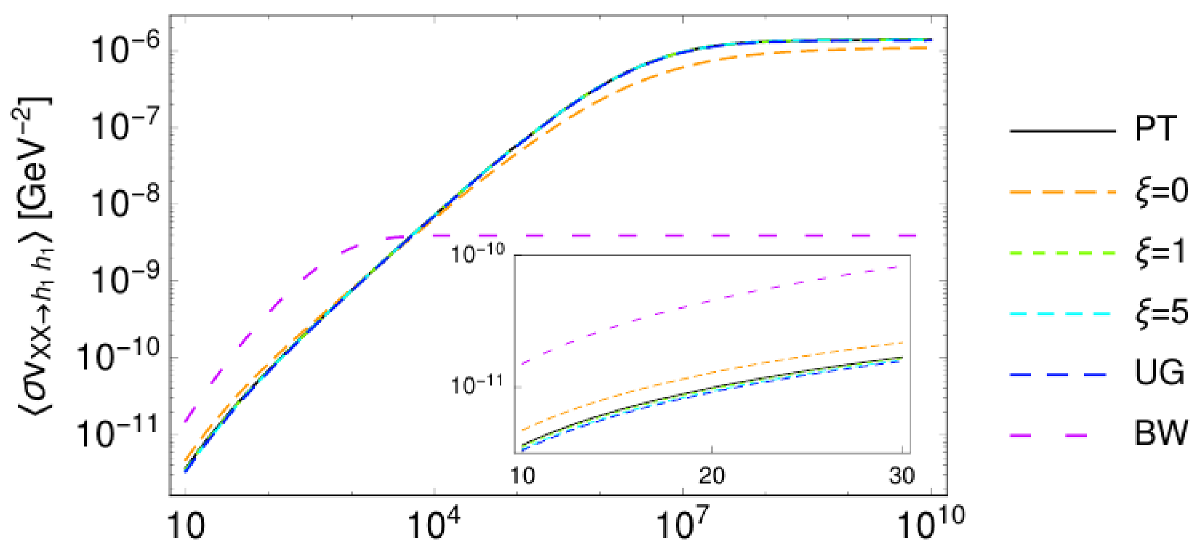
$$iA^{XX\to \bar{f}f}_{\mu\nu} = \sum_{ij} (V^{XXh_i}_{\mu\nu} + \hat{V}^{XXh_i}_{\mu\nu}) i\widehat{\Lambda}_{ij} V^{h_j \bar{f} f}$$

Condition that guaranties good high-energy behaviour of the process with resummed propagator

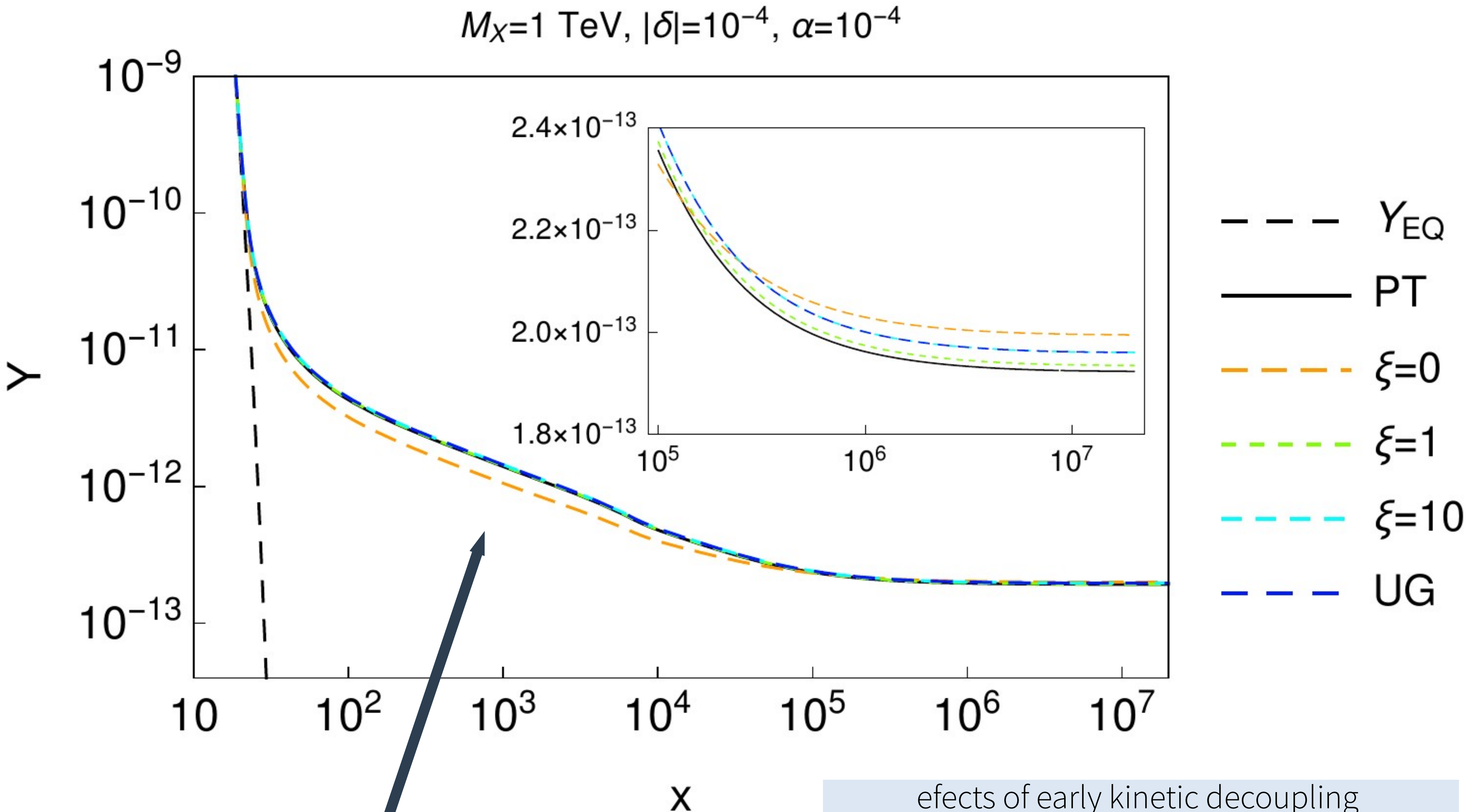
$$p_1^\mu p_2^\nu \widehat{\Gamma}^{h_i XX}_{\mu\nu}(q,p_1,p_2) = i g_x M_X \, R_{2j} \widehat{\Delta}^{-1}_{ji}(q^2) + \mathcal{O}\big[\ln(s/M_X^2)\big]$$

Thermally averaged cross-sections





Relic density calculation



prolonged period of effective annihilation

efects of early kinetic decoupling
Included *[1705.10777]*

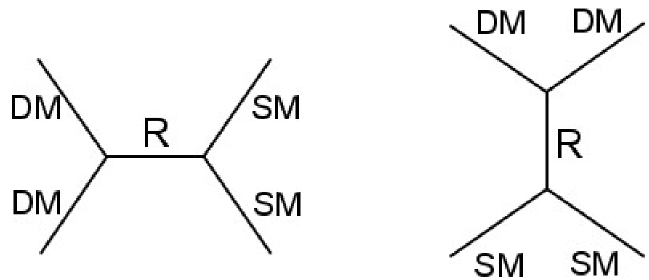
Summary

- resonance region is a viable part of many otherwise strongly constraint dark matter model
- the Breit-Wigner approximation fails if mediator couples dominantly to the dark matter state
- relativistic treatment of resonant amplitude requires proper resummation technique
- pinch technique provides a method respecting gauge invariance and unitarity what results in proper behaviour near the resonance and in the high energy limit

BACKUP

Kinetic decoupling – simplified picture

Scatterings on the abundant relativistic SM states → thermal equilibrium



- relic abundance requires small coupling between DM and SM
- scattering process is not resonantly enhanced

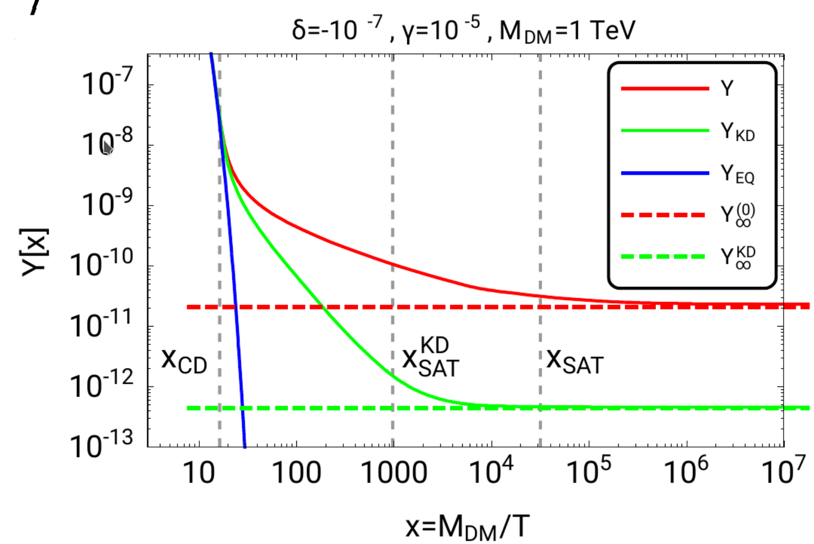
Comparision of the Hubble rate to scattering rate

$$H(T_{kd}) \sim \Gamma_{\rm scat}(T_{kd}) \Rightarrow x_{kd}\left(\frac{\max[\delta,\gamma]^{3/2}}{10^{-6}}\right)^{\frac{1}{4}} \Longrightarrow T_{KD} \sim T_{CD}$$

Kinetic and chemical decoupling temperatures are comparable

$$T_{\rm DM} = \begin{cases} T_{\rm SM}, & \text{for } T \geq T_{\rm KD} \ = T_{\rm CD} \\ T_{\rm SM}^2/T_{\rm KD}, & \text{for } T < T_{\rm KD} \ = T_{\rm CD} \end{cases}$$

non-relativistic expanding gas



Kinetic decoupling

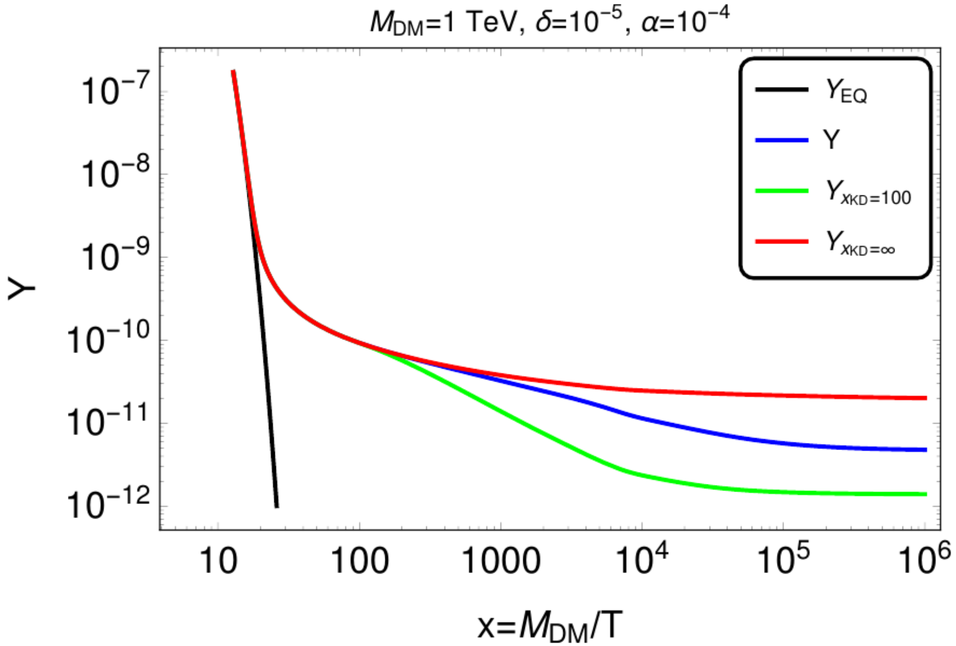
Second moment of Boltzmann equation $\int p^2 L[f] = \int p^2 C[f]$

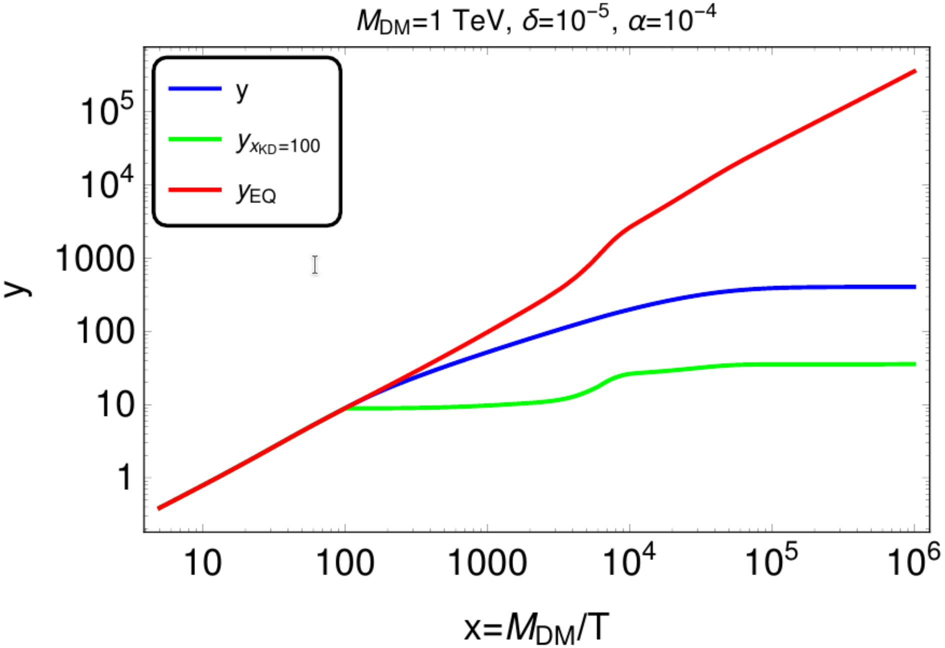
Temperature parameter $T_{DM} \propto \int p^2 f(p) d^3p \qquad y \equiv \frac{M_{DM} T_{\rm DM}}{s^{2/3}}$

Coupled set of Boltzmann equations

$$\frac{dY}{dx} = -\frac{1 - \frac{x}{3}\frac{g'_{*s}}{g_{*s}}}{Hx} s \left(Y^2 \langle \sigma v_{\rm rel} \rangle_{x_{DM}(y)} - Y_{EQ}^2 \langle \sigma v_{\rm rel} \rangle_x \right)$$

$$\frac{dy}{dx} = -\frac{1 - \frac{x}{3}\frac{g'_{*s}}{g_{*s}}}{Hx} \left[2 M_{DM} c(T)(y - y_{EQ}) - sy \Big(Y \left(\langle \sigma v_{\rm rel} \rangle_{x_{DM}} - \langle \sigma v_{\rm rel} \rangle_2 |_{x_{DM}} \right) - \frac{Y_{EQ}^2}{Y} \left(\langle \sigma v_{\rm rel} \rangle_x - \frac{y_{EQ}}{y} \langle \sigma v_{\rm rel} \rangle_2 |_x \right) \Big) \right]$$





Background field gauge

In the conventional formalism directly the fields appearing in the classical Lagrangian are quantized. A gauge-fixing term is added to $\mathcal{L}_C$ which breaks the explicit gauge invariance.

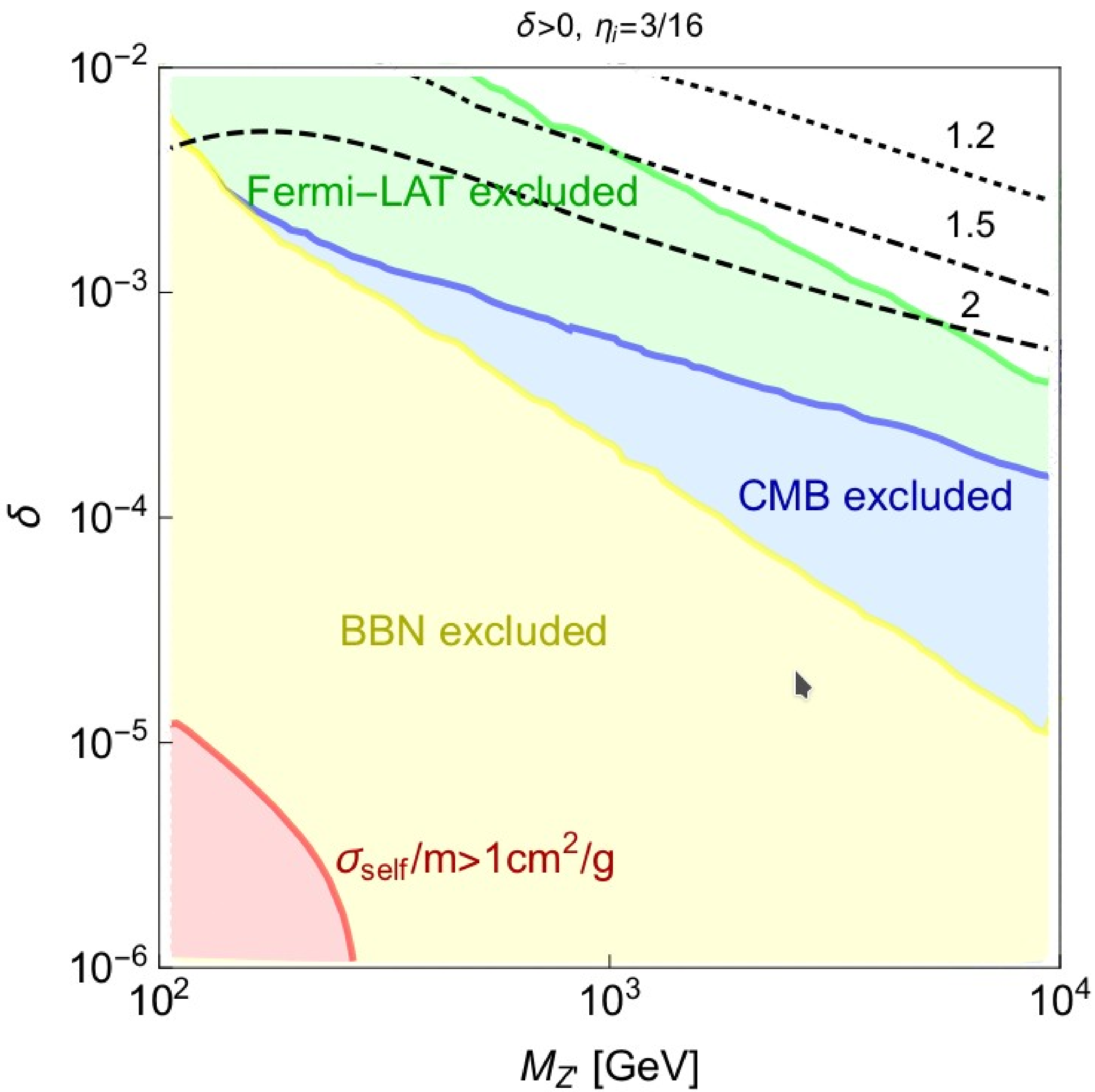
Instead, when going from the classical to the quantized theory in the BFM [1,2], the fields $\hat{V}$ of $\mathcal{L}_C$ are split into classical background fields $\hat{V}$ and quantum fields V,

$$\mathcal{L}_C(\hat{V}) \to \mathcal{L}_C(\hat{V}+V). \tag{14}$$

Gauge fixing breaks the invariance of only quantum fields

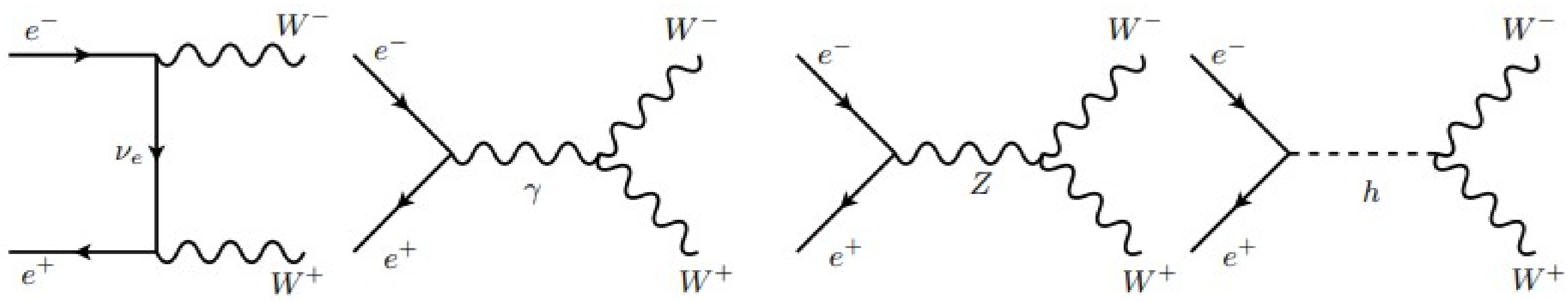
$$\mathcal{L}_{GF} = -\frac{1}{2\xi_Q^W}\left[(\delta^{ac}\partial_\mu + g_2\varepsilon^{abc}\hat{W}_\mu^b)W^{c,\mu} - ig_2\xi_Q^W\frac{1}{2}(\hat{\Phi}_i^\dagger\sigma_{ij}^a\Phi_j - \Phi_i^\dagger\sigma_{ij}^a\hat{\Phi}_j)\right]^2 - \frac{1}{2\xi_Q^B}\left[\partial_\mu B^\mu + ig_1\xi_Q^B\frac{1}{2}(\hat{\Phi}_i^\dagger\Phi_i - \Phi_i^\dagger\hat{\Phi}_i)\right]^2,$$

Bounds in the parameter space



- mixing angle set by relic density constraint
- maximal dark gauge coupling satisfying perturbative unitarity
- enhancement of low-velocity cross-section → strong bounds from indirect searches
- effects of early kinetic decoupling modify relic density by up to a factor of 2 in the allowed region

Unitarity



Neglecting the Higgs contribution the amplitude grows as s^1/2 violating the unitarity

$$\frac{1}{s-m_h^2} \Rightarrow \frac{1}{s-m_h^2+i\mathrm{Im}\Pi(s)}$$

The s^2 term is proportional to

$B_0(s, M_W^2, M_W^2) - B_0(s, \xi_W M_W^2, \xi_W M_W^2).$

and vanish for $s \gg M_W^2$ and $s \gg \xi_W M_W^2$

$$\Pi_{HH}^{(WW)}(s,\xi_W) = \frac{\alpha_w}{4\pi}\left[\left(\frac{s^2}{4M_W^2} - s + 3M_W^2\right) B_0(s, M_W^2, M_W^2) + \frac{M_H^4 - s^2}{4M_W^2} B_0(s, \xi_W M_W^2, \xi_W M_W^2)\right].$$

Unitarity restoration can be arbitraly delayed