The scalar field on Einstein-Maxwell background with cosmological constant

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To explore and understand the realm of the gravitational physics in different dimensions, finding different types of exact solutions to Einstein gravity (especially in the presence of matter fields) is of great importance.

Few examples of solutions:

Black hole solutions with different non-trivial horizons in higher dimensions, such as 3-sphere horizon, black rings with 2-sphere times 1-sphere horizon, black saturn and black lens in five dimensions.

Solutions to higher dimensional Einstein-Maxwell Theory which are asymptotically locally flat, de Sitter and anti-de-Sitter with non-vanishing NUT charges.

Convoluted-like solutions in Einstein-Maxwell Theory in six and higher dimensions

Dyonic and solitonic solutions, braneworld cosmologies and string theory extended solutions.

Inclusion of the dilaton field as the simplest matter fields to Einstein-Maxwell theory provides the simplest system of gravity-force-matter.

A lot of interest to find and explore the solutions and physical properties of solutions in Einstein-Maxwell-dilaton theory with/without cosmological constant and also extended theories with Chern-Simons term or Liouville potential.

Moreover, in the context of generalized Freund-Rubin compactification
(FR compactification refers to solutions of 11d supergravity that are compactified on a 7-sphere to four-dimensional AdS; these solutions are vacuum of M-theory and are dual to a 3d CFT of N M2 branes in flat space in large limit of N),
with a non-zero cosmological constant and a dilaton field, the Einstein-Maxwell-dilaton theory possesses two different coupling constants:
dilaton-Maxwell coupling
dilaton-cosmological constant coupling

Inspired by the above facts and also solutions in the low energy limit of heterotic string theory/M-theory with NUT twist, we consider the Einstein-Maxwell-dilaton theory with two coupling constants and find exact new analytical solutions.

Einstein-Maxwell-dilaton theory with two different non-zero coupling constants a and b

$$S = \frac{1}{16\pi}\int d^5x\sqrt{-g}\left\{R - \frac{4}{3}(\nabla\phi)^2 - e^{-4/3a\phi}F^2 - e^{4/3b\phi}\Lambda\right\}$$

The theory with two different coupling constants, arises in the context of generalized Freund-Rubin compactification with a cosmological constant and the dilaton field. The Einstein-Maxwell theory is special case of theory where both coupling constants vanish.

Varying the action with respect to the metric tensor:

$$R_{\mu\nu} - \frac{1}{3}\Lambda g_{\mu\nu}e^{4/3b\phi} - \left(F_{\mu}^{\lambda}F_{\nu\lambda} - \frac{1}{6}g_{\mu\nu}F^2\right)e^{-4/3a\phi} - \frac{4}{3}\nabla_{\mu}\phi\nabla_{\nu}\phi = 0,$$

Varying the action with respect to the gauge tensor:

$$\nabla_\mu (e^{-4/3a\phi} F^{\mu\nu}) = 0$$

Varying the action with respect to the dilaton field:

$$\nabla^2 \phi + a/2e^{-4a\phi/3} F^2 - b/2e^{4/3b\phi} \Lambda = 0$$

We consider the metric ansatz:

$$ds_5^2 = -\frac{1}{H(r)^2} dt^2 + R(t)^2 H(r) ds_n^2$$

time-dependent

$$ds_n^2 = V(r)(dr^2 + r^2 d\Omega^2) + \frac{(d\psi + n\cos\theta d\phi)^2}{V(r)}$$

$$V(r) = 1 + \frac{n}{r}$$

ds_n^2 is the exact solution to the vacuum Einstein equations in four dimensions. The presence of NUT charge leads to the existence of the Misner-Dirac strings running on the z-axis and the solution is asymptotically locally flat; a very non-trivial solution in four dimensions beside the Schwarzschild solution. The coordinate ψ parameterizes the Hopf fibration of a circle over the sphere.

The Misner-Dirac string can be removed by restricting the range of coordinate ψ.

Gauge field ansatz:

$$A_t(t,r) = \alpha R(t)^2 (F(r) - \beta)$$

constants

Dilaton field ansatz:

$$\phi(t,r) = -\frac{3}{4a}\ln\{R(t)^{\gamma}H(r)^{\delta}\}$$

constants

Maxwell equations:

$$\nabla_\mu(e^{-4/3a\phi}F^{\mu\nu}) = 0, \xrightarrow{\nu = t} F(r) = F_1 + h\int\frac{dr}{H(r)^{\delta+2}r^2}$$

$$\xrightarrow{\nu = r} \gamma = -4$$

Gravity equation:

$$\mathcal{G}_{\mu\nu} = 0 \xrightarrow{\mu = t \qquad \nu = r} \delta = a^2$$

Dilaton field:

$$\phi(t,r) = -\frac{3}{4a}\ln\left\{\frac{H(r)^{a^2}}{R(t)^4}\right\}$$

Other diagonal Einstein equations:

coupled non-linear differential equations for both metric functions

$$\bar{\mathcal{G}}_{tt} = -2\alpha^2 h^2 H(r)^{-2a^2} R(t)^4 a^2 - 36V(r)H(r)^{-a^2+5}R(t)^4 r^4\left(\frac{d}{dt}R(t)\right)^2 + 2\Lambda R(t)^{2\frac{2b+3a}{a}}H(r)^{-a^2-ab+3}V(r)r^4a^2$$
$$- 12H(r)^{-a^2+5}R(t)^5V(r)r^4a^2\left(\frac{d^2}{dt^2}R(t)\right) - 3H(r)^{-(a-1)(a+1)}R(t)^4r^4a^2\frac{d^2}{dr^2}H(r) + 3R(t)^4(H(r))^{-a^2}r^4a^2\left(\frac{d}{dr}H(r)\right)^2$$
$$- 6H(r)^{-(a-1)(a+1)}R(t)^4r^3a^2\frac{d}{dr}H(r), \tag{A1}$$

$$\bar{\mathcal{G}}_{rr} = 12R(t)^5H(r)^{-a^2+5}V(r)^2r^4\frac{d^2}{dt^2}R(t) + 36R(t)^4H(r)^{-a^2+5}V(r)^2r^4\left(\frac{d}{dt}R(t)\right)^2 - 6R(t)^4H(r)^{-a^2+1}V(r)r^4\frac{d^2}{dr^2}H(r)$$
$$- 6R(t)^4H(r)^{-a^2+2}r^4\frac{d^2}{dr^2}V(r) - 12R(t)^4H(r)^{-a^2}V(r)r^4\left(\frac{d}{dr}H(r)\right)^2 - 12R(t)^4H(r)^{-a^2+1}V(r)r^3\frac{d}{dr}H(r)$$
$$- 12R(t)^4H(r)^{-a^2+2}r^3\frac{d}{dr}V(r) - 8\Lambda R(t)^{2\frac{2b+3a}{a}}H(r)^{-a^2-ab+3}V(r)^2r^4 + 8\alpha^2h^2H(r)^{-2a^2}R(t)^4V(r)$$
$$- 9r^4V(r)R(t)^4H(r)^{-a^2}a^2\left(\frac{d}{dr}H(r)\right)^2, \tag{A2}$$

$$
\begin{aligned}
\bar{\mathcal{G}}_{\theta\theta} = {} & 6\frac{d^2}{dt^2}R(t)R(t)^5H(r)^{-a^2+5}V(r)^3r^4 + 18R(t)^4H(r)^{-a^2+5}V(r)^3r^4\left(\frac{d}{dt}R(t)\right)^2 - 3R(t)^4H(r)^{-a^2+1}V(r)^2r^4\frac{d^2}{dr^2}H(r) \\
& - 3R(t)^4H(r)^{-a^2+2}V(r)r^4\frac{d^2}{dr^2}V(r) + 3R(t)^4H(r)^{-a^2}V(r)^2r^4\left(\frac{d}{dr}H(r)\right)^2 + 3R(t)^4H(r)^{-a^2+2}r^4\left(\frac{d}{dr}V(r)\right)^2 \\
& - 6R(t)^4H(r)^{-a^2+1}V(r)^2r^3\frac{d}{dr}H(r) - 6R(t)^4H(r)^{-a^2+2}V(r)r^3\frac{d}{dr}V(r) \\
& - 3R(t)^4H(r)^{-a^2+2}n^2 - 4\Lambda R(t)^{2\frac{2b+3a}{a}}H(r)^{-a^2-ab+3}V(r)^3r^4 - 2R(t)^4H(r)^{-2a^2}\alpha^2h^2V(r)^2.
\end{aligned} \tag{A3}
$$

$$
\bar{\mathcal{G}}_{\phi\phi} = \cos^2\theta\bar{\mathcal{G}}^{(1)}_{\phi\phi} + \bar{\mathcal{G}}^{(2)}_{\phi\phi}
$$

$$
\begin{aligned}
\bar{\mathcal{G}}^{(1)}_{\phi\phi} = {} & 3R(t)^4H(r)^{-(a-1)(a+1)}V(r)^4r^6\frac{d^2}{dr^2}H(r) + 6R(t)^4H(r)^{-(a-1)(a+1)}V(r)^4r^5\frac{d}{dr}H(r) - 3R(t)^4H(r)^{-a^2+2}n^2r^4\left(\frac{d}{dr}V(r)\right)^2 \\
& - 6R(t)^5H(r)^{-a^2+5}V(r)^5r^6\frac{d^2}{dt^2}R(t) + 6R(t)^4H(r)^{-a^2+2}V(r)^3r^5\frac{d}{dr}V(r) + 4R(t)^{2\frac{3a+2b}{a}}H(r)^{-a^2-ab+3}\Lambda V(r)^5r^6 \\
& + 3R(t)^4H(r)^{-a^2+2}V(r)^3r^6\frac{d^2}{dr^2}V(r) - 3R(t)^4H(r)^{-a^2+2}V(r)^2r^6\left(\frac{d}{dr}V(r)\right)^2 - 18R(t)^4H(r)^{-a^2+5}V(r)^5r^6\left(\frac{d}{dt}R(t)\right)^2 \\
& + 3R(t)^4H(r)^{-a^2+2}n^2V(r)^2r^2 - 3R(t)^4H(r)^{-a^2}V(r)^4r^6\left(\frac{d}{dr}H(r)\right)^2 - 3R(t)^4H(r)^{-(a-1)(a+1)}V(r)^2n^2r^4\frac{d^2}{dr^2}H(r) \\
& - 6R(t)^4H(r)^{-(a-1)(a+1)}V(r)^2n^2r^3\frac{d}{dr}H(r) + 2R(t)^4h^2H(r)^{-2a^2}r^2V(r)^4\alpha^2 + 3R(t)^4H(r)^{-a^2}V(r)^2n^2r^4\left(\frac{d}{dr}H(r)\right)^2 \\
& + 3R(t)^4H(r)^{-a^2+2}V(r)n^2r^4\frac{d^2}{dr^2}V(r) + 18R(t)^4H(r)^{-a^2+5}V(r)^3n^2r^4\left(\frac{d}{dt}R(t)\right)^2 \\
& + 6R(t)^4H(r)^{-a^2+2}V(r)n^2r^3\frac{d}{dr}V(r) + 6R(t)^5H(r)^{-a^2+5}V(r)^3n^2r^4\frac{d^2}{dt^2}R(t) - 2h^2H(r)^{-2a^2}R(t)^4V(r)^2\alpha^2n^2 \\
& - 4\Lambda R(t)^{2\frac{3a+2b}{a}}H(r)^{-a^2-ab+3}V(r)^3n^2r^4 + 3R(t)^4H(r)^{-a^2+2}n^4.
\end{aligned} \tag{A5}
$$

$$\bar{\mathcal{G}}^{(2)}_{\phi\phi} = -6R(t)^4H(r)^{-(a-1)(a+1)}V(r)^4r^5\frac{d}{dr}H(r) - 3R(t)^4H(r)^{-(a-1)(a+1)}V(r)^4r^6\frac{d^2}{dr^2}H(r) - 2R(t)^4h^2H(r)^{-2a^2}r^2V(r)^4\alpha^2$$
$$+ 3R(t)^4H(r)^{-a^2}V(r)^4r^6\left(\frac{d}{dr}H(r)\right)^2 - 3R(t)^4H(r)^{-a^2+2}n^2V(r)^2r^2 + 18R(t)^4H(r)^{-a^2+5}V(r)^5r^6\left(\frac{d}{dt}R(t)\right)^2$$
$$- 6R(t)^4H(r)^{-a^2+2}V(r)^3r^5\frac{d}{dr}V(r) - 3R(t)^4H(r)^{-a^2+2}V(r)^3r^6\frac{d^2}{dr^2}V(r) + 3R(t)^4H(r)^{-a^2+2}V(r)^2r^6\left(\frac{d}{dr}V(r)\right)^2$$
$$- 4R(t)^{2\frac{3a+2b}{a}}H(r)^{-a^2-ab+3}\Lambda V(r)^5r^6 + 6R(t)^5H(r)^{-a^2+5}V(r)^5r^6\frac{d^2}{dt^2}R(t), \tag{A4}$$

$$\bar{\mathcal{G}}_{\psi\psi} = 6R(t)^5H(r)^{-a^2+5}V(r)^3r^4\frac{d^2}{dt^2}R(t) + 18R(t)^4H(r)^{-a^2+5}V(r)^3r^4\left(\frac{d}{dt}R(t)\right)^2 + 3R(t)^4H(r)^{-a^2+2}V(r)r^4\frac{d^2}{dr^2}V(r)$$
$$- 3R(t)^4H(r)^{-a^2+1}V(r)^2r^4\frac{d^2}{dr^2}H(r) - 3R(t)^4H(r)^{-a^2+2}r^4\left(\frac{d}{dr}V(r)\right)^2 + 3R(t)^4H(r)^{-a^2}V(r)^2r^4\left(\frac{d}{dr}H(r)\right)^2$$
$$+ 6R(t)^4H(r)^{-a^2+2}V(r)r^3\frac{d}{dr}V(r) - 6R(t)^4H(r)^{-a^2+1}V(r)^2r^3\frac{d}{dr}H(r)$$
$$+ 3R(t)^4H(r)^{-a^2+2}n^2 - 4\Lambda R(t)^{2\frac{3a+2b}{a}}H(r)^{-a^2-ab+3}V(r)^3r^4 - 2R(t)^4H(r)^{-2a^2}\alpha^2h^2V(r)^2, \tag{A6}$$

The only non-diagonal Einstein equation

$$
\begin{aligned}
\bar{\mathcal{G}}_{\psi\phi} = & -3R(t)^4 H(r)^{-a^2+2} n^2 - 6R(t)^5 H(r)^{-a^2+5} r^4 V(r)^3 \frac{d^2}{dt^2} R(t) - 18R(t)^4 H(r)^{-a^2+5} V(r)^3 r^4 \left(\frac{d}{dt} R(t)\right)^2 \\
& - 3R(t)^4 H(r)^{-a^2+2} V(r) r^4 \frac{d^2}{dr^2} V(r) + 3R(t)^4 H(r)^{-(a-1)(a+1)} V(r)^2 r^4 \frac{d^2}{dr^2} H(r) + 3R(t)^4 H(r)^{-a^2+2} r^4 \left(\frac{d}{dr} V(r)\right)^2 \\
& - 3R(t)^4 H(r)^{-a^2} V(r)^2 r^4 \left(\frac{d}{dr} H(r)\right)^2 - 6R(t)^4 H(r)^{-a^2+2} V(r) r^3 \frac{d}{dr} V(r) + 6R(t)^4 H(r)^{-(a-1)(a+1)} V(r)^2 r^3 \frac{d}{dr} H(r) \\
& + 4\Lambda R(t)^{2\frac{3a+2b}{a}} H(r)^{-a^2-ab+3} V(r)^3 r^4 + 2R(t)^4 H(r)^{-2a^2} \alpha^2 h^2 V(r)^2. \qquad \text{(A7)}
\end{aligned}
$$

The field equation for Dilaton

$$
\begin{aligned}
& 36V(r) R(t)^4 H(r)^{-a^2+3} r^4 \left(\frac{d}{dt} R(t)\right)^2 + 12V(r) R(t)^5 H(r)^{-a^2+3} r^4 \frac{d^2}{dt^2} R(t) + 3R(t)^4 H(r)^{-a^2-1} r^4 a^2 \frac{d^2}{dr^2} H(r) \\
& - 3R(t)^4 H(r)^{-a^2-2} r^4 a^2 \left(\frac{d}{dr} H(r)\right)^2 + 6R(t)^4 H(r)^{-a^2-1} a^2 r^3 \frac{d}{dr} H(r) + 4bR(t)^{2\frac{3a+2b}{a}} H(r)^{-a^2-ab+1} \Lambda V(r) r^4 a \\
& + 2a^2 \alpha^2 h^2 H(r)^{-2a^2-2} R(t)^4 = 0. \qquad \text{(A8)}
\end{aligned}
$$

Combining these equations with the goal of eliminating one of the two functions lead to a differential equation for the time-dependent metric function $R(t)$ with the solutions:

$$R(t) = (R_0 t + R_1)^m$$

where $$m = -\frac{a^2-2ab-2}{4(ab+1+b/a)}$$

However, the other equations imply a time dependent cosmological constant or a zero cosmological constant, unless

$$ab = -2$$

The differential equation for $R(t)$ then changes to

$$\left(\frac{dR}{dt}\right)^2 \left(a^3 - 2a - \frac{8}{a}\right) - R\left(\frac{d^2R}{dt^2}\right)(a^3 + 2a) = 0,$$

with solutions

$$R(t) = R_0 t^{\frac{a^2}{4}}$$

where we choose the constant in solution to be zero

The differential equation for $H(r)$ becomes

$$\left(\frac{dH}{dr}\right)^2 a^2 r + 2r\left(\frac{d^2H}{dr^2}\right)H(r) + 4\left(\frac{dH}{dr}\right)H(r) = 0$$

with solutions

$$H(r) = \left(\frac{r+h}{r}\right)^{\frac{2}{2+a^2}}$$

Moreover $F(r) = F_1 + h\int \frac{dr}{H(r)^{\delta+2} r^2} \longrightarrow F(r) = \frac{r}{r+h}$

We also find:

$$\alpha^2 = \frac{3}{2+a^2},$$

$$\Lambda = \frac{3a^2}{8}(a^2 - 1)$$

The cosmological constant can be zero, negative or positive

So, we find the solutions to the five-dimensional Einstein-Maxwell-dilaton as:

$$ds_5^2 = -(\frac{r}{r+h})^{\frac{4}{2+a^2}} dt^2 + t^{\frac{a^2}{2}} (\frac{r+h}{r})^{\frac{2}{2+a^2}} (\frac{r+n}{n})$$
$$\times \{dr^2 + r^2 d\Omega^2 + \frac{n^2(d\psi + n\cos\theta d\phi)^2}{(r+n)^2}\}$$

$$A_t(t,r) = R_0^2 \sqrt{\frac{3}{2+a^2}} t^{\frac{a^2}{2}} (\frac{r}{r+h} - \beta)$$

$$\phi(t,r) = -\frac{3}{4a} \ln\{\frac{(\frac{r+h}{r})^{\frac{2a^2}{2+a^2}}}{R_0^4 t^{a^2}}\}$$

In asymptotically region $r \to \infty$.

$$ds_5^2 = -dt^2 + t^{\frac{a^2}{2}}\{dr^2 + r^2 d\Omega^2 + (d\psi + n\cos\theta d\phi)^2\}$$

For a fixed time slice, it represents the fibration of a circle over a 2-spher. The Ricci scalar of the asymptotic metric is divergent at $t = 0$ due to vanishing of the spatial section of the asymptotic metric. Rescaling the asymptotic metric by the asymptotic value of the conformal factor $\lim_{r\to\infty} e^{-4/3a\phi}$

the Ricci scalar approaches

$t^{a^2-2}(\frac{1}{4}a^4 + 2a^2)$ which is finite or zero as long as a^2 is greater or equal to 2

Novelty of 5d solutions: Our solution can not be uplifted to higher dimensions

It is possible to uplift the d-dimensional solutions to Einstein-Maxwell-dilaton theory to (d+1)-dimensional solutions of Einstein gravity with a cosmological constant, only for special coupling constants

$$a = \pm\sqrt{\frac{3(d-1)}{d-2}} \text{ and } b = \mp\sqrt{\frac{3}{(d-1)(d-2)}}$$

Example of uplifting from five to six dimensions:

$$ds_6^2 = e^{+\frac{4}{3}b\phi} ds_5^2 + e^{-4b\phi}(dw + 2A_t dt)^2$$

for special coupling constants

5d solution to EMD

$$b = \mp\tfrac{1}{2} \text{ and } a = \pm 2$$

Our condition for the coupling constants $ab = -2$

Two main reasons why the uplifting doesn't work for the solutions:

$$ds_5^2 = -(\frac{r}{r+h})^{\frac{4}{2+a^2}} dt^2 + t^{\frac{a^2}{2}} (\frac{r+h}{r})^{\frac{2}{2+a^2}} (\frac{r+n}{n}) \times \{dr^2 + r^2 d\Omega^2 + \frac{n^2(d\psi + n\cos\theta d\phi)^2}{(r+n)^2}\}$$

First: The 5d metric is not diagonal due to presence of NUT charge; A carefully analysis of the uplifting procedure shows that it works only if the lower-dimensional metric is diagonal.

Second: The rescaling of the 5d metric by conformal factor $e^{+\frac{4}{3}b\phi}$ **(in the uplifted metric) yields a very singular metric for all values of the coupling constants a anc** $b = \frac{-2}{a}$ **.**

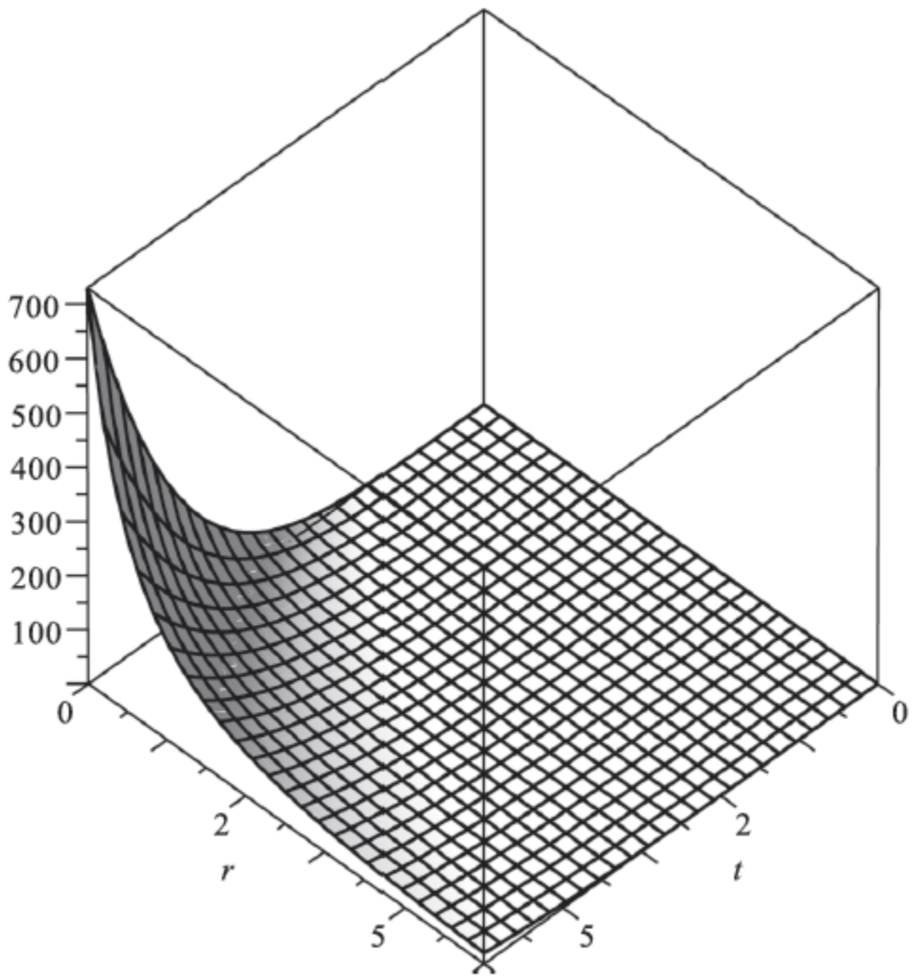
Moreover our solutions can not be uplifted to (5+d) dimensional Einstein-Maxwell theory with a cosmological constant.
In fact some solutions of 5d Einstein-Maxwell-dilaton theory can be lifted to solutions to to (5+d) dimensional Einstein-Maxwell theory with a cosmological constant:

$$\mathbf{S} = \frac{1}{16\pi}\int d^{5+d}x\sqrt{-\mathbf{g}}\{\mathbf{R} - F^2 - \Lambda\}$$

if the coupling constants are equal to: $a = b = \sqrt{\frac{d}{(3+d)}}$

In this case, the uplifted metric is the summation of the 5d metric multiplied by a conformal factor and a d-dimensional constant curvature space multiplied by another conformal factor. Both conformal factors can depend on coordinates of the 5d metric.

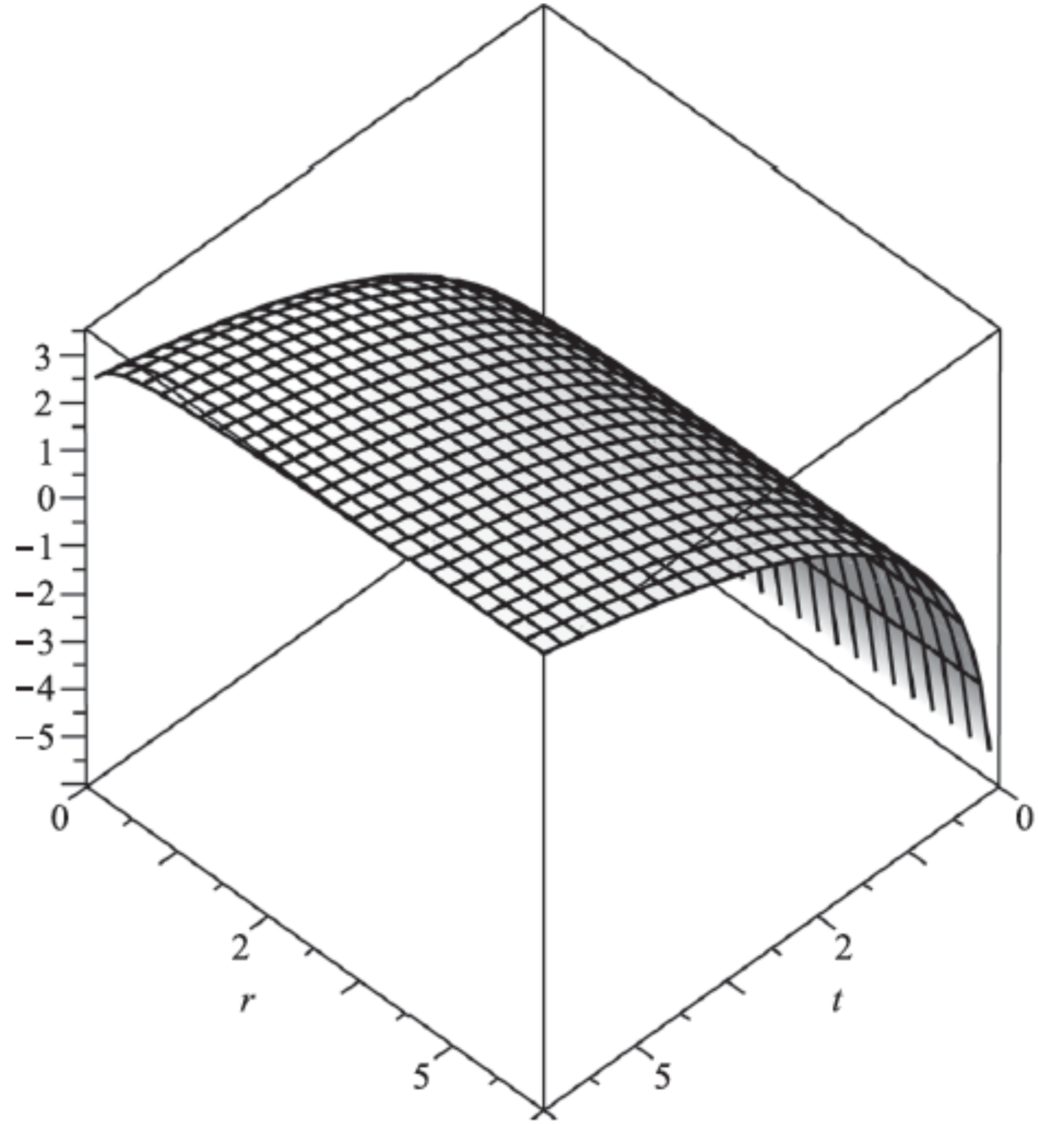
These two reasons together mean, we may need a new form of ansatz for the 6d metric, that the rescaling of the 5d metric is done by a conformal factor proportional to a and not b, when the lower-dimensional metric is non-diagonal.



Maxwell tensor:

$$F_{tr} = \sqrt{\frac{3}{2+a^2}}\frac{h}{(r+h)^2}\, t^{\frac{a^2}{2}}.$$

Dilaton field:



We may expect for $b \to 0$ or equivalently $a \to -\infty$, the solutions reduce to asymptotically dS solutions to the theory, however this expectation is not valid the Ricci scalar and the cosmological constant are not proportional.

Einstein-Maxwell-dilaton theory with $a = b$

depends on both time and r

$$ds_5^2 = -\frac{1}{H(t,r)^2}dt^2 + R(t)^2 H(t,r) ds_n^2,$$

$$A_t(t,r) = \frac{\alpha}{R(t)^{a^2}}(F(t,r) - \beta)$$

We consider the same ansatz for the dilaton field as previous case

$$\phi(t,r) = -\frac{3}{4a}\ln\{R(t)^\gamma H(r)^\delta\}$$

The Maxell equations and gravity equations

$$F(t,r) = F_1(t) + F_2(t)\int \frac{dr}{r^2 H(t,r)^{2+\delta}}$$

$$\gamma = 2a^2,\ \delta = a^2$$

$$F_2(t) = hR(t)^{-2-a^2}$$

$$\phi(t,r) = -\frac{3}{4a}\ln\{R(t)^{2a^2}H(t,r)^{a^2}\}$$

+,- or zero

Other equations:

$$H(t,r) = \left\{1 + \frac{h}{rR(t)^{2+a^2}}\right\}^{\frac{2}{2+a^2}}$$

$$\Lambda = \frac{3(4-a^2)}{2a^4}$$

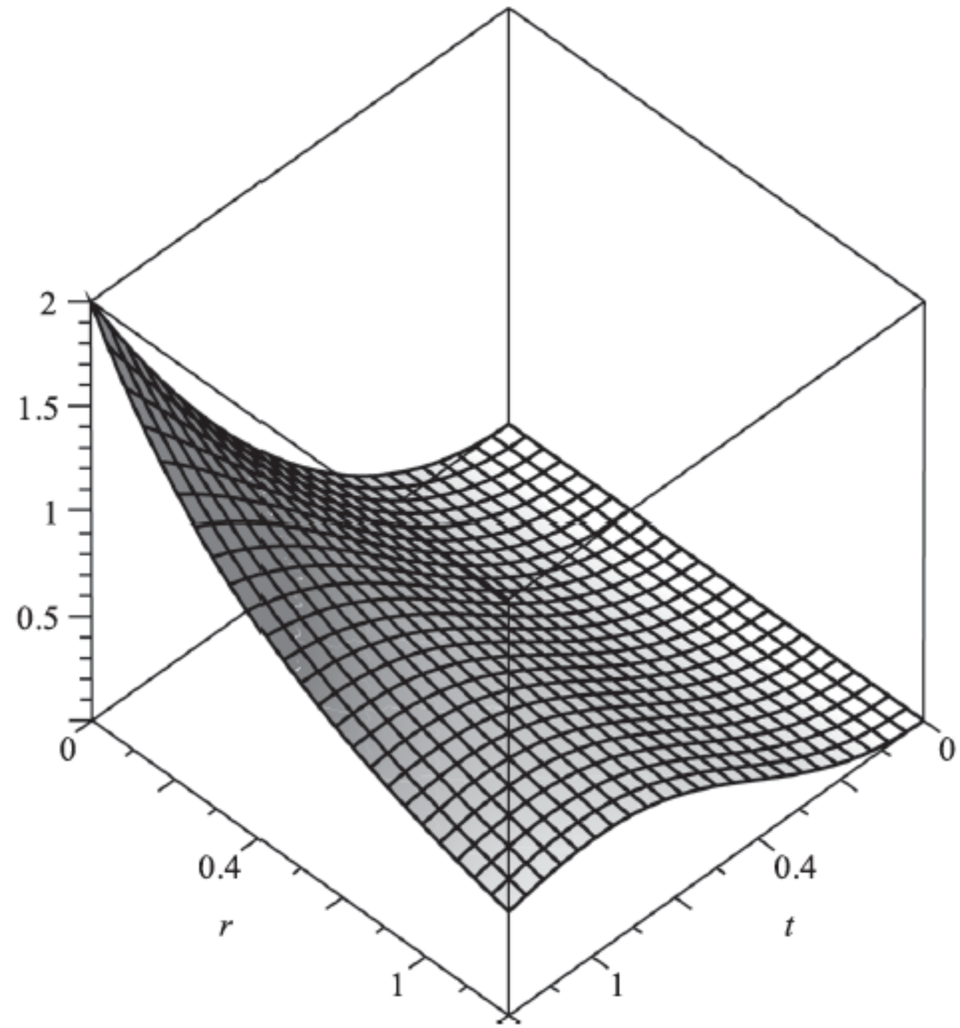
$$R(t) = R_0 t^{\frac{1}{a^2}}$$

$$F(t,r) = \frac{rR(t)^{2+a^2}}{h+rR(t)^{2+a^2}}$$

$$\alpha^2 = \frac{3}{2+a^2}$$

Maxwell tensor:

$$F_{tr} = \alpha h \frac{1}{(tr + t^{-2a^{-2}} h)(t^{\frac{a^2+2}{a^2}} r + h)}$$



The 5d metric:

$$ds_5^2 = t^{\frac{4}{a^2}} r^{\frac{4}{2+a^2}} \left(h + r t^{\frac{2+a^2}{a^2}}\right)^{\frac{-4}{2+a^2}} dt^2 + \frac{r+n}{r^{\frac{4+a^2}{2+a^2}}} \left(h + r t^{\frac{2+a^2}{a^2}}\right)^{\frac{2}{2+a^2}}$$
$$\times \left\{ dr^2 + r^2 d\Omega^2 + \frac{(d\psi + n\cos\theta d\phi)^2}{V(r)^2} \right\}$$

Asymptotically at $r \to \infty$, **the metric reduces to**

$$-dt^2 + t^{2/a^2}\{dr^2 + r^2 d\Omega^2 + (d\psi + n\cos\theta d\phi)^2\}.$$

with a bolt singularity at $r = 0$

Moreover, the solutions can be uplifted to (5+d)-dimensional solutions to Einstein-Maxwell theory with a positive cosmological constant where

$$d = \frac{3a^2}{1-a^2} \geq 1$$

$$ds^2_{5+d} = e^{2\sqrt{\frac{4d}{9(d+3)}}\phi(t,r)} ds^2_5 + e^{\frac{-6}{d}\sqrt{\frac{4d}{9(d+3)}}\phi(t,r)} d\vec{y} \cdot d\vec{y},$$

$$\vec{y} = (y_1, \ldots, y_d)$$

$$\phi(t,r) = -\frac{3}{4a} \ln\{R(t)^{2a^2} H(t,r)^{a^2}\}.$$

Example: $a = \frac{1}{2} \longrightarrow d = 1$ $\quad \Lambda = 90 \quad \alpha^2 = \frac{4}{3}$

$$ds_6^2 = \frac{1}{\sqrt{R(t)}H(t,r)^{9/4}V(r)} \{-V(r)dt^2 + R(t)^2 H(t,r)^3 V(r)^2 dr^2 + R(t)^2 H(t,r)^3 V(r)^2 r^2 d\theta^2$$
$$+ R(t)^2 H(t,r)^3 V(r)^2 r^2 \sin^2\theta d\phi^2 + R(t)^2 H(t,r)^3 d\psi^2 + 2n \cos\theta R(t)^2 H(t,r)^3 d\psi d\phi$$
$$+ n^2 \cos^2\theta R(t)^2 H(t,r)^3 d\phi^2 + R(t)^2 H(t,r)^3 V(r) dy^2\}.$$

We simply can not go to the limiting case a=0, as this make the dilaton field not well defined.

Einstein-Maxwell-dilaton theory with $a = b$ **and equal to zero.**

The dilaton decouples from he theory

Metric ansatz: $$ds_5^2 = -\frac{1}{H(t,r)^2}dt^2 + R(t)^2 H(t,r) ds_n^2,$$

Gauge field ansatz: $$A_t(t,r) = \alpha(F(t,r) - \beta)$$

Equations of motion lead to:

$$H(t,r) = 1 + \frac{h}{R(t)^2 r}$$

$$R(t) = R_0 e^{\gamma t} \qquad \Lambda = 6\gamma^2$$

Moreover:

$$F(t,r) = \frac{1}{H(t,r)}$$

$$\alpha^2 = \frac{3}{2}$$

The metric is indeed asymptotically dS

$$-dt^2 + e^{\gamma t}\{dr^2 + r^2 d\Omega^2 + (d\psi + n\cos\theta d\phi)^2\}$$

Compard to case, where a=b

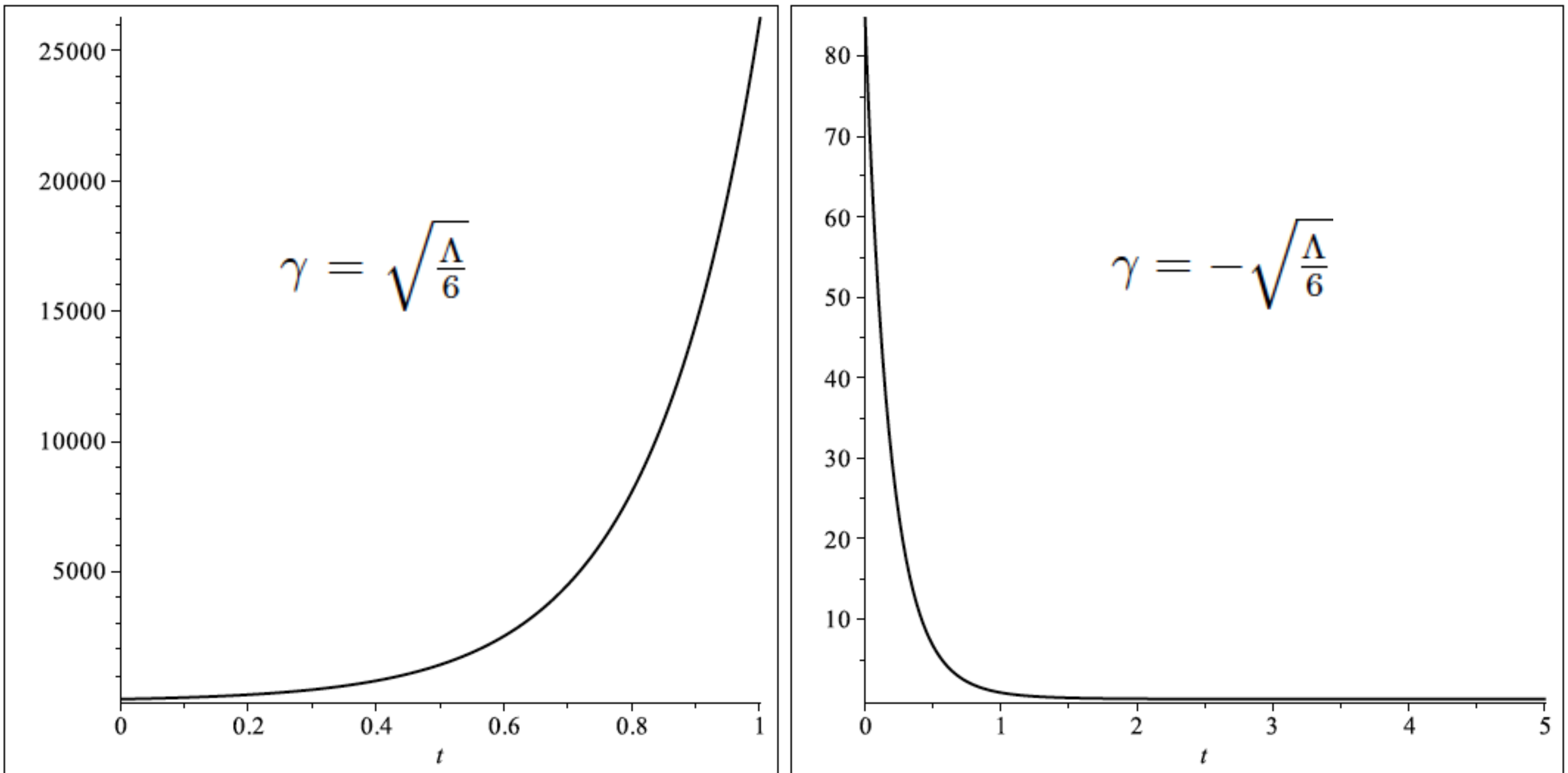
$$-dt^2 + t^{2/a^2}\{dr^2 + r^2 d\Omega^2 + (d\psi + n\cos\theta d\phi)^2\}$$

and the case where a is not equal to b

$$ds_5^2 = -dt^2 + t^{\frac{a^2}{2}}\{dr^2 + r^2 d\Omega^2 + (d\psi + n\cos\theta d\phi)^2\}$$

The hypersurfaces of constant time are expanding or shrinking patches of dS, while in former cases, they are only expanding patches of a non-static spacetime.

c-function



Very special case: a=0 with no Maxwell field:

The theory reduces to Einstein-dilaton theory with cosmological constant

$$ds^2 = -dt^2 + R(t)^2 ds_n^2$$

$$\begin{aligned} \phi(t) &= \tfrac{3}{4b} \ln t^B \\ R(t) &= R_0 t^A \end{aligned}$$

We find $A = \frac{1}{b^2}$ and $B = -2 \quad \Lambda = \frac{3}{2}\frac{4-b^2}{b^4}$

Thank you for your attention !