

Vacuum expectation values and spontaneous CP violation in multi-Higgs potentials

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Invariant approach to CP in the scalar sector

IdMV, King, Luhn, Neder

<http://arxiv.org/abs/1603.06942>

<http://arxiv.org/abs/1704.06322>

<http://arxiv.org/abs/1706.?????>

See also:

Branco, Rebelo, Silva-Marcos

<http://arxiv.org/abs/hep-ph/0502118>

Davidson, Haber

<http://arxiv.org/abs/hep-ph/0504050>

Gunion, Haber

<http://arxiv.org/abs/hep-ph/0506227>

Standard form

$$V = \phi^{*a} Y_a^b \phi_b + \phi^{*a} \phi^{*c} Z_{ac}^{bd} \phi_b \phi_d$$

Symmetry invariance (e.g. $SU(2)_L$) is encoded in Y, Z

Complex conjugation

Complex conjugation raises/lower indices:

$$\begin{aligned}\phi_a &\mapsto (\phi_a)^* \equiv \phi^{*a} \\ \phi^{*a} &\mapsto (\phi^{*a})^* \equiv \phi_a\end{aligned}$$

$V = V^*$ therefore

$$(Y_b^a)^* = Y_a^b$$

and

$$(Z_{bd}^{ac})^* = Z_{ac}^{bd}$$

General CP transformation (X)

$$\begin{aligned}\phi_a &\mapsto \phi^{*a'} X_{a'}^a \\ \phi^{*a} &\mapsto X_a^{\dagger a'} \phi_{a'}\end{aligned}$$

E.g. for a triplet one may have:

$$X_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$(\phi_1, \phi_2, \phi_3) \mapsto X_{23}(\phi^{*1}, \phi^{*2}, \phi^{*3}) = (\phi^{*1}, \phi^{*3}, \phi^{*2})$$

Basis change (V)

$$\begin{aligned}\phi_a &\mapsto V_a^{a'} \phi_{a'} \\ \phi^{*a} &\mapsto \phi^{*a'} V^{\dagger a}_{a'}\end{aligned}$$

$$\begin{aligned}Y_a^b &\mapsto V_a^{a'} Y_{a'}^{b'} V^{\dagger b}_{b'} \\ Z_{ac}^{bd} &\mapsto V_a^{a'} V_c^{c'} Z_{a'c'}^{b'd'} V^{\dagger b}_{b'} V^{\dagger d}_{d'}\end{aligned}$$

Basis invariants (no X , no V)

Basis invariants: contract all the indices
(this cancels V and X dependence)

Examples

$$Z_{ab}^{ab} \text{ and } Z_{ba}^{ab}$$

Of 24 possibilities with two Z , just two independent ones, e.g.:

$$Z_{bd}^{ab} Z_{ac}^{cd} \text{ and } Z_{cd}^{ab} Z_{ab}^{cd}$$

Many invariants are equivalent, or products of smaller invariants

From basis invariants to CPIs

Many basis invariants are CP-even (all examples so far)

Construct **CP-odd invariant** (CPI)

$$\mathcal{I} = I - I^*$$

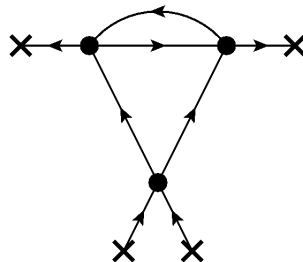
Complex conjugation swaps upper and lower indices:
diagrams and other technique help find cases with $I \neq I^*$

SCPI

Spontaneous CP-odd invariant (SCPI) involves the VEVs

$$\mathcal{J} = J - J^*$$

$$J^{(3,2)} \equiv Z_{a_4 a_5}^{a_1 a_2} Z_{a_2 a_6}^{a_3 a_4} Z_{a_7 a_8}^{a_5 a_6} v_{a_1} v_{a_3} v^{*a_7} v^{*a_8} =$$

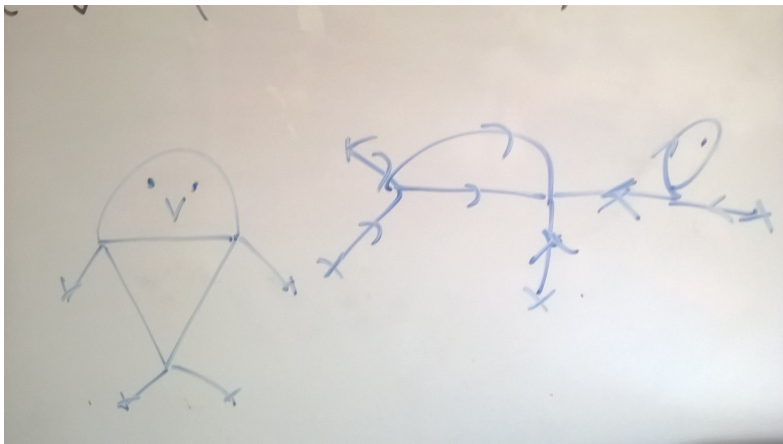


The real (well, complex) Penguin diagram!

$$J^{(3,2)} \equiv Z_{a_4 a_5}^{a_1 a_2} Z_{a_2 a_6}^{a_3 a_4} Z_{a_7 a_8}^{a_5 a_6} v_{a_1} v_{a_3} v^{* a_7} v^{* a_8} =$$



Whiteboard...



SCPI for $V(H_1, H_2)$ with CP_0

$$\mathcal{J}_{CP_0}^{(3,2)} = F(a_1, a_2, b, b', c_1, c_2, d, \langle h_{i,j} \rangle) [\langle h_{2,1} \rangle \langle h_{1,1} \rangle^* + \langle h_{2,2} \rangle \langle h_{1,2} \rangle^* - h.c.]$$

SCPV depends on **relative phase** between $\langle H_1 \rangle$ and $\langle H_2 \rangle$
(note $\langle h_{1,1} \rangle = \langle h_{2,1} \rangle = 0$ for charge preserving VEVs)

SCPI for $V_{\Delta(27)}(\varphi)$

$$\begin{aligned}\mathcal{J}^{(3,2)} &= \frac{1}{4}(\textcolor{red}{d}^{*3} - \textcolor{red}{d}^3)Q(|v_i|) \\ &+ \frac{1}{2}(dd^{*2} - 2d^*s^2 + d^2s)(v_2v_3v_1^{*2} + v_1v_3v_2^{*2} + v_1v_2v_3^{*2}) \\ &- \frac{1}{2}(d^2d^* - 2ds^2 + d^{*2}s)(v_2^*v_3^*v_1^2 + v_1^*v_3^*v_2^2 + v_1^*v_2^*v_3^2) .\end{aligned}$$

Impose trivial CP, CP_0 : forces $\textcolor{red}{Arg}(d) = 0$

SCPI for $V_{\Delta(27)}(\varphi)$ with CP_0

$$\mathcal{J}_{CP_0}^{(3,2)} = \frac{1}{2}(d^3 - 2ds^2 + d^2s) \\ \left[(v_2 v_3 v_1^{*2} + v_1 v_3 v_2^{*2} + v_1 v_2 v_3^{*2}) - (v_2^* v_3^* v_1^2 + v_1^* v_3^* v_2^2 + v_1^* v_2^* v_3^2) \right]$$

VEVs: $\omega \equiv e^{i2\pi/3}$ is a calculable phase

$\langle \varphi \rangle = (1, \omega, \omega^2)$ conserves CP

$\langle \varphi \rangle = (\omega, 1, 1)$ violates CP with a calculable phase:

Spontaneous Geometrical CP violation

New minima

Using a method of decreasing symmetry, found minima for potentials with two triplets of $\Delta(3n^2)$ and $\Delta(6n^2)$ (with $n = 2, 3$ and $n > 3$)

For one triplet of $A_4, S_4, \Delta(27), \Delta(54)$, reproduce results in I.P. Ivanov, C.C. Nishi

<https://arxiv.org/abs/1410.6139>

For $\Delta(6n^2)$ with $n > 3$ we get

$$v_1(1, 0, 0), \quad v_2(1, 1, 0), \quad v_3(1, 1, 1)$$

as follows...

Minima for one triplet of $\Delta(6n^2)$, $n > 3$

$$V_0(\varphi) \equiv -m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$$V_0 = V_{U(3)} + V_{\Delta(6\infty^2) \times U(1)}.$$

$U(3)$ - single orbit connected by the continuous symmetry transformations

Orbit split

$$V_{\Delta(6\infty^2)\times U(1)} = s(\varphi_1\varphi^{*1}\varphi_1\varphi^{*1} + \varphi_2\varphi^{*2}\varphi_2\varphi^{*2} + \varphi_3\varphi^{*3}\varphi_3\varphi^{*3})$$

splits the single $U(3)$ orbit into 3 classes of directions

$$\left\{ \begin{pmatrix} e^{i\eta} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ e^{i\eta} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ e^{i\eta} \end{pmatrix} \right\}, \left\{ \begin{pmatrix} e^{i\eta} \\ e^{i\zeta} \\ 0 \end{pmatrix}, \text{permut.} \right\}, \left\{ \begin{pmatrix} e^{i\eta} \\ e^{i\zeta} \\ e^{i\theta} \end{pmatrix}, \text{permut.} \right\}.$$

Minima for two VEVs of $\Delta(6n^2)$, $n > 3$

For two triplets, method reveals some candidate VEVs like

$$\begin{aligned} (e^{i\eta}, 0, 0), (e^{i\eta'}, 0, 0) &\rightarrow (1, 0, 0), (1, 0, 0) \\ (e^{i\eta}, e^{i\zeta}, e^{i\theta}), (e^{i\eta'}, e^{i\zeta'}, e^{i\theta'}) &\rightarrow (1, 1, 1), (1, e^{i\zeta'}, e^{i\theta'}). \end{aligned}$$

For last case, the phases are physical.

Minimize phase-dependent part of potential to fix the phases.

Minima with calculable phases

Found VEVs with calculable phases

$$(1, 1, 1), (1, \omega, \omega^2) \text{ and } (1, 1, 1), (1, \omega^2, \omega).$$

Do they SCPV?

Use SCPI to test for SCPV

$$\begin{aligned}
 \mathcal{J}^{(3,2)} = & -\frac{1}{16} \tilde{s}_2 [\tilde{r}_2 (-4s - 4s' + 2\tilde{s}_1 - \tilde{s}_2 + 3\tilde{r}_2) - \tilde{s}_3^2] W_{CP_0} \\
 & - \frac{1}{8} i \tilde{s}_3 [\tilde{s}_3^2 - 3\tilde{r}_2^2] W_{CP_{23}} \\
 & - \frac{1}{16} i \tilde{s}_2 \tilde{s}_3 [-4s - 4s' + 2\tilde{s}_1 - \tilde{s}_2] (\dots)
 \end{aligned}$$

where

$$W_{CP_0} \equiv [(v_1 v_1'^* v_2' v_2^* + v_2 v_2'^* v_3' v_3^* + v_3 v_3'^* v_1' v_1^*) - h.c.]$$

New SGCPV

$$W_{CP_0} \equiv [(v_1 v_1'^* v_2' v_2^* + v_2 v_2'^* v_3' v_3^* + v_3 v_3'^* v_1' v_1^*) - h.c.]$$

$$W_{CP_0}^{(3,2)}[v(1, 1, 1), v'(1, \omega, \omega^2)] = 3(\omega - \omega^2)v^2 v'^2 \neq 0$$

Found new cases of SGCPV in $\Delta(3n^2) \times CP_0$, for 6HDM

Summary

- Developed formalism for CPIs and SCPIs
- Methods for explicit and spontaneous CP violation are valid for any potential when brought to standard form
- Verified the CP properties of 3HDM and 6HDM symmetric under $\Delta(3n^2)$ and $\Delta(6n^2)$ groups, for new minima and found new cases with SGCPV

Thank you

Dziekuje

Standard form

$$V = \phi^{*a} Y_a^b \phi_b + \phi^{*a} \phi^{*c} Z_{ac}^{bd} \phi_b \phi_d$$

For n Higgs doublets $H_{i\alpha} = (h_{i,1}, h_{i,2})$, where $\alpha = 1, 2$ denotes the $SU(2)_L$ index and i goes from 1 to n

$$\phi = (\varphi_1, \varphi_2, \dots, \varphi_{2n-1}, \varphi_{2n}) = (h_{1,1}, h_{1,2}, \dots, h_{n,1}, h_{n,2})$$

Invariance under Symmetry (e.g. $SU(2)_L$) is encoded in the Y and Z tensors

Potential 2HDM

$$\begin{aligned}
 V(H_1, H_2) = & m_1^2 H_1^\dagger H_1 + m_{12}^2 e^{i\theta_0} H_1^\dagger H_2 + m_{12}^2 e^{-i\theta_0} H_2^\dagger H_1 + m_2^2 H_2^\dagger H_2 + \\
 & + a_1 \left(H_1^\dagger H_1 \right)^2 + a_2 \left(H_2^\dagger H_2 \right)^2 \\
 & + b \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_2 \right) + b' \left(H_1^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) + \\
 & + c_1 e^{i\theta_1} \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_1 \right) + c_1 e^{-i\theta_1} \left(H_1^\dagger H_1 \right) \left(H_1^\dagger H_2 \right) + \\
 & + c_2 e^{i\theta_2} \left(H_2^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) + c_2 e^{-i\theta_2} \left(H_2^\dagger H_2 \right) \left(H_1^\dagger H_2 \right) + \\
 & + d e^{i\theta_3} \left(H_1^\dagger H_2 \right)^2 + d e^{-i\theta_3} \left(H_2^\dagger H_1 \right)^2 .
 \end{aligned}$$

Potentials $\Delta(3n^2)$, A_4 , $\Delta(27)$

$$V_0(\varphi) = V_{\Delta(3n^2)}(\varphi) = -m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$$V_{A_4}(\varphi) = V_0(\varphi) + c \left(\varphi_1 \varphi_1 \varphi^{*3} \varphi^{*3} + \varphi_2 \varphi_2 \varphi^{*1} \varphi^{*1} + \varphi_3 \varphi_3 \varphi^{*2} \varphi^{*2} \right) \\ + c^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_3 + \varphi^{*2} \varphi^{*2} \varphi_1 \varphi_1 + \varphi^{*3} \varphi^{*3} \varphi_2 \varphi_2 \right) ,$$

$$V_{\Delta(27)}(\varphi) = V_0(\varphi) + d \left(\varphi_1 \varphi_1 \varphi^{*2} \varphi^{*3} + \text{cycl.} \right) \\ + d^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_2 + \text{cycl.} \right)$$

Potential $\Delta(3n^2)$

$$V_{\Delta(3n^2)}(\varphi) = -m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$V_{\Delta(3n^2)}(\varphi)$ is invariant under both CP_0 and CP with X_{23}

Potential A_4

$$V_{A_4}(\varphi) = V_0(\varphi) + c \left(\varphi_1 \varphi_1 \varphi^{*3} \varphi^{*3} + \varphi_2 \varphi_2 \varphi^{*1} \varphi^{*1} + \varphi_3 \varphi_3 \varphi^{*2} \varphi^{*2} \right) \\ + c^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_3 + \varphi^{*2} \varphi^{*2} \varphi_1 \varphi_1 + \varphi^{*3} \varphi^{*3} \varphi_2 \varphi_2 \right) ,$$

$V_{A_4}(\varphi)$ is invariant under CP with X_{23} for any c

Potential A_4 , self-conjugate under X_{23}

$$V_{A_4}(\varphi) = V_0(\varphi) + c \left(\varphi_1 \varphi_1 \varphi^{*3} \varphi^{*3} + \varphi_2 \varphi_2 \varphi^{*1} \varphi^{*1} + \varphi_3 \varphi_3 \varphi^{*2} \varphi^{*2} \right) \\ + c^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_3 + \varphi^{*2} \varphi^{*2} \varphi_1 \varphi_1 + \varphi^{*3} \varphi^{*3} \varphi_2 \varphi_2 \right) ,$$

$V_{A_4}(\varphi)$ is invariant under CP with X_{23} for any c
 $(\phi_1, \phi_2, \phi_3) \mapsto X_{23}(\phi^{*1}, \phi^{*2}, \phi^{*3}) = (\phi^{*1}, \phi^{*3}, \phi^{*2})$

Potential $\Delta(27)$

$$V_{\Delta(27)}(\varphi) = V_0(\varphi) + d \left(\varphi_1 \varphi_1 \varphi^{*2} \varphi^{*3} + \text{cycl.} \right) \\ + d^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_2 + \text{cycl.} \right)$$

$V_{\Delta(27)}(\varphi)$ is CPV

Invariant under both CP_0 and CP with X_{23} only if d is real.

Potential $\Delta(27)$, doublets

$$\begin{aligned}
 V_0(H) = & -m_h^2 \sum_{i,\alpha} h_{i\alpha} h^{*i\alpha} + \sum_{i,j,\alpha,\beta} \left[r_1 (h_{i\alpha} h^{*i\alpha}) (h_{j\beta} h^{*j\beta}) + r_2 (h_{i\alpha} h^{*i\beta}) (h_{j\beta} h^{*j\alpha}) \right] \\
 & + s \sum_{i,\alpha,\beta} (h_{i\alpha} h^{*i\alpha}) (h_{i\beta} h^{*i\beta})
 \end{aligned}$$

$$V_{\Delta(27)}(H) = V_0(H) + \sum_{\alpha,\beta} \left[d \left(h_{1\alpha} h_{1\beta} h^{*2\alpha} h^{*3\beta} + \text{cycl.} \right) + \text{h.c.} \right]$$

2 generation example

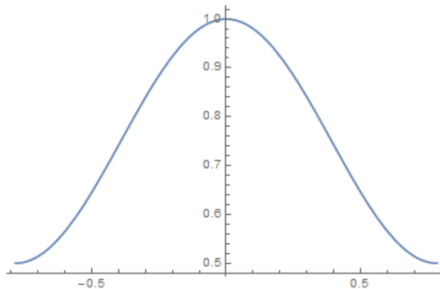
Extrema of $|\varphi_1|^4 + |\varphi_2|^4$ for fixed magnitude v :

Positive: $\propto (1, 1)/\sqrt{2} \rightarrow V \sim +2v^4/4$

Negative: $\propto (0, 1) \rightarrow V \sim -v^4$

2 generation example plot

```
Plot[Cos[ $\theta$ ] ^ 4 + Sin[ $\theta$ ] ^ 4, { $\theta$ , - $\pi/4$ ,  $\pi/4$ }]
```



```
Minimize[Cos[ $\theta$ ] ^ 4 + Sin[ $\theta$ ] ^ 4,  $\theta$ ]
```

```
{Cos[2 ArcTan[ $1 - \sqrt{2}$ ]] ^ 4 + Sin[2 ArcTan[ $1 - \sqrt{2}$ ]] ^ 4, { $\theta \rightarrow -2$  ArcTan[ $1 - \sqrt{2}$ ]]}
```

```
FullSimplify[Pi / 4 == -2 ArcTan[ $1 - \sqrt{2}$ ]]
```

```
True
```

Minima for two triplets of $\Delta(6n^2)$

$$(e^{i\eta}, 0, 0), (e^{i\eta'}, 0, 0) \rightarrow (1, 0, 0), (1, 0, 0)$$

$$(e^{i\eta}, 0, 0), (0, e^{i\eta'}, 0) \rightarrow (1, 0, 0), (0, 1, 0)$$

$$(e^{i\eta}, 0, 0), (e^{i\eta'}, e^{i\zeta'}, 0) \rightarrow (1, 0, 0), (1, 1, 0)$$

$$(e^{i\eta}, 0, 0), (0, e^{i\eta'}, e^{i\zeta'}) \rightarrow (1, 0, 0), (0, 1, 1)$$

$$(e^{i\eta}, 0, 0), (e^{i\eta'}, e^{i\zeta'}, e^{i\theta'}) \rightarrow (1, 0, 0), (1, 1, 1)$$

$$(e^{i\eta}, e^{i\zeta}, 0), (0, e^{i\zeta'}, e^{i\theta'}) \rightarrow (1, 1, 0), (0, 1, 1)$$

$$(e^{i\eta}, e^{i\zeta}, 0), (e^{i\eta'}, e^{i\zeta'}, 0) \rightarrow (1, 1, 0), (1, e^{i\zeta'}, 0)$$

$$(e^{i\eta}, e^{i\zeta}, 0), (e^{i\eta'}, e^{i\zeta'}, e^{i\theta'}) \rightarrow (1, 1, 0), (1, e^{i\zeta'}, 1)$$

$$(e^{i\eta}, e^{i\zeta}, e^{i\theta}), (e^{i\eta'}, e^{i\zeta'}, e^{i\theta'}) \rightarrow (1, 1, 1), (1, e^{i\zeta'}, e^{i\theta'}).$$