PQ field with small self-coupling: evolution in early universe and implications for ALP DM

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based on PK, M. Olechowski[†], JCAP 06 (2023) 043

Scenario: non-trivial evolution of $\Phi_{PQ} \equiv \Phi$ in the early universe

- ("saxion") $S \equiv |\Phi|/\sqrt{2}$ obtains large field value S_i during inflation
  (stochastic inflationary fluctuations) [Starobinsky, Yokoyama, 94'] see talk by O. LEBEDEV
- $\Phi \sim$ homogeneous with large initial value
  $\Rightarrow$ ALP can contribute to both CDM and WDM
- one can start with the usual Mexican hat PQ potential... :

$$V(\Phi) = \lambda_\Phi \left(|\Phi|^2 - \frac{f_a^2}{2} \right)^2 = \frac{\lambda_\Phi}{4} \left(S^2 - f_a^2 \right)^2$$

... $S_i \gg f_a \Rightarrow$ warm axions can be produced after onset of S oscillations
(parametric resonance) e.g. [Harigaya, Ibe, Kawasaki, Yanagida, 15'] see talk by C. LIN

$\lambda_\Phi \lesssim 10^{-20}$(DM isocurvature) $\Rightarrow$ one should consider corrections to $V(\Phi)$

e.g. non-renormalizable corrections explicitly breaking $U(1)_{PQ}$ used as source of kinetic misalignment mechanism e.g. [Co, Harigaya, 19'], [Co, Hall, Harigaya, 19'], [Co, Hall, Harigaya, Olive, Verner, 20']

- In the talk we investigate other types of corrections...

Radiative potential (Coleman-Weinberg, CW)

- Gildener-Weinberg

$$\lambda_{\Phi}(\mu) = 0$$

$$\mathcal{L} \supset -\sum_i \frac{1}{2}\lambda_{\Phi\phi_i} |\Phi|^2 \phi_i^2 - \sum_j y_j \Phi \overline{\psi}_j \psi_j$$

$$V(\Phi) = \frac{1}{64\pi^2} \sum_{\text{scalars}} M_{\phi_i}^4 \left[\ln\left(\frac{M_{\phi_i}^2}{\mu^2}\right) - \frac{3}{2}\right] - \frac{4}{64\pi^2} \sum_{\text{fermions}} M_{\psi_j}^4 \left[\ln\left(\frac{M_{\psi_j}^2}{\mu^2}\right) - \frac{3}{2}\right]$$

$$M_{\phi_i}^2 = m_i^2 + \lambda_{\Phi\phi_i} |\Phi|^2 \, , \qquad M_{\psi_j}^2 = y_j^2 |\Phi|^2$$

For simplicity we will assume:

- $$y_j = y, \qquad \lambda_{\Phi\phi_i} = \lambda, \qquad m_i^2 = m^2, \quad (m^2/\mu^2 \lesssim e)$$

1. Geometric correction: relevant ∼only during inflation(≈ de Sitter)
   The radiative potential receives corrections from curvature dependent invariants

   [Markkanen, Nurmi, Rajantie, Stopyra, 18'], [Hardwick, Markkanen, Nurmi, 19']

$$V(\Phi) = \frac{1}{64\pi^2} \sum_{\text{bosons}} \left\{ M_{\phi_i}^4 \left[\ln\left(\frac{\left|M_{\phi_i}^2\right|}{\mu^2} \right) - \frac{3}{2} \right] + \frac{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}}{90} \ln\left(\frac{\left|M_{\phi_i}^2\right|}{\mu^2} \right) \right\}$$
$$- \frac{4}{64\pi^2} \sum_{\text{fermions}} \left\{ M_{\psi_j}^4 \left[\ln\left(\frac{\left|M_{\psi_j}^2\right|}{\mu^2} \right) - \frac{3}{2} \right] - \frac{\frac{7}{8}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + R_{\mu\nu}R^{\mu\nu}}{90} \ln\left(\frac{\left|M_{\psi_j}^2\right|}{\mu^2} \right) \right\}$$

$$M_{\phi_i}^2 = m^2 + \lambda \left|\Phi\right|^2 + \left(\xi - \frac{1}{6}\right) R \,, \qquad M_{\psi_j}^2 = y^2 \left|\Phi\right|^2 + \frac{1}{12} R$$

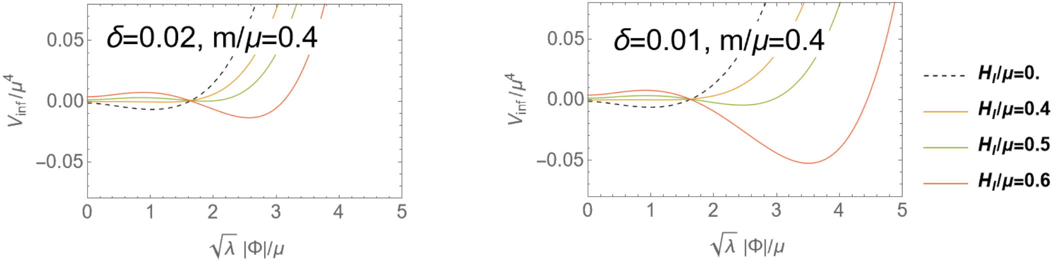
where $\mathcal{L} \supset -\sum_{\text{scalars}} \frac{1}{2}\xi R \phi_i^2$

2. Thermal corrections, relevant at ∼earlier post-inflationary stage
   due to ϕ_i and/or ψ_j in thermal bath (thermalization, thermal corrections to $V(\Phi)$)

the goal is to check influence of 1., 2. on *axion* contribution to CDM and WDM

Interesting dependence of $V_{inf}(\Phi)$ on H_I, if spectrum of particles is similar to supersymmetric one

$$N_s = 4N_f, \qquad y^2 = (1-\delta)\lambda, \qquad \delta \lesssim 1$$



$$P_{\rm eq}(S) \propto S \exp\left(-\frac{8\pi^2}{3}\frac{V(S)}{H_I^4}\right) \quad \to \quad S_i \sim S_{inf,min}$$

$$S_{inf,min} \approx \frac{H_I}{\sqrt{\delta\lambda}}, \qquad \frac{H_I^2}{S_{inf,min}^2} \sim O(\delta\lambda) \lesssim 10^{-8} \text{ (DM isocurvature)}$$

We will assume $H_I \gtrsim m \Rightarrow$ therefore geometric corrections imply enhancement of S_i

$\xi_{PQ} = 0$, $\xi < 1/4$ soon after inflation $V(\Phi) \sim$ "the dashed curve" $(+\Delta V_T)$

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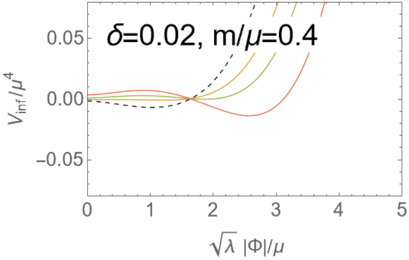

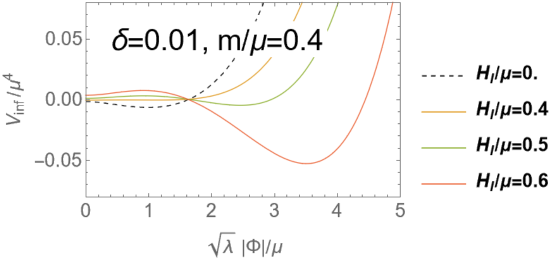

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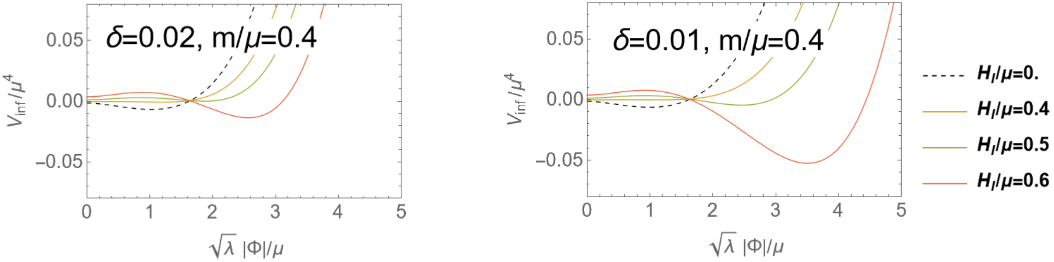

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$\xi_{PQ} = 0$, $\xi < 1/4$ soon after inflation $V(\Phi) \sim$ "the dashed curve" $(+\Delta V_T)$

- We will assume ϕ_i and/or ψ_j are populated in thermal bath (unless too heavy):
  - thermal effect crucial from the very beginning...
    onset of oscillations ($H \sim 3m_{S,\text{ eff}}$) due to thermal mass
  - .. or only later
    onset of oscillations due to zero-temperature mass

  We will focus of the former case:
  - full potential can be approximated by a quadratic one $\frac{1}{2}\frac{\alpha}{24}T^2S^2$
    $\Rightarrow$ (s)axion production due to parametric resonance very strongly suppressed ($\dot{\omega}_k/\omega_k^2 \lesssim 1$)
  - resonant production of ϕ_i and/or ψ_j also ineffective

The picture:

- at early stages S oscillates in $\sim$quadratic potential, no resonant production
- the oscillation amplitude redshifts...

The picture breaks down around $\widetilde{T}$ such that $\partial^2 V_{full}(S=0)/\partial S^2 \approx 0\ldots$

$$\widetilde{T}^2 \approx \frac{3}{4\pi^2} \frac{N_s}{n_{\text{eff}}\left(\widetilde{T}\right)} \ln\left(\frac{e\mu^2}{m^2}\right) m^2$$

when T approaches $\widetilde{T}$, full potential develops global minimum at $S \neq 0$
$\Rightarrow$Further evolution of S determined by $A_S(\widetilde{T})/f_a$:

numerical simulations necessary

- scenario "A": $\boldsymbol{A_S(\widetilde{T}) \gg f_a}$ ...
- scenario "B": $\boldsymbol{A_S(\widetilde{T}) \sim f_a}$ ...
- scenario "C": $\boldsymbol{A_S(\widetilde{T}) \ll f_a}$:
    - S oscillations decay very quickly due to tachyonic instability
      warm (s)axions produced

      [Felder, Garcia-Bellido, Greene, Kofman, Linde, Tkachev, 00'], [Felder, Kofman, Linde, 01']

	λ	δ	m/μ	μ [GeV]	H_I [GeV]	n_{CW}	n_{CW+G}	n_{CW+T}	n_{CW+T+G}
P_1	10^{-7}	0.1	0.1	$\mathbf{10^9}$	$\mathbf{10^{11}}$	0.042	0.20	$4.57 \cdot 10^5$	$4.57 \cdot 10^5$
P_2	10^{-7}	0.1	0.1	$\mathbf{10^{10}}$	$\mathbf{10^{13}}$	34	195	$4.33 \cdot 10^5$	$3.18 \cdot 10^5$
P_3	10^{-7}	0.1	0.1	$\mathbf{10^{12}}$	$\mathbf{10^{13}}$	56	223	$4.28 \cdot 10^5$	$3.04 \cdot 10^5$
P_4	10^{-7}	0.1	**0.1**	10^{11}	10^{13}	41.6	206	$4.3 \cdot 10^5$	$3.1 \cdot 10^5$
P_5	10^{-7}	0.1	**0.5**	10^{11}	10^{13}	41.9	207	$5.0 \cdot 10^3$	$2.2 \cdot 10^3$
P_6	10^{-7}	0.1	**0.7**	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	**0.8**	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	**0.03**	0.5	10^{11}	10^{13}	84	$6.9 \cdot 10^2$	$1.9 \cdot 10^3$	$3.0 \cdot 10^3$
P_9	10^{-7}	**0.01**	0.5	10^{11}	10^{13}	$1.6 \cdot 10^2$	$2.0 \cdot 10^3$	$1.2 \cdot 10^3$	$8.9 \cdot 10^3$
P_{10}	10^{-6}	**0.01**	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	**0.001**	0.5	10^{11}	10^{13}	36	$1.1 \cdot 10^3$	370	940
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$n_{CW} \lesssim n_{CW+G}$: $n_{CW+G}/n_{CW} \sim O(5)$ for $\delta = 0.1$ and $n_{CW+G}/n_{CW} \sim O(30)$ for $\delta = 0.001$

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$\mathrm{P}_{1\div 6,10,12\div 14}$– scenario C2, n_{CW+T}, $n_{CW+T+G} \ggg n_{CW}$

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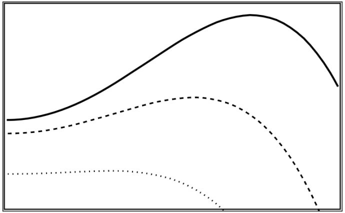
$P_{1\div6,10,12\div14}$– scenario C2, strong dependence of n_{CW+T}, n_{CW+T+G} on m/μ

- C2: **barrier between global minimum and $\boldsymbol{S} = 0$** (for some temperatures)
  the global minimum of the full potential has a non-negligible depth at phase transition
  - non-negligible number of particles may be produced



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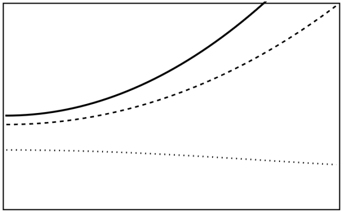
P_7– scenario C1, n_{CW+T}, $n_{CW+T+G} \approx 0$

- C1: **no barrier between global minimum and $S = 0$**
  full potential changes with temperature and it is very shallow at $T \sim \widetilde{T}$
  - the amount of warm axions very small



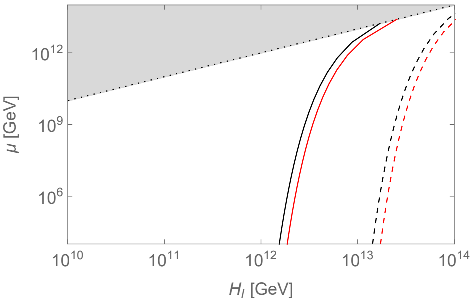
- Non-trivial evolution of Φ_{PQ} $\Rightarrow$ very rich and interesting WDM phenomenology (very small self-coupling)
- $\Rightarrow$Significant role is played by various corrections: radiative&&geometric, thermal
- If H_I is not too small, V_{inf} develops a new deep minimum at large Φ
  (i) DM isocurvature relaxed (ii) very long inflation might not be needed
  especially interesting are models in which ϕ_i,ψ_j have a quasi-susy spectrum
- Post-inflationary evolution of Φ may be quite complicated

Thermal corrections $\Rightarrow$
(i) the resonant production (s)axions strongly suppressed...
(ii) ...later produced due to the tachyonic instability
(iii) n_a orders of magnitude smaller or bigger than w/o thermal corrections
(iv) no extra ingredients... necessary for thermalization (of saxion remnants)

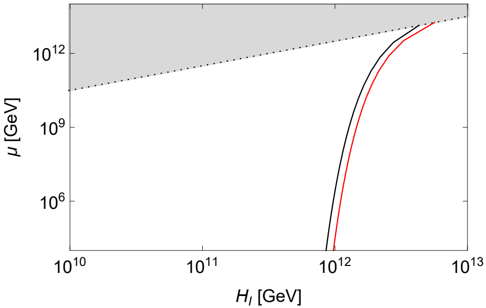
CDM: oscillating S carries the initial PQ phase θ_i...

BACK UP

MD($\lambda = 10^{-7}$, $\delta = 0.1$)



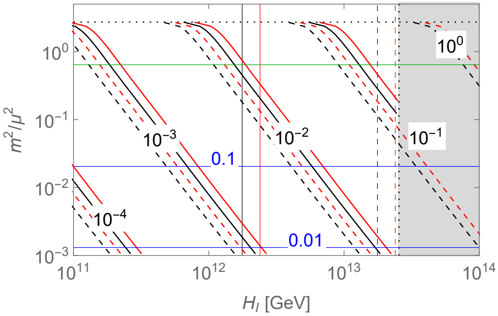
RD($\lambda = 10^{-10}$, $\delta = 10^{-2}$)



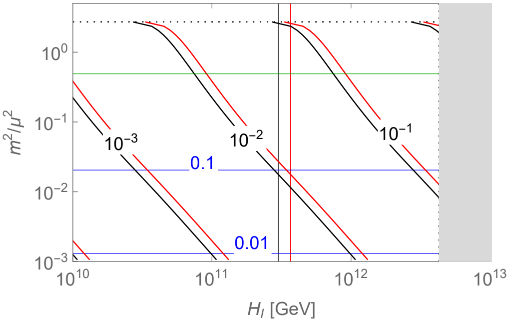
For parameters to the left from the black/red solid lines, the S oscillations start due to the thermal mass

Some examples – lines of constant $A_S(\widetilde{T})/S_{\min,0}$

$\mathrm{MD}(\lambda = 10^{-7},\ \delta = 0.1)$



$\mathrm{RD}(\lambda = 10^{-10},\ \delta = 10^{-2})$



$\Rightarrow$ scenario B typically only when both H_I and m/μ are $\sim$maximal allowed

$\Rightarrow$ scenario C typically when either H_I or m/μ is smaller

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P_5	10^{-7}	0.1	**0.5**	10^{11}	10^{13}	41.9	207	$5.0 \cdot 10^3$	$2.2 \cdot 10^3$
P_6	10^{-7}	0.1	**0.7**	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	**0.8**	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	**0.03**	0.5	10^{11}	10^{13}	84	$6.9 \cdot 10^2$	$1.9 \cdot 10^3$	$3.0 \cdot 10^3$
P_9	10^{-7}	**0.01**	0.5	10^{11}	10^{13}	$1.6 \cdot 10^2$	$2.0 \cdot 10^3$	$1.2 \cdot 10^3$	$8.9 \cdot 10^3$
P_{10}	10^{-6}	**0.01**	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	**0.001**	0.5	10^{11}	10^{13}	36	$1.1 \cdot 10^3$	370	940
P_{12}	$\mathbf{10^{-7}}$	0.1	0.2	10^{10}	10^{12}	1.3	6.5	$8.4 \cdot 10^4$	$8.3 \cdot 10^4$
P_{13}	$\mathbf{10^{-8}}$	0.1	0.2	10^{10}	10^{12}	23	120	$2.6 \cdot 10^5$	$2.5 \cdot 10^5$
P_{14}	$\mathbf{10^{-9}}$	0.1	0.2	10^{10}	10^{12}	$4.2 \cdot 10^2$	$2.1 \cdot 10^3$	$7.9 \cdot 10^5$	$5.8 \cdot 10^5$

P_8, P_9, P_{11}– scenario B, $n_{CW+T}, n_{CW+T+G} \gg n_{CW}$

	λ	δ	m/μ	$\mu\,[\mathrm{GeV}]$	$H_I\,[\mathrm{GeV}]$	n_{CW}	n_{CW+G}	n_{CW+T}	n_{CW+T+G}
P_1	10^{-7}	0.1	0.1	$\mathbf{10^9}$	$\mathbf{10^{11}}$	0.042	0.20	$4.57 \cdot 10^5$	$4.57 \cdot 10^5$
P_2	10^{-7}	0.1	0.1	$\mathbf{10^{10}}$	$\mathbf{10^{13}}$	34	195	$4.33 \cdot 10^5$	$3.18 \cdot 10^5$
P_3	10^{-7}	0.1	0.1	$\mathbf{10^{12}}$	$\mathbf{10^{13}}$	56	223	$4.28 \cdot 10^5$	$3.04 \cdot 10^5$
P_4	10^{-7}	0.1	**0.1**	10^{11}	10^{13}	41.6	206	$4.3 \cdot 10^5$	$3.1 \cdot 10^5$
P_5	10^{-7}	0.1	**0.5**	10^{11}	10^{13}	41.9	207	$5.0 \cdot 10^3$	$2.2 \cdot 10^3$
P_6	10^{-7}	0.1	**0.7**	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	**0.8**	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	**0.03**	0.5	10^{11}	10^{13}	84	$6.9 \cdot 10^2$	$1.9 \cdot 10^3$	$3.0 \cdot 10^3$
P_9	10^{-7}	**0.01**	0.5	10^{11}	10^{13}	$1.6 \cdot 10^2$	$2.0 \cdot 10^3$	$1.2 \cdot 10^3$	$8.9 \cdot 10^3$
P_{10}	10^{-6}	**0.01**	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	**0.001**	0.5	10^{11}	10^{13}	36	$1.1 \cdot 10^3$	370	940
P_{12}	$\mathbf{10^{-7}}$	0.1	0.2	10^{10}	10^{12}	1.3	6.5	$8.4 \cdot 10^4$	$8.3 \cdot 10^4$
P_{13}	$\mathbf{10^{-8}}$	0.1	0.2	10^{10}	10^{12}	23	120	$2.6 \cdot 10^5$	$2.5 \cdot 10^5$
P_{14}	$\mathbf{10^{-9}}$	0.1	0.2	10^{10}	10^{12}	$4.2 \cdot 10^2$	$2.1 \cdot 10^3$	$7.9 \cdot 10^5$	$5.8 \cdot 10^5$

$\mathrm{P}_8, \mathrm{P}_9, \mathrm{P}_{11}$ – scenario B, geometric correction increase n : $n_{CW+T} \lesssim n_{CW+T+G}$

	λ	δ	m/μ	μ [GeV]	H_I [GeV]	n_{CW}	n_{CW+G}	n_{CW+T}	n_{CW+T+G}
P_1	10^{-7}	0.1	0.1	$\mathbf{10^9}$	$\mathbf{10^{11}}$	0.042	0.20	$4.57 \cdot 10^5$	$4.57 \cdot 10^5$
P_2	10^{-7}	0.1	0.1	$\mathbf{10^{10}}$	$\mathbf{10^{13}}$	34	195	$4.33 \cdot 10^5$	$3.18 \cdot 10^5$
P_3	10^{-7}	0.1	0.1	$\mathbf{10^{12}}$	$\mathbf{10^{13}}$	56	223	$4.28 \cdot 10^5$	$3.04 \cdot 10^5$
P_4	10^{-7}	0.1	**0.1**	10^{11}	10^{13}	41.6	206	$4.3 \cdot 10^5$	$3.1 \cdot 10^5$
P_5	10^{-7}	0.1	**0.5**	10^{11}	10^{13}	41.9	207	$5.0 \cdot 10^3$	$2.2 \cdot 10^3$
P_6	10^{-7}	0.1	**0.7**	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	**0.8**	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	**0.03**	0.5	10^{11}	10^{13}	84	$6.9 \cdot 10^2$	$1.9 \cdot 10^3$	$3.0 \cdot 10^3$
P_9	10^{-7}	**0.01**	0.5	10^{11}	10^{13}	$1.6 \cdot 10^2$	$2.0 \cdot 10^3$	$1.2 \cdot 10^3$	$8.9 \cdot 10^3$
P_{10}	10^{-6}	**0.01**	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	**0.001**	0.5	10^{11}	10^{13}	36	$1.1 \cdot 10^3$	370	940
P_{12}	$\mathbf{10^{-7}}$	0.1	0.2	10^{10}	10^{12}	1.3	6.5	$8.4 \cdot 10^4$	$8.3 \cdot 10^4$
P_{13}	$\mathbf{10^{-8}}$	0.1	0.2	10^{10}	10^{12}	23	120	$2.6 \cdot 10^5$	$2.5 \cdot 10^5$
P_{14}	$\mathbf{10^{-9}}$	0.1	0.2	10^{10}	10^{12}	$4.2 \cdot 10^2$	$2.1 \cdot 10^3$	$7.9 \cdot 10^5$	$5.8 \cdot 10^5$

$P_{1\div6,10,12\div14}$– scenario C2, geometric correction decreases n : $n_{CW+T} \gtrsim n_{CW+T+G}$

- thermal corrections neglected – (s)axions produced by parametric resonance:
  - $n_{CW+G} \gtrsim n_{CW}$

$$n_{CW} \propto \delta^{-5/8} \lambda^{-5/4} H_I^{3/2} T^3, \qquad n_{CW+G} \propto \delta^{-1} \lambda^{-5/4} H_I^{3/2} T^3$$

  dependence on m,μ very weak

- thermal corrections accounted –(s)axions produced by tachyonic instability:
  - scenario C more natural than B and especially A
  - C$\rightarrow$ $C1$(no barrier), $C2$(barrier for some T)
  - scenario C1 : $n_{CW+T}, n_{CW+T+G} \approx 0$
  - scenario C2 : $n_{CW+T}, n_{CW+T+G} \ggg n_{CW}$

$$n_{CW+T}, n_{CW+T+G} \propto \lambda^{-1/2} \left(\frac{m}{\mu}\right)^{-2} T^3$$

  dependence on δ, μ and H_I much weaker, $n_{CW+T+G} \lesssim n_{CW+T}$

- dedicated numerical computations for $C1 - C2$ transition region, scenario B ...

scenario D: early thermalization : $T_{\rm th} > \widetilde{T}$

- interactions with thermal plasma – modification to EOM:

$$\ddot{S} + (3H + \Gamma_{\rm th})\,\dot{S} + \frac{\partial V_{\rm eff}}{\partial S} = 0$$

$$\Gamma_{\rm th}^{(\phi)} \sim \frac{\lambda^2 S^2}{\alpha_{\rm th} T}\,, \qquad\qquad \Gamma_{\rm th}^{(\psi)} \sim y^2 \alpha_{\rm th} T$$

$$\Rightarrow \qquad T_{\rm th} \sim \sqrt{\frac{45}{4\pi^3 g_*}}\,\alpha_{\rm th} \lambda M_{Pl}$$

early thermalization if:

$$\frac{m^2}{\mu^2} \ln\left(\frac{e\mu^2}{m^2}\right) \lesssim \frac{15 n_{\rm eff}\left(\widetilde{T}\right)}{\pi g_* N_s}\,\alpha_{\rm th}^2 \lambda^2 \frac{M_{Pl}^2}{\mu^2}$$

$\alpha_{\rm th} = \alpha_s \sim 0.1 \Rightarrow$ RHS bigger than 1, if $\mu \lesssim \lambda \cdot 10^{17}$ GeV

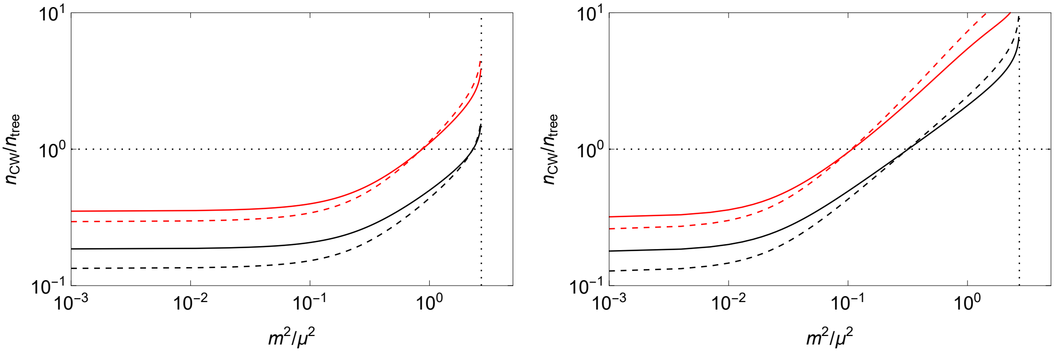

Figure: The ratio of densities of warm axions produced via a parametric resonance with (n_{CW}) and without (n_{tree}) radiative corrections taken into account as a function of m^2/μ^2. The relevant parameters are fixed as: $\delta = 0.1$ (left panel) and $\delta = 0.01$ (right panel); $H_I/\mu = 5$ (red curves) and $H_I/\mu = 10^3$ (black curves). The solid (dashed) lines correspond to situation when the axions are produced after (before) the end of the reheating process.

- We will focus on
  - CW with only geometric corrections ($CW+G$)
  - CW with only thermal corrections ($CW+T$)
  - CW with both geometric and thermal corrections ($CW+T+G$)

  n_{CW}, n_{CW+G}, n_{CW+T} and n_{CW+T+G} are rescaled to a common T (in units T^3)

n_{CW} vs n_{tree}: $n_{CW}/n_{tree} \lesssim 1$ depending on m/μ, δ and H_I/μ

- Two different approximations:
  - $$n_{CW}, n_{CW+G} \sim \frac{1}{2}\frac{V_{CW}}{m_S}\bigg|_{S=S_i} \qquad \text{(parametric resonance)}$$
  - $$n_{CW+T}, n_{CW+T+G} \sim \frac{1}{2}\frac{\Delta V_{tot}}{m_S}\bigg|_{T\sim\widetilde{T}} \qquad \text{(tachyonic instability)}$$

  $\Delta V_{tot}=$(available potential energy), $m_S=$(mass at the global minimum)

- Cold axions produced via the conventional misalignment mechanism
- $n_{a,\ cold}$ often determined by stochastic processes during inflation...
  ...despite $V_{full}(\Phi)$ is in the unbroken phase for some time

saxion keeps oscillating and carries the initial PQ phase θ_i

- in the end one should compare

$$\rho_{a,\,warm} + \rho_{a,\,cold} \leftrightarrow \rho_{DM,\ observed}$$

$\Rightarrow$ extra flexibility:
if $\rho_{a,\,warm}$ too small (or vanishing...), often possible to complement with $\rho_{a,\,cold}$ (choice of θ_i)

Number densities (n.d.) of ALP WDM n_i (in units T^3):

1. radiative corrections: n_{CW} vs n_{tree}: $n_{CW}/n_{tree} \lesssim 1$ (depending on m/μ, δ and H_I/μ)
   - n_{CW} $\equiv$(n.d. of warm ALP for "pure" CW potential)
   - n_{tree} $\equiv$(n.d. of warm ALP for the *corresponding* Mexican hat potential)
2. We will take n_{CW} as the reference and compare it with:
   - n_{CW+G} $\equiv$(CW with only geometric corrections)
   - n_{CW+T} $\equiv$(CW with only thermal corrections)
   - n_{CW+T+G} $\equiv$(CW with both geometric and thermal corrections)

n_{CW}, n_{CW+G}, n_{CW+T} and n_{CW+T+G} are rescaled to a common T

- We use the full thermal potential

$$V_T(\Phi) = \frac{T^4}{2\pi^2}\left[\sum_{\text{bosons}} J_+\left(\frac{M_{\phi_i}}{T}\right) + 4\sum_{\text{fermions}} J_-\left(\frac{M_{\psi_j}}{T}\right)\right]$$

$$J_\pm(y) = \pm\int_0^\infty x^2 \ln\left[1 \mp \exp\left(-\sqrt{x^2+y^2}\right)\right] \mathrm{d}x$$

Number densities (n.d.) of ALP WDM n_i:

1. radiative corrections: n_{CW} vs n_{tree}: $n_{CW}/n_{tree} \lesssim 1$ (depending on m/μ, δ and H_I/μ)
   - n_{CW} $\equiv$(n.d. of warm ALP for "pure" Gildener-Weinberg potential)
   - n_{tree} $\equiv$(n.d. of warm ALP for the *corresponding* Mexican hat potential)
2. We will take n_{CW} as the reference and compare it with:
   - n_{CW+G} $\equiv$(CW with only geometric corrections)
   - n_{CW+T} $\equiv$(CW with only thermal corrections)
   - n_{CW+T+G} $\equiv$(CW with both geometric and thermal corrections)

n_{CW}, n_{CW+G}, n_{CW+T} and n_{CW+T+G} are rescaled to a common T (and in units T^3)