

*Workshop on Non-Standard Dark Matter*  
— Warsaw, Poland — 5 June 2016

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# Flavor-Mixed DM in Cosmology

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M. Medvedev, University of Kansas

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# Outline

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## Cosmology

- small-scale problems in CDM

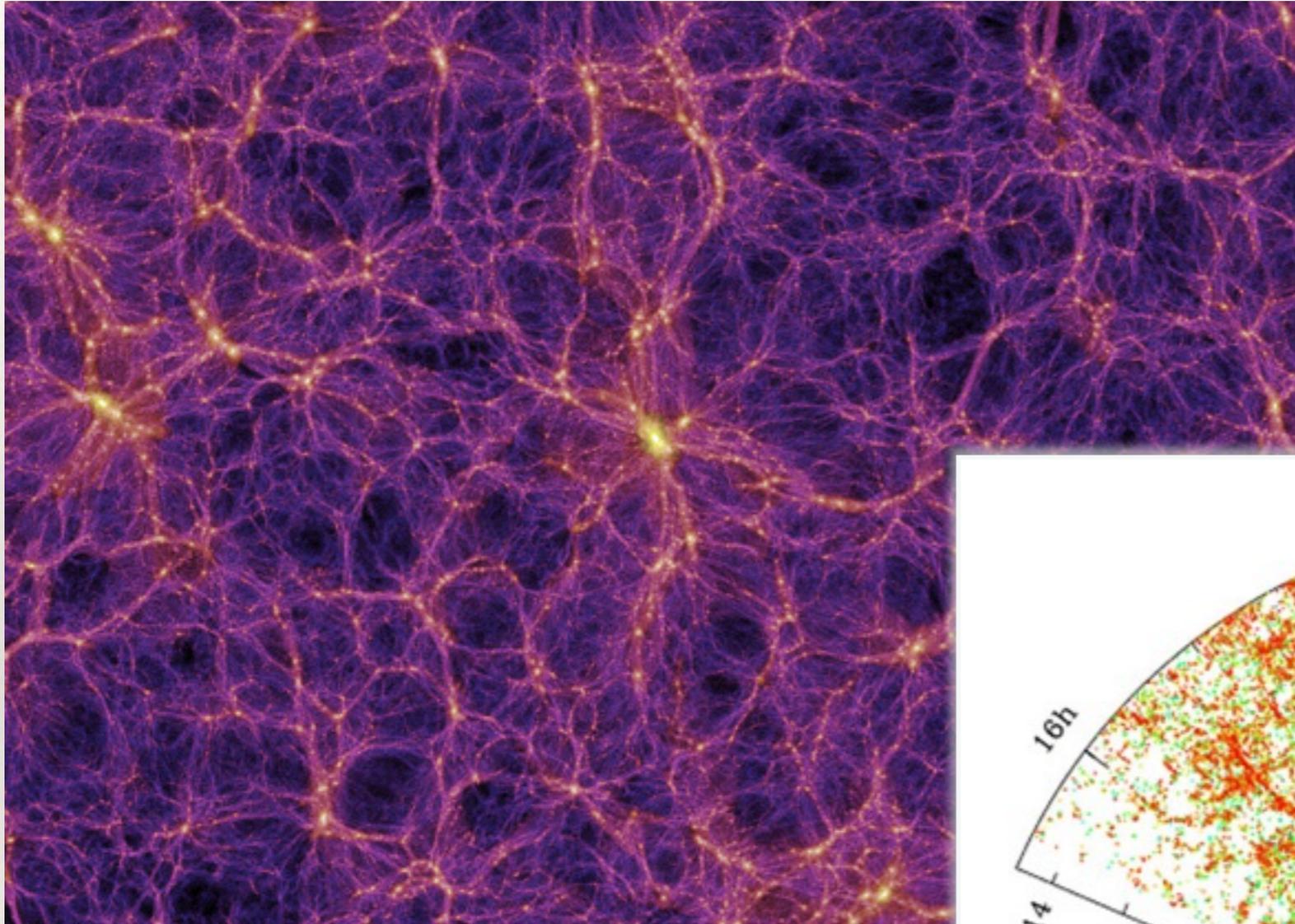
## Quantum physics

- quantum evaporation

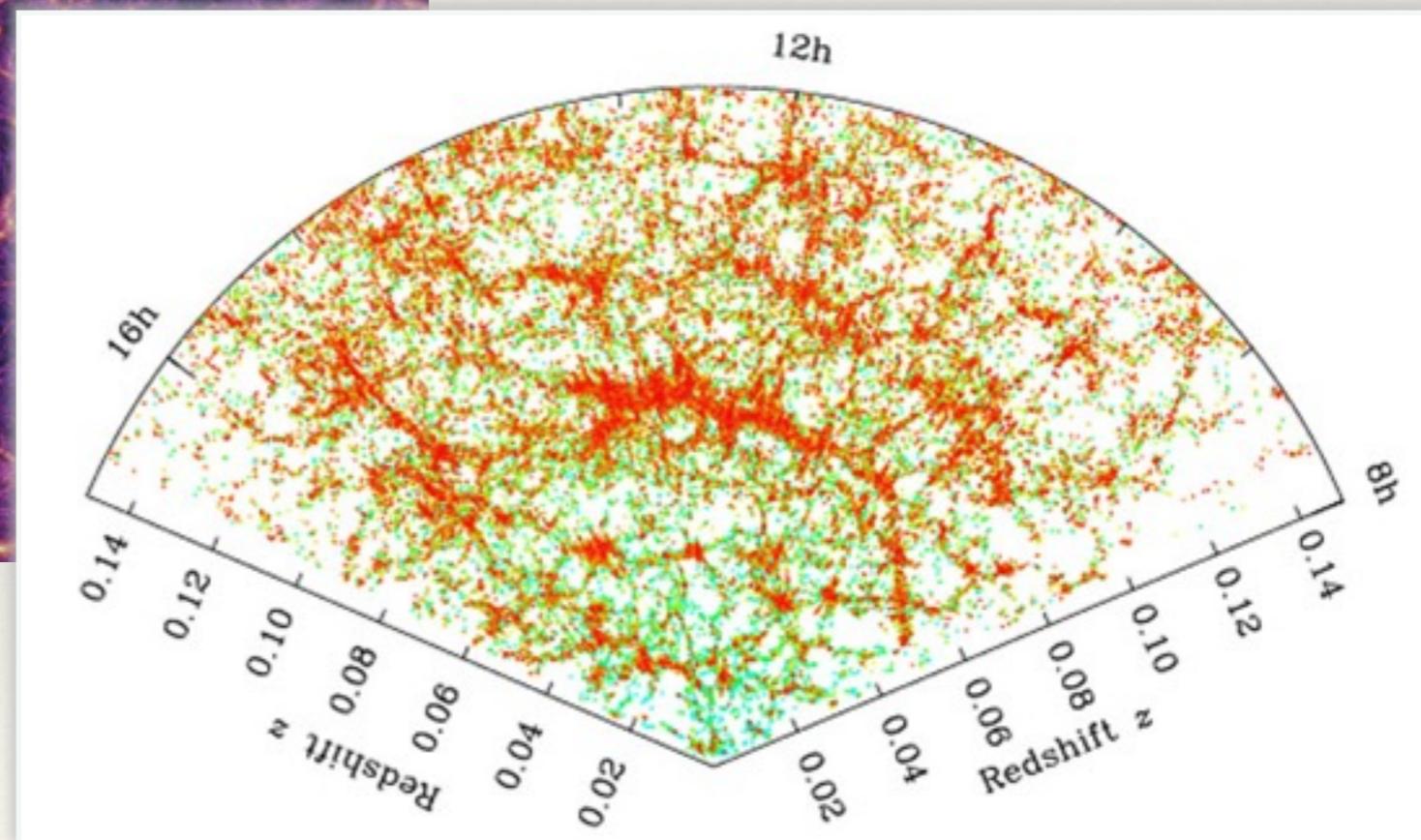
## Cosmology again

- 2-component dark matter (2cDM)

# Triumph on large scales



$\Lambda$ CDM reproduces  
Large-Scale Structure  
of the universe

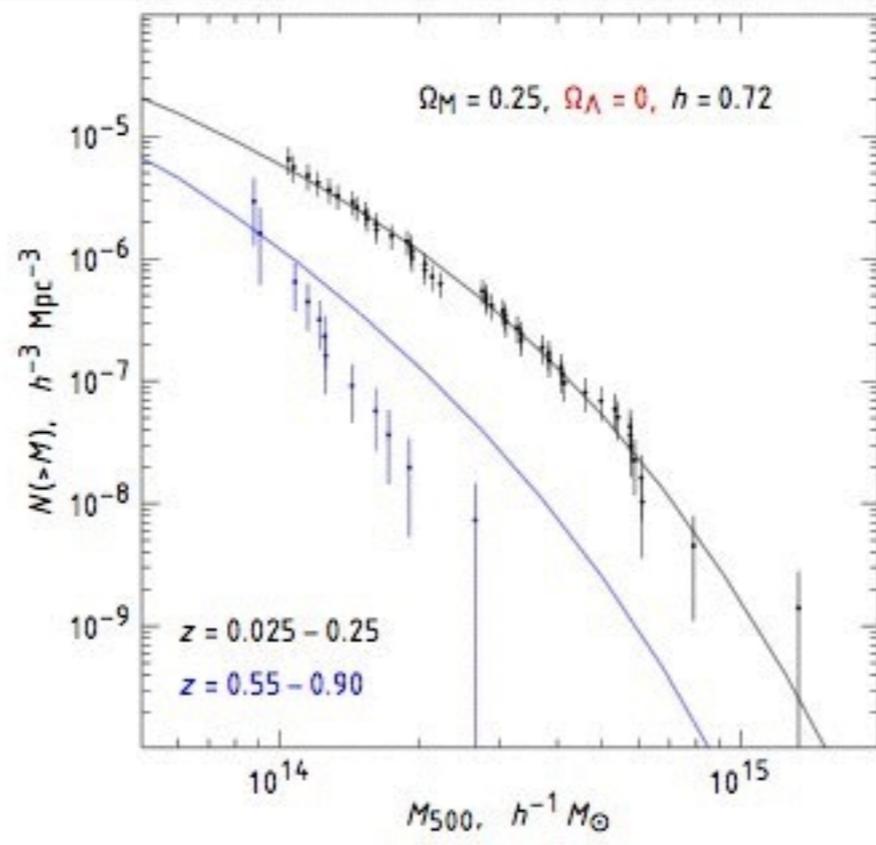
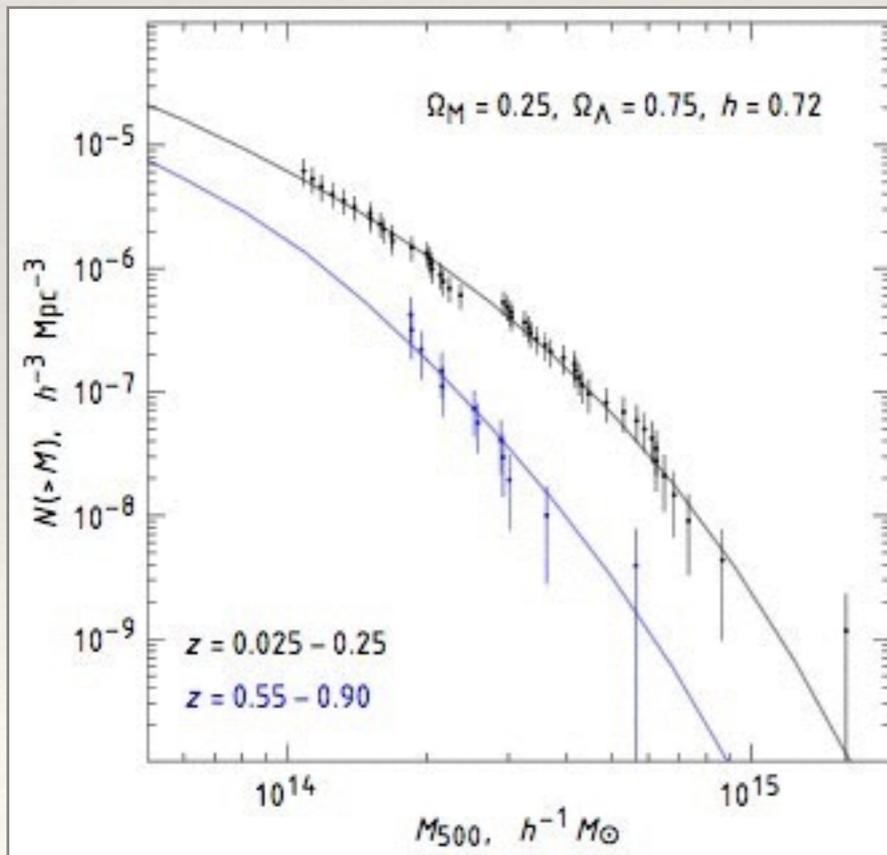
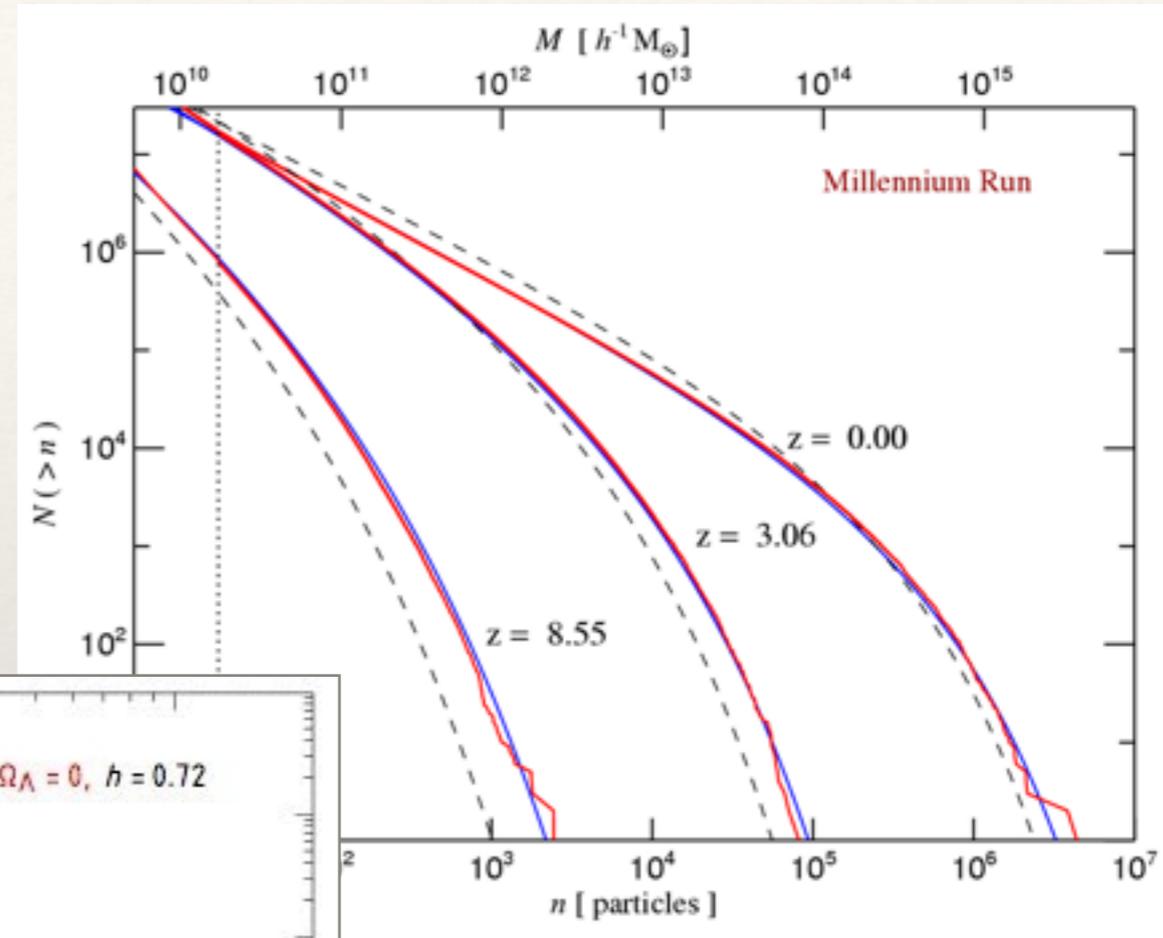


*Millenium simulation*

*SDSS survey*

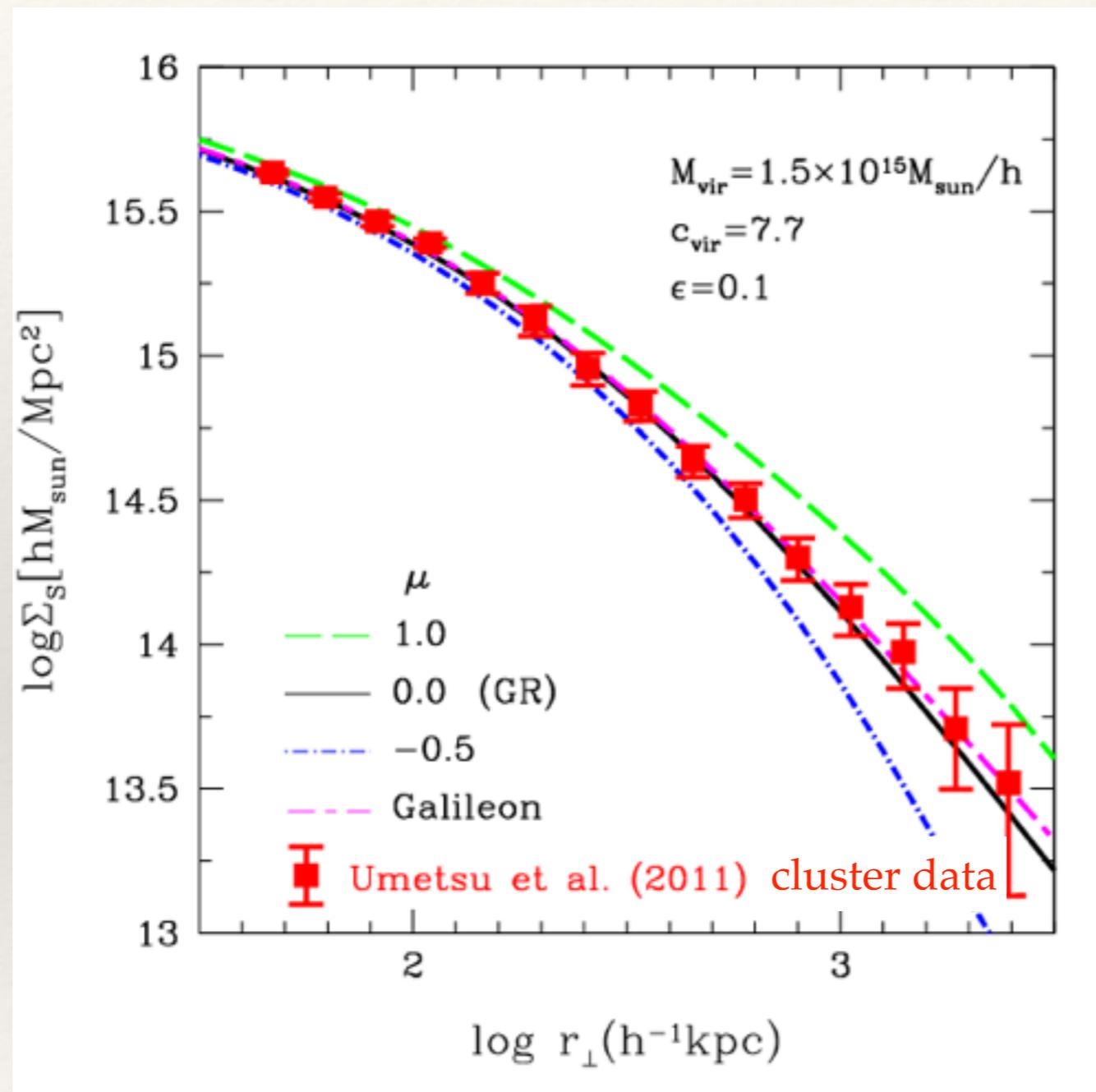
# $\Lambda$ CDM on large scales - halo counts

- Cumulative distribution of Dark Matter halos by mass fits cluster data
- Halo counts yields cosmological parameters



Vikhlinin et al

# $\Lambda$ CDM on large scales - density profiles

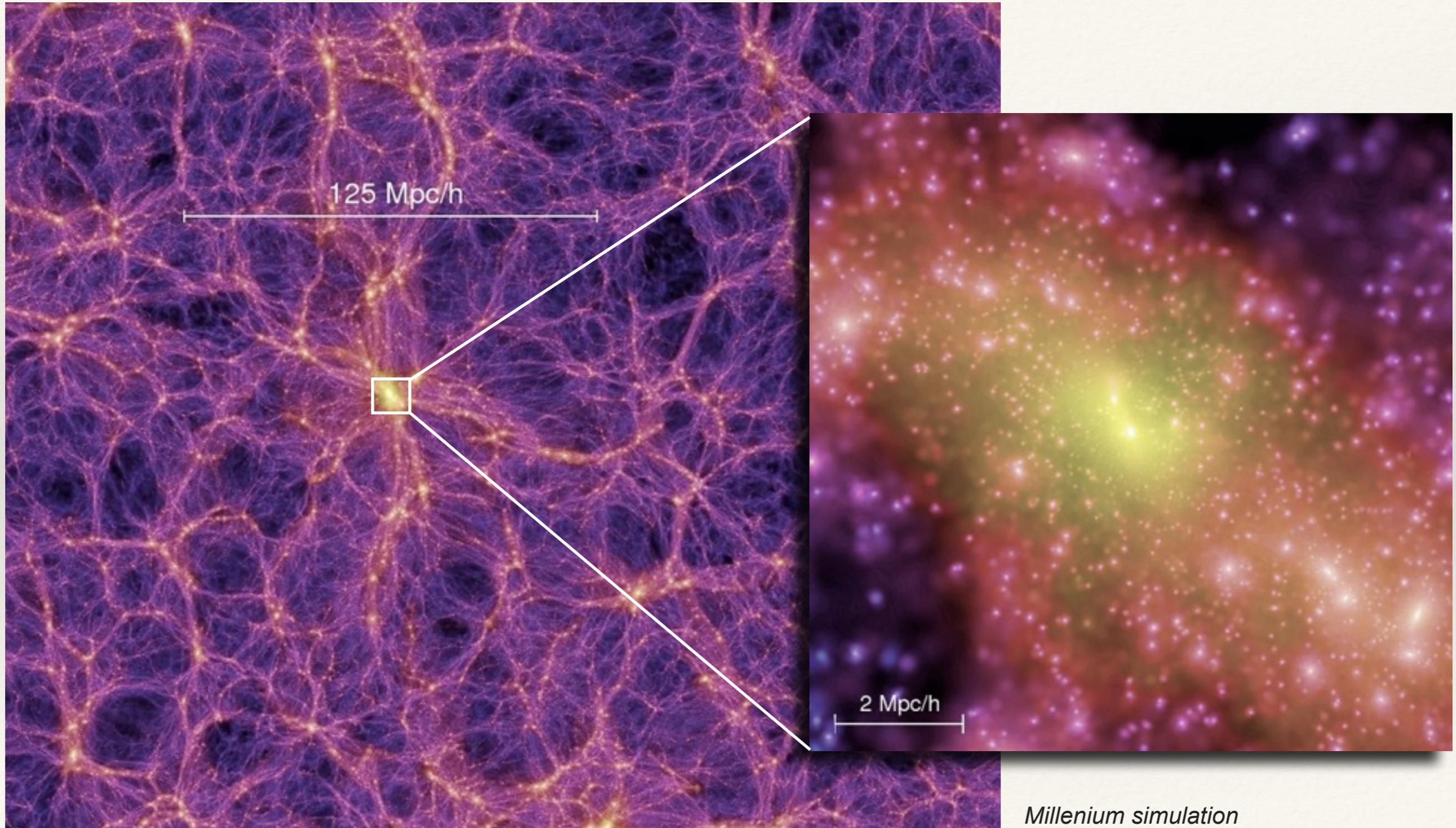


- Dark Matter density profiles in halos described by Navarro-Frenk-White (NFW) profile fit galaxy cluster data

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...but too much small-scale stuff

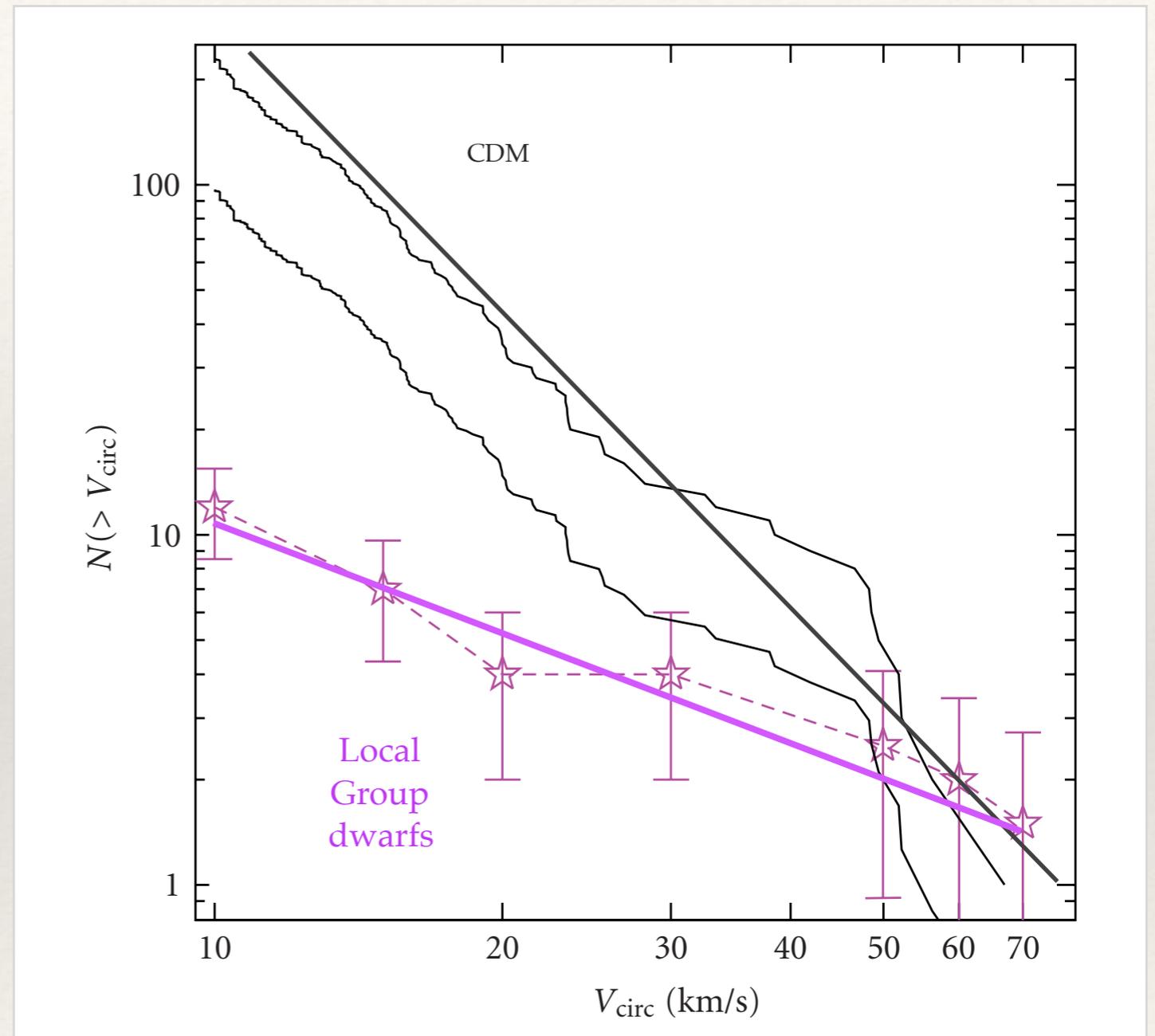
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*Millenium simulation*

# $\Lambda$ CDM problems – small scales

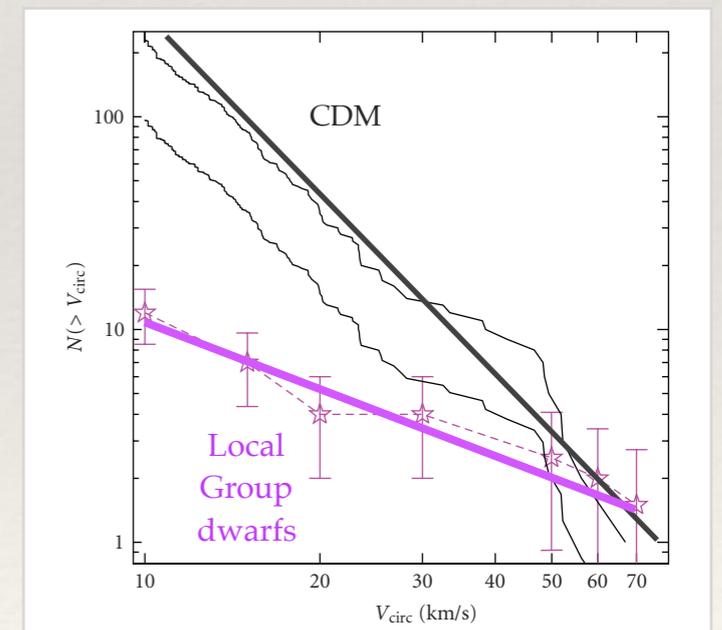
- substructure problem  
(missing satellites)



Klypin+ 1999  
Kravtsov 2010

# $\Lambda$ CDM problems – small scales

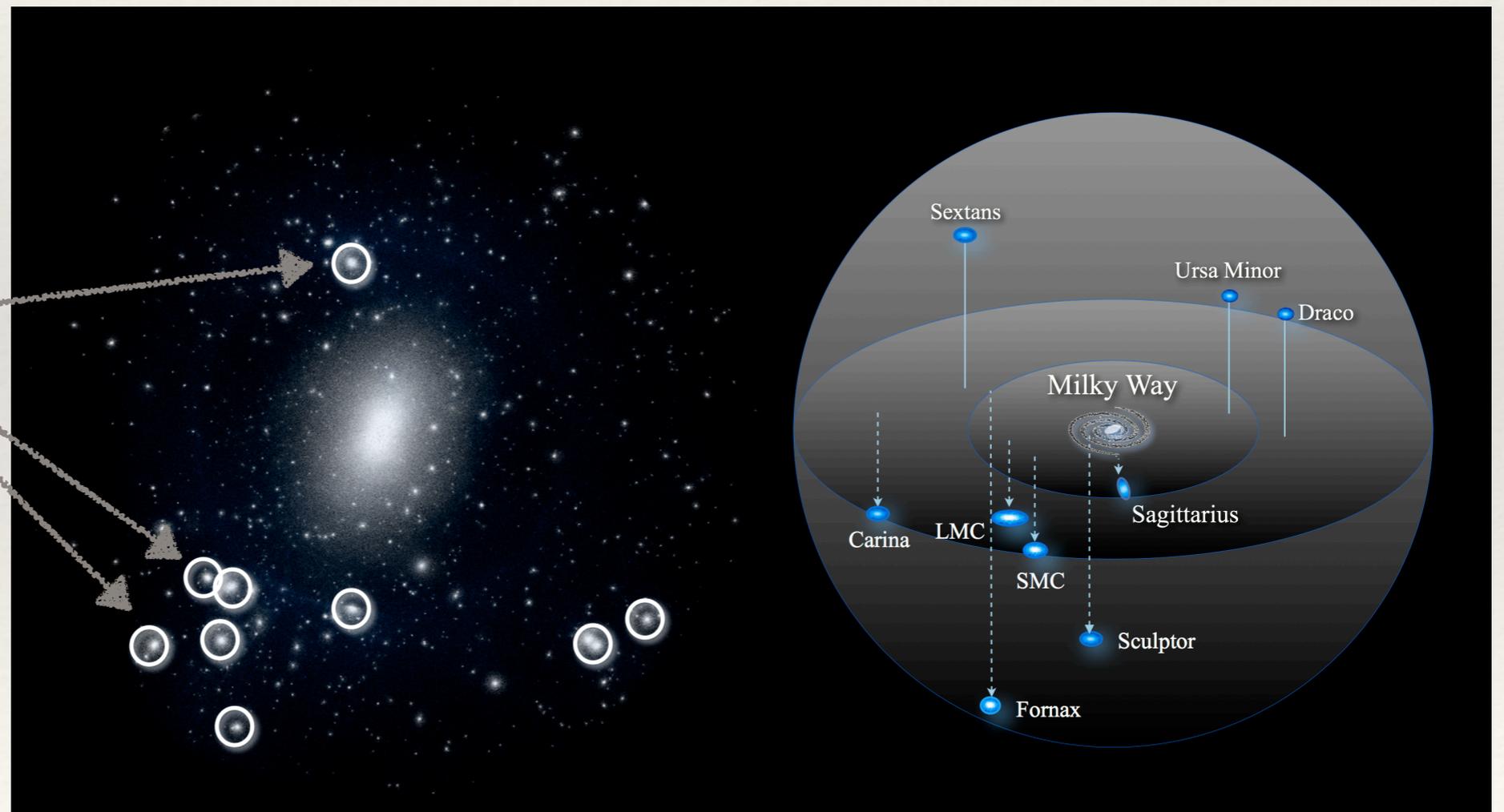
- substructure problem  
(missing satellites)



# $\Lambda$ CDM problems – small scales

- substructure problem  
(missing satellites)
- too-big-to-fail problem

*None of these  
most massive  
are observed*

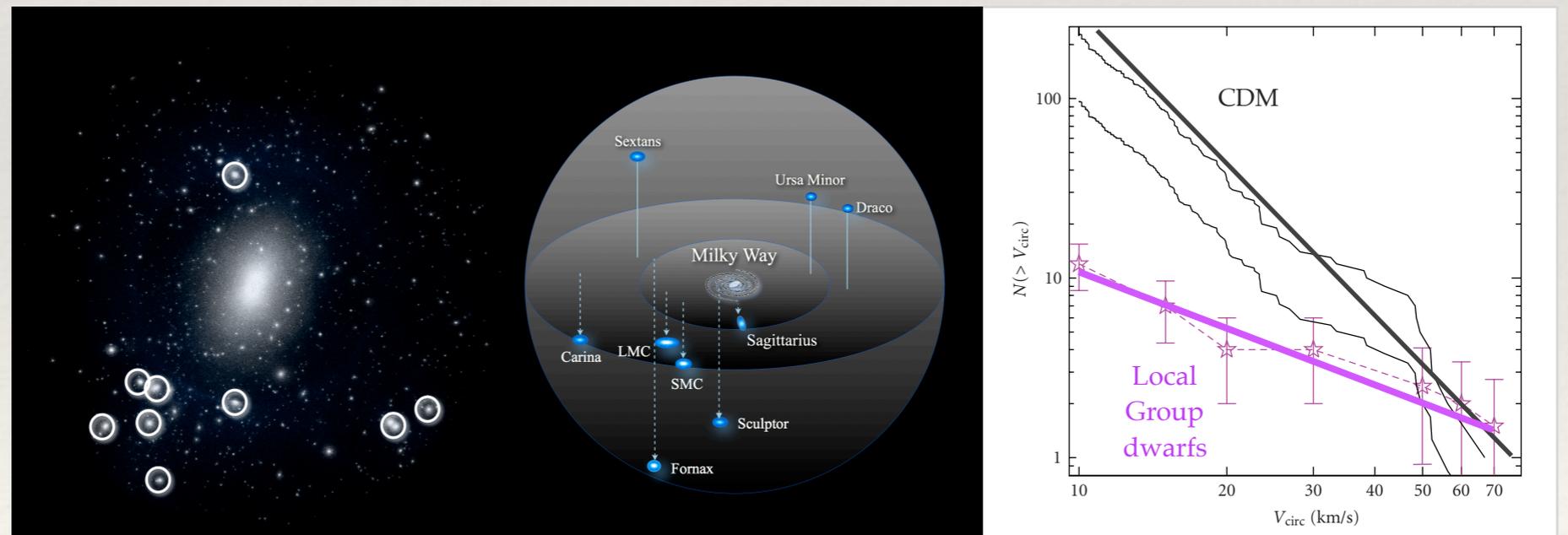


from Weinberg et al 2013  
by Garrison-Kimmel, Boylan-Kolchin & Bullock

Yniguez et al. 2013

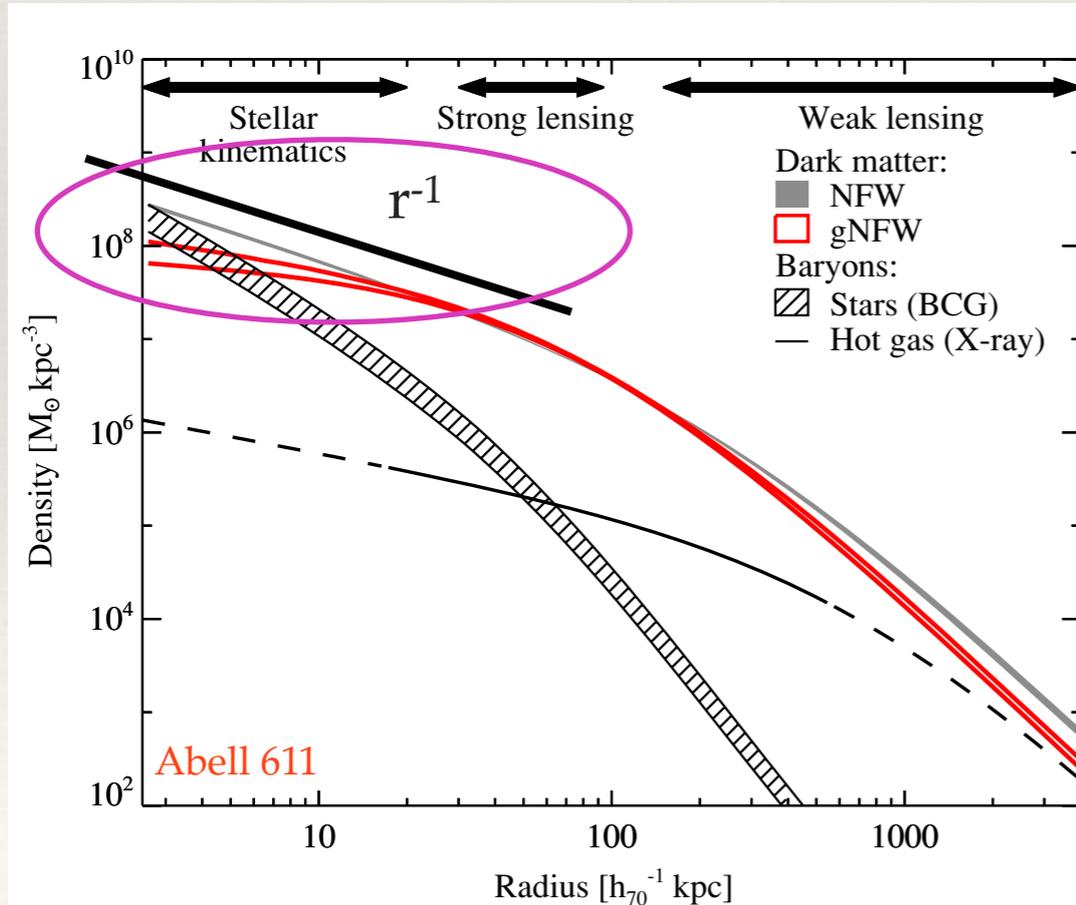
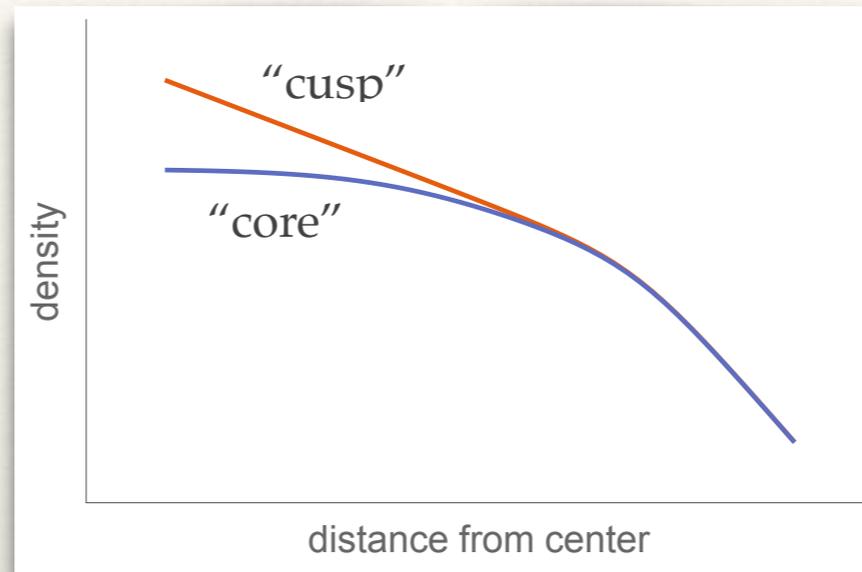
# $\Lambda$ CDM problems – small scales

- substructure problem  
(missing satellites)
- too-big-to-fail problem

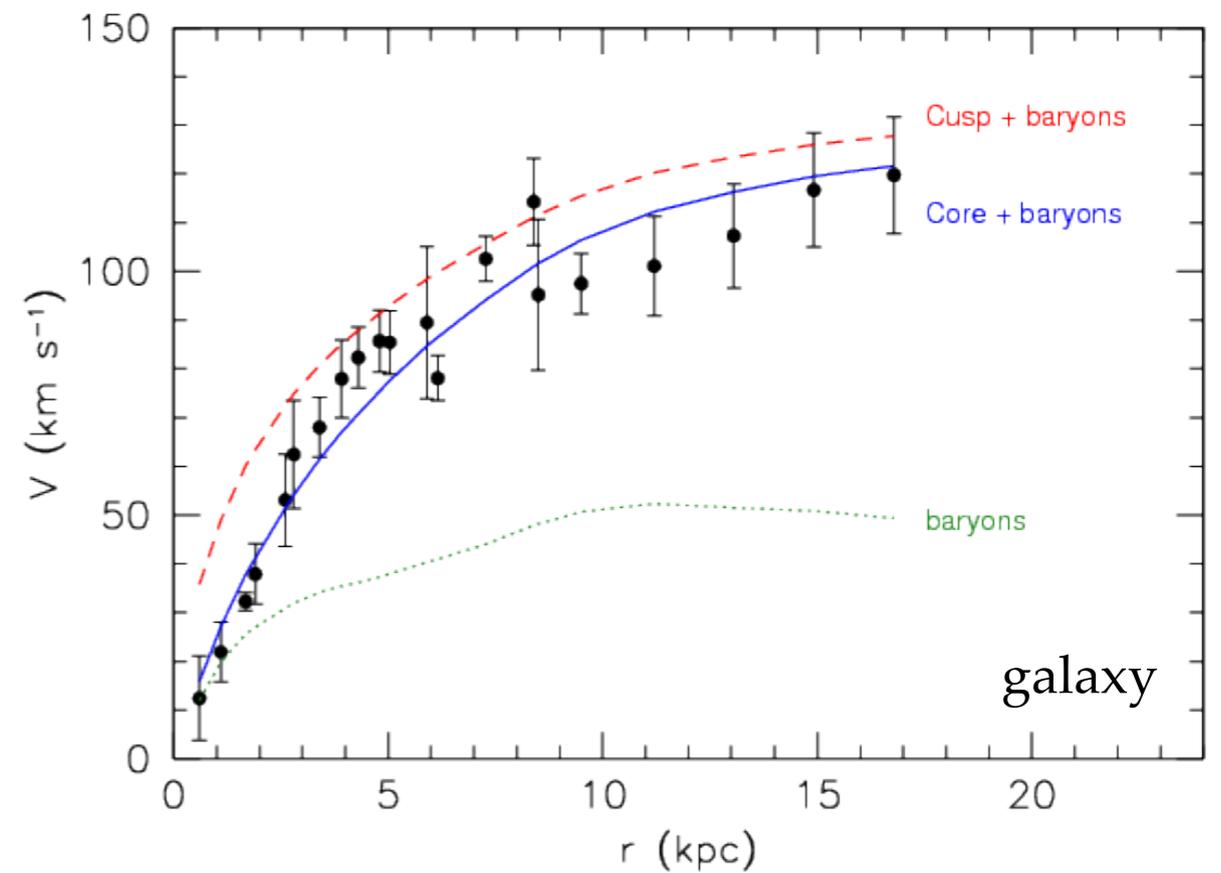


# $\Lambda$ CDM problems – small scales

- substructure problem (missing satellites)
- too-big-to-fail problem
- **core/cusp problem**

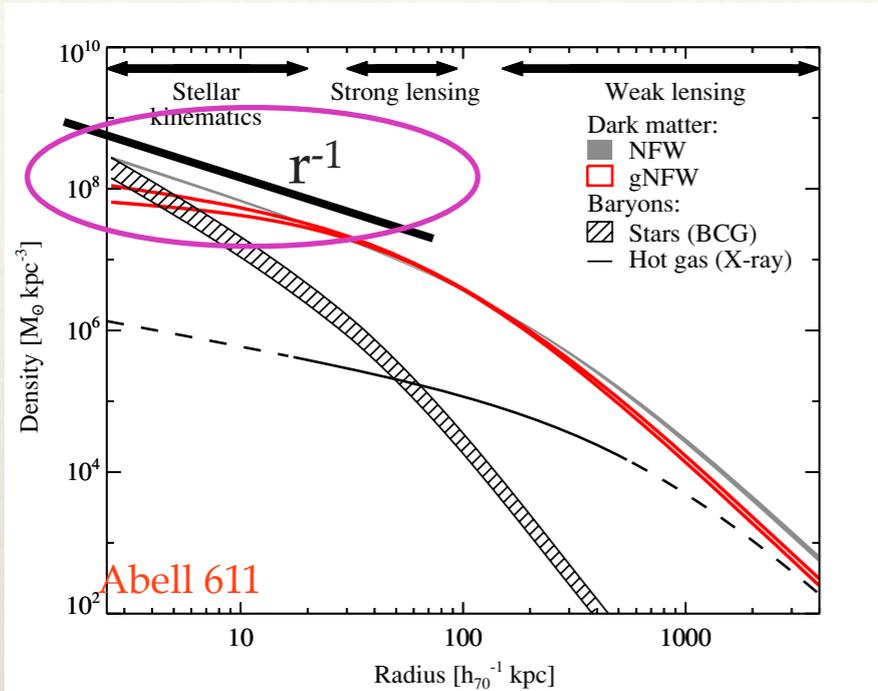


Newman+ 2009

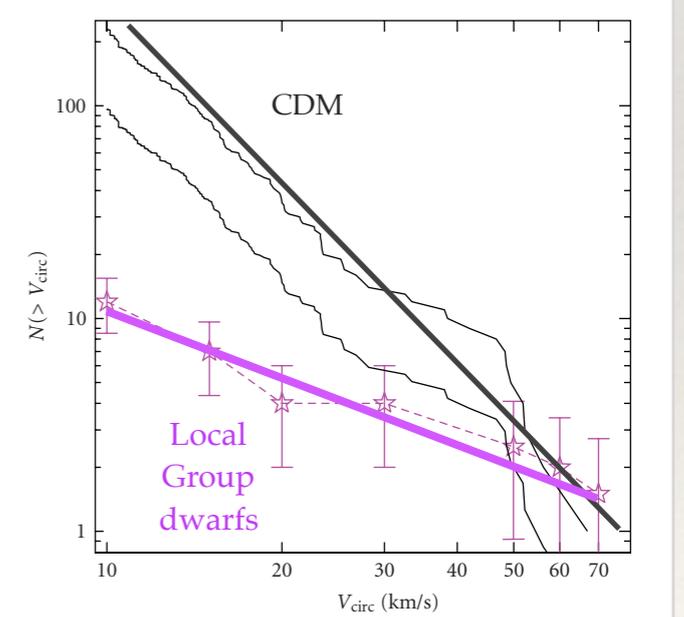
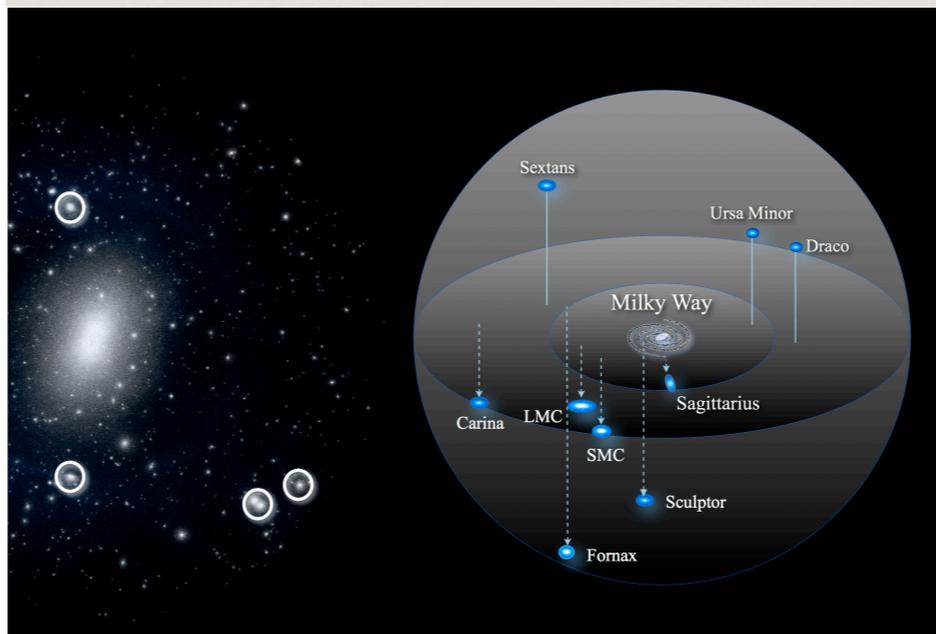
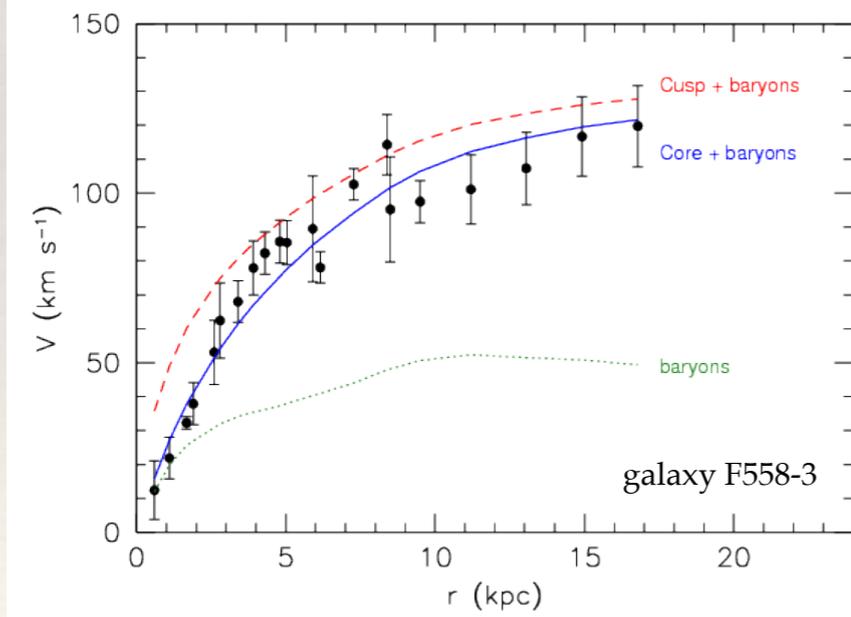


Kuzio de Naray et al. 2008

# $\Lambda$ CDM problems – small scales



- substructure problem (missing satellites)
- too-big-to-fail problem
- core / cusp problem



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# Possible solutions

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- **Baryonic physics**
  - NS feedback
  - outflows
  - modified star formation
  
- **Dark Matter physics**

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# Possible solutions

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- **Baryonic physics**

- NS feedback
- outflows
- modified star formation



**Inconclusive**

- **Dark Matter physics**

# Dark Matter

Dark Matter

$m$

"just" a parameter

# Dark Matter

**m**

"just" a parameter

**T**

WDM - cannot solve satellites  
and cusp problem simultaneously:  
different scales

# Dark Matter

$m$

"just" a parameter

$T$

WDM - cannot solve satellites  
and cusp problem simultaneously:  
different scales

$\sigma$

SIDM - solves cusp problem only;  
cannot affect satellite counts

Dark Matter

$m$

$T$

$\sigma$

Dark Matter

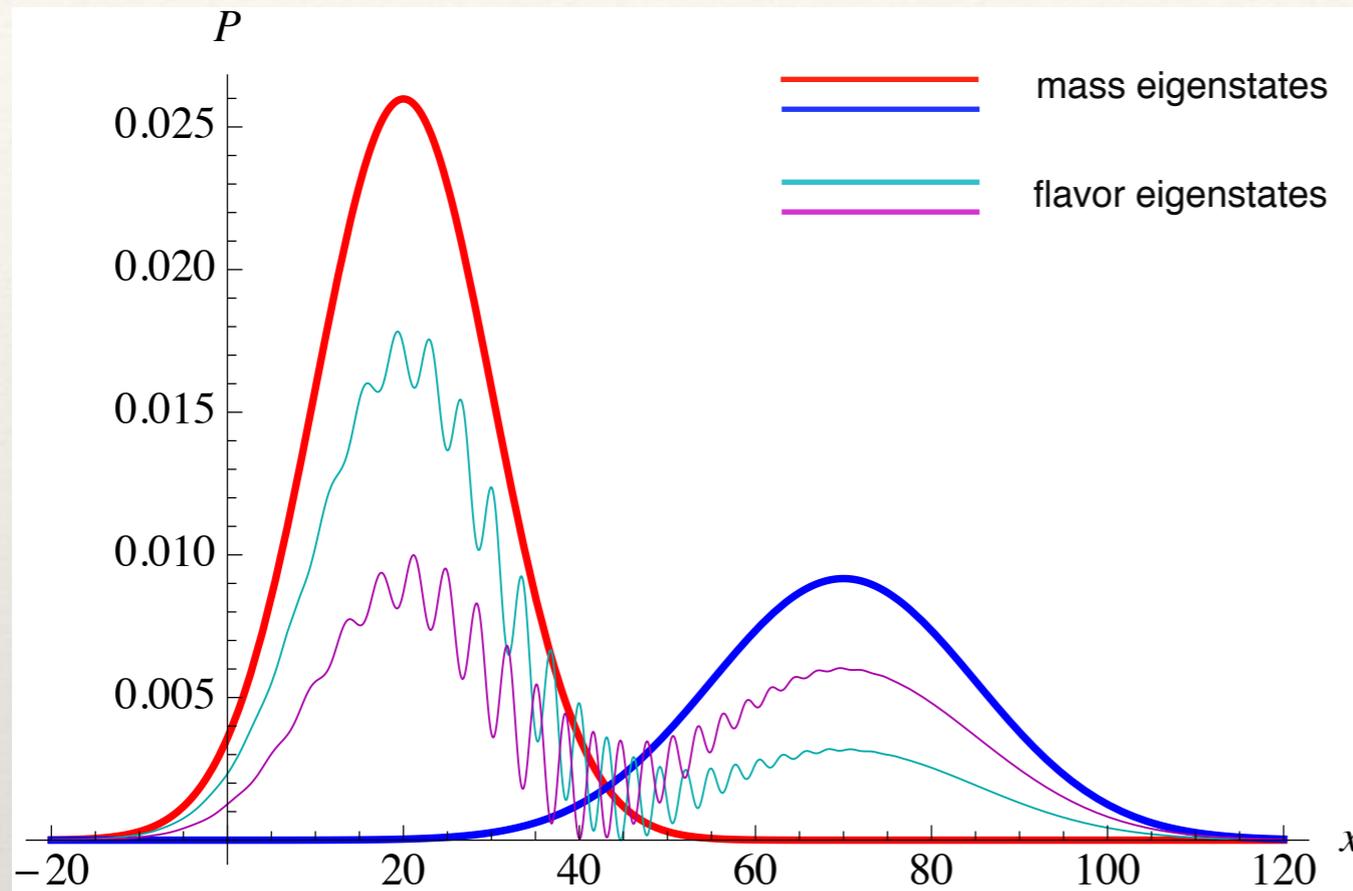
$m$

$T$

$\sigma$

flavor

# 2-component particle



B. Pontecorvo  
*Zh. Teor. Exp Fiz (1957); Soviet JETP (1958)*

$$\begin{pmatrix} |\text{flavor}_1\rangle \\ |\text{flavor}_2\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\text{mass}_{\text{heavy}}\rangle \\ |\text{mass}_{\text{light}}\rangle \end{pmatrix}$$

Flavor is a quantum property that allows a particle to have several masses altogether, at the same time and vice versa

# Illustrative model

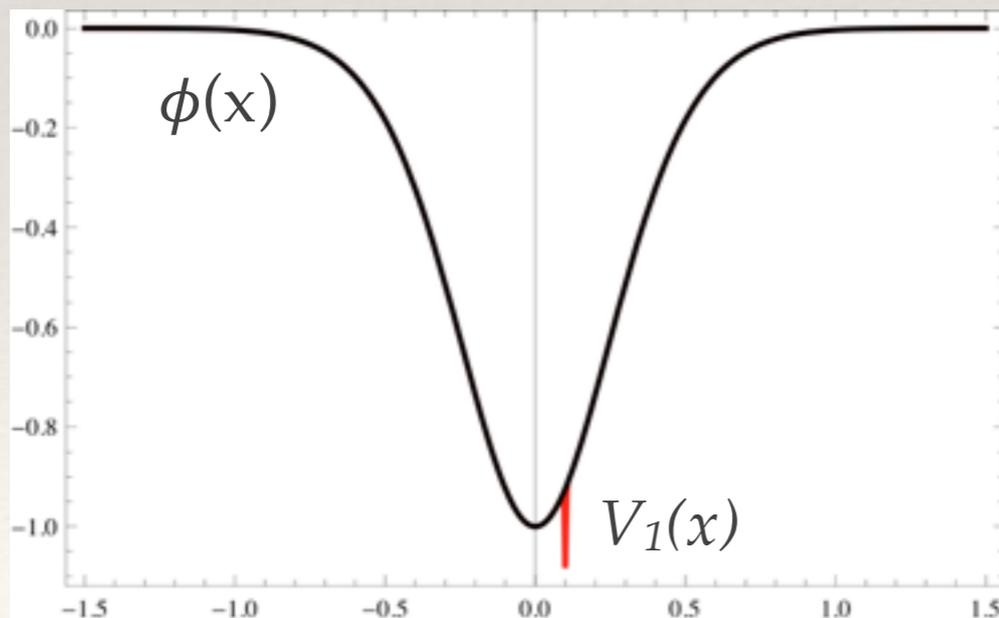
Schrödinger equation

$$i\partial_t \begin{pmatrix} m_h(x, t) \\ m_l(x, t) \end{pmatrix} = \left[ \underbrace{\begin{pmatrix} -\partial_{xx}^2/2m_h & 0 \\ 0 & -\partial_{xx}^2/2m_l - \Delta m \end{pmatrix}}_{H_{\text{free}}} + \underbrace{\begin{pmatrix} m_h\phi(x) & 0 \\ 0 & m_l\phi(x) \end{pmatrix}}_{H_{\text{grav}}} + \underbrace{\begin{pmatrix} V_{hh} & V_{hl} \\ V_{lh} & V_{ll} \end{pmatrix}}_V \right] \begin{pmatrix} m_h(x, t) \\ m_l(x, t) \end{pmatrix}$$

$H_{\text{free}}$

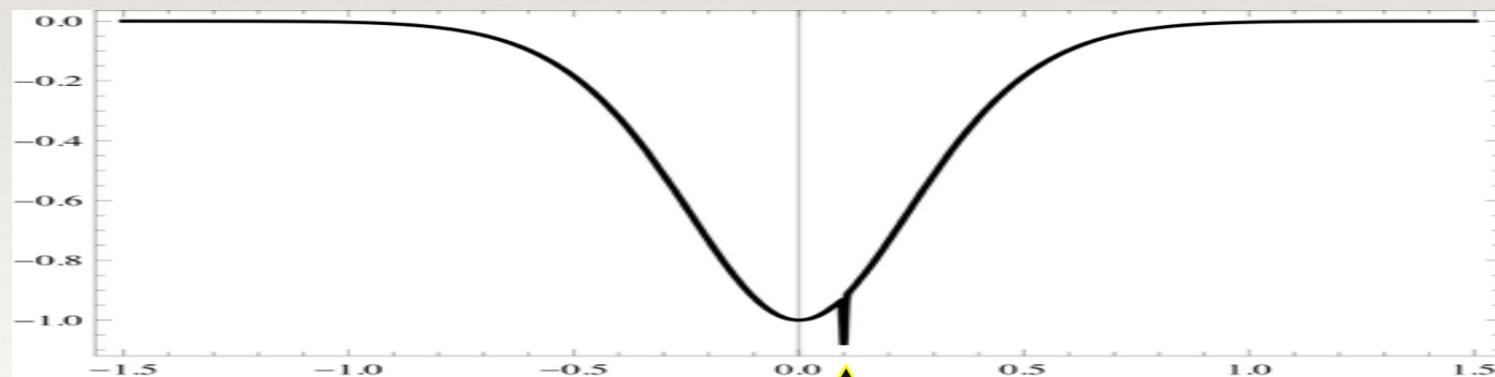
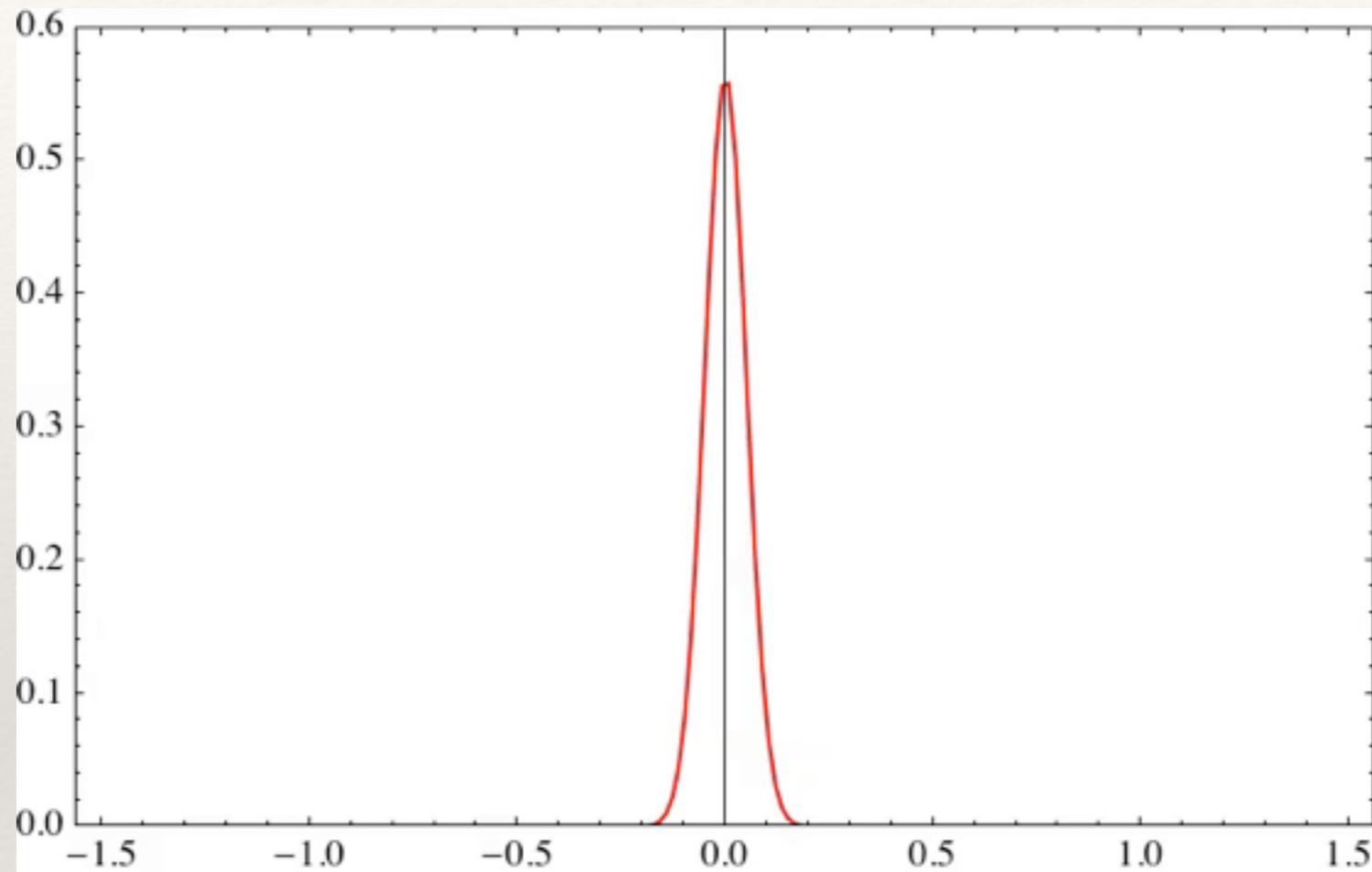
$H_{\text{grav}}$

$V$



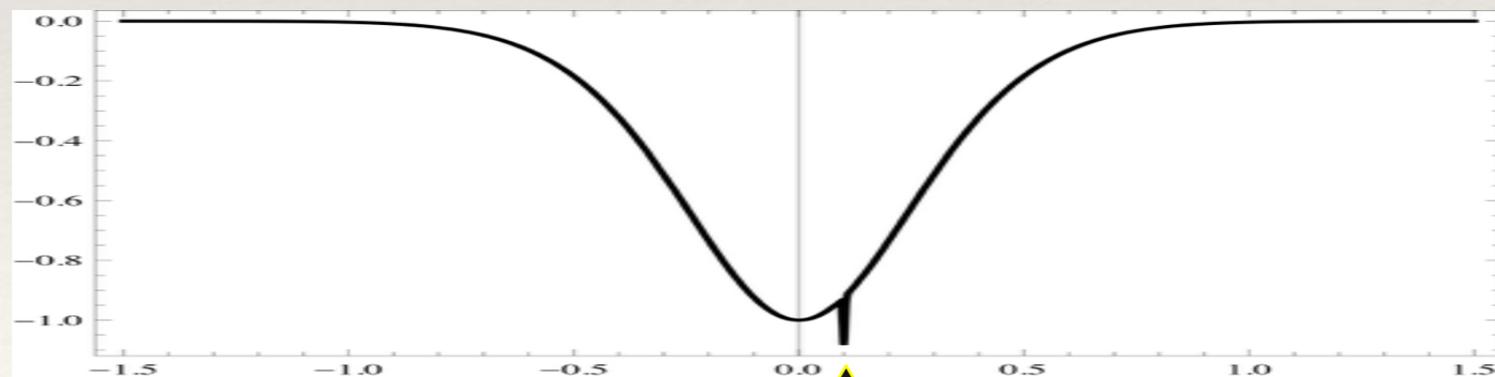
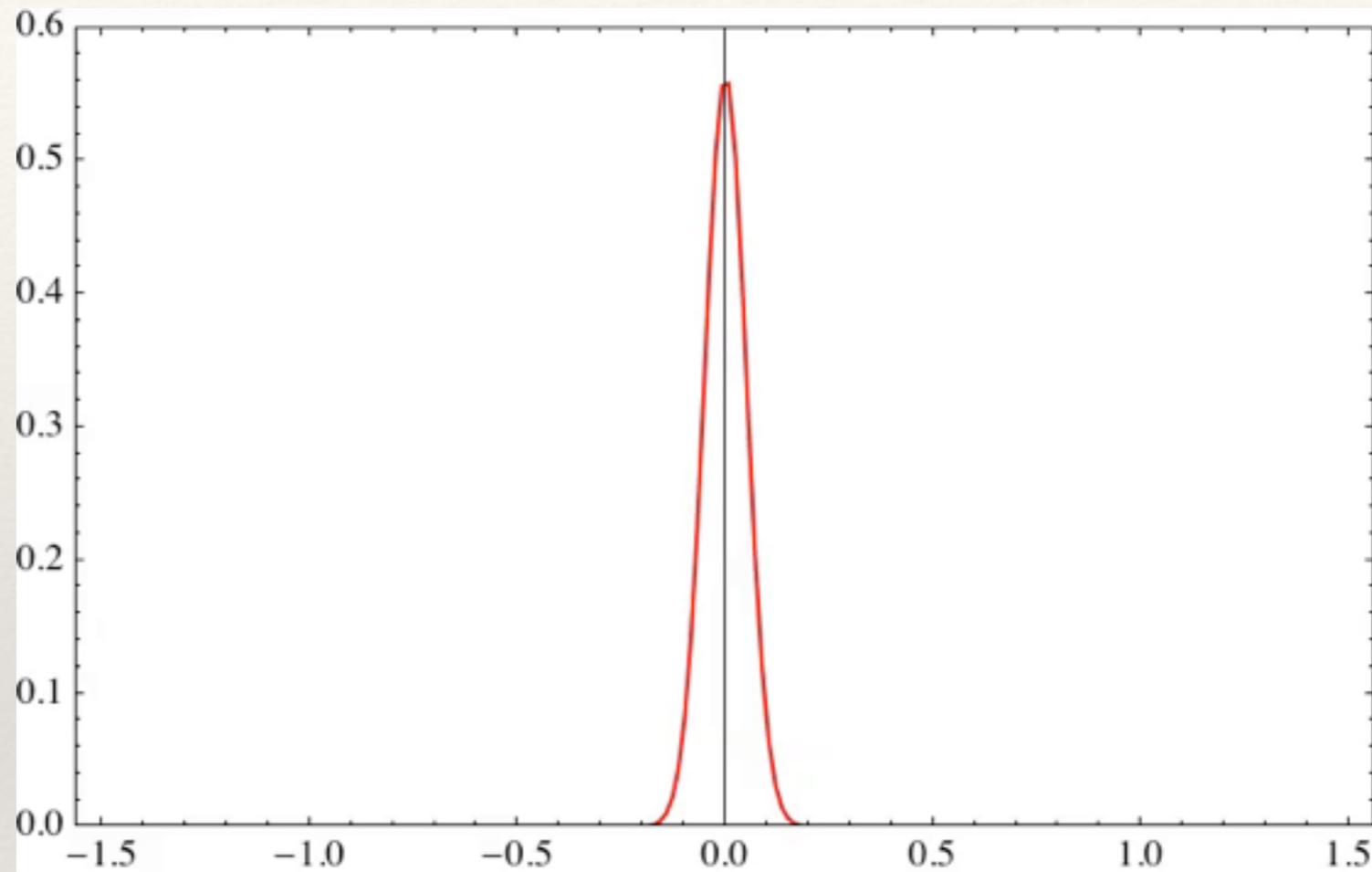
$$\begin{pmatrix} V_{hh} & V_{hl} \\ V_{lh} & V_{ll} \end{pmatrix} = U \begin{pmatrix} V_1 & 0 \\ 0 & 0 \end{pmatrix} U^\dagger$$

# No flavor mixing case



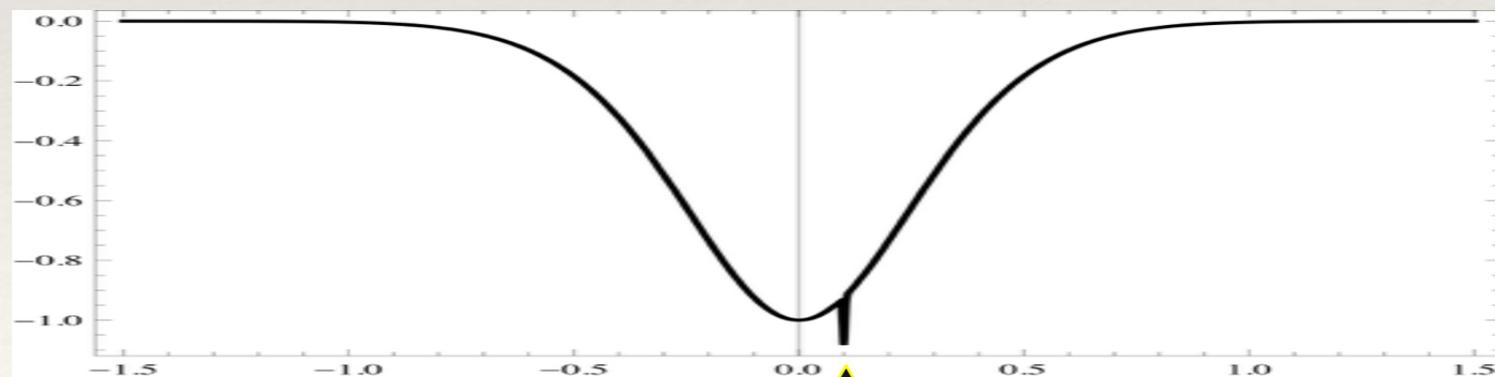
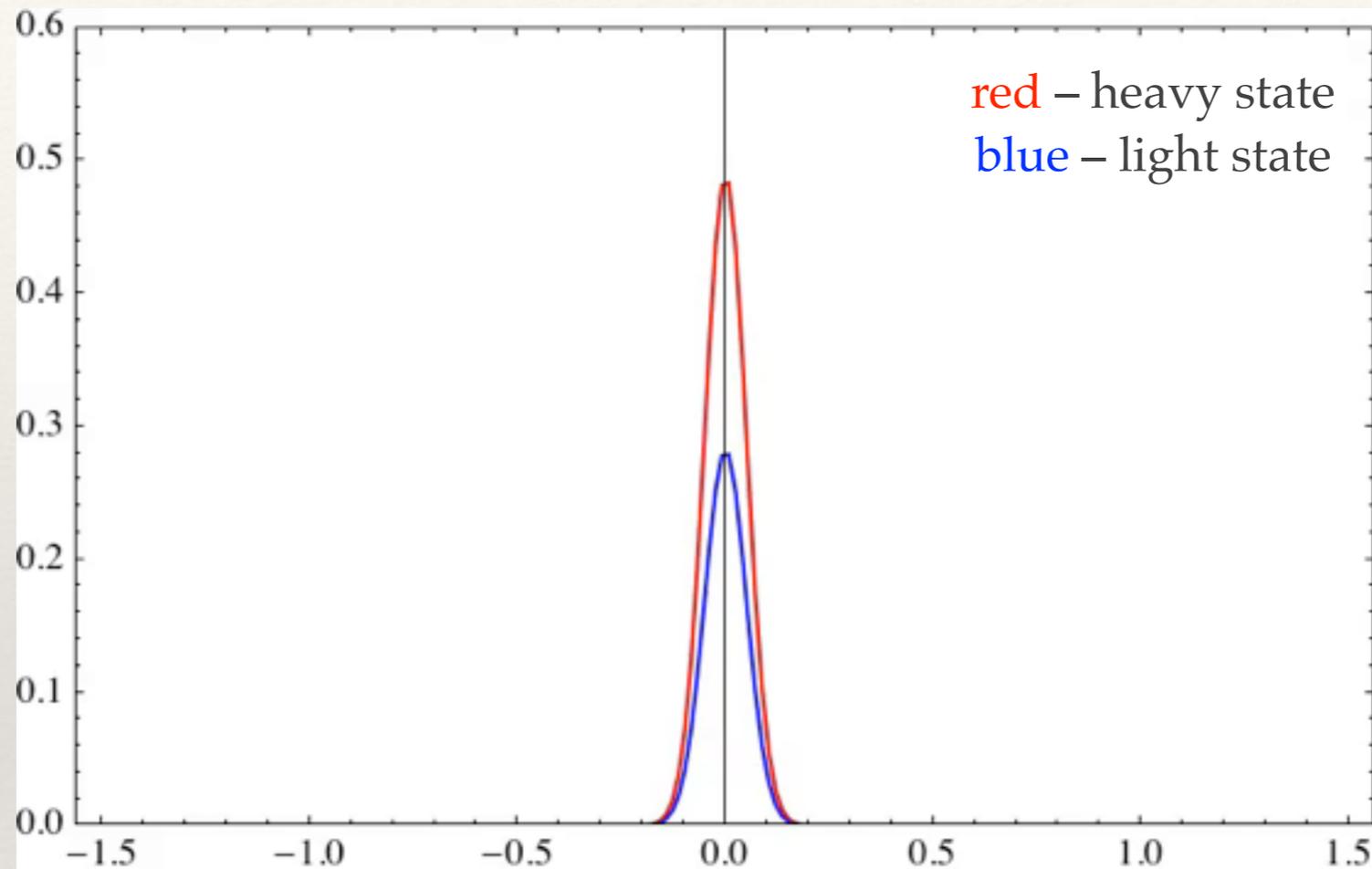
scatterer

# No flavor mixing case



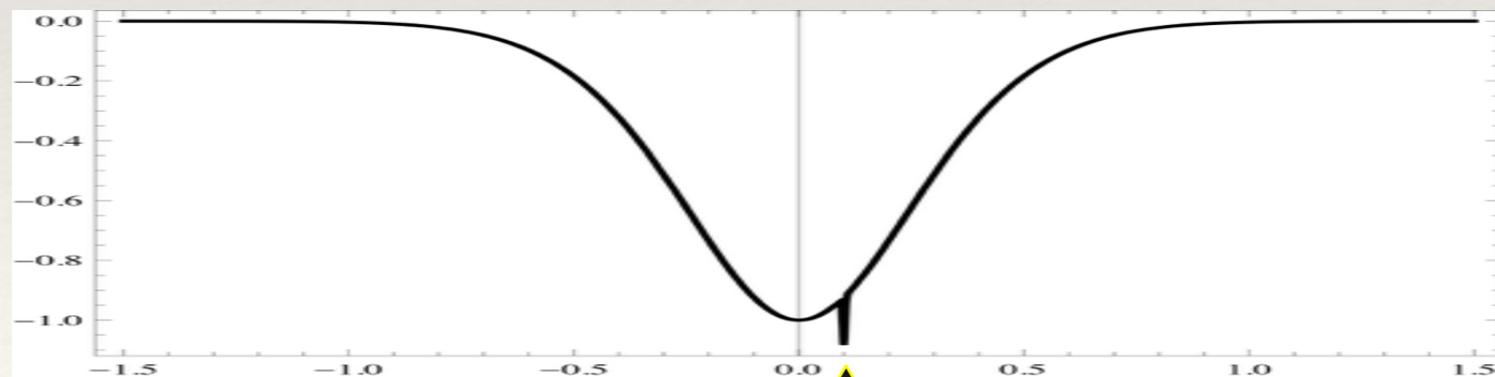
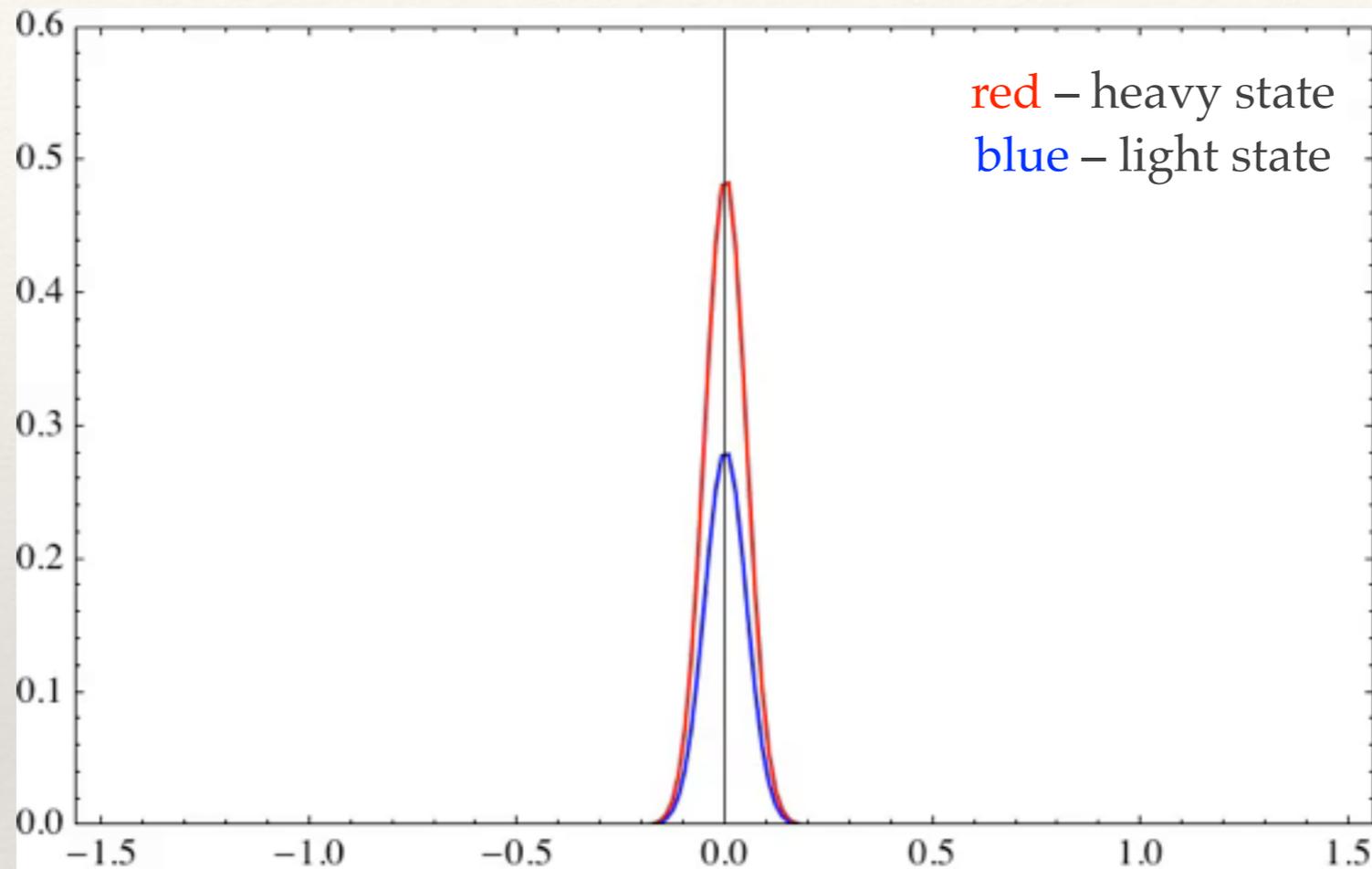
scatterer

# With flavor mixing



scatterer

# With flavor mixing

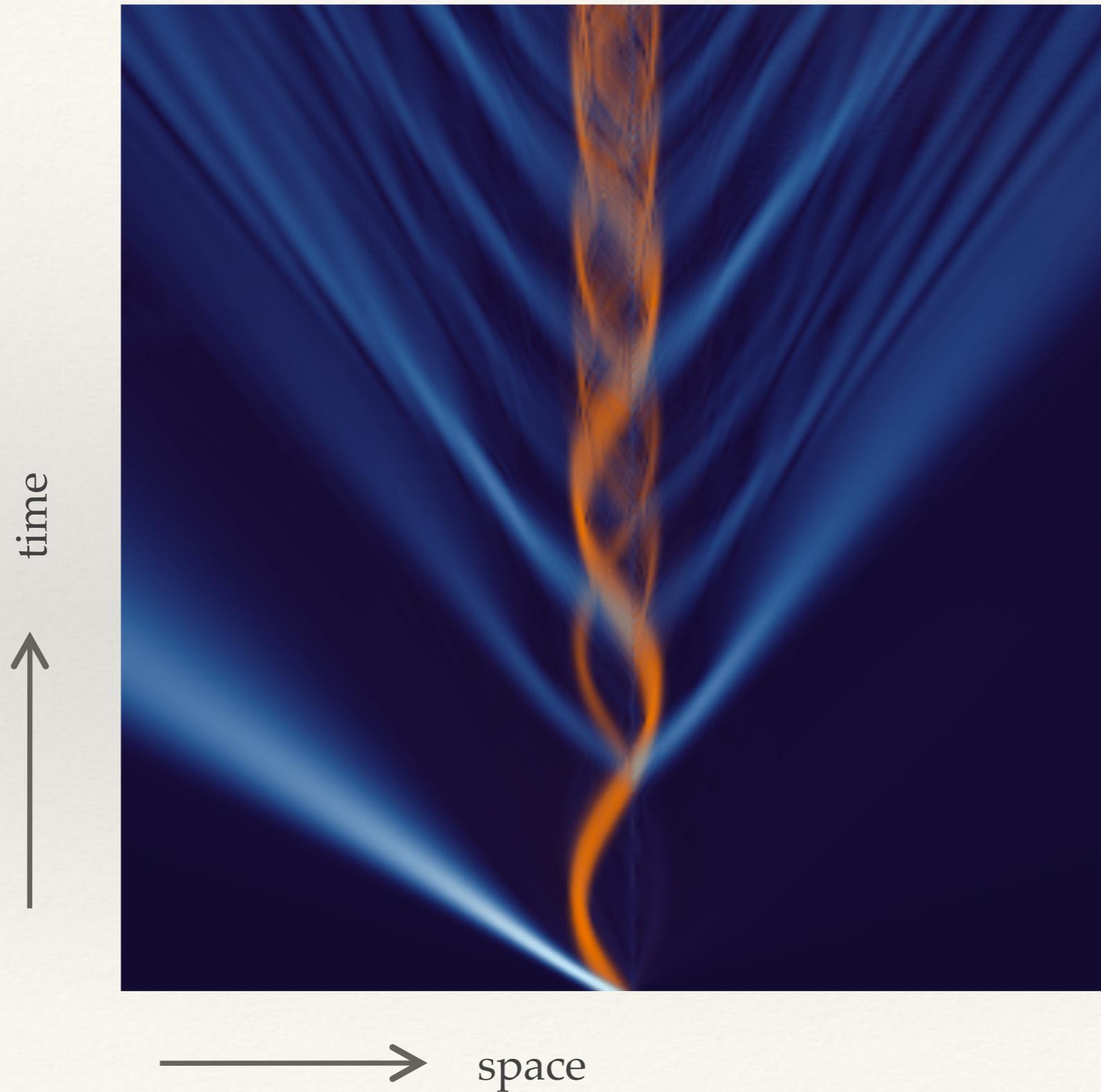


scatterer

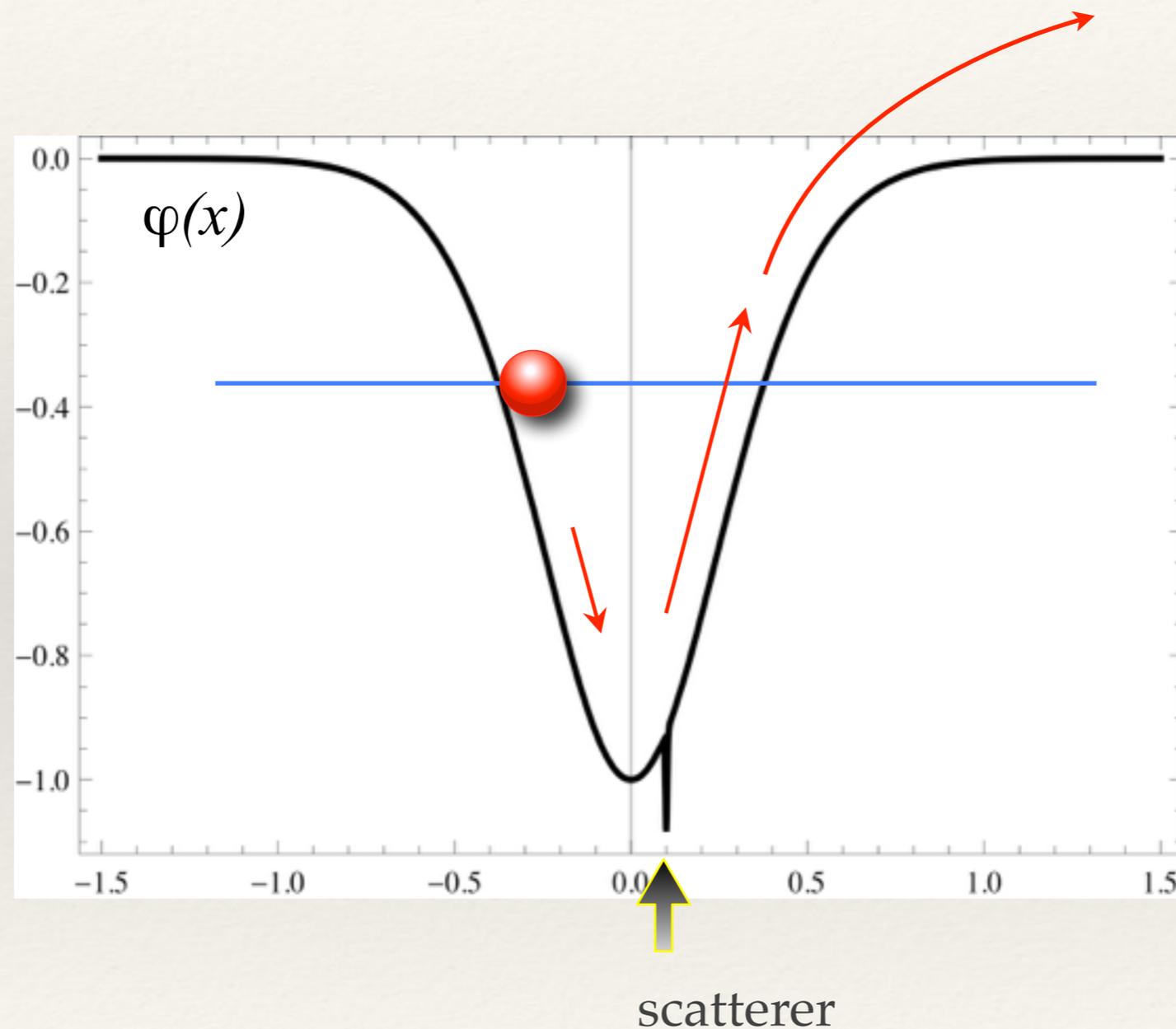
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# Space-Time diagram

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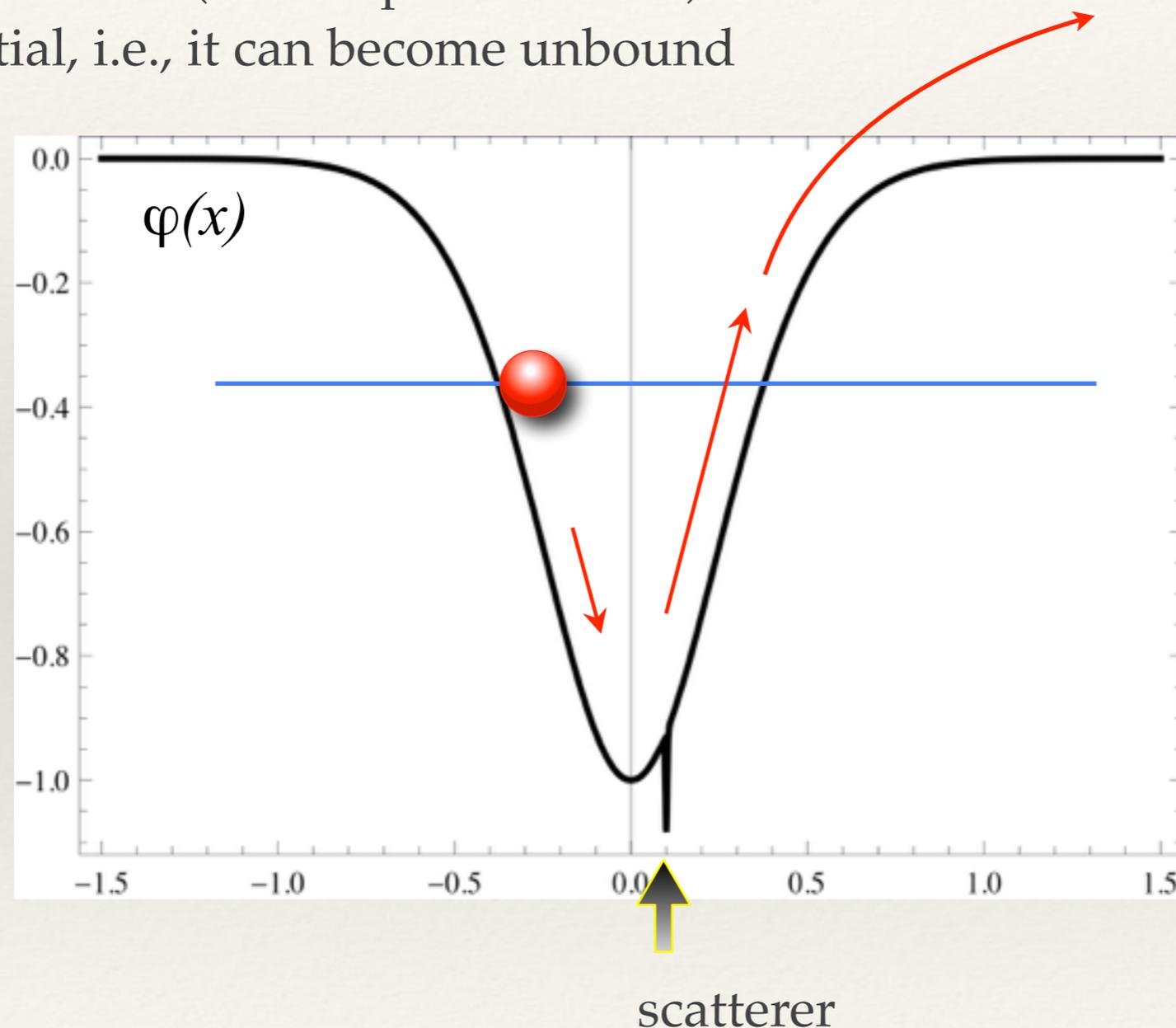


# A trapped particle + scattering + flavor mixing



# A trapped particle + scattering + flavor mixing

The particle can leak out of (or “evaporate” from) the gravitational potential, i.e., it can become unbound



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# “Münchhausen effect”

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Baron von Münchhausen lifted himself (and his horse) out of the mud by pulling on his own pigtail.

It is one of the “true” stories from *“The Surprising Adventures of Baron Munchhausen”* by R.E. Raspe

# 2-component flavor-mixed DM (2cDM)

Dark Matter — stable 2-component mixed particle

Neutralinos  
Sterile neutrinos  
Axion+photon

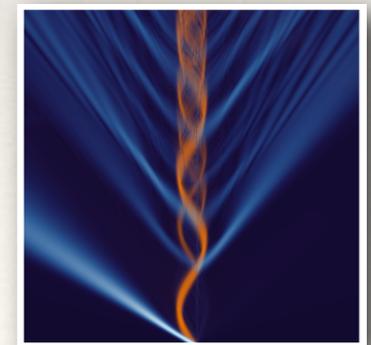
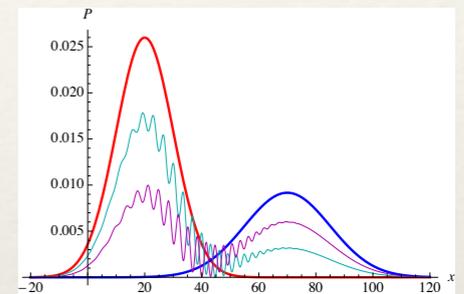
...

DM halos — self-gravitating ensembles of mass eigenstates

$|h\rangle, |l\rangle$

Mass-conversions and quantum evaporation

$$|h\rangle + |l\rangle \rightarrow |l\rangle + |l\rangle$$

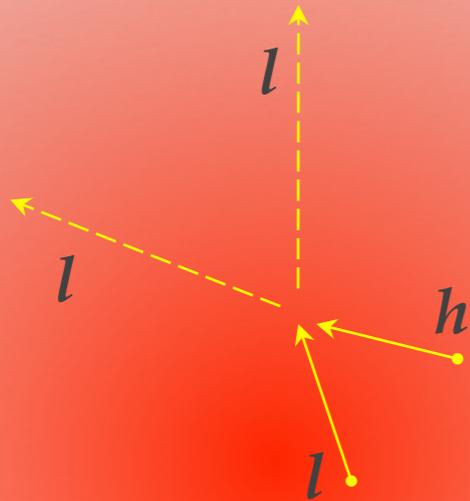


# DM halos

Energy conservation:

$$2 \frac{1}{2}(m_l v_l^2) = (m_h - m_l) c^2$$

$$v \sim v_{kick} = c (\Delta m/m_l)^{1/2}$$



if  $v_{kick} \gg v_{escape}$

dwarf halos destroyed

if  $v_{kick} \ll v_{escape}$

central cusps softened

# Technical: Interaction of 2-comp particles

Wave-functions

$$|ff\rangle \equiv \begin{pmatrix} \alpha\alpha \\ \alpha\beta \\ \beta\alpha \\ \beta\beta \end{pmatrix} \equiv \begin{pmatrix} \alpha_1\alpha_2(\mathbf{x}_1, \mathbf{x}_2, t) \\ \alpha_1\beta_2(\mathbf{x}_1, \mathbf{x}_2, t) \\ \beta_1\alpha_2(\mathbf{x}_1, \mathbf{x}_2, t) \\ \beta_1\beta_2(\mathbf{x}_1, \mathbf{x}_2, t) \end{pmatrix} \quad |mm\rangle \equiv \begin{pmatrix} hh \\ hl \\ lh \\ ll \end{pmatrix} \equiv \begin{pmatrix} h_1h_2(\mathbf{x}_1, \mathbf{x}_2, t) \\ h_1l_2(\mathbf{x}_1, \mathbf{x}_2, t) \\ l_1h_2(\mathbf{x}_1, \mathbf{x}_2, t) \\ l_1l_2(\mathbf{x}_1, \mathbf{x}_2, t) \end{pmatrix}$$

Mixing

$$|ff\rangle = U_2 |mm\rangle$$

$$U_2 \equiv U \otimes U = \begin{pmatrix} \cos^2 \theta & -\cos \theta \sin \theta & -\cos \theta \sin \theta & \sin^2 \theta \\ \cos \theta \sin \theta & \cos^2 \theta & -\sin^2 \theta & -\cos \theta \sin \theta \\ \cos \theta \sin \theta & -\sin^2 \theta & \cos^2 \theta & -\cos \theta \sin \theta \\ \sin^2 \theta & \cos \theta \sin \theta & \cos \theta \sin \theta & \cos^2 \theta \end{pmatrix}$$

Interaction

$$\tilde{V} = \begin{pmatrix} V_{\alpha\alpha} & 0 & 0 & 0 \\ 0 & V_{\alpha\beta} & 0 & 0 \\ 0 & 0 & V_{\beta\alpha} & 0 \\ 0 & 0 & 0 & V_{\beta\beta} \end{pmatrix}$$

$$V = U_2^\dagger \tilde{V} U_2 = \begin{pmatrix} A & E & E & D \\ E & C & D & F \\ E & D & C & F \\ D & F & F & B \end{pmatrix}$$

$$A = \frac{1}{8} [3V_{\alpha\alpha} + 2V_{\alpha\beta} + 3V_{\beta\beta} + 4(V_{\alpha\alpha} - V_{\beta\beta}) \cos 2\theta + (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta}) \cos 4\theta],$$

$$B = \frac{1}{8} [3V_{\alpha\alpha} + 2V_{\alpha\beta} + 3V_{\beta\beta} - 4(V_{\alpha\alpha} - V_{\beta\beta}) \cos 2\theta + (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta}) \cos 4\theta],$$

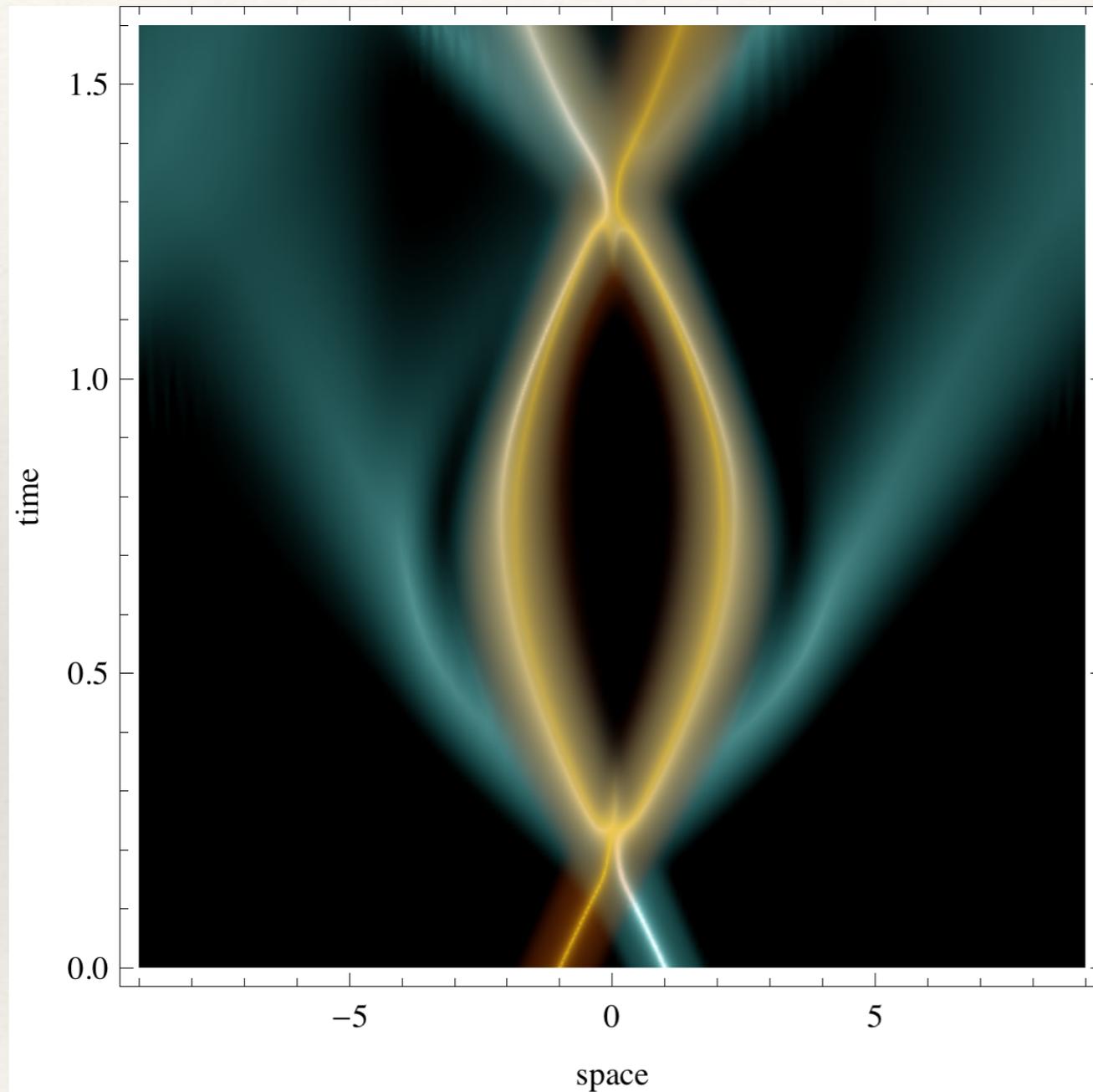
$$C = \frac{1}{8} [V_{\alpha\alpha} + 6V_{\alpha\beta} + V_{\beta\beta} - (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta}) \cos 4\theta],$$

$$D = \frac{1}{4} [V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta}] \sin^2 2\theta,$$

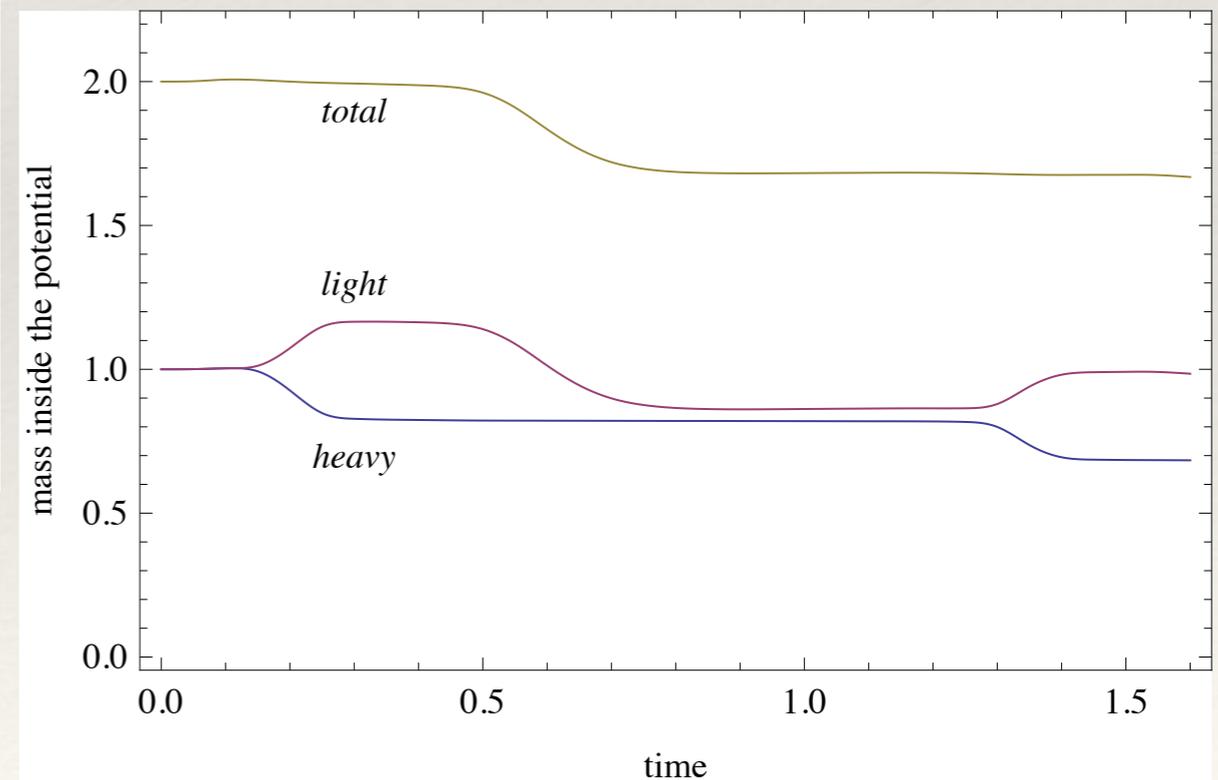
$$E = -\frac{1}{4} [V_{\alpha\alpha} - V_{\beta\beta} + (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta}) \cos 2\theta] \sin 2\theta,$$

$$F = -\frac{1}{4} [V_{\alpha\alpha} - V_{\beta\beta} - (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta}) \cos 2\theta] \sin 2\theta,$$

# Complete evaporation of 2-comp. particles



$$|h\rangle + |l\rangle \rightarrow |l\rangle + |l\rangle$$



# Implementation

- ❖ Gadget, 50 Mpc/h box, standard  $\Lambda$ CDM cosmology
- ❖ At each step:
  - ◆ Pairs of nearest neighbors are identified
  - ◆ Densities of each species are found at each particle location
  - ◆ Conversion probabilities are calculated
  - ◆ Monte-Carlo module is used for conversions
  - ◆ Energy-momentum is manifestly conserved in every interaction
- ❖ 2 free parameters:  $\sigma(v)/m$  [with  $\sigma \propto (v/v_k)^{-1}$ ] and  $\Delta m/m$  [or  $v_k = c(2\Delta m/m)^{1/2}$ ]

$$P_{s_i t_i \rightarrow s_f t_f} = (\rho_{t_i}/m_{t_i}) \sigma_{s_i t_i \rightarrow s_f t_f} |\mathbf{v}_{t_i} - \mathbf{v}_{s_i}| \Delta t \Theta(E_{s_f t_f})$$

$$\sigma_{s_i t_i \rightarrow s_f t_f} = \sigma_{s_i}(v) = \sigma (v/v_0)^{-a}$$

$a = 1$

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# No change on large scales

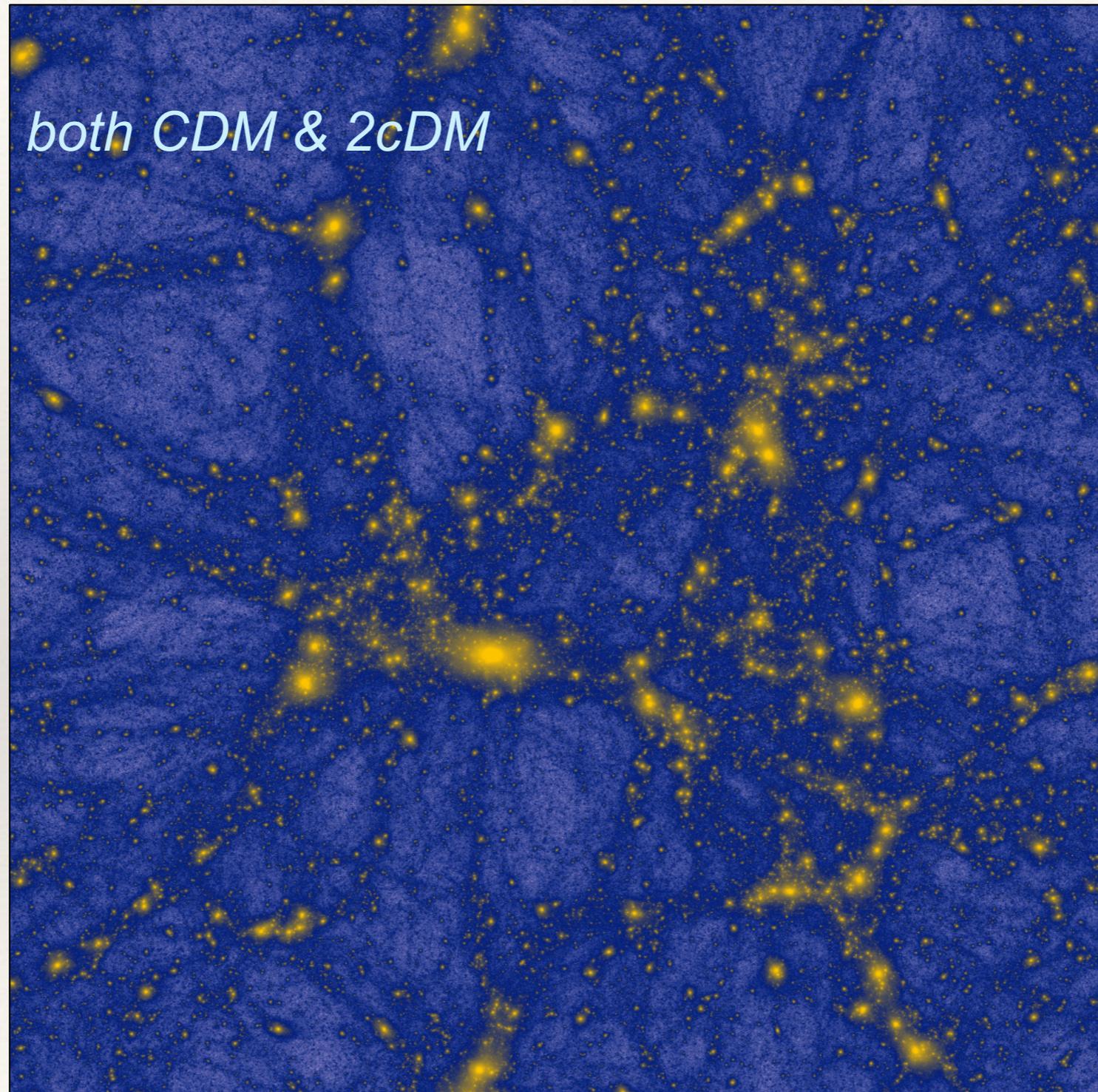
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*CDM*

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# No change on large scales

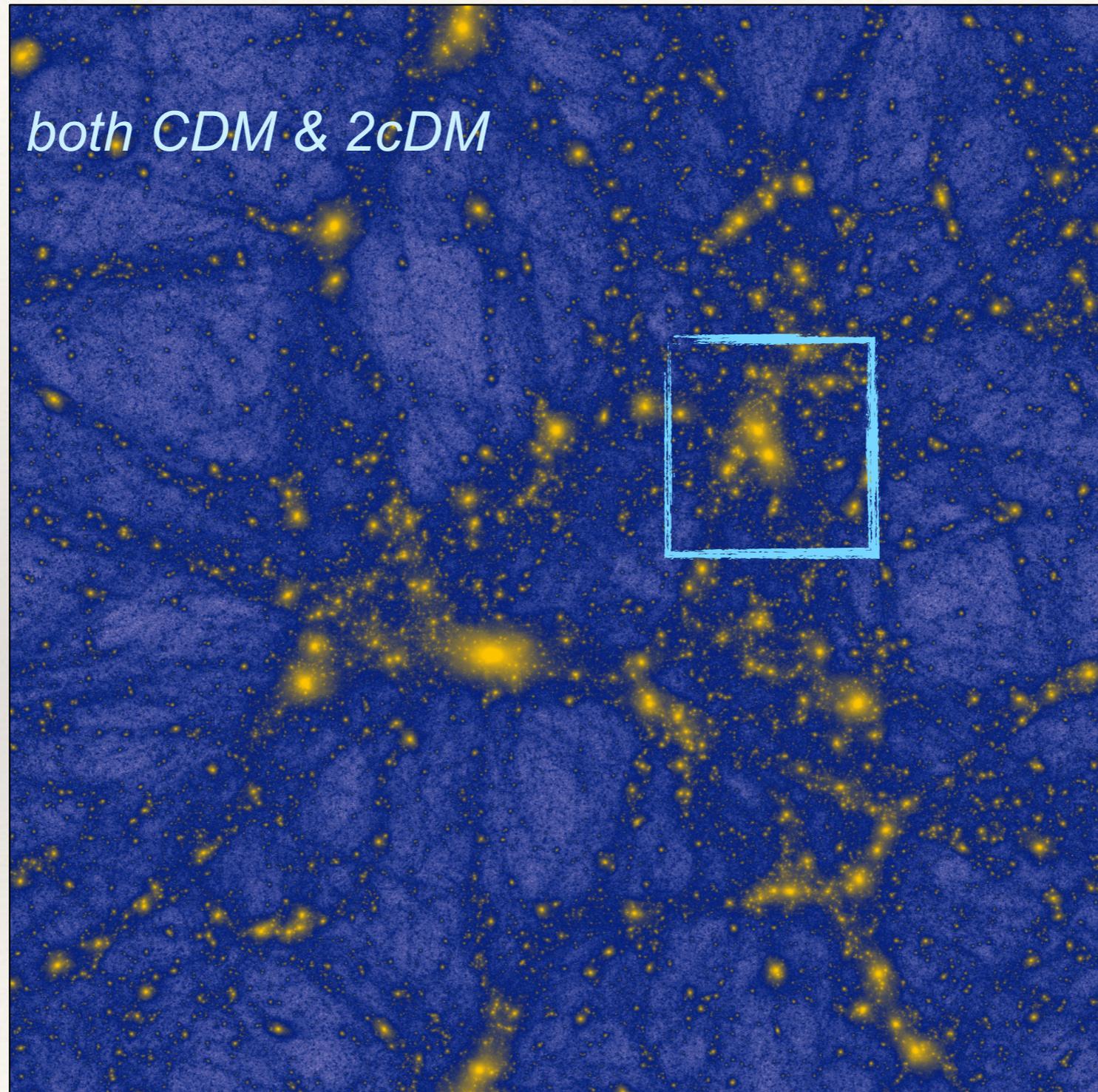
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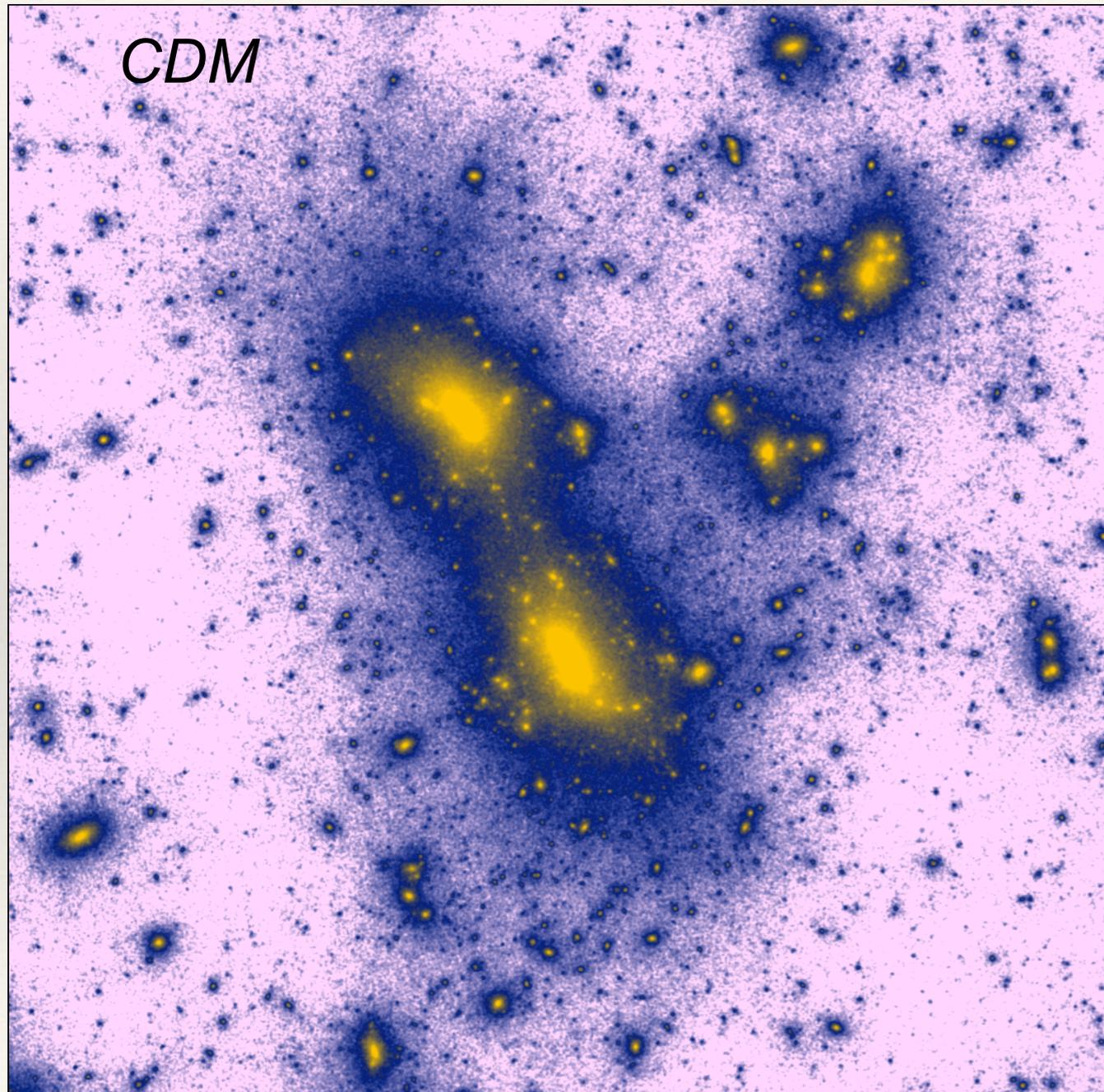
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# Less substructure on small scales

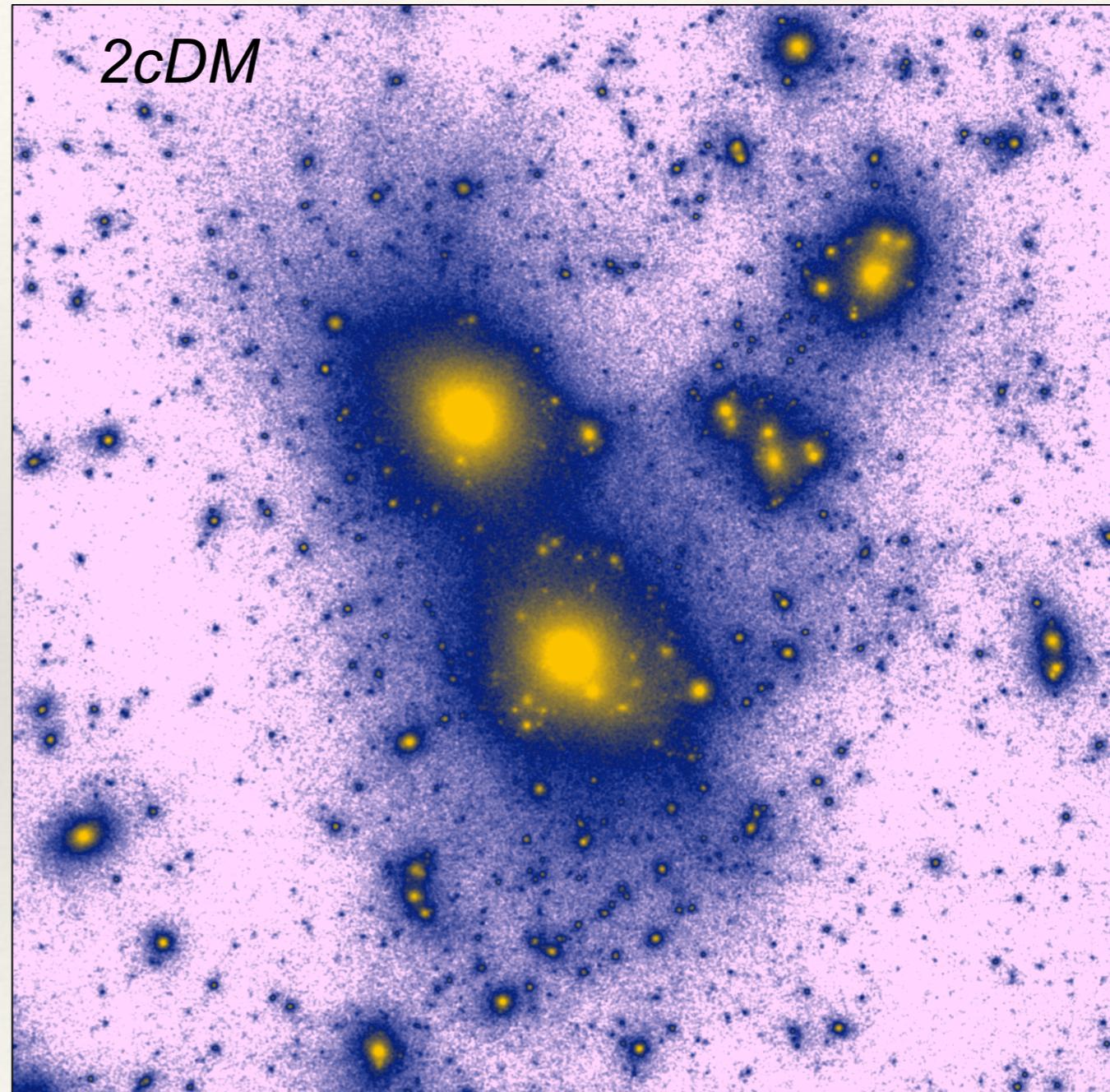
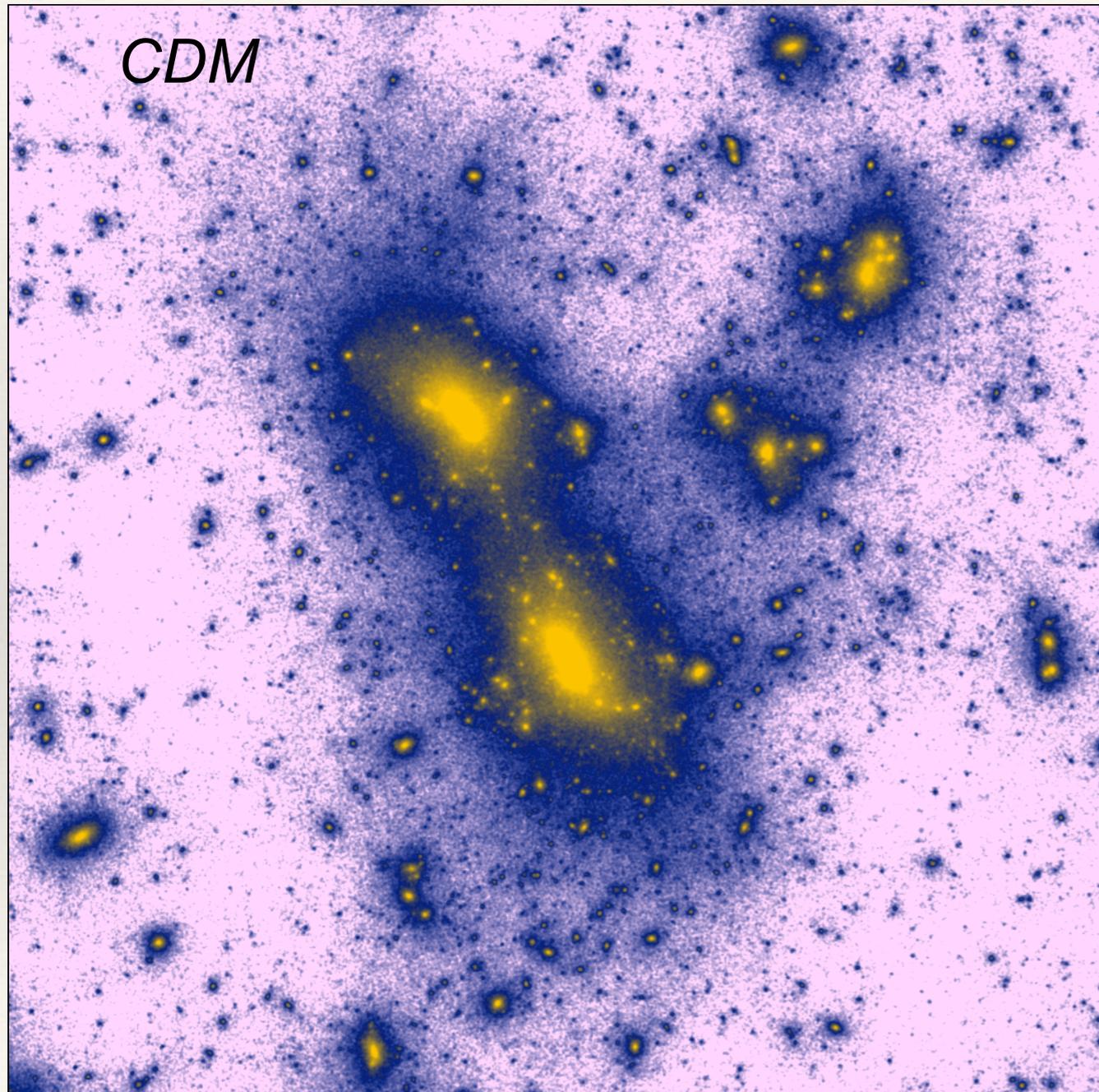
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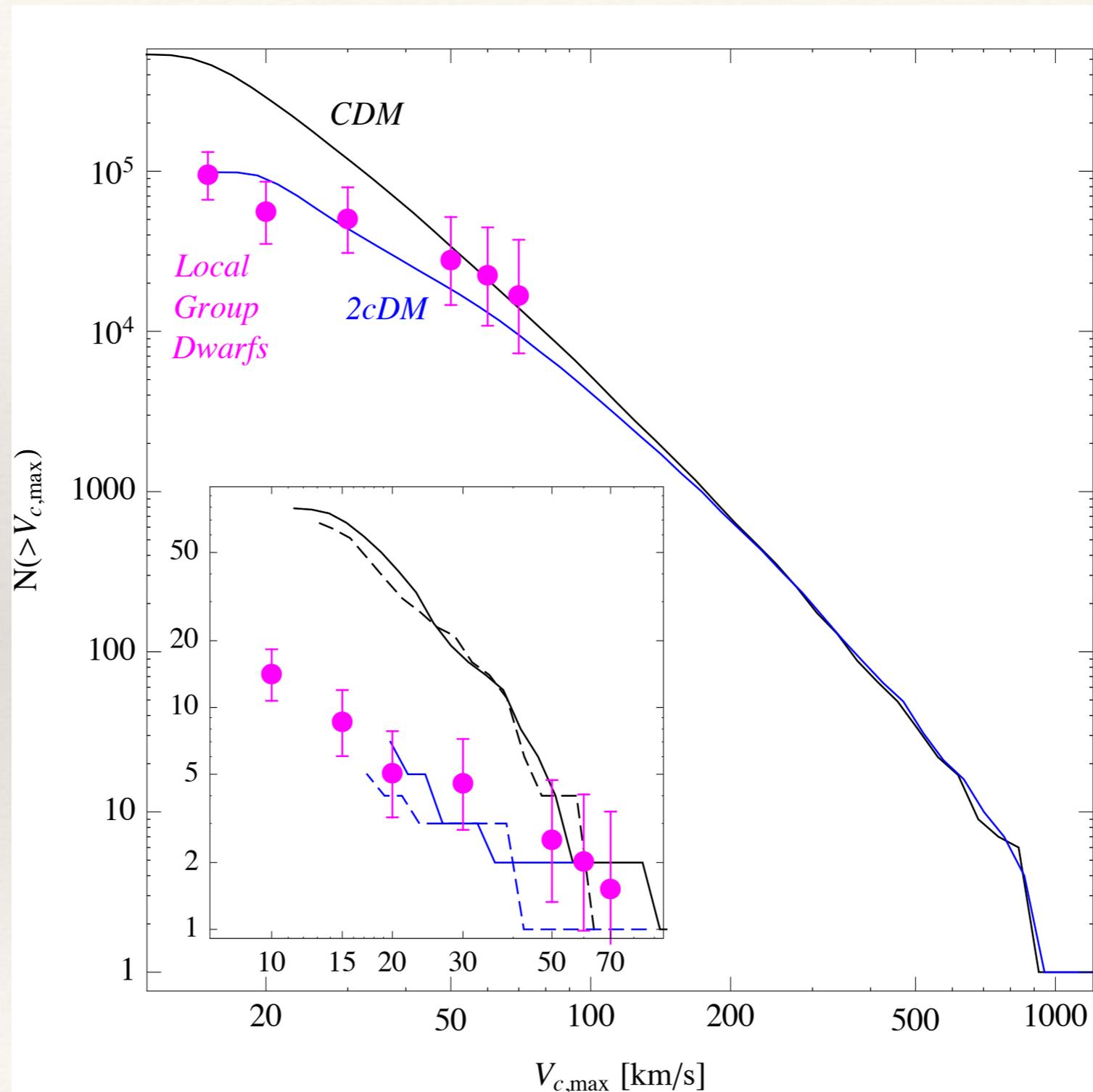
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# Less substructure on small scales

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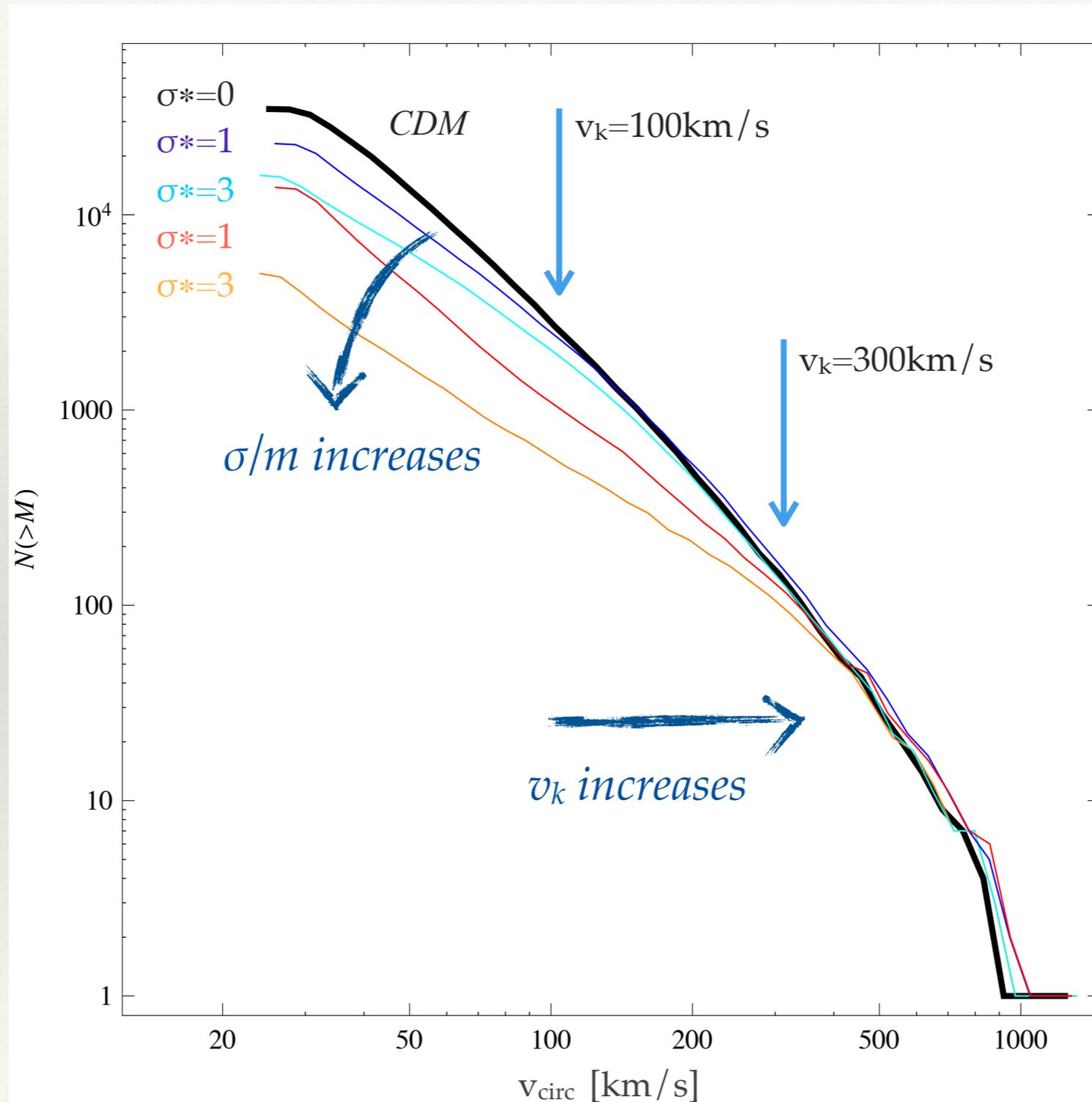


# Velocity function



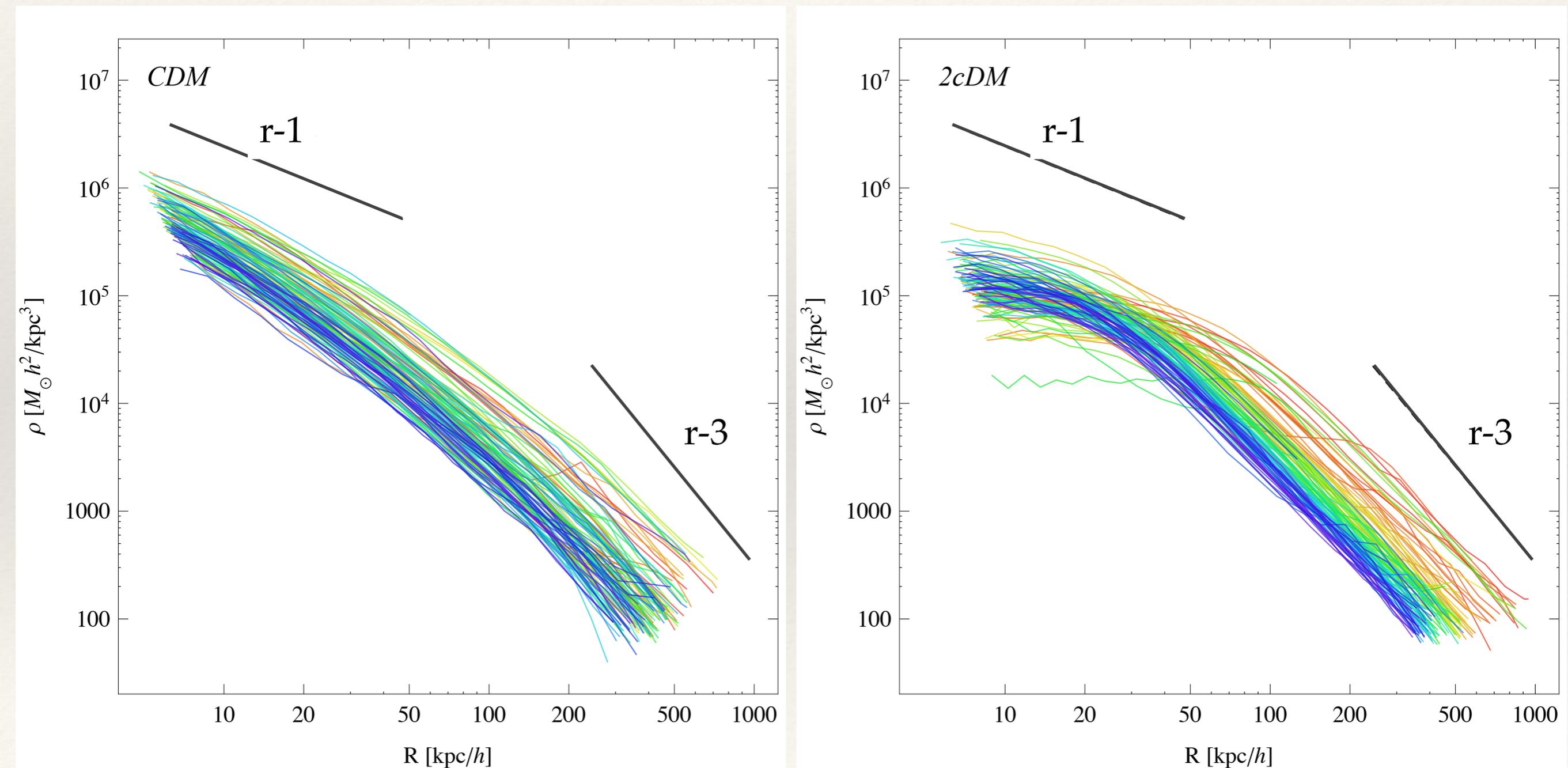
$\sigma_* = 0.75 \text{ cm}^2/\text{g}$   
 $v_k = 50 \text{ km/s}$   
 $\Delta m/m = 10^{-8}$

# Key parameters

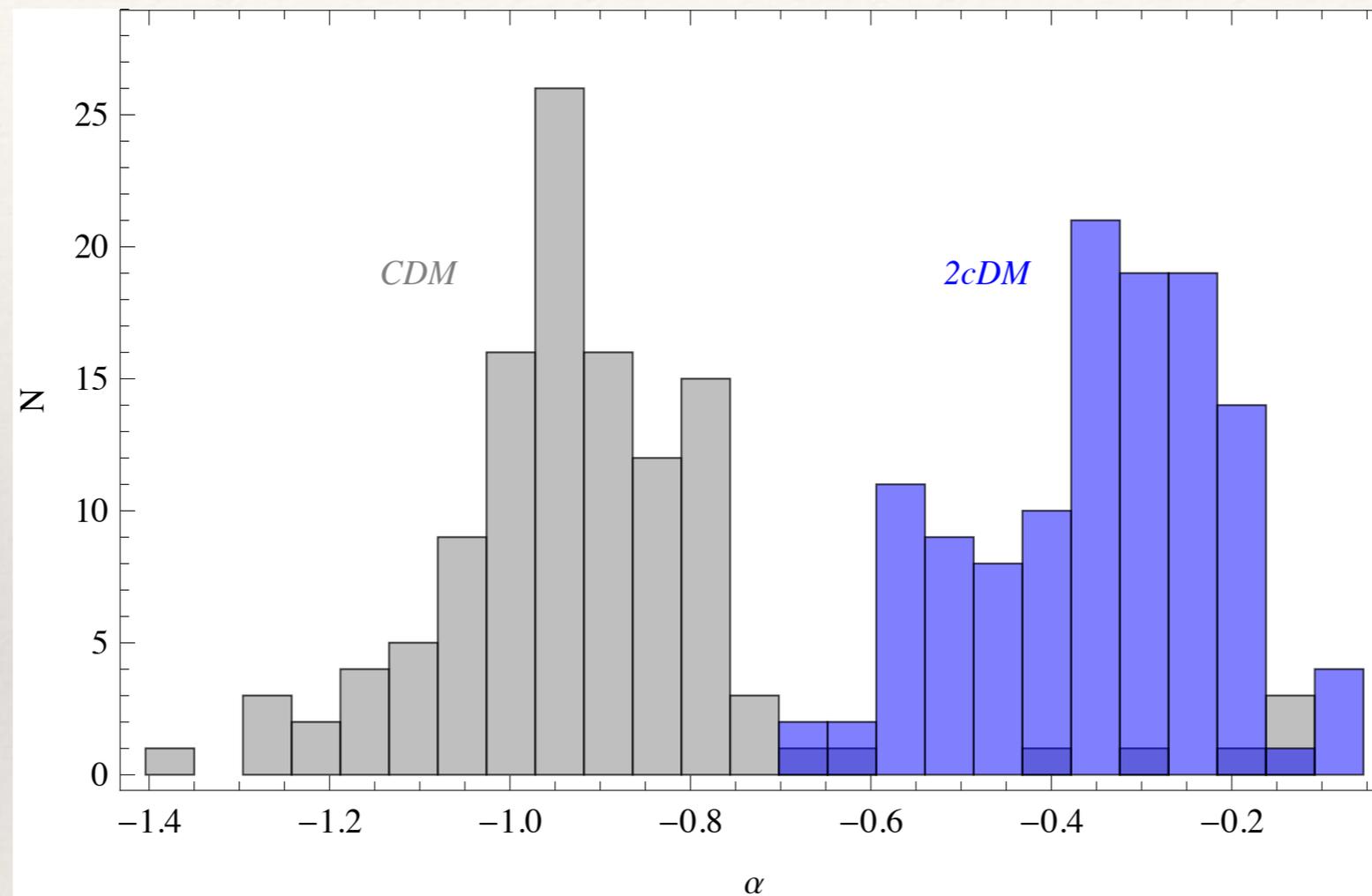


$$v_k = c(2\Delta m / m)^{1/2}$$
$$\sigma^* = \sigma / m$$

# Density profiles



# Slopes of density profiles

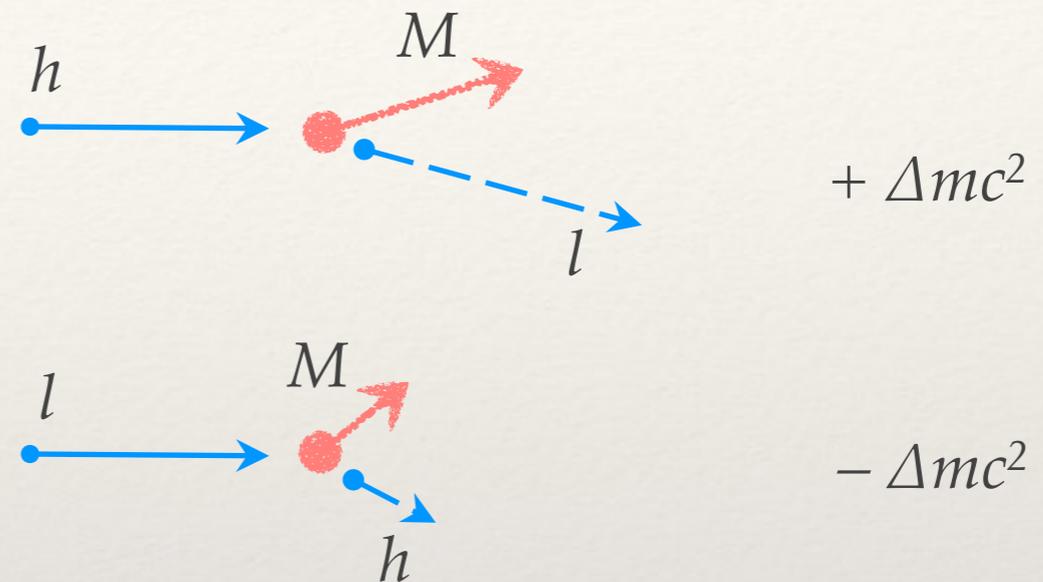


We fit density profiles with function  
 $\rho = r^\alpha / (1 + r^\beta)$  and evaluating  $\alpha$  at  $r = 7 \text{ kpc}/h$

# 2cDM predictions

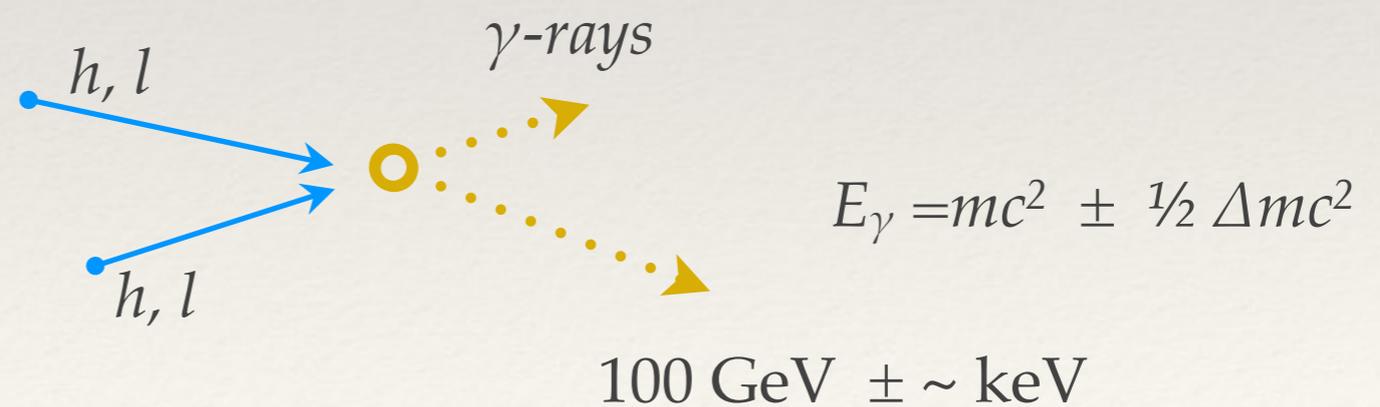
## direct detection

“inelastic recoil”



## indirect detection

“ $\gamma$ -ray annihilation line triplet”



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# 2cDM vs SIDM

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$\sigma_{\text{conv}}(\mathbf{v})$  — determines slope

$\sigma_{\text{scatt}}(\mathbf{v})$  — determines core

$\Delta\mathbf{m}/\mathbf{m}$  (or  $\mathbf{v}_k$ ) — determines break

$\sigma(\mathbf{v})$  — determines cores

-- nothing can reduce substructure

-- gravi-thermal collapse

- stronger cusps  $r^{-2}$  unless fine tune  $\sigma(\mathbf{v}) - H_0$

How robust is the model?

# How robust is the 2cDM model?

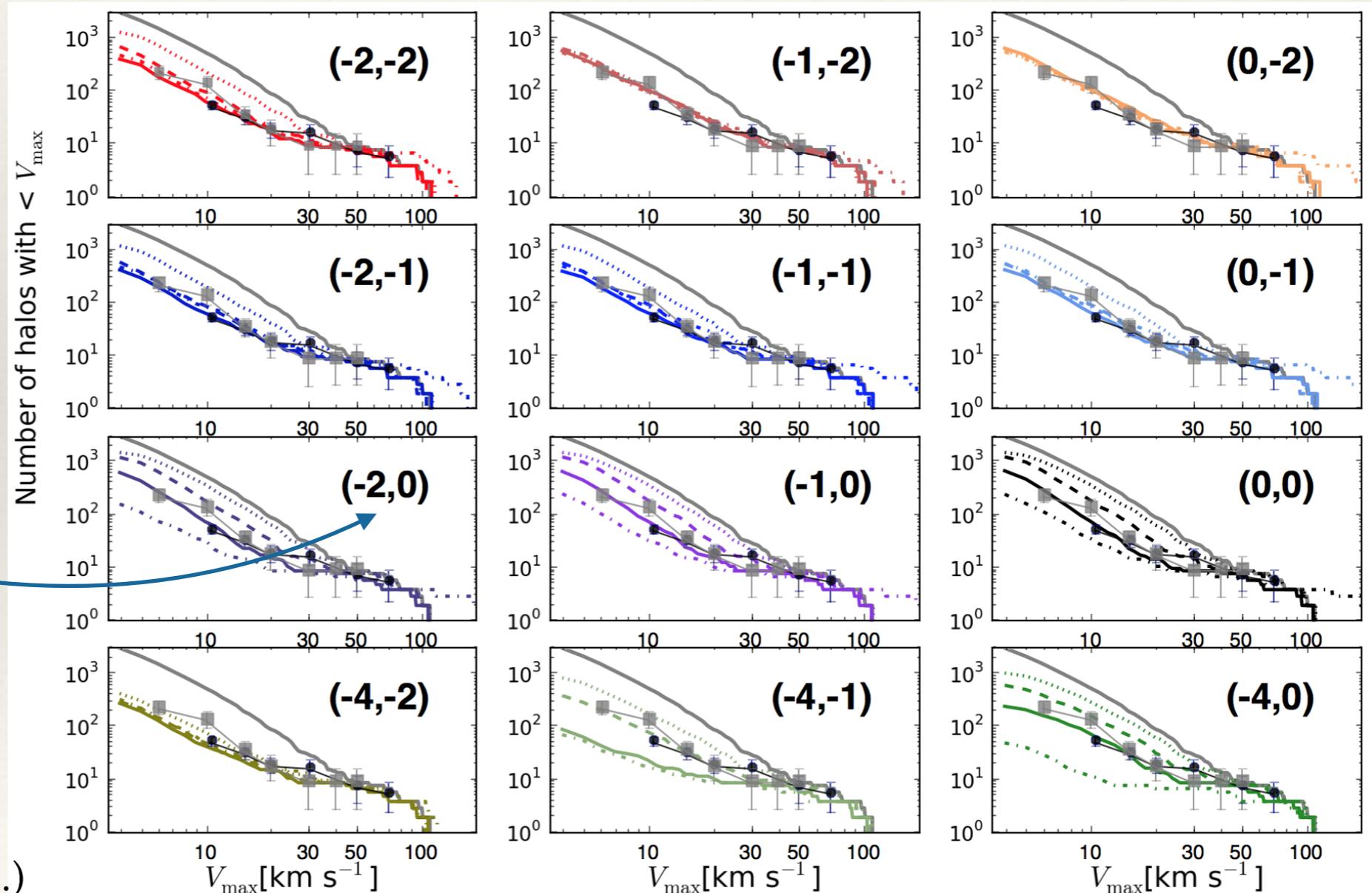
## SUBSTRUCTURE

$$\sigma(v) = \begin{cases} \sigma(v/v_0)^{a_s} & \text{for scattering} \\ \sigma(v/v_0)^{a_c} & \text{for conversion} \end{cases}$$

$\sigma = (0.01, 0.1, 1)$  and 10,

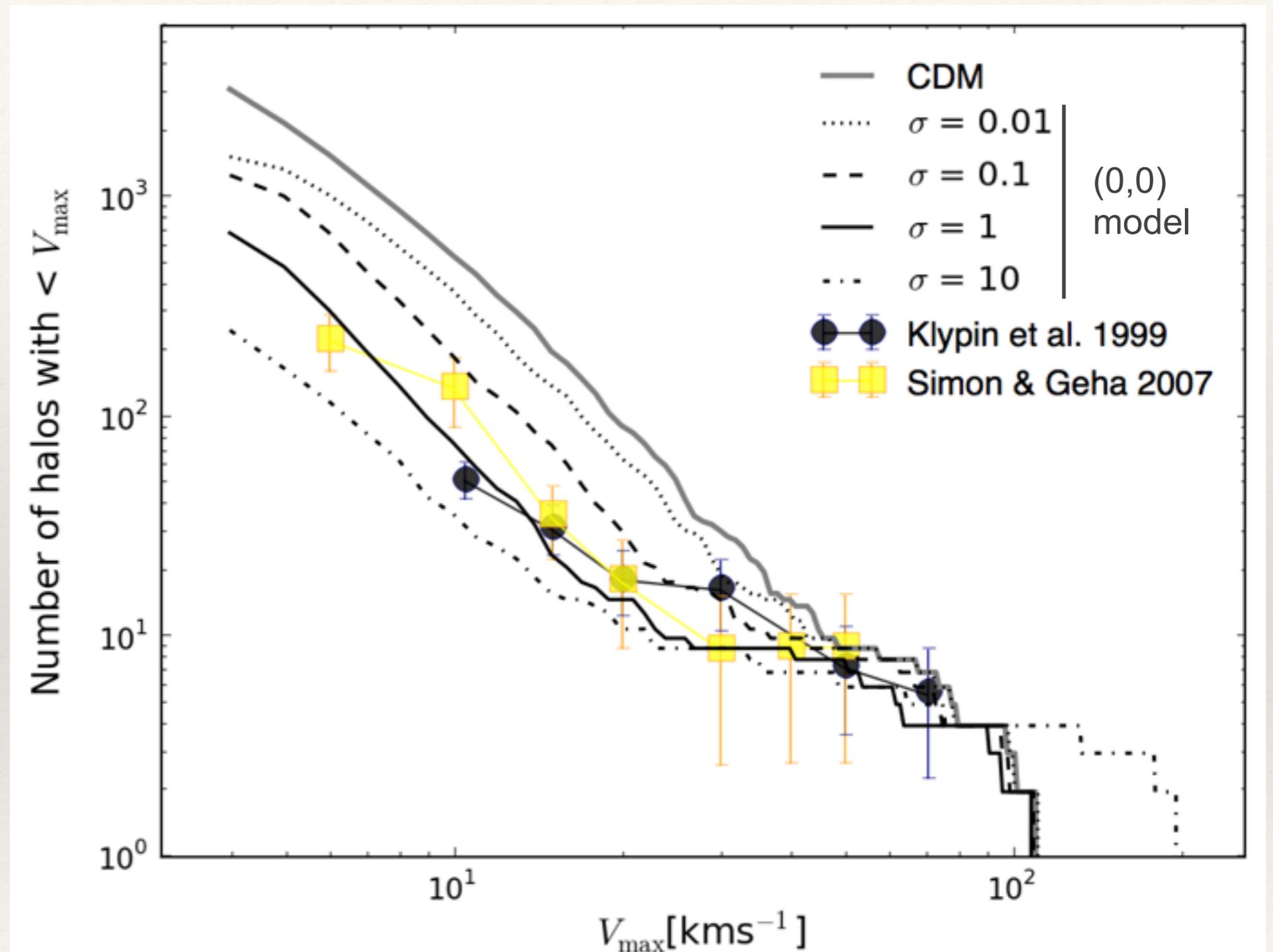
*consistent with observations*

$(a_s, a_c)$



(Todoroki & MM, in prep.)

# Just one example



$M_{\text{vir}} \sim 10^8 M_{\odot}$

$M_{\text{vir}} \sim 10^{11} M_{\odot}$

(Todoroki & MM, in prep.)

# How robust is the 2cDM model?

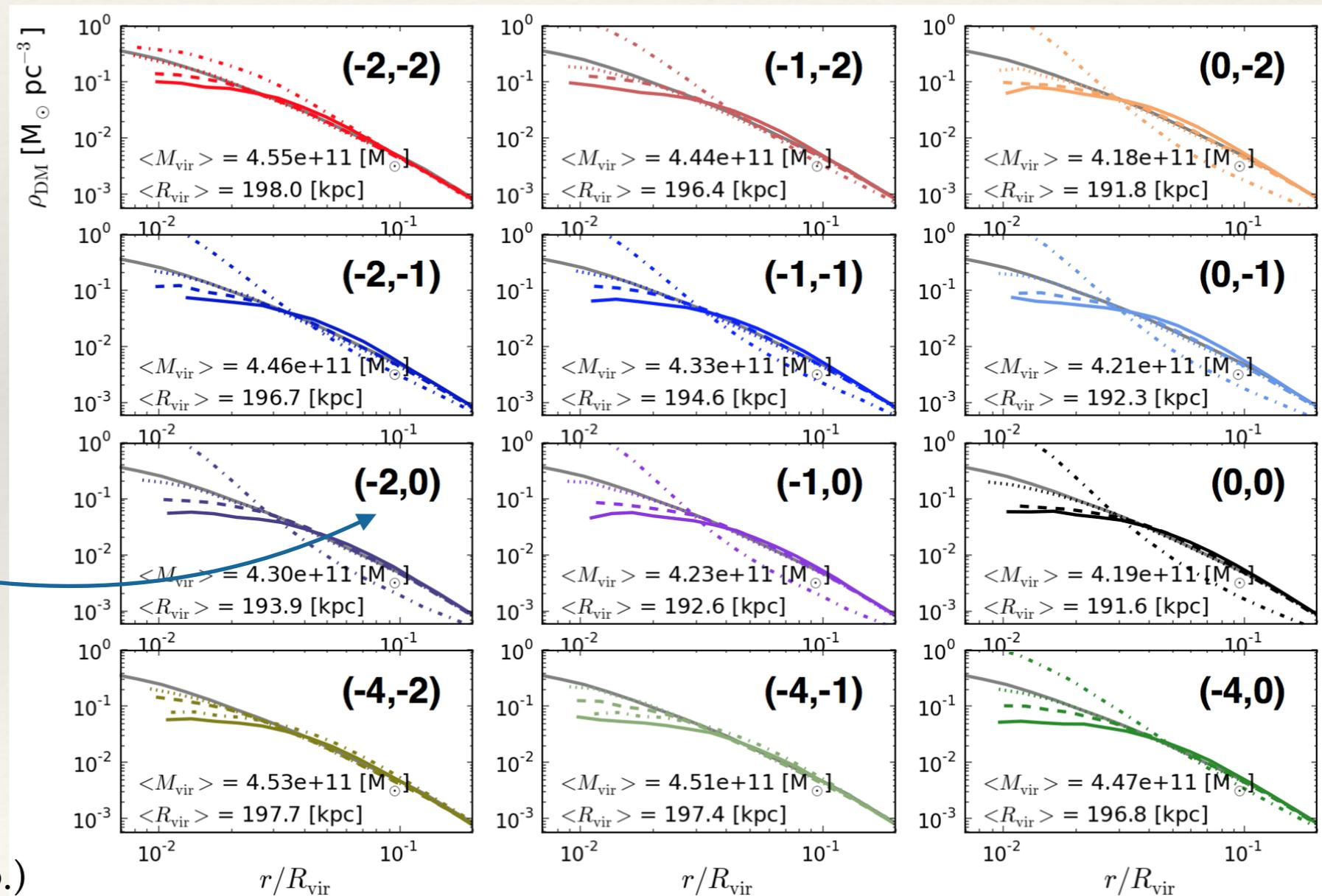
## PROFILES

$$\sigma(v) = \begin{cases} \sigma(v/v_0)^{a_s} & \text{for scattering} \\ \sigma(v/v_0)^{a_c} & \text{for conversion} \end{cases}$$

$\sigma = (0.01, 0.1, 1)$  and 10,

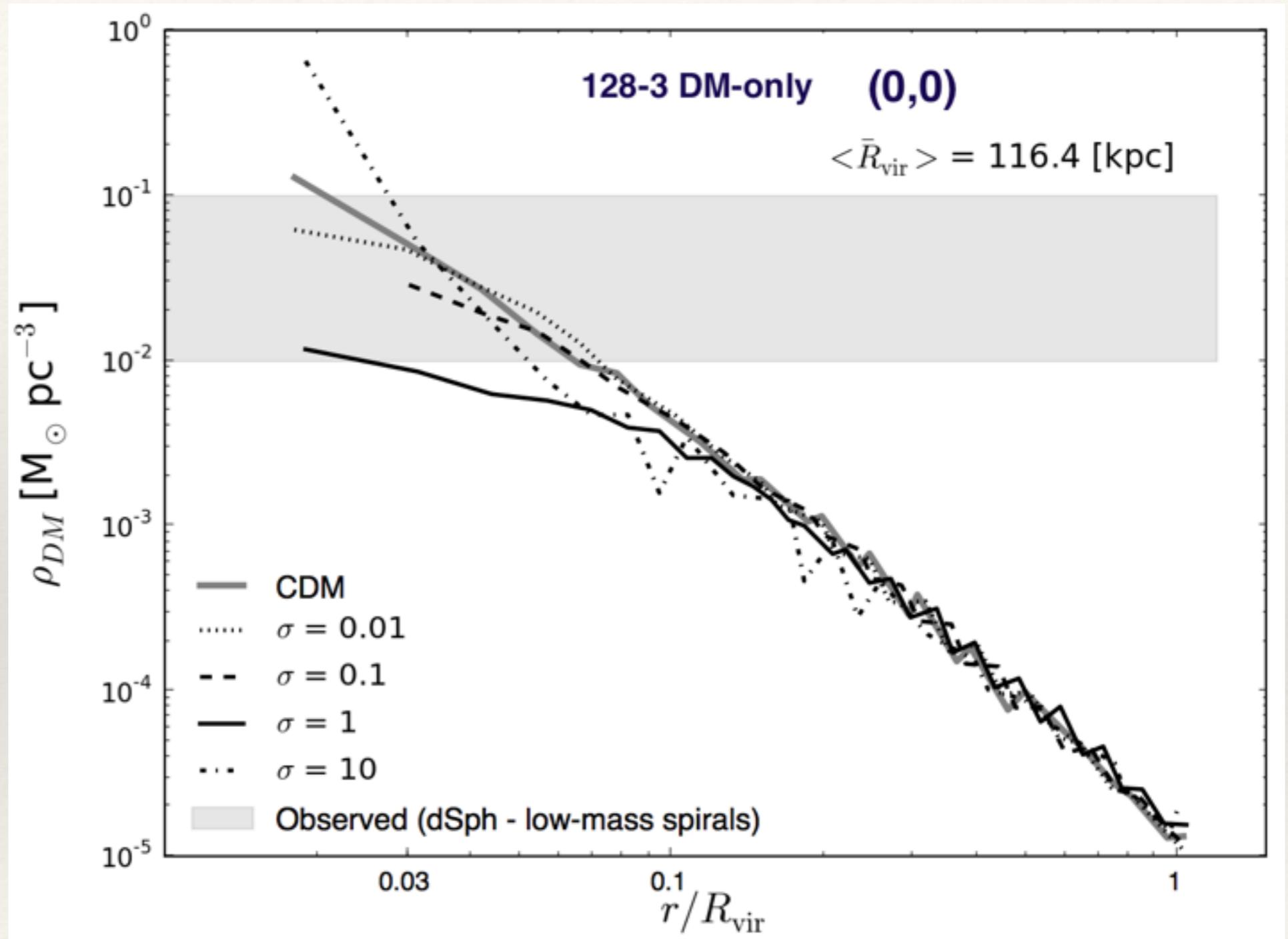
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# Just one example

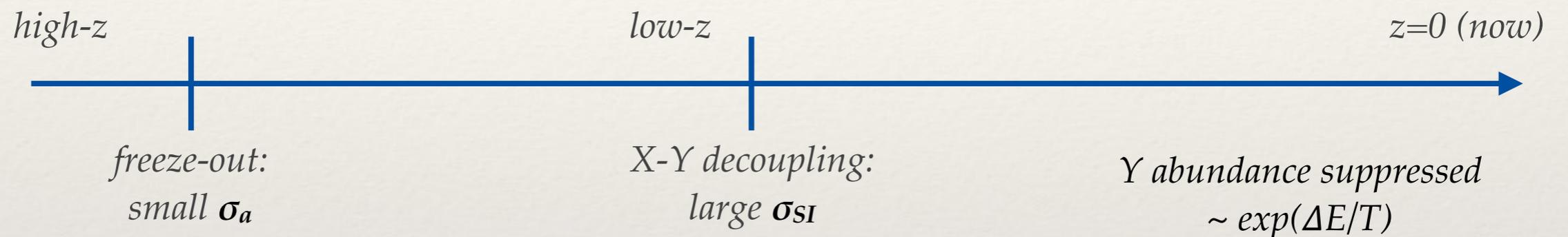


# 2cDM vs others

2cDM looks like any multi-species / composite DM -- allows "reactions"  $Y \rightarrow X$

excited, inelastic, exothermal DM,...

early universe "catastrophe"



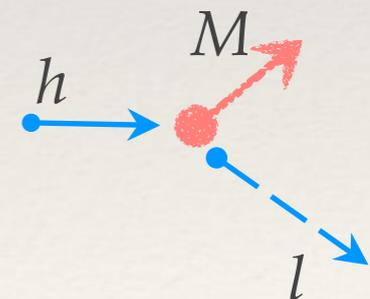
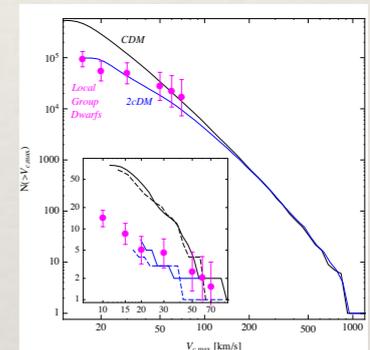
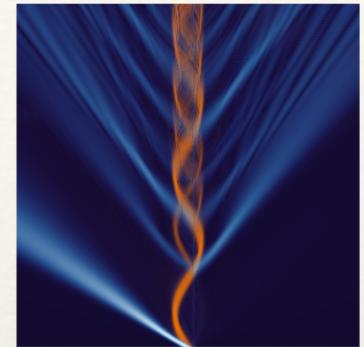
Not a problem for 2cDM: conversions do not occur before structure formation starts (needed to separate mass states)

$$\sigma_{\text{conv}}^{\text{fst}} \sim (\Delta m/m)^4 \sigma_{\text{conv}}$$

before structure formation after structure formation

# Conclusions

- ❖ “quantum evaporation” – a new effect (MVM 2010)
- ❖ 2cDM:
  - ✦ can solve all small-scale problems simultaneously
  - ✦  $\sigma(v) \sim 0.1 \dots 0.01$  – consistent with all constraints
  - ✦  $\Delta m / m \sim 10^{-8} \iff v_k \sim 50\text{-}100 \text{ km/s}$
- ❖ 2cDM predicts
  - ✦ break in mass function: suppression at  $M_{\text{vir}} \leq 10^{10} M_{\odot}$
  - ✦ *inelastic* recoils with  $\Delta E \sim \Delta mc^2$  in direct detection DM
  - ✦  $\gamma$ -ray annihilation line *triplet* with  $\Delta E_{\gamma} = \frac{1}{2} \Delta mc^2$ ,  
(if  $m_{\chi} \sim 100 \text{ GeV}$ , then  $\Delta E_{\gamma} \sim \Delta m \sim \text{keV}$ )
  - ✦ direct detection DM recoil may depend on target species

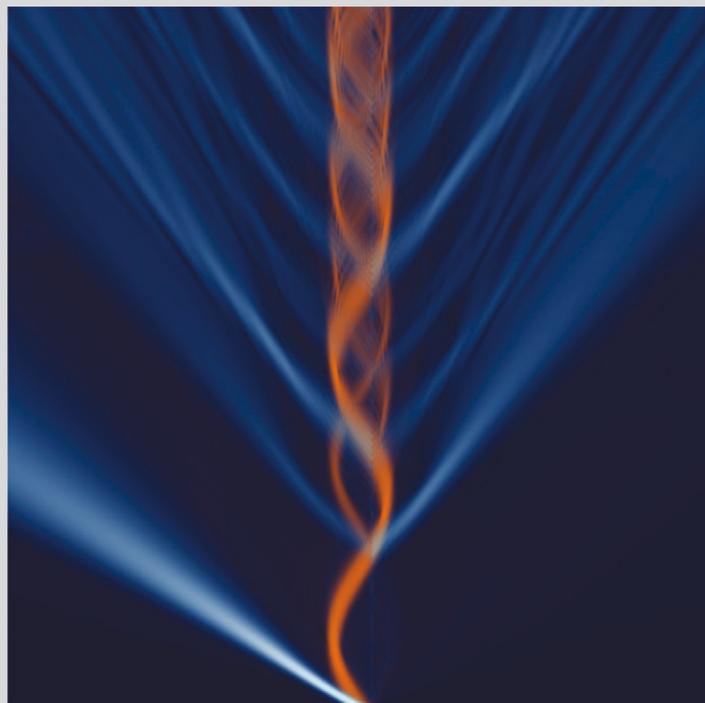


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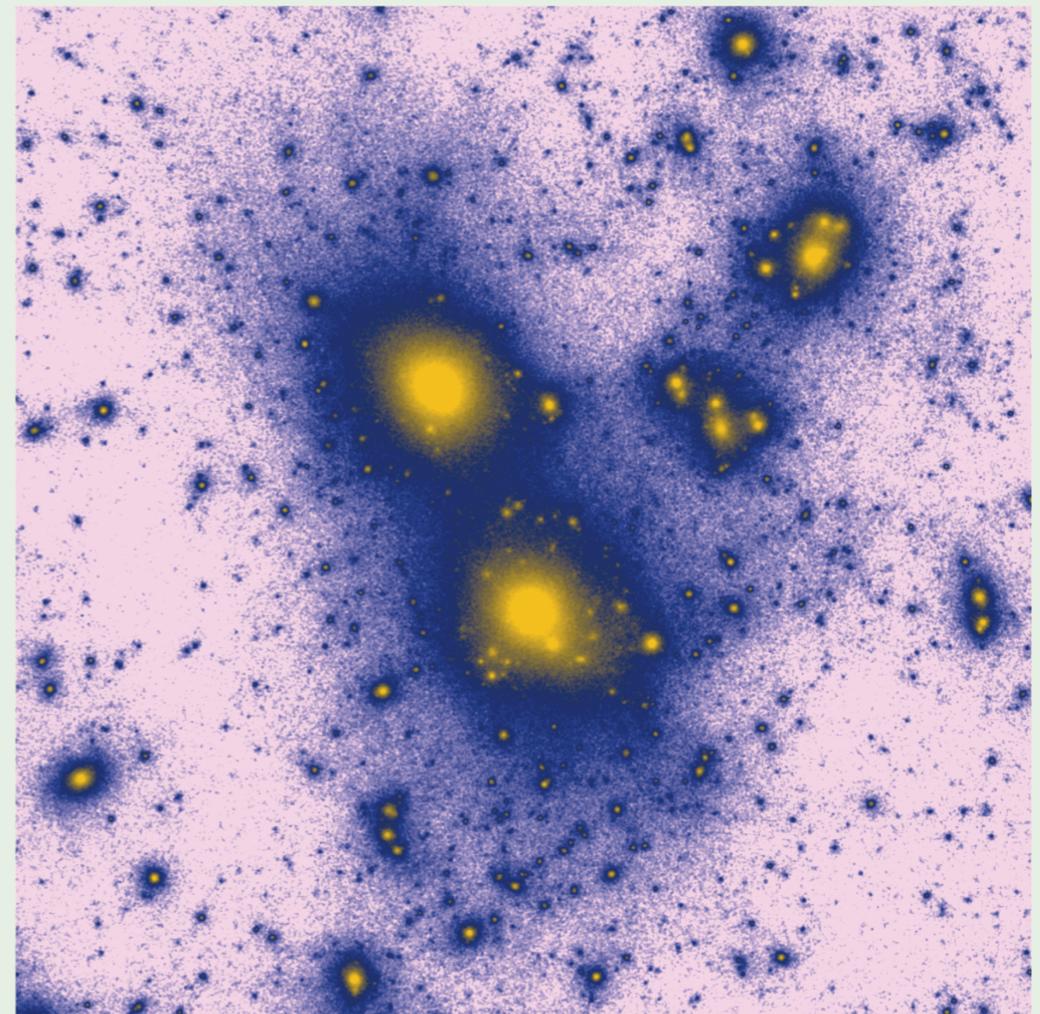
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7

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